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PRESERVICE MATHEMATICS TEACHERS' ACHIEVEMENT AND EVALUATION OF MATHEMATICAL MODELLING

Burçak BOZ-YAMAN

Abstract: The aim of the study was to understand preservice mathematics teachers' improvement of their knowledge of mathematical modeling and knowledge for teaching modeling post-intervention. A total of 57 preservice mathematics teachers voluntarily participated in the study, which lasted for a period of five weeks. Data sources included three separate measurements; Mathematical Modeling Test, a questionnaire about the meaning of mathematical modeling, and model-eliciting activities. A quantitative and qualitative data analysis approach was employed in order to better understand the intervention achievement. The Mathematical Modeling Test data was examined with paired sample t-test, and the other two datasets were examined using thematic analysis to identify the preservice teachers' pedagogical content knowledge through mathematical modeling and their performance at each step of the mathematical modeling. Data analysis revealed that although the preservice teachers increased their mathematical modeling knowledge, they experienced difficulties during the interpretation and adaptation of the mathematical modeling results into the real life situations.

Key words: mathematical modeling; preservice mathematics teachers; knowledge on content; knowledge on pedagogy, teacher certification program

1. Introduction

Mathematical modeling has gained importance as a cognitively demanding mathematical activity since it links classroom mathematics with real life and engages students in describing situations mathematically, making sense of quantities, constructing representations, contextualizing mathematical expressions, and refining the models they develop (Lesh and Doerr 2003; Stillman and Galbraith 2011). Borromeo Ferri (2010b) stated that mathematical modeling is a compulsory competency and transitions processes back-and-forth between reality and mathematics. The National Council of Teachers of Mathematics (NCTM) underlined the importance of mathematical modeling by stating

> Instructional programs from prekindergarten through grade 12 should enable all students to create and use representations to organize, record and communicate mathematical ideas; select apply and translate among mathematical representations to solve problems, use representations to model and interpret physical, social, and mathematical phenomena. (NCTM 2000, p. 67)

Similarly, in Turkish high school mathematics curriculum, mathematical modeling is considered as a dynamic method which helps to see and more easily identify relationships of real life problems, to explain those relations using mathematical terms, and to classify, generalize and deduce a result (Millî Eğitim Bakanlığı [Turkish Ministry of National Education, MoNE] 2013). Additionally, the Common Core State Standards for Mathematics (CCSSM) highlight the use of mathematical modeling as "Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace" (Common Core State Standards for Mathematics [CCSSI] 2010, p. 7).

Despite an increased interest in mathematical modeling, its use is quite limited in everyday mathematics teaching (Blum 2015). According to Blum (2015), mathematical modeling is a cognitively demanding activity with several competencies involved; making the teaching and learning of mathematical modeling not an easy job. Therefore, integrating modeling tasks into instructional

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practices represents a challenging demand for teachers, such as in the selection of meaningful real world tasks, balancing teacher support with independent student work, managing unpredictable student responses, and the encouragement of multiple solution methods (Blum and Borromeo Ferri 2009). These teaching competencies should form part of teacher education programs and preservice teachers should be provided with the opportunity to develop content and pedagogical content knowledge about modeling (Anhalt and Cortez 2016; Cetinkaya et al. 2016). Darling-Hammond (2000) asserted that a key factor to student achievement was dependent on teachers' content knowledge and knowledge about teaching modeling. Therefore, for better teaching and learning of mathematical modeling, preservice teachers' content knowledge and pedagogy of modeling should be improved through teacher education programs.

In recent years, many research studies (Anhalt and Cortez 2016; Aydogan Yenmez, Erbas, Alacaci, Cakiroglu and Cetinkaya 2017; Cetinkaya et al. 2016; Doerr and English 2006) have been conducted with the goal of investigating the teaching and learning process of mathematical modeling. These studies can mostly be grouped into two camps, with the effective teaching of mathematical modeling in primary, secondary and tertiary instruction (Aydogan et al. 2017; Doerr and English 2006), and how teacher education programs in universities can be developed through teaching mathematical modeling to preservice teachers (Anhalt and Cortez 2016; Cetinkaya et al. 2016; Eraslan 2012; Holmquist and Lingefjärd 2003; Riede 2003). Borromeo Ferri and Blum (2010b) emphasized that during the process of becoming a mathematics teacher, mathematical modeling should form a significant part of preservice teacher education, at both a theoretical and practical level. Various studies have stated that pedagogical competencies and knowledge are necessary for the teaching of mathematical modeling in the classroom (e.g., Borromeo Ferri and Blum 2010a; Doerr 2007; Doerr and English 2006; Doerr and Lesh 2011; Kaiser and Stender 2013). Besides knowledge about mathematical modeling; opinions and perspectives of teachers/preservice teachers about using mathematical modeling in the mathematics classrooms or how to use it during teaching mathematics are another important topic (Ferri and Blum 2009; Akgun 2015). Ferri and Blum (2009) examined preservice mathematics teachers' perspectives about modeling process and found out their difficulties while experiencing the modeling activities. Similarly, Akgun (2015) investigated preservice mathematics teachers' opinions on using mathematical modeling in the mathematics courses and he revealed that the participants thought that classroom management was difficult during mathematical modeling lessons and it was time consuming comparing with the other mathematics lessons. Mathematical classrooms with mathematical modeling are mostly constructed as group working with high amount of "cognitive activation" (Borromoe Ferri 2018, p.78) rather than absorbing knowledge through teacher's action. Therefore, in this kind of highly cognitively active classrooms teachers should deal with each groups' work, examine their solving process and use pupils' mistakes constructively for next step of the discussion. According to Baumert and Kunter (2013) in the cognitively high demanding classrooms, teachers need to demonstrate qualified classroom management and orchestration of the learning opportunities.

The other important issue for teaching and learning mathematical modeling is technology usage. Confrey et al. (2010) stated that digital technologies are more visible during the mathematical modeling process in the educational studies. According to Geiger (2011) technology is not a tool for exploration, development of a model or it is used for validation however, technology makes use of mathematical ideas after the mathematical model is developed. Most of the studies (i.e., Greefrath 2011; Lingerfjäd 2013; Villarreal, Esteley and Smith 2018) recommend to provide future teachers opportunities for experiencing technology integrated mathematical modeling process during their preservice education. Yet, digital technologies have potential advantages to the teaching and learning modeling, teachers' dispositions toward both technologies and mathematical modeling manipulate these advantages.

According to Anhalt and Cortez (2016) to become a teacher who can use modeling process requires careful integration of mathematical modeling into teacher education programs. Yet it is not enough, preservice teachers' background knowledge and perspectives also be concerned. Therefore, preservice teachers' knowledge, perspectives and teaching competencies should be examined and be improved. Teaching competency (ability to plan and perform modeling lessons and knowledge of appropriate interventions during pupils' modeling processes) is one of the listed competencies proposed by Borromeo Ferri and Blum (2010b). Teachers need to developed teaching competencies for mathematical modeling to use modeling in their classrooms. Borromeo Ferri (2018) identified four teaching competencies (theoretical, task, instruction and diagnostic dimensions) that a teacher has possessed for the mathematical modeling lessons. Theoretical dimension involves teacher's knowledge of modeling cycles, about goals for modeling and about types of modeling tasks. Task dimension contains teacher's ability to solve, analyze and create modeling tasks. Instruction dimension involves teacher's ability to plan and execute modeling lessons and knowledge of appropriate interventions while students' modeling processes. In the last dimension, diagnostic dimension. Teacher has ability to identify phases in student's modeling process and diagnose their difficulties during such processes. In the current study, preservice teachers' conceptual understanding and solutions of modeling tasks are examined and these two concepts are matched with two teaching competencies from the Borremo Ferri's (2018) list, that is theoretical and task dimensions. Moreover, diagnostic dimension also considered by examining preservice teachers' perspectives about interventions, support and feedback while teaching modeling tasks to their students and also recognizing the difficulties and mistakes of their students.

