



EXAMINING PRE-SERVICE TEACHERS' VIEWS ON REASONING, DEVELOPING REASONING AND IMPORTANCE OF REASONING SKILLS IN GEOMETRY

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Abstract: The aim of this study is to examine pre-service mathematics teachers' views on reasoning, importance of reasoning in geometry and development of reasoning skills. The study was designed in the phenomenological (phenomenological) design, one of the qualitative research methods. The study group of the study was conducted on 61 pre-service teachers who were studying in the elementary school mathematics teaching program of a university in the Southeastern Anatolia region of Turkey. In the study, a semi-structured interview form, developed by the researcher to collect data, was used to determine the opinions of pre-service teachers on the reasoning, types of reasoning and the importance of reasoning skills in geometry. The data obtained through this form were analyzed by content analysis method. When the research results are examined; It was determined that the pre-service teachers did not have sufficient knowledge about reasoning and the components of reasoning skills. The pre-service teachers stated that reasoning skills are important in geometry. In addition, the pre-service teachers expressed important and valuable views on what can be done to improve reasoning skills. Suggestions are presented depending on the results of the study.

Key words: Reasoning, pre-service teachers, geometry, reasoning game.

1. Introduction

Mathematics is knowledge obtained by reasoning (Rohana, 2015). Mathematics education performs a more important function than teaching operations and providing computational skills in daily life, and provides important supports such as thinking, linking events, reasoning, making predictions, problem solving that enable us to survive in the life war that is getting more and more complicated with each passing day (Umay, 2003). The reasoning made by mathematicians involves the way to associate or form an idea or concept with the next concept or idea (Brodie, 2010). One of the main goals of mathematics education is to find logical answers to the question "why" or to help improve reasoning (Danişman & Erginer, 2017). Among the skills that mathematics should teach students are reasoning, association, modeling, problem solving and communication. These skills can be developed by learning basic mathematical concepts and ways of acquiring mathematical knowledge (Baki, 2018).

Reasoning is the process of thinking and reaching a rational conclusion by considering all factors (Umay & Kaf, 2005). According to Lithner (2008), reasoning is the exercises requested from students in the classroom environment, tests, group work, etc. Thom (2011) states that reasoning involves "generating, applying and evaluating assumptions as well as verifying our thoughts and actions while dealing with mathematics." Rastogi (1983) found that one of the main factors in the success of mathematics is reasoning. Various classifications for evaluating students' reasoning skills have been proposed in the literature. These dimensions include interrelated and improved versions of each other. TIMSS categorized reasoning skills as analyzing, generalizing, making connections, making decisions, and finally solving non-routine problems (TIMMS, 2003). Reasoning ability is a method of measuring mental capacity. It includes the ability to solve problems and reach answers, that is, to reach solutions logically and to generalize (Fathima & Rao, 2008). There can be reasoning at all levels of mathematical understanding, and mathematical reasoning can be used at all levels of difficulty in solving non-routine problems (Jäder, Sidenvall, & Sumpter, 2017). Human beings, who are distinguished from other

