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THE EFFECT OF ERROR-BASED ACTIVITIES ON THE REFLECTIVE THINKING SKILLS OF PRE-SERVICE ELEMENTARY EDUCATION MATHEMATICS TEACHERS

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Abstract: The purpose of the current study is to investigate the effect of error-based activities on the reflective thinking processes of pre-service elementary education mathematics teachers. To this end, qualitative research method was employed. The current study was carried out with the participation of six pre-service elementary education mathematics teachers attending a university in Turkey in the spring term of the 2017-2018 academic year. In order to collect the data in the study, the participating pre-service teachers were asked to write diaries and face-to-face interviews were conducted with them. The collected data were analyzed by coding them on the basis of the pre-service teachers' responses to the questions asked to them in line with the pre-determined conceptual framework. The obtained data have revealed that the error-based activities had positive effects on the pre-service teachers' thinking processes. In addition to this, the pre-service teachers' reflective thinking processes positively changed over the week

Key words: error-based activity, reflective thinking skill, pre-service elementary education mathematics teacher

1. Introduction

In order to learn mathematics meaningfully and to use it in real life, it is necessary to learn mathematics as a language and to use the names and objects of the language of mathematics such as numbers, measurements, data, patterns effectively. To do so, it is important to be able to capitalize on abilities such as creating associations between topics and concepts, establishing mathematical communication, solving problems by choosing and trying strategies and reasoning in the process of learning mathematics (Heemsoth & Heinze, 2016). The accomplishment of all these depends on the thinking process of the individual. In particular, problem solving and reasoning processes cannot be performed effectively without reflective thinking skill, one of the higher-order thinking skills.

Thinking, especially reflective thinking or inquiry, is essential for both teachers and students. Most people agree that reflection is at the core of learning (Lundquist, 1999). Reflective thinking is defined by Dewey as "an active, permanent and careful evaluation of any belief or assumed form of knowledge in the light of the reasons supporting it and the consequences it might yield" (Dewey, 1933). According to this definition, reflective thinking can also be thought of as a meaning-making process through which the student goes from one experience to another and develops a deeper understanding of the relationships between different experiences and ideas.

The individual reflects on his/her own thinking and experiences while reasoning in the learning process and this process of reflection leads to the formation of actions. Only in this way he/she can develop problem solving and construct new knowledge. The process of learning mathematics involves abilities such as creating associations between topics and concepts, establishing mathematical communication, solving problems by choosing and trying strategies and reasoning. In order to make these processes effective, it is necessary to have higher-order thinking skills. Another dimension of meaningful learning in mathematics is the reflective thinking skill, which is accepted as one of the higher-order thinking skills (Erdoğan and Şengül, 2018; Kramarski Weiss and Sharon, 2013; Albayrak, Şimşek and Yazıcı, 2018; Agustan Juniati, and Siswono, 2017; Odafe, 2008).

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In our minds, the construction of new knowledge takes place through the reflections made on prior knowledge. Reflection becomes the most evident when new experiences contradict our previous understanding. What is important here is the effective use of reflection on prior experiences to make sense of new information (Lundquist, 1999). In addition, with the reflection made during learning, misinterpretations and errors occur when what needs to be known is structurally inconsistent with what was previously known (Hartnett and Gelman, 1998; as cited in Santagata, 2005).

Errors should be seen as potential opportunities to provide negative information (Kuhn, 1970; Lakatos, 1976; Kline, 1980). In this respect, errors play a much more fundamental role in the growth of a discipline than is thought. Studies have shown that errors and difficulties are the best opportunity for brain development (Boaler, 2016; Moser, Schroder, Heeter, Moran, and Lee, 2011; Tall, 1990). This kind of research conducted on the brain and errors is very important for teachers and parents (Boaler, 2016).