As a result, the current study investigates the evolution of preservice teachers' content knowledge and pedagogy of mathematical modeling. The four research questions were formulated in order to guide the study:

1. Is there any change of preservice mathematics teachers' achievement level on building a mathematical model for the modeling questions after modeling activities intervention?

2. Based on the modeling cycle (understanding, mathematising, interpreting, validating), in which part of mathematical modeling activities are preservice mathematics teachers' more successful?

3. As future teachers, what are the pedagogical perspectives of preservice teachers on teaching mathematical modeling activities?

4. As future teachers, in which part of teaching mathematical modeling processes, they thought that they will have difficulties according to their own modeling experiences?

2. Theoretical Framework

Knowledge about mathematical modeling and pedagogical knowledge on mathematical modeling are two main perspectives in the current study. Schmidt (2011) stated that knowledge about mathematical modeling tasks is an important competency for teachers. According to Bostic (2012), "a model-eliciting activity (MEA) is one type of rich task that can be used to teach through problem solving" (p. 262). Model-eliciting activities are examined through modeling process frameworks. There are various mathematical modeling process frameworks to be found in the literature (e.g., Blomhoj and Jensen 2006; Blum and Leiß 2007; Borromeo Ferri 2006; Galbraith and Stillman 2006). Although researchers have approached mathematical modeling from different perspectives, it is widely accepted that mathematical modeling is a cyclical process (Anhalt and Cortez 2016; Lesh and Doerr 2003). As shown in Figure 1, the modeling cycle adopted in the current study involves the processes of understanding the task, simplifying the task, mathematising (constructing a mathematical model), working mathematically, interpreting and validating (Borromeo Ferri 2006). This modeling cycle helps to get a better understanding of mathematical modeling and deeper view into each phases of the modeling process. The cycle presents the steps of the real world and mathematics world with connection processes with numbers 1 to 6 (see Figure 1). Since the cycle of modeling given in the Figure 1 is named as "Diagnostic modeling cycle" and it is used in teacher education (Borromeo Ferri 2007), it is preferred for the current study.

In this model real situation represents the real life problem that is going to be examined through modeling process. This problem is taken from the reality. When the understanding process is emerged the mental representation of the situation is occurred. The understanding process depending on personal experiences. In order to obtain real model a simplification is done and a mental picture is structured. When one can simplify and structure the task, real model is occurred and this helps to make assumptions about the situation. By using mathematising and extra-mathematical knowledge that is the knowledge is not given in the problem, one can produce mathematical model. The mathematical model is in the mathematics world and needs mathematical approach and competencies. As a result of working mathematically the mathematical results are occurred. These results must be interpreted concerning the real problem to get real results. The real results let someone bring back from mathematics to reality. The real results are validated through mental representation and real model. These processes continue cyclically. This model was used in the current study and, as Borromeo Ferri and Blum (2010b) stated, theoretical competency was aimed at by sharing it with the participant preservice teachers as a model that represents the mathematical modeling cycle. Doerr and English (2003) asserted that students' skills in developing and refining their models by criticizing one another's assumptions and claims and by obtaining explanations and justification for problem solutions were developed by working on modeling exercises.

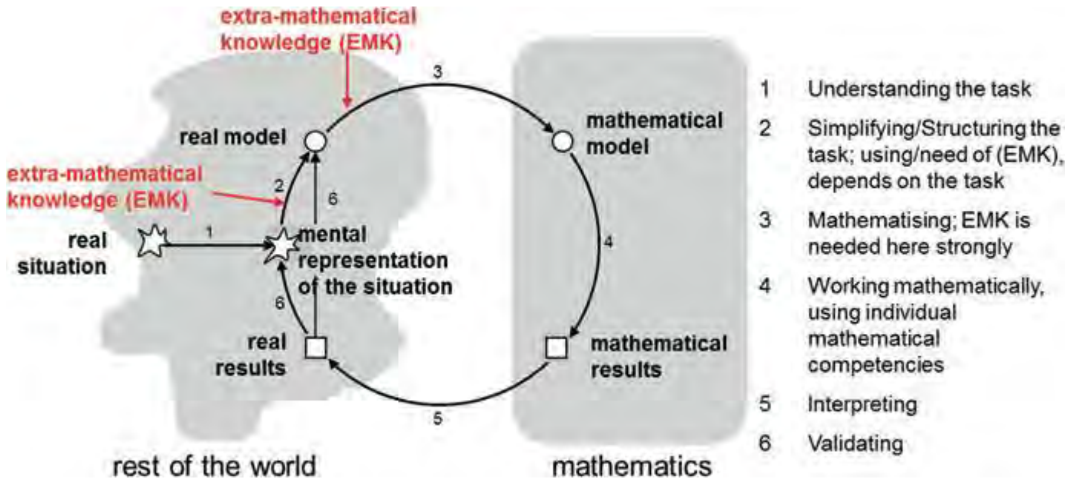

Figure 1. *The modeling cycle by Blum and Leiß (2007) as adapted and presented by Borromeo Ferri (2006 p.92)*

The need for further research describing competencies to use modeling in teaching has been voiced by a number of researchers (e.g., Blum 2015; Doerr 2007; Doerr and Lesh 2011; Kaiser 2014) and further research is needed in the field of teacher education within mathematical modeling. Therefore, in the current study, preservice mathematics teachers' pedagogical perspectives on mathematical modeling were examined through modeling activities. In a similar manner, Cetinkaya et al. (2016) designed a course-size research in order to investigate preservice mathematics teachers' development on mathematical modeling in the context of real life problems, and also the pedagogical principles and strategies needed for the teaching of mathematics through modeling. In their study, Cetinkaya et al. (2016) of three teaching semesters, 25 preservice secondary mathematics teachers were exposed to the mathematical modeling process as learners in order to learn how to solve and manage mathematical modeling activities, and as preservice teachers to teach mathematics through modeling activities. According to the findings, the preservice mathematics teachers developed ideas about the nature of mathematical modeling, as well as the relationship between mathematical modeling and meaningful understanding of pedagogy. Moreover, the researchers revealed that the preservice teachers realized the changing roles of teachers during mathematical modeling implementation and the diversity of students' thinking.

Similarly, Anhalt and Cortez (2016) worked with 11 preservice mathematics teachers attending a modelling lessons in their teacher preparation program in the USA. The preservice teachers were engaged with several mathematical modeling problems through reading, discussions and reflection. According to findings preservice teachers broadened and deepened their conceptual understanding of mathematical modeling. The researchers underlined that more targeted research is necessary to address the development of future teachers' knowledge for teaching mathematical modeling, that is pedagogy of mathematical modeling (Anhalt and Cortez 2016). Pedagogy of mathematical modeling is explained as presenting appropriate pedagogical perspectives that teachers use effectively for mathematical modeling to teach mathematics or for teaching mathematical modeling.