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creatures with the ability to think as a requirement of their creation, take into account all factors and reach a rational conclusion by using the power of thought, by analyzing and discovering situations they have not encountered before, making logical predictions and assumptions, reaching some conclusions by justifying their thoughts and achieving them. can explain these results (Umay, 2003). To summarize, the concept of reasoning means making a conclusion from the propositions, facts and judgments, and making sure of these propositions or judgments (Altıparmak & Öziş, 2005). One of the most used fields of reasoning is undoubtedly mathematics. Because while teaching geometry, algebra, probability, numbers and many other subjects within mathematics; It also teaches basic skills such as reasoned thinking, exploring patterns, predicting, reasoning, and reaching conclusions (Umay, 2003). From an epistemological perspective, reasoning is the building block of mathematics (Steen, 1999). Mathematical reasoning is the ability to reason about these objects by using mathematical objects (Brodie, 2010). In other words, it is the process of obtaining new knowledge by using the unique tools of mathematics such as definition and symbols, and thinking styles such as deduction and induction, based on the available information (Ministry of National Education [MoNE], 2013a). The purpose of mathematics education is to enable students to increase their self-confidence in mathematical reasoning, producing mathematical thinking and defending their thoughts. (MoNE, 2013a). In mathematics, facts are reached not by experiment or observation, but by reasoning that underlies all rules and operations in mathematics (Umay & Kaf, 2005). Reasoning is also defined as the process of organizing evidence, information and thoughts together in order to draw conclusions about what is complete or true (Leighton, 2003). In another definition, reasoning is expressed as a mental process performed to create new information from old information (Rips, 1994). As can be seen from these definitions, reasoning skill can be qualified as a prerequisite for making sense of and explaining information. Even if it is at an advanced level, if the basis of a thought cannot be justified, if it is not based on knowledge, if it is not rational, it is not possible to accept it as reasoning (Umay, 2003). In general, mathematical reasoning is the process of drawing conclusions based on evidence or stated assumptions (NCTM, 2009). It is the process of mathematical reasoning to draw conclusions about some ideas based on facts obtained through logical and critical thinking in solving mathematical problems (Rohana, 2015).

More research is needed to discover the definition of the basic cognitive processes that make up mathematics (Sprigler & Alsup, 2003). However, in the literature review, it was seen that there were not enough studies on this subject.

Although the scope of the present study is limited, in the field of mathematics, it will contribute to how, where, when and for what purpose pre-service mathematics teachers can use their mathematical reasoning skills in their teaching activities. Since the study focused on the importance of reasoning and reasoning skills of pre-service teachers and taking opinions about the suggestions for the development of this skill, it is thought that it contains important signs on these issues. This situation reveals the importance of the study. In this context, the general purpose of the study is to examine the opinions of pre-service mathematics teachers about the importance of reasoning, the importance of reasoning skills in geometry, and the suggestions for its development. In line with this general purpose, answers were sought for the following sub-problems.

- 1) What are the meanings that pre-service teachers attribute to the definition of reasoning?
- 2) What are the opinions of the pre-service teachers about the reasoning games?
- 3) What are the pre-service teachers' views on the importance of reasoning in geometry?
- 4) What are the pre-service teachers' suggestions on how to improve reasoning skills?

2. Method

2. 1. Research Model

In this study, one of the qualitative research methods, a phenomenological study design was used. The phenomenological pattern focuses on phenomena that we are aware of but do not have a deep and detailed understanding (Yıldırım & Şimşek, 2013). The main purpose of phenomenological studies is to bring personal experiences of a phenomenon to a more general level (Creswell, 2007).

2. 2. Research Group

The study was carried out with 61 pre-service teachers studying at the elementary school mathematics teaching department of a state university in the Southeastern Anatolia Region. While choosing the participants, it was paid attention that they had taken the lessons in which reasoning skills were explained. In this respect, criterion sampling was chosen from purposeful sampling methods in the selection of the research group. Because in criterion sampling, observation units can be formed from people, objects or situations with certain qualities (Büyüköztürk et al., 2016).