Teachers are encouraged to use errors as a starting point in teaching any subject. The reason for this is that students shouldn't acquire common misconceptions about the subject to be covered (Ingram, Baldry & Pitt, 2015). Santagata (2005) suggests that students and teachers should value errors and move from seeing them as learning failures towards seeing them as learning outcomes. If students view errors as failures and avoid making them, their brains don't grow and they miss out on opportunities for improvement. Therefore, teachers should regard errors as an opportunity for students' brain development and learning. Accordingly, students should be involved in activities that might result in errors. In particular, it is recognized that errors can be a powerful tool for diagnosing learning difficulties and therefore for overcoming them (Heinze, 2018). Studies using this interpretation of the role of errors show the importance of increasing awareness of individual differences and difficulties in learning mathematics and the inability to correct errors by explaining the same subject over and over again.

Recognizing the importance of errors, teachers see them as a sign of students' need to reconstruct their knowledge (Santagata, 2005). On the other hand, reflective thinking develops students' weaknesses by helping them realize what and why they do in learning any subject and concept and in problem solving on the basis of their errors and misconceptions (Agustan et al., 2017; Betne, 2009). Seen from this perspective, it would be most natural for students to resort to reflective thinking when faced with error-based activities because they have experienced a situation contrary to their previous experiences. In fact, in the current study, it was observed that the pre-service teachers constructed new knowledge after evaluating their previous experiences they had habitually with reflective thinking through new experiences including error-based activities. In this connection, the problem of the study was worded as follows;

How do error-based activities affect the reflective thinking skills of pre-service elementary education mathematics teachers?

Theoretical Framework

The findings obtained in the current study to determine the effect of error-based activities on the reflective thinking skills of pre-service teachers were analyzed according to the components of reflective thinking proposed by Kember, Leung, Jones, Loke, McKay, Sinclair, Tse, Webb, Wong, Wong, and Yeung (2000) and the interactions between the explanations given for these components by Kember et al. (2000) and the explanations made by the pre-service teachers about their reflective thinking practices after the implementation of error-based activities were determined. The four sub-dimensions in the reflective thinking scale developed by Kember et al. (2000) are shown in Table 1 below.

Table 1: *Four components of reflective thinking*

Components	Characteristics
Habitual action	Doing without thinking, doing with previous knowledge, doing with the known formula, doing only with what the teacher has said
Understanding	Understanding the required question, internalizing the required question, thinking about the required question, understanding the content of the required question

Reflection	Thinking of a formula different from the given one about the question, thinking of another way, trying to develop an alternative way, trying alternative ways
Critical reflection	Changing your perspective, not having fixed ideas, knowing that there is no single solution to a question, discovering other ways of thinking and the errors previously believed to be correct.

2. Method

The purpose of the current study is to investigate the reflective thinking skills of pre-service elementary education mathematics teachers through error-based activities. To this end, the current study was designed as a case study. Case study is an approach that involves an in-depth analysis of a limited system, using multiple data sources to collect systematic information about how it works (Chmiliar, 2010). In the study, a setting was prepared to conduct error-based activities every week with six pre-service teachers. After the weekly activities, questions were given to them to respond in their diaries. These questions were regularly answered by the participants every week. After this regular application was completed, semi-structured interviews were conducted with the pre-service teachers. The study lasted for three weeks. Each week, three error-based activity questions were shared with the pre-service teachers and then discussed. Each application lasted nearly for two hours.

2. 1. Participants

The study was conducted with 6 third-year pre-service teachers attending the department of elementary education mathematics teaching of a state university in the spring term of the 2017-2018 academic year. In the selection of the participants, the criterion sampling method, one of the purposive sampling methods, was used. The criteria used in the determination of the participants were “voluntary participation” and “having different achievement levels”. The participants were selected according to their grade point averages at the end of the fall term of the third year; two pre-service teachers with the grade point average in the range of 3.5-3.0, two pre-service teachers with the grade point average in the range of 3-2.5 and two pre-service teachers with the grade point average in the range of 2.5-2.