3. Methodology

Johnson and Onwuegbuzie's (2004) dominant/sequential design of the mixed method approach is employed in the study. In this design, the qualitative part of the study is more dominant than the quantitative, and both parts sequentially follow each other. Preservice teachers are exposed an intervention about mathematical modeling activities for 12 weeks. During and after this intervention their achievement levels on mathematical modeling activities, perceptions of conceptual understanding on mathematical modeling and perspectives of pedagogy of mathematical modeling are examined. The quantitative part of the current study involved an achievement test, Mathematical Modeling Test (MMT) (see Appendix) containing three modeling problems in order to categorize the participants based on their mathematical modeling success, and to identify their achievement level prior to the intervention. The problems were related to linear function, quadratic function, and trigonometric function, respectively. The problems were adapted from a study by Bukova-Güzel and Uğurel (2010), and were applied as a pretest and posttest in the current study. The quantitative results added perspective to the qualitative results in order to understand the level of improvement in constructing mathematical modeling for the modeling tasks.

The qualitative part of the study concerned the conceptual understanding of preservice teachers on mathematical modeling and perspectives on mathematical modeling pedagogy. Therefore, as the second data source, the written questionnaire, consisted of a single question, "What is mathematical modeling?" asked at the end of the intervention and third data source is participants' worksheets, collected during the intervention. The qualitative part of the study concerned the examination of the participants' worksheets through the process of modeling tasks and answers about both modeling problems and pedagogy of modeling through teaching questions. The "teaching questions" are asked to preservice teachers for examining the mathematical modeling activities from a teacher's perspective. The activity worksheets, included mathematical modeling activities about different function families (linear, polynomial, exponential, logarithmic, and trigonometric), providing participants with the opportunity to apply the mathematical modeling cycle. The mathematical modeling activities were collated from the related literature and redesigned within the worksheets. Among the 12 activities used in the program, five were chosen for the current study, the content of each problems and teaching questions are presented in Table 1.

In the first three mathematical modeling activities presented in Table 1, data was provided to the preservice teachers in advance. In the other two mathematical modeling activities, the preservice teachers gathered data by way of observation or collection. The teaching questions were asked towards the middle of the semester, since at that point, the preservice teachers should understand about what mathematical modeling is and also they should have learned about teaching mathematics as well as how to analyze students thinking and difficulties since they were taking Practicum course at the same time. Therefore, another reason for selecting five activities for the analysis among the 12 mathematical modeling activities was that these specific five each involved teaching questions related to different function families.

Table 1. *Mathematical Modeling Activities*

Name of the Mathematical Modeling Activity	Real life content of Mathematical Modeling Activity	Mathematical content of Mathematical Modeling Activity	Source of Mathematical Modeling Activity	Example of some Teaching Questions
Which Gasoline Is Better?	Examine two kinds of gasoline (97 and 95 octane) according to price and the distance that a car could travel for each fuel type.	Linear function	Fendel, Resek, Alper and Fraser (2005)	Why your students would have a difficulty in matematising step?

Selling Magazine	A company examines how much of an increase could be applied to the price of a magazine so that customers would still be willing to buy it, despite the increase.	Parabola	Aydın, Asma and Erbaş (2008)	How can you help your students in their difficulty in the validating step of the modeling?
Curing Poison Ivy	A patient with an acute poison ivy rash is given a five-day "prednisone taper." A daily dosage of the drug prednisone is administered. According to daily decay rate, calculate the prednisone left within the body.	Polynomial function	Almgren Kime, Clark and Michael (2011 p. 587)	Why a teacher would ask his/her students to give two different ways while working mathematically in the modeling process?
Too Hot To Handle, Too Cold To Enjoy	Examine the relationship between the time taken to drink a cup of hot coffee and its temperature.	Exponential function	National Council of Teachers of Mathematics (NCTM). (n.d.)	What would be the instructional positive and negative aspects of implementing the activity in the classroom?
High High Mountains	Measure a mountain's height using a clinometer. Writing a formula about angle measured and height of the mountain.	Trigonometric function	Produced by the researcher/author and a colleague	[How can you be sure the formula you produced calculates the heights of each mountain you wanted to measure?] In the previous question what would be the teacher's aim?

3. 1. Participants and Settings

The participants of the current study were 57 preservice high school mathematics teachers enrolled to a teacher certification program during the 2015-2016 academic year. They are also the fourth year students of Mathematics Department. The education in Mathematics Department last four years yet in teacher certification program period takes 14 weeks. The Teacher Certification Program is one of two routes available to becoming a high school mathematics teacher in Turkey. The two options are a four-year undergraduate program and a postgraduate certification program, both offered by Faculties of Education (Erbilgin and Boz, 2013). The postgraduate program usually takes one academic year to complete. However, at some universities, including the institution of the author, the certification program has been shortened to 14 weeks (two terms of seven weeks).

The participants are selected through convenient sampling since they are enrolled in Methods of Teaching Mathematics course whose instructor is the author of the current study. She collaborated with a colleague who is an instructor of the Instructional Technology course in order to integrate mathematical modeling into the contents of both courses. The overall goal being to increase preservice high school mathematics teachers' mathematical knowledge for teaching (Ball, Thames and Phelps 2008). As part of both courses, the preservice teachers solve modeling problems at the high school level, and analyze the modeling tasks from the teachers' pedagogical perspective.

Mathematical modeling is the focus of the Methods of Teaching Mathematics and Instructional Technology courses since it encompasses a wide range of mathematical processes. The researcher and her colleague who was also a mathematics educator with doctorate degree taught their courses through the same perspective of mathematical modeling activities. The two colleagues were arranged teaching procedure to conduct the modeling activities and they decided to prepare the modeling activities cooperatively. For twelve weeks they regularly met to discuss the modeling activities that they were going to implement in their classrooms. After implementation of each week they gathered again to examine and unpack the learning opportunities that the preservice teachers had during the lessons.

Mathematical modeling is considered as a pedagogical vehicle to teach mathematical concepts in a meaningful way; adopting Lesh and Doerr's (2003) perspective. Accordingly, the course activities aim to equip preservice teachers with the knowledge and skills to be able to use modeling to teach mathematical topics. The preservice mathematics teachers are introduced to the concept of mathematical modeling and engage in mathematical modeling activities in order to improve their content and pedagogical content knowledge. There were six modeling activities used for each of the mathematics education courses, making 12 modeling activities in total. Activity sheets were given to the preservice teachers at the beginning of each week and the groups then discussed the task and the related questions. They were provided graphical calculator or computer access during the courses. Groups used these technological tools for drawing graphs, checking for calculations or for some activities collecting and presenting data. In the groups at least one of the preservice teacher was expert on using technological tools that was supported to them. Groups shared their ideas and solutions for each question on the activity sheets. During this process the instructor only lead the discussion and asked them leading questions only. After solving the modeling questions, the groups discussed the "teaching questions," which were prepared to examine and predict the preservice teachers' reactions to teaching mathematical modeling activities in their future teaching career. These procedures were followed for both courses by the two mathematics educators. The modeling questions were unpacked through questions prepared based on Borromeo-Ferri (2006) modeling cycle. In the activity sheets based on the cycle, each problem was to be analyzed and examined for understanding and simplifying the task, a mathematical model constructed during the mathematising process, mathematical works examined and obtain a mathematical result, and the result interpreted based on the context of the problem. Besides solving modeling problems, the preservice teachers were required to answer teaching questions designed to elicit their knowledge about teaching mathematical modeling. In these hypothetical scenarios, the preservice teachers were made to think about how a teacher could support students during the activity of mathematical modeling, how they would analyze the modeling activities from a teaching perspective, and when evaluating a modeling activity from a teacher's perspective, when a model should be revised or remain unchanged. These questions were designed based on the selected teaching competencies; such as, knowledge of appropriate interventions during the students' modeling process, balancing teacher support with independent student work, encouragement of multiple solution methods, ability to plan and perform modeling lesson; from the related literature (Borromeo Ferri and Blum 2010a; Doerr 2007; Doerr and English 2006; Doerr and Lesh 2011; Kaiser and Stender 2013).