2. 3. Data Collection Tool and Process

In the research, a semi-structured student interview form, developed by the researcher as a data collection tool, was used to examine in depth the opinions of pre-service mathematics teachers about the importance of reasoning, the importance of reasoning skills, what should be done to develop reasoning skills, and the importance of reasoning in geometry. The reason for using this form is because it provides access to in-depth information (Büyüköztürk et al., 2016). A draft form consisting of five questions suitable for the purpose of the study was prepared with a comprehensive literature review. It was paid attention that the questions in the draft form were easy, understandable and answerable by the students in accordance with the purpose of the research, but they did not contain guiding expressions. This form was submitted to two instructors who are experts in mathematics education and qualitative research for construct validity. In this process, in order to evaluate the opinions of the experts, an expert evaluation form was prepared for each question in the form, in which the categories "suitable", "partially suitable" and "not suitable" were prepared. Based on the feedback received from this form, it was decided that three questions were not appropriate and were removed from the form. Then, the form was applied to thirty-two pre-service mathematics teachers, excluding the research sample, with a pilot study. In this process, the questions that were not suitable in terms of grammar and comprehensibility were corrected and the final semi-structured interview form consisting of three open-ended questions was created. In the interview form, there are three open-ended questions about the pre-service teachers' opinions about the importance of reasoning skills, the things to do to develop reasoning skills, and the importance of reasoning in geometry. While preparing the form, expert academicians' opinions were consulted in qualitative research methods. In line with the opinions and suggestions received, the data collection tool was finalized and applied to the participants. Interviews with participants lasted 15-20 minutes on average. In the analysis of the data, the participants were shown with coding as T1, T2, T3,..., T61.

2. 4. Data Analysis

Content analysis was used in the analysis of the data obtained. Content analysis ensures that the obtained raw data are made sense of, creating a certain framework and organizing after the emerging situation becomes clear, revealing and concretizing the codes and categories (Patton, 2002). In the creation of codes and categories, the data was created primarily the codes and categories that could be encountered by the author. Afterwards, codes and categories were created by examining them separately by two researchers experienced in mathematics education. As a result of the analysis, the researchers came together to compare the codes and categories, and the codes and categories with different opinions were finalized. Miles and Huberman (1994) call the similar codes as Consensus and the dissociated codes as Disagreement, and propose the formula of $\text{Consensus Percentage} = \text{Consensus} / (\text{Consensus} + \text{Disagreement}) * 100$ for coder reliability. In the study, the agreement percentage of the codes obtained by the researchers was found to be .85. The Miles-Huberman reliability formula value being .70 and above shows that the coding is reliable (Yıldırım & Şimşek, 2013). The answer given by a participant to a question mostly occurred in situations that could be included in more than one category, although it was mostly classified into one category. Some of the participants did not answer some questions. This situation was evaluated under the "no answer" category in the study.

3. Findings

The question was asked “What does reasoning mean to you?” in order to find out what meaning the participants attributed to mathematical reasoning. When the answers were examined, it was determined that there were many different opinions and they were categorized. The responses about reasoning were grouped under 15 categories. The total category frequency was determined as 82. In Table 1, the categories created for the opinions of the participants are presented.

Table 1. Participants' Opinions on definition of Reasoning

| Categories | f |
|--|----|
| Generating ideas with thinking skills | 18 |
| Developing solutions | 14 |
| Thinking logically | 9 |
| Systematic solution | 8 |
| Looking at daily events from different angles | 7 |
| Problem solving strategy | 6 |
| Deriving new propositions from existing propositions | 3 |
| Judgment | 3 |
| Expressing abstract concepts better | 3 |
| Making the problem understandable | 3 |
| Examining with details | 2 |
| Overcoming difficulties | 2 |
| No answer | 2 |
| Predicting | 1 |
| Total | 82 |

According to Table 1, the opinions of the participants about reasoning are concentrated in the categories of "generating ideas with thinking skills" and "developing solutions". In addition to this, the other categories are respectively; thinking with a logical approach, systematic solution, looking at daily events from different angles, problem solving strategy, obtaining new propositions from existing propositions, judgment, expressing abstract concepts better, making the problem understandable, examining with details, overcoming difficulties, predicting and mathematical literacy. The opinions of the participants given below, with the most repeated categories at the top.