The preservice teachers worked in nine groups of three or four throughout the seven-week term in each mathematics education course. These groups were formed by considering the quartiles of scores obtained from the mathematical modeling pretest results so as to construct heterogeneous groups, which remained the same for each course. Therefore, in each group there were preservice teachers with low pretest scores and high pretest scores. The reason behind the preference for group work during the lessons was that research studies in the literature have claimed that modeling activities are better examined through group working (e.g., Ikeda, Stephens and Matsuzaki 2007). In each group working session, the participants held mathematically-based discussion in order to obtain multiple views of the mathematical situation and for productive arguments about producing the best-fit model for the real life situations.

3. 2. Data Analysis

In the quantitative part of the study; MMT was applied as a pretest and posttest; a total of 33 out of 57 preservice teachers took both tests. The MMT data were analyzed using paired sample t-test. Each modeling question of the MMT was graded according to a rubric produced by the researcher. The rubric involves four criteria: analyzing the problem (determining the variables and assumptions), creating a mathematical model (graph, formula etc.), the mathematical solution, and reflection (interpreting the solution, revising the model if necessary). According to the rubric presented in Table 2, each question was graded with 0-3 points.

Table 2. *Rubric for Evaluating Questions of Mathematical Modeling Test.*

Criteria	0 Point	1 Point	2 Points	3 Points
Criteria 1 Identifying variables and concepts related with the problem to solve it.	No variables or concepts related to the problem could be identified precisely.	Some visual or verbal explanation exist but they contain no identifiable variables or concepts.	Some mathematical variables or concepts for solving the problem are unidentifiable.	Variables and concepts related to the problem are identified precisely.
Criteria 2 Constructing a mathematical model (e.g., graph, algebraic equation).	Model not constructed correctly or constructed model is incorrect.	Constructed model relates to the problem, but is impractical to use.	Constructed model fits the problem, but contains computational errors.	Model used is both correct and precisely constructed.
Criteria 3 Solving the problem mathematically.	Problem not solved, or is completely wrong.	Problem solving process involves mistakes, pointing to a deficient solution.	Problem solution method is true, but contains mistakes.	Problem is solved completely.
Criteria 4 Interpreting results from a problem solution and adapting to real life situations.	No result obtained, result could not be interpreted, or completely wrong interpretation.	Obtained result accepted as true without any reason given.	Obtained result adapted to problem situation and interpreted with explanation, but contains some missing points.	Obtained results correctly interpreted, and adapted to real life situation.

The other data source, the written questionnaire (What is mathematical modeling?), was analyzed according to thematic inductive approach (Braun and Clarke 2006). The thematic approach helps to identify patterns of meaning among a dataset and it straddles the line between inductive versus deductive reasoning. An inductive thematic approach is a "bottom-up" approach in which themes and codes are derived from the content of the data. Whilst difficult to ignore the theory while coding data, the researcher gives priority to data-based meanings rather than the theory itself. Therefore, the written questionnaire was coded using a "bottom-up" approach and codes were assigned by considering the mathematical modeling cycle (Borromeo Ferri 2006).

The researcher and a colleague independently coded the responses of the preservice teachers, and the consistency of the codes was determined using the method [consensus/(agreement+disagreement)* 100] (1994, Miles and Huberman). 90% inter-rater reliability was reported. The researchers then came together and discussed the part (10%) that was coded differently, until full agreement (100%) was achieved. Direct quotes from the preservice teachers' responses are used to support the codes and themes.

The preservice teachers' classroom works (responses to the worksheet questions) were analyzed from two perspectives: 1) Preservice teachers' success in completing the mathematical modeling cycle; and 2) Preservice teachers' perspectives about how to teach modeling and its pedagogical aspect. A total of 45 worksheets (nine groups of five worksheets) were coded by the author and another mathematics educator using the rubric developed for the MMT. For the first perspective, the scores obtained the rubric were converted into percentages. The percentages are calculated based on the highest score (12 points) one can get from a modeling activity based on rubric given in Table 2. The percentages preferred to use for understanding achievement level's comparisons among the phases of modeling cycle. A percentage between 0-33 is considered low, 34-67% as medium, and 68-100% as a high level of achievement for mathematical modeling.

Table 3 provides information about each data source, the type of analysis applied, and for what purpose the data was included in the study. Using multiple data sources enhanced data credibility by enabling the researcher to compare information, and to search for and confirm patterns.

Table 3. *Data Sources.*

Data source	Type of analysis	Aim
Mathematical Modeling Test (MMT)	Quantitative approach (paired sample t-test)	To identify the difference between pre and post intervention achievement
Written questionnaire: What is mathematical modeling?	Thematic inductive analysis	To identify what preservice teachers conceptualized about mathematical modeling
Preservice teachers' classwork on modeling activities	Thematic analysis of teaching questions	To identify preservice teachers' pedagogical content knowledge through mathematical modeling
	Descriptive analysis of mathematical modeling	To identify preservice teachers' performances for each step of the mathematical modeling cycle

The preservice teachers' classroom works examined through the teaching questions. The answers of these questions are analyzed by using open coding method (Miles and Huberman 1994). Codes with similar dimensions were collapsed into groups which eventually formed the themes. 3 themes are emerged; these are how preservice teachers support students learning, analyzing mathematical modeling activities from the teachers' perspective, and evaluating worksheet questions from the teachers' perspective.

3. 3. Trustworthiness

In the qualitative research studies to ensure the reliability trustworthiness examination is crucial (Lincoln and Guba 1985). Triangulation, using three different measurement tools, is used to verify credibility. During the data gathering my colleague and I periodically gathered and discussed the activity sheets and findings obtained from the activity sheets. Through the weekly meeting I could examine the whole research process from multiple perspectives. In these meeting intercoder agreement is obtained and so dependability is secured. Moreover, transferability is guaranteed by giving a thick description of the research processes by explaining the procedure of the mathematical modeling implementation in the lessons. As a researcher and instructor of this study, I am aware of my bias and values that brought to the research process. These values and biases are under control through the weekly meeting discussions with my colleague, thus confirmability is ensured.

Limitation of the study is that interview with preservice teachers or endorsement was not used. The data from activity worksheets and written questionnaire are interpreted through the observation of the instructor who is the researcher in this case. In order to strengthen the transferability of the findings that emerged during data analysis, the presentation of the findings will contain some excerpts from preservice teachers' answers.

4. Results

One of the purpose of this study is that examining the change in preservice mathematics teachers' content knowledge of mathematical modeling. The MMT results provided an answer for that change. A paired sample t-test was conducted to compare scores of MMT from pre-test and posttest. There was a significant difference in scores for pretest ($M=12.58$, $SD= 4.66$) and posttest ($M=20.45$, $SD= 7.02$) results; $t(32)=-0.8300$, $p<.000$. These results suggest that the intervention of mathematical modeling does have increased the success of the preservice teachers' mathematical modeling achievement. Specifically, this study suggests that when preservice teachers are engaged with modeling activities with performing reflecting on the activities, they could develop their achievement.

When deeper analysis is conducted on performance of preservice teachers' achievement on modeling activities, it appears that they had difficulties in different part of the modeling problems for each mathematical modeling activities. In Table 4 these findings are summarized.

Table 4. *Achievement Level Results for Each Mathematical Model Activities*

Names of the Mathematical Model Activities	Which Gasoline Is Better?	Selling Magazine	Curing Poison Ivy	Too Hot To Drink, Too Hot To Handle	High High Mountains
Criteria 1 Identifying variables and concepts related with the problem to solve it.	85%	83%	74%	67%	63%
Criteria 2: Constructing a mathematical model	89%	73%	89%	86%	93%
Criteria 3: Solving the problem mathematically	48%	50%	56%	33%	74%
Criteria 4: Interpreting results from a problem solution and adapting to real life situations	67%	29%	48%	61%	71%

The preservice mathematics teachers' responses to the mathematical modeling questions were analyzed using the previously described rubric and presented in the Table 4 above. From the five worksheets, the groups obtained high and medium achievement levels (85%, 83%, 74%, 67%, and 63%, respectively) for the first criteria which is related with the analyzing the problems and identifying the variables and high achievement level (89%, 73%, 89%, 86%, and 93%, respectively) for the second criteria which requires constructing the mathematical model. However, the preservice teachers obtained low, medium, and high achievement levels (48%, 50%, 56%, 33%, and 74%, respectively) for the third criteria where the solution of the problem is conducting and for the fourth criteria (67%, 29%, 48%, 61%, and 71%, respectively) in which the results of the solution interpreted and adapted for the real life.

These results indicated that the preservice mathematics teachers were good at analyzing problems and creating mathematical models, but experienced difficulties in finding the mathematical solutions and interpreting the results or revising the model. In the following subsection, one of the modeling tasks is examined deeply in order to understand the preservice mathematics teachers' actions on the modeling tasks. The reason for the selection of this particular modeling task is because of the low achievement percentages.

4. 1. "Selling Magazine" task and preservice mathematics teachers' responses to each step:

The Selling Magazine task involves a polynomial function. In the task it is expected that changing of the variable in a quadratic formula is realized by the students, and a model constructed in order to explain the turning point of the polynomial function within the context of the problem. The problem given was as follows:

> The retail price of the Mathematical Thought Magazine, which is published quarterly with current sales of around 25,000, is 5.50 TL (Turkish Lira). However, due to the increase in production and paper costs, it has become inevitable to increase the retail price of the magazine. In order to better understand the negative impact of the increase on magazine sales, a survey was conducted among its readership. Accordingly, it is expected that every 0.50 TL increase in the magazine's retail price will result in 1,250 fewer magazine sales. If you were the managers of the magazine, what retail price would you set?

The problem was detailed in the activity sheet, with five questions asked based on the four criteria in order to evaluate the content knowledge about modeling processes of the preservice mathematics teachers. For the first criteria, "What variables are you going to use to construct the model?" was asked. Most of the groups answered this question correctly, and as can be seen from Table 4, the success was found to be 83%. For the same question, the following explanations can be given as to the incorrect or incomplete answers: "We determined the variables according to the amount of rise" [Group-6, there is no specific x or y variable explanations therefore, achievement level labeled as %0]."x= How many times a rise was given" [Group-9, There was no y variable explanation therefore, achievement level labeled as %33]

For the second criteria, most groups first examined the pattern of each price increase versus the sales decrease, and then tried to write the polynomial function. For example, in Figure 2, Group-1 established a four-column pattern, with the first column representing the number of magazine sales decreasing for each retail price rise. The second column shows the magazine retail price changes, and the third shows the amount of money realized from the magazine sales. The fourth column shows the income variance based on the retail price change, which is the net result seen by the magazine's owner.

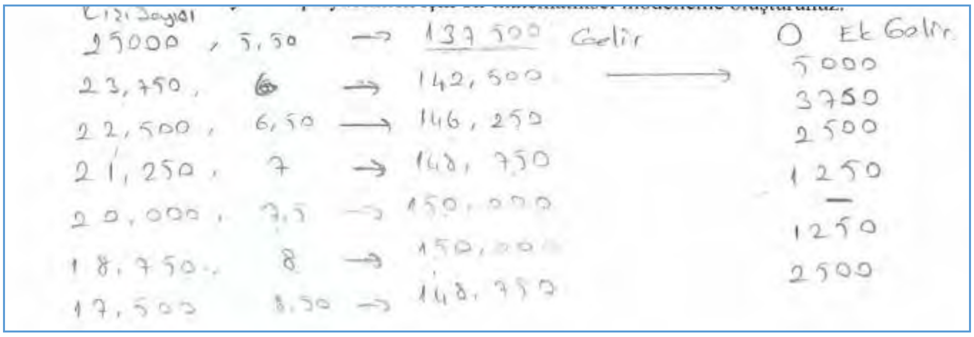

Figure 2. *Group-1's Solution*

Although the four column examination is very informative it is not helpful to construct the formula for the function. Therefore, Group-1 did not construct the model (achievement level labeled as %83) of polynomial function since they concentrated on the additional income for each price rise that the magazine owner would receive. This pattern did not lead them to produce a polynomial function, which is dependent on the relationship between the retail price increase and the resultant income.

On the other hand, Group-5 first identified the variables (x= 25,000 the number of magazine sales; p=5.50 TL is the current retail selling price per magazine; Income= x.p, k: is the amount of each price rise). They constructed a relationship among the amount of the price rise, the total number of magazines sold, and the resultant increased retail price of the magazine (see Figure 3). According to these relationships, Group-5 produced a "G function" (the achievement level labeled as %100) dependent on k as the amount of the price rise.

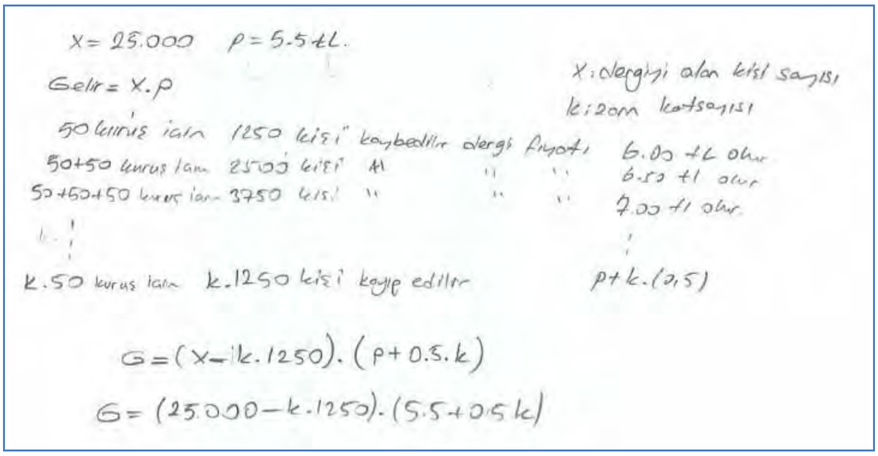

Figure 3. *Group-5's Solution*

For the third criteria, the groups' performances were observed as being 50% successful. That is, half of the groups could not solve the problem, even though they obtained the mathematical model. For example, as can be seen in Figure 4, Group-4 (the achievement level labeled as %33) examined the relationship between the retail price of the magazine and the number of magazines sold. At a retail price of 7.50 TL and 8.00 TL, they found it generated the same income level of 150,000 TL. According to this examination, although they constructed the correct model, they could not find the right answer; which was 7.75 TL as the new retail price of the magazine.