T3: *Being able to comment on the problems we encounter means something to me as reasoning.*

T14: *It is to be able to look at some events differently in daily life. If we think of mathematics, since some of them are abstract, reasoning offers the opportunity to express abstract concepts better.*

T22: *With the help of the information I know, it provides the opportunity to comment on the new problems I will encounter and to produce solutions. Actually, we can call it prediction.*

T34: *Reasoning is a way of thinking that develops a little higher order thinking skill of the individual and produces a little reflection of this skill in daily life and shows its effect in this sense.*

T37: *Reasoning is the way of thinking that enables to think about the solution by examining the smallest details of the problem and to seek new solutions. We can also call it judgment.*

T42: *Provides practicalization on a subject. We learn in what ways it can be solved and, in addition, how much it saves in terms of time.*

T46: *It is a situation that I generally use in problem solving. It is a method that allows me to solve many questions without memorizing many rules, and provides an indirect way to learn many lessons easier and permanently. It has benefited me a lot in terms of success. I take advantage in any situation.*

T51: *It means deriving a new proposition from the related situations and propositions of our mind. For example, Gauss's first collection of n terms is the best example of reasoning in my opinion.*

As can be seen in the examples of participant views, there are opinions that fall into more than one category. For example, the opinions expressed by the participants with the codes T14, T34, T37, T46 can be included in more than one category. Given as an example of categories; T3 participant's view is generating ideas with thinking skills, T14 participant's view is to be able to express abstract concepts better and to look at daily events from different angles, T22 participant's view is to developing solutions, T34 participant's view is to generate ideas with the ability to think and to look at daily events from different angles, T37 participant's view is to developing solutions, reasoning and examining with details, T42 participant's view is to developing solutions, T46 participant's view is to developing solutions, problem solving strategy and generating ideas with thinking skills, T51 participant's view is to deriving new propositions from existing propositions categories.

In order to determine whether the participants have information about reasoning games, they were asked the question "Do you have any information about Reasoning Games? Which games can you give examples?". The responses of the participants about reasoning games were collected under 15 categories. Table 2 includes these categories.

Table 2. *Participants' opinions on reasoning games*

| Categories | f |
|-------------------|----|
| Chess | 28 |
| Have no idea | 13 |
| Sudoku | 7 |
| Checkers | 6 |
| Bridge | 5 |
| Backgammon | 2 |
| Rubik's cube | 2 |
| Puzzle | 2 |
| Intelligence game | 1 |
| Tangram | 1 |
| Mangala | 1 |
| Mental games | 1 |
| Nine men's morris | 1 |
| Crossword | 1 |
| Tower | 1 |
| Total | 72 |

When Table 2 is examined, it is seen that the majority of the participants used the game of chess as a reasoning game. On the other hand, it is seen that the number of pre-service teachers who stated that they do not have information about reasoning games is quite high compared to other categories. Other categories formed by the responses received from pre-service teachers are; sudoku, checkers, bridge, backgammon, rubik's cube, puzzle, intelligence game, tangram, mangala, mental games, nine men's morris, crossword and tower. Below are sample participants' opinions, especially in the most repeated categories.

T51: Reasoning games enable students to achieve gains such as creative thinking, critical thinking, processing ability, entrepreneurship, and communication. The most important and favorite reasoning games for me are chess and sudoku.

T19: It is necessary to be able to think very differently while playing reasoning games. Games such as chess, sudoku, bridge can be given as examples of reasoning games.

T5: Reasoning games are activities that teach student learning. Especially these games like sudoku need to be more active in math lessons.

T29: Reasoning games are games that can be solved with a certain logic. For example; like chess, checkers and tower games.

T23: It is a type of game that benefits the development of the mind like chess.

T13: Brain games are a kind of reasoning game. Reasoning games are games that reinforce our knowledge.

T45: I can give an example of the game of chess. Reason is guided by guessing the moves of the opponent.

T49: Chess is a reasoning game. Because when we move the stones, we move by thinking.

T60: Reasoning games are games that ensure that mathematics and strategies are well known and interventions are made accordingly. Chess, checkers, backgammon and bridge are reasoning games.

T2: These are games that are a type of winning the game by using mental high-level skills. For example, bridge is a reasoning game.