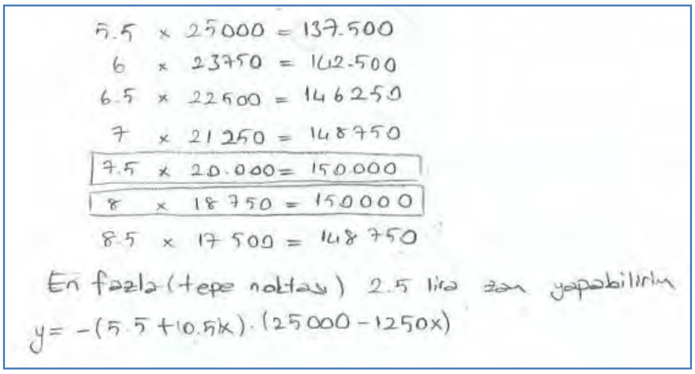

Figure 4. *Group-4's Solution*

Another incorrect answer was given by Group-8. In Figure 5, it can be seen that they utilized a valid mathematical model; however, the graph of the function which represented their mathematical model was a quadratic function, where y intercepted at the point 1,375,000 when the multiplication (5.50 TL x 25,000 sales) was computed. However, Group-8 (the achievement level labeled as % 0) drew a graph (see Figure 6) which did not intercept the y axis at that point.

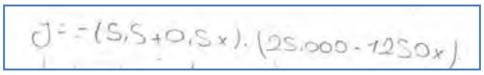

Figure 5. *Group-8's Mathematical Model*

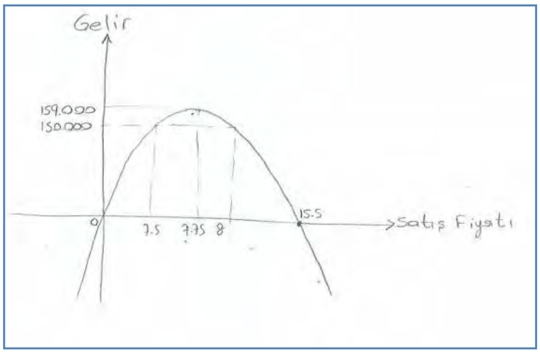

Figure 6. *Group-8's Graph*

For the last criteria, the groups' success percentage was found to be very low (29%). Most of the groups could not find the result and as a consequence they could not perform any interpretation of the result. Among the groups who found the result, some were able to interpret it considering the context of the problem. For example, Group-7 (the achievement level labeled as %100) drew the graph of a function that correlated with the mathematical model given in Figure 7. Although the graph was slightly incorrect at the point 264,006 (this point should be 150,156.25 at the y value of the turning point), the x value of the turning point of the parabola (4.5) was correctly plotted. According to their graph and answer, they interpreted the result as follows,

> 137,500 TL is the starting point on the y axis, which is the total income from selling the magazine at the starting point. After 4.5 retail price rise hikes, the income from the magazine sales starts to decline; so that fewer sales means decreasing income.

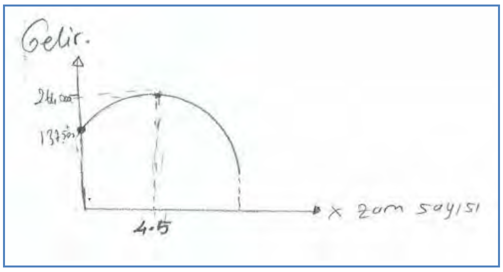

Figure 7. *Group-7's Graph*

Among the criteria, the last one is the most compelling. The preservice mathematics teachers even constructed a proper mathematical model for the problem assigned to them, although they experienced some difficulties in explaining the meaning of the result in the context of the problem. This difficulty was observed for all of the mathematical modeling activities.

4. 2. Pedagogy of Mathematical Modeling

The examined data emerged three themes. These themes and related codes are summarized in the Table 5 below.

Table 5. *Analysis of Preservice Teachers' Responses to Teaching Questions*

Themes	Codes	Examples from worksheets
Theme 1 How preservice mathematics teachers support student learning	Teacher-centered approach Scaffolding strategies	Assisting students by providing too much direction, explaining the correct answers, giving different examples Make students collect more data, let students help each other, connect with previous topics, asking open-ended questions
Theme 2 Analyzing mathematical modeling activities from the teachers' perspective	Strength of modeling activities Connections Motivational tool Student-centeredness Effective learning Weakness of modeling activities Readiness level Classroom management Distraction with technology	Strength of modeling activities Real life relations, math connection between representations, math connections between topics Learn a topic based on need Learning by doing Retention, conceptual understanding, challenging Weakness of modeling activities Prerequired knowledge, prier mathematical knowledge, classroom achievement level Time and control issues Technology distract students' attention
Theme 3 Evaluating worksheet questions from the teachers' perspective	Identifying mathematical purpose (graph reading) Identifying pedagogical purpose (multiple solutions)	Graph reading, producing formula, solving questions Comprehending multiple solutions, producing different perspective to solve questions, use multiple representations

In the first theme the answers of the preservice teachers complied with two perspectives of teacher behavior in the classroom. These are "teacher-centered approach" and "scaffolding strategies". In these codes preservice teachers explain their behaviors in their future classroom when they are going to be a teacher with modeling lesson.

In the second theme, preservice teachers' answers collapsed into two opposite perspectives in usefulness of mathematical modeling activities in mathematics classrooms. These are strength and weakness of modeling activities which means how a mathematical modeling lesson effect and help learning of mathematics in the classroom. The codes were "connections," "motivational tools," "student-centeredness," "effective learning," are clustered in strength of the mathematical modeling and "readiness level", classroom management," and "distracting with technology" are constructed the weakness of the modeling activities.

In the last theme the answers related with the preservice teachers' evaluation ability for teaching materials. The questions about how preservice teachers evaluate the modeling questions as a teacher and how they interpret the reason behind the modeling questions in pedagogical perspectives are constructed the last theme with two sub codes. Preservice teachers' answers about the aim of the mathematical modeling question gathered in "identifying the mathematical purpose" and "identifying pedagogical purposes". In the first one preservice teachers mostly observed the mathematical context of the modeling questions but in the second code preservice teachers realized that these modeling activities with teaching questions also help them think about their future students' mathematical thoughts and mental structures about the mathematics.

According to themed analysis, for Theme 1 (How preservice mathematics teachers support student learning), at times the preservice teachers expressed a teacher-centric approach by stating too much direction or explaining exact answers. For example, in response to a question asking what learning difficulties a student could have in creating a model and how to assist such a student for the activity (High High Mountains), Group-1 answered that "The student could have difficulty with finding the period. In that case we could provide the formula to help." In this answer it can be seen that the preservice teachers did not think that they should lead the student for finding or constructing the formula; instead, they suggested providing their student with the formula without further deductive reasoning. On the other hand, in some other few cases, the groups showed a scaffolding approach to support the students' learning such as asking students to create a new representation or connecting the current topic with previous topics. For instance, for the same question, Group-5 answered that "We requested students to create a table to explore the tendency of the data." In this answer, the preservice teachers presented an expected reaction for a mathematics teacher.

For Theme 2 (Analyzing mathematical modeling activities from the teachers' perspective), the seven codes describe the strengths and weaknesses of the modeling activities during the teaching and learning, according to the preservice mathematics teachers' perspectives. For instance, Group-4 stated that the modeling activities were a real life connection of the mathematics topics because of the context of each problem. These kinds of phrases were coded as "connections" and was classified under the group "strengths of modeling activities." Another group explained their similar thoughts as "connection with real life makes mathematics learning meaningful for the students therefore modeling problems is a strong tool for mathematics classrooms"

The Group-4 continued, stating that "students could collect data by themselves" which was coded as "student-centeredness" whereas "students could reach conclusions, make generalizations and interpretations" was coded as "effective learning" and "teachers could increase motivation by using technology" was coded as "motivational tools." So far, these codes were all grouped under "strengths of modeling activities." Preservice teachers claimed that students could make some generalization and interpretations during the modeling tasks and the goal of the mathematics learning is being able to create generalizations for the mathematical thoughts. Since they put forward their thoughts in this way these were classified as strengthened of the modeling activities. Moreover, preservice teachers stated that motivation for learning mathematics is an important component for the lesson. Therefore, according to answers of Group-2, in modeling lesson using technology or collecting data or investigating different ways to solve the problems are all effective ways of motivating the students. These perspectives are all strengthen of modeling activities for mathematics classroom. On the other hand, according to other groups of preservice teachers using modeling activities may have some disadvantages.

According to Group-3, "classroom management may be difficult when we use these activities" and "while using technology in these activities students' attention may get lost and this makes difficult to control the students". These kinds of answers were coded as "classroom management." Some groups believe that teachers may have difficult times when they use mathematical modeling problems in the mathematics lessons, because a teacher may have a poor classroom management. According to another group, the modeling activities were examined through the students' background, since they stated that "these activities would be difficult for students if they do not know how to write a function or do not know the parabolic function, for example. So that students should know these concepts beforehand." This quote was coded as "readiness level" and was classified under "weaknesses of modeling activities." Because preservice teachers believe that mathematical modeling lessons are not good for all students. Because these problems contain a lot of background information and if a student does not have one of these information according to preservice teachers these students had difficulties to solve problems. Therefore, mathematical modeling had some disadvantages.

Another critic for the modeling activities was about using technology during the activities. The groups stated that technology should be used in a balanced way; that is, using technology should not be placed ahead of the main aim of the activity. It was said that "technology should be used for more mathematical observation and dynamic features of the program should be used for analyzing the functions in the activities. We mean that technology helps with mathematical observations." From this quote it can be seen that using technology is considered most of the time as a helpful tool, but that excessive usage or placing technology at the core of the activity was taken as an obstacle to modeling activities. Therefore, technology is considered in weakness and also strengthened of the modeling activities. The category is depending on how the technology is used by the students and teachers. For Theme 3 (Evaluating worksheet questions from the teachers' perspective), the preservice teachers' responses were clustered under two codes. These were "identifying mathematical purpose" and "identifying pedagogical purpose." In several worksheets, the preservice mathematics teachers were asked to identify the possible purpose of a question from the worksheet. The majority of the groups identified "reading graphs," "working with representations," and "multiple solution methods" as possible teacher goals in asking the questions. In some cases, they were unable to make deep observations about the mathematical pedagogical aspect of the teacher's question. For example, in the third mathematical modeling activity (Curing Poison Ivy), the worksheet included a question asking to make observations about the graphs of polynomials when the degree was even or odd. The next question asked preservice mathematics teachers to determine the teacher's purpose for asking the degree of the function and related observation. Group-9 answered as "he/she may ask this question because in order to help students read the graph and think visually." Many groups responded in a similar way, missing the purpose of examining the regularity in the graphs and making generalizations. This answer showed that the preservice teachers thoughts on teaching were dependent on superficial comments about concepts, rather than their comprehensive understanding and orchestrating a learning environment through connections and logical reasoning.

Another observation about the preservice teachers' development of conceptions for teaching modeling was produced from the written questionnaire. Analysis of the written questionnaire revealed that comprehension of mathematical modeling collapsed into four categories. These were "mathematizing," "modeling cycle," "problem solving," and "misunderstanding." In the mathematizing category, 16 (28%) preservice teachers (out of 57) conceptualized mathematical modeling as translating real life situations into mathematical language such as visual, graphic, or algebraic representations. For example, one of the preservice teachers wrote that "modeling is representing a mathematical topic based on a real life situation by using mathematical terminology."

A total of 13 (23%) preservice teachers' answers were classified as modeling cycle involving the identification of variables and assumptions, constructing a model, mathematical solution, and interpreting and revising the model. In this category, the preservice teachers described each step of the modeling cycle. A total of 16 (28%) preservice teachers limited the definition of modeling to the problem-solving process. For example, one preservice teacher defined modeling as "solving a real life problem with mathematical expressions." Some of the preservice teachers added Polya's problem-solving steps to their definitions. Twelve (21%) of the preservice teachers defined modeling incorrectly. Some of the definitions included representing a mathematical topic by "using real life examples to make a mathematical topic more concrete," thereby defining mathematical modeling inversely. Some preservice teachers viewed modeling as a mathematics activity. For example, one such definition stated that "mathematical modeling was a type of a student-centered activity aimed to develop students' multiple perspectives rather than narrow thinking." The results of the written questionnaire showed that preservice teachers mostly understood mathematical modeling as mathematizing. This result was consistent with their achievement in the modeling activities.

According to Table 4, the preservice teacher groups were mostly successful in Criteria 2 which is constructing a mathematical model. For this part of the modeling activity, the preservice teachers were required to write a mathematical function which represented the given real life situation. Since they were very successful in this part of the modeling activities, they mostly thought that mathematical modeling meant writing a formula which explained real life, so that the process of mathematical modeling was all about mathematizing according to them.

5. Discussion and Conclusion

Mathematical knowledge needed for teaching mathematics has been the focus of many teacher educators' research endeavors (e.g., Ball et al. 2008) since student learning is closely related to teacher knowledge (Hill, Rowan and Ball 2005). Mathematics teachers should have knowledge on the content (mathematics, more specifically mathematical modeling in the current study), as well as knowledge of the pedagogical tools. Data analysis indicated that although the preservice teachers increased their content knowledge of mathematical modeling, they had some difficulties in accomplishing some steps of the modeling cycle, solving the model and interpreting the solution. The solution of a mathematical model involves solving the task based on real life context. The preservice teachers struggled to think critically and make decisions about real life situations. One possible reason behind this struggle could be that solving a mathematical modeling question requires not only mathematical knowledge, but also extra-mathematical knowledge (Blum 2015). The difficulty with interpreting the result and revising the model could be related to the preservice teachers' lack of experience with mathematical modeling. Most of the preservice teachers were unable to accurately define mathematical modeling, a finding that was aligned to previous studies found in the literature (Çiltaş and Işık 2013; Delice and Kertil 2015; Eraslan 2012). This revealed that preservice teachers in Turkey are perhaps not usually provided with opportunities to learn mathematical modeling during their K-12 school life, and therefore possess limited knowledge on the subject.