T4: These are games that improve problem solving and analysis skills such as Sudoku.

As can be seen in the examples of participant opinions, there are opinions that fall into more than one category. For example, the opinions expressed by the participants with the codes T19, T29, T51, T60 can be included in more than one category. T19, T23, T45, T49, T51, T60 participants' views Chess, T2, T19 and T60 participants' views are bridge, T4, T5, T19, T51 participants' views are sudoku, T29 and T60 participants' views are checkers, T29 participant's opinion is tower, T60 participant's opinion is given as an example of backgammon categories.

In order to reveal the opinions of the participants about the importance of reasoning in geometry, the question "Explain your thoughts on the importance of reasoning in geometry?" was posed. When the answers given by the participants were examined, it was determined that the opinions were gathered under 11 categories. In Table 3, the categories created for the opinions of the participants are presented.

Table 3. Participants' opinions on the importance of reasoning in geometry

| Categories | f |
|--------------------------------------|----|
| Seeing well and effective processing | 19 |
| Positive and useful | 13 |
| Necessary | 7 |
| No answer | 6 |
| Develops abstract thinking skills | 5 |
| Provides versatile thinking skills | 4 |
| Reaching results faster | 4 |

| | |
|---|----|
| Finding solutions in different ways | 3 |
| Making geometry more understandable with analytical thinking method | 3 |
| Exploring | 2 |
| No importance | 1 |
| Total | 67 |

According to Table 3, the majority of the participants answered as "seeing well and effective processing" and "positive and useful". Some of the participants did not respond by stating that there was no information on the subject. In addition, one participant stated that reasoning in geometry is not important. Other categories formed by the responses received from the participants are; develops abstract thinking skills, provides multi-directional thinking skills, reaching results faster, finding solutions in different ways, making geometry more understandable with analytical thinking method, exploring. Below are sample participants' opinions, especially in the most repeated categories.

T29: *The ability to reason in geometry allows us to see the similarity edge relations more quickly, especially in polygons, by rotating shapes, enlarging or reducing them to certain dimensions.*

T45: *Considering the fact that formulas are formed from formulas, I can say that the success of the student with high reasoning skills in geometry lesson increases.*

T17: *Geometry is the phenomenon of seeing. If our perspective is wide, geometry becomes more understandable when approached with analytical thinking method.*

T26: *I think there is not much reasoning in geometry. Geometry is just seeing and understanding the subject.*

T54: *Since geometric thinking is spatial thinking, it contributes to students' versatile thinking skills. When used interactively with other lessons, positive feedback is obtained.*

T11: *Using the ability to reason on shapes in geometry, we find the result more quickly.*

T14: *Geometry is all about seeing. The child solves with reasoning skills in what way to use what he knows*

T31: *Because geometry is an abstract lesson, it is actually a lesson in which reasoning skills are very important.*

T46: *With reasoning, we do not need to memorize the cosine theorem, many questions can be made by lowering the perpendicular. Many problems can be solved this way. It is very difficult to find out which method to use in which question in geometry. Helps the student to overcome these difficulties.*

T28: *Geometry means using the mind. So reasoning is an important factor*

As can be seen in the examples of participant opinions, there are opinions that fall into more than one category. For example, the opinion of the T46 participant can be included in more than one category. T14 and T29 participants' opinions are seeing well and effective, processing T28, T45 and T46 participants' opinions are positive and useful, T11 and T46 participants' opinions reach the result faster, T26 participant's opinion is not important, T17 participant's opinion makes geometry more understandable with analytical thinking method. The opinion of the T54 participant is given as an example of the categories that provide multi-dimensional thinking skills.

In order to get the participants' views on what can be done to improve reasoning skills, this question has been asked: "What kinds of activities / practices / exercises etc. can be done to improve reasoning

skills?" When the answers given by the participants were examined, it was determined that the opinions were gathered under 13 categories. In Table 4, the categories created for the opinions of the participants are presented.