Analysis of the preservice teachers' knowledge on teaching mathematical modeling revealed that they had positive opinions about teaching modeling activities to their future students. They thought that the activities would help high school students to make mathematical connections, to engage in mathematical inquiry, and to increase their motivation for learning mathematics. However, the preservice teachers expressed some concerns about managing activity-based lessons, and also in using technology within those lessons.

A further aspect of the pedagogy of mathematical modeling is the knowledge of classroom management in the process of teaching through modeling. In order to efficiently build up the classroom setting for group work and to closely observe each group's modeling processes, teachers must possess the appropriate knowledge and abilities. (Lesh and Doerr 2003; Lingefjärd and Meier 2010; Zawojewski, Lesh and English 2003). However, according to many studies (Beeth and Adadan 2006; Harding and Hbaci 2015; Reupert and Woodcock 2010) preservice teachers struggles with the classroom management in their field experience. Aligned with these studies, in the current study preservice teachers posit that classroom management could be a problem in the mathematical modeling lessons because of the cooperative learning groups. They underlined their concern about classroom management because they did not experience about cooperative learning in mathematics classroom neither as a student nor as a teacher. Although the preservice teachers had concerns about classroom management, they demonstrated a transitional approach from teacher-centeredness to student-centeredness in their findings about pedagogy of mathematical modeling. In Borromeo Ferri and Blum's (2010b) study managing a student-centered lesson with group work emerged as one of the four themes which is the similar findings of the currents study. In that study, the researchers suggested that the ability to perform a modeling lesson was one of the pedagogical content knowledge components of mathematical modeling (Borromeo Ferri and Blum 2010b). Similarly, Cetinkaya et al. (2016) claimed that fostering competencies of preservice teachers to teach mathematical modeling in way that promotes student inquiry and discovery is a critical teacher education domain. In the current study, the opinions of the preservice teachers were transferred from a teacher-centered to a student-centered classroom, but since the duration of the program was very short at only seven weeks, the transformation of their teaching approach was incomplete. In response to criticism from opponents of teacher education, Darling-Hammond (2006) stated that more recent weak teacher education programs under-prepare teachers in shorter time periods. Turkey has experience of a similar teacher education context.

Another concern for the preservice teachers was related to their using technology in activity-based lessons. This concern was based on their lack of knowledge in using technology as a pedagogical tool. However, teachers should have both general knowledge about using technology in teaching mathematics, and knowledge about which technologies to use and how it may be useful for a particular modeling task (Lesh and Caylor 2007; Pead, Ralph and Muller 2007). Some of the preservice teachers embraced technology in mathematical modeling activities, yet some seemed unable to use it to conduct effective mathematical observations whilst others viewed technology as a distractor to classroom management. This finding leads to the consideration of Technological Pedagogical Content Knowledge (TPACK) related to teachers of mathematics. Since they lacked knowledge in this domain, the preservice teachers considered technology as a distractor to their lessons. Although the preservice teachers in the certificate program had taken a class which involved the topics of TPACK, since the duration of the study was very short, the preservice teachers were unable to take the opportunity to improve themselves in these concepts.

One suggestion based on the results of the current study is that preservice teachers should be provided with more opportunity to improve their content knowledge in mathematical modeling. They could take additional courses that focus on mathematical modeling during their undergraduate education. Then, in their postgraduate teaching certificate program, mathematical modeling teaching competencies should form part of their education as preservice teachers. Preservice teachers should be provided with opportunities to develop content knowledge and knowledge about teaching modeling starting from their preservice years. According to Borromeo Ferri (2014), mathematical modeling cannot be expected to transfer naturally from learning mathematics, but rather it must be learned specifically. Therefore, courses focusing on mathematical modeling that are designed to enhance preservice teachers' content knowledge and competencies related to the teaching of mathematical modeling within teacher education programs are needed. The results of the current study are promising since, even within a short timeframe, there were improvements seen in the preservice teachers' content knowledge and knowledge about teaching; however, some domains still required further improvement. Therefore, another important suggestion is that the duration and quality of teacher education (certificate) programs should be extended so that the preservice teachers can engage in more varied modeling activities, and be given the opportunity to reflect on their own teaching, and to analyze student thinking in modeling lessons.

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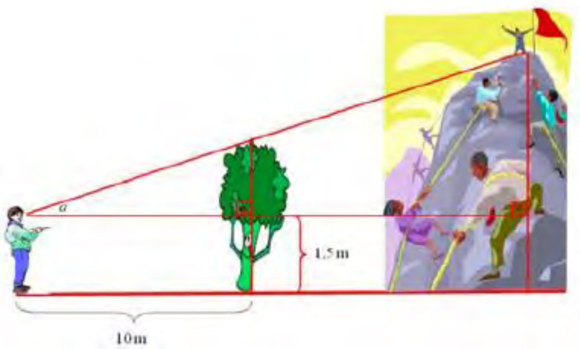
Appendix

Mathematical Modeling Test (MMT)

Problem 1: The contained with the capacity of 500 lt. contains 20 liters at the beginning. You are going to examine filling the container with different fountains flowing at different speeds. Please answer the following questions:

a) This container is going to be filled by fountain by flowing 2 liters per a minute. Identify a mathematical model of the water accumulated in the contained depending on time. Using the constructed model decide that how much second should be needed for filling the container.
b) Find a mathematical model for the amount of accumulated water in the container by using a fountain speed 1 liter per a minute. After that draw a graph of this model. Similarly find another model for an amount of water in the contained filled by a fountain speed 10 liter per a minute. Draw the graph of this model on the same coordinate system with the previous one.
c) Compare models that you found in a and b in order to similarity and differences. Decide which concept of mathematical model is related with the increasing or decreasing of filling time.
d) Write a generalization about how the water should be flow from the fountain for a minimum or a maximum time to fill the container.

Problem 2: Jack wants to calculate the length of the tree. He stands in front of the tree and look at it by α degree through the eye level. In this position he realized that he could see the top of the mountain. Please answer the related questions:



a) According to you how Jack can calculate the length of the tree? Produce a method for calculating the length of the tee.
b) What should be α when the tree has 11.5 meters or smaller than 11.5 meters?
c) How α be changed when he approaches the tree?
d) With the given information decide that whether Jack could find the height of the mountain. Please give a mathematical explanation.

Problem 3: A transport company wants to determine whether a large caravan can be moved along a highway that runs under a parabolic curve bridge. The base width of this bridge is 12 meters and its height from the center is 6 meters. Answers the following questions related with this situation.

a) Draw a figure that is going to help to show the width and height of the bridge and also useful for solution of the problem.
b) Decide whether the caravan with 9-meter width and 3.2-meter long is suitable for passing through the highway under the bridge.
c) Explain briefly what mathematical judgments you have made as to whether this caravan is suitable for passing under the bridge.

Author

Burçak BOZ YAMAN, Muğla Sıtkı Koçman University, Muğla, Turkey, e-mail: burcak@mu.edu.tr

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