Table 4. *Participants' opinions on improving reasoning skills*

| Categories | f |
|---|----|
| Game activities | 20 |
| Intelligence game | 12 |
| Activities to encourage research and thinking | 11 |
| No answer | 10 |
| Various problems | 9 |
| Daily life | 4 |
| Material | 5 |
| Group work | 4 |
| Readiness | 2 |
| Reasoning should be given as a course | 3 |
| Learning by doing and living | 2 |
| Association | 3 |
| Drama | 1 |
| Total | 85 |

When Table 4 is examined, most of the participants stated their views as "game activities", "intelligence games" and "activities to encourage research and thinking". Some of the participants did not respond by stating that there was no information on the subject. The reason that there are two separate categories as game activities and intelligence games, the participants mentioned general games as game activities. Other categories formed by the responses received from the participants are; various problems, daily life, material, group work, readiness, reasoning should be given as a course, learning by doing and living, drama. Below are sample participants' opinions, especially in the most repeated categories.

T10: *We can make them think about lessons and subjects by connecting them with games and daily life.*

T15: *Activities should be carried out in accordance with the principle of concrete to abstract, vitality and most importantly the principle of suitability for the student (age, interest, need, gender).*

T21: *The teacher can bring the topic to the class with the help of a material, and enable the students to reason about these issues and the problems that the subject involves, and reach conclusions.*

T51: *I think it should definitely be given as a course. It should be developed with intelligence games. Students should be encouraged to research and think. I think more effective individuals will be raised in this way.*

T22: *Activities that require a strategy should be prepared and organized. Problems should not be solved and should be made by the student.*

T29: *Chess, checkers and math games should be played. Students should be actively directed to establish logical relationships between numbers using abstract shapes.*

T17: *Students should be offered activities that they can do with group work, and practices should be made in which they can communicate and affect each other in cooperation.*

T40: *First of all, we have to get the information discovered. We should not give the student the information, but let them discover it with clues at the end of a game activity or research activity.*

T46: *We must encourage students to come up with new ideas. We should increase the importance of problem-based mathematics education, especially mathematical problem solving activities. We should direct the question of how else can we solve a problem to the student. We should encourage you to relate a topic told to another topic or topics. In addition to these, we must differentiate their point of view.*

As can be seen in the examples of participant views, there are opinions that fall into more than one category. For example, opinions expressed by T10, T15, T21, T22, T40, T46, T51 participants can be included in more than one category. T10, T29 and T40 participants' opinions, game activities, T51 participant's opinion intelligence games, T15, T22, T40, T46, T51 participants' opinions, activities to encourage research and thinking, T21, T22, T46 participants' opinions on various problems, T10 participant's opinion on daily life, The opinion of the T15 participant, the readiness, the opinion of the T21 participant should be given as the material, the opinion of the T51 participant as the reasoning lesson, and the opinion of the T17 participant is given as an example of the group work categories.

4. Conclusion, Discussion and Suggestions

In the study, the opinions of pre-service mathematics teachers on Reasoning, Developing Reasoning and Importance of Reasoning Skills in Geometry were examined in depth. The qualitative data collected with the semi-structured interview form was analyzed by content analysis. In the analysis process, categories and frequency distributions were evaluated in line with the answers given by the pre-service teachers to each question in the interview form. When the answers given by the participants to the question "What does reasoning mean to you?" are examined, it is possible to reach some results. Considering the categories formed by the views of pre-service mathematics teachers, there is no indication that they understand mathematical reasoning at a sufficient level. However, as far as pre-service teachers generally perceive or learn informally, they are aware that mathematical reasoning is a very important skill. The state of consciousness about a subject is closely related to the knowledge. When the opinions received from the pre-service teachers are examined, it can be said that there is an awareness, even if they do not have full knowledge, in the light of the experiences gained by the pre-service teachers. This result obtained from the study is similar to the findings of the study by Loong, Vale, Bragg, and Herbert (2013), that although some teachers can tell the meaning of reasoning, some of them use ambiguous expressions such as confusing concepts with problem solving and reasoning skills. In another study, Herbert, Vale, Bragg, and Loong (2015) complied with their teachers' views on mathematical reasoning. Öz and Işık (2017) stated in their study that they perceive the mathematical reasoning skills of pre-service teachers as associating subjects with daily life, being able to offer different solutions to problems, accessing new information based on known information, problem solving, and various types of thinking. It can be said that there are similar results in this aspect. When it comes to reasoning skills, we come across games that require some reasoning skills. Reasoning games play an important role in developing reasoning skills (MoNE, 2013b). For this reason, the participants were asked "Do you have any information about Reasoning Games? Which games can you give an example? " The question has been posed. Considering the categories, it is seen that the preservice teachers expressed some games that closely related to their reasoning skills. Considering the games here, it is an important result that "chess" has an important place. Chess is a game of strategy. So it is a game built on moves and strategies. When there is an explanation for the purpose of a move in the game of chess, we cannot talk about reasoning. Therefore, it does not seem possible to say much about the other components of the reasoning mentioned here. The same is not true for sudoku. Because in sudoku the solution always has a hash. Sudoku is defined as a game that improves reasoning skills (MoNE, 2013b). Considering the categories, it can be said that pre-service teachers do not pay attention to other subtitles other than explaining the purpose in reasoning. This is significantly related to the definition of thinking, which, in general, is purposeful, but may not show a mathematical feature. In addition, it can be said that pre-service teachers do not have any interest in games that require reasoning. Geometry is a way of doing mathematics that also takes visual thinking or visualization into account. In this respect,

visualization as a type of explaining a problem is important for reasoning. For this purpose, the answers given by the pre-service mathematics teachers to the question “Please explain your thoughts on the importance of reasoning skills in geometry” may contain important information for reasoning. Considering the categories of reaching the results faster, good vision, effective processing, and reaching solutions in different ways, it is seen that the pre-service teachers think of reasoning as problem-solving. It is seen that expressions that provide multi-directional thinking skills and provide abstract thinking skills are concepts related to thinking skills. The category of making geometry more understandable by analytical thinking method describes reasoning. However, the frequency of this category is very low. On the other hand, the fact that they find it positive and useful and that they find it necessary can be shown as evidence of an intuitive attitude as well as the awareness of the pre-service teachers of such a deficiency. According to the categories formed by the opinions of the pre-service mathematics teachers, the opinion that the use of reasoning skills in geometry increases the success is dominant. This result and the results obtained in the studies of Lutfiyya (1998) and Baghdad and Saban (2014) are similar. Undoubtedly, one of the most important goals of societies today is to raise individuals who can think creatively and put these thoughts into practice. This aim makes it compulsory for learners to show improvement in their reasoning skills. For this purpose, any opinion that will provide improvement is important. For this purpose, and considering the above results, what kind of activities / practices / exercises etc. on the development of mathematical reasoning skills of secondary school students were asked to pre-service teachers. Explain question is essentially important. When the answers given by the pre-service mathematics teachers to this question are examined, it can be said that game activities have an important place. The association of topics with daily life will create a positive perception of the usefulness of reasoning in daily life, beyond mathematics being an abstract phenomenon that needs to be memorized. Thus, the utilitarian approach of human nature will positively affect the attitude and motivation towards mathematics. Opinions that this situation will increase the interest and success in mathematics are dominant. This result is in line with the results of the study conducted by Kocaman (2017). In addition, as this study shows, there is still a need for a deeper investigation of reasoning in terms of learning groups. If we make an evaluation even on the existing components, there is a need for such studies, since various subcomponents of these components may arise. In order to achieve this goal, the researchers' suggestion is that some improvements to be made on the basis of curriculum and course contents will contribute to the mathematics understanding and success of the pre-service teachers and therefore our student population.

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