2. 2. Data collection tools

The data collection tools are reflective diaries written by the pre-service teachers after each application and semi-structured interviews conducted with each pre-service teacher. Each of the pre-service teachers was given two questions to respond in their reflective dairies after each weekly regular application. In each application conducted weekly in a three-week period with the pre-service teachers, three error-based activity questions were discussed with the pre-service teachers. In each week when error-based activities were implemented, one question on algebra, geometry and proof each, thus, a total of three questions, were given to the participating pre-service teachers. The error-based activities were taken from the book “Error-Based Activities in Mathematics Teaching” by Konyalıoğlu and Gedik (2015). The pre-service teachers were asked to answer two questions given to them after each application to respond in their reflective diaries. These questions are given below:

- Can you explain your thinking processes during the application conducted in this week? How were you before the application? How were you during the application?
- What do you think about whether the application conducted directed you to thinking styles different from your previous thinking habits?

The questions prepared to be used in the interviews to be conducted with the pre-service teachers after the completion of the application are given below.

1. When you saw the questions first in the application conducted, what did you think about the questions and their solutions?
2. What kind of change did you observe in your viewpoint of the questions and their solutions during the application process?

3. What kind of change did you observe in your thinking processes arising from your participation in the application at the end of the application?

2. 3. Data analysis

In the analysis of the collected data, the reflective thinking components framework developed by Kember et al. (2000) was taken as the basis. The explanations made by the pre-service teachers in their reflective diaries and during the face-to-face interviews were coded on the basis of the 4 components in this framework. If the pre-service teacher used phrases such as "thinking in a result-oriented manner", "using memorised approaches" and "doing as known previously", it was coded as the component of habitual action. If the pre-service teacher used the phrases such as "making efforts", "recognising" and "thinking" in his/her explanations, it was coded as the component of understanding. If the pre-service teacher used the phrases such as "analysing the root of the question", "consistency between the root of the question and its solution", "approaching the activity from different perspectives" in his/her explanations, it was coded as the component of reflection. Finally, if the pre-service teacher used the phrases such as "necessity of questioning" and "recognition of the mistakes previously committed" in his/her explanations, it was coded as the component of critical reflection. In order to contribute to the better understanding of according to which component, the expressions in the explanations of the pre-service teachers were coded are presented in the table below.

Table 2. *Coding of the pre-service teachers' expressions according to the components of reflective thinking*

CODES	HABITUAL ACTION	UNDERSTANDING	REFLECTION	CRITICAL REFLECTION
EXPRESSIONS USED BY THE PRE-SERVICE TEACHERS IN THEIR EXPLANATIONS	Thinking in a result-oriented manner	Making efforts	Analysing the root of the question	Necessity of questioning
	Memorizing	Recognizing	Consistency between the root of the question and its solution	Recognition of the errors committed before
	Doing as known previously	Thinking	Approaching the activity from different perspectives	
			Thinking in different ways	

The findings regarding each week of the three-week application process carried out with the pre-service teachers and the data obtained from the face-to-face interviews are both given with the necessary explanations and presented in tables for better understanding. The findings obtained are supported with direct quotations from the pre-service teachers' explanations.

3. Findings

The findings obtained in the current study conducted with pre-service teachers were presented separately in order to show the change in each pre-service teacher during the three-week application process, taking into account the components of reflective thinking stated in the conceptual framework section. The findings obtained from each pre-service teacher for each week were given in tables within the framework of the determined codes. Tables were created in order to determine in which codes decreases occurred and in which codes increases occurred in the pre-service teachers' explanations about the questions related to the application. The error-based activity questions were solved for the pre-service teachers for

three weeks. After the application, they were given two questions to respond. The responses given by PT1 to these questions are presented according to the determined codes in the table below.

3. 1. Findings Obtained for the Pre-service Teacher PT1 for Each Week

Table 3. *Responses given by PT1 each week*

CODES	1st Week	2nd Week	3rd Week
Habitual action	Memorizing, doing as known previously		
Understanding	Recognizing, thinking	Recognizing, thinking	Recognizing, thinking
Reflection		Thinking in different ways	Approaching the activity from different perspectives, thinking in different ways
Critical reflection		Recognizing the errors previously committed	Necessity of questioning

As can be seen in Table 3, the pre-service teacher adopted the constructs of habitual action and understanding towards the application questions related to the first week error-based activity. When the pre-service teacher's responses to these questions were examined, it was found that she used the phrases such as "memorizing", "doing as known previously" and also stated that she had started thinking more about the next questions as a result of the recognition of previous errors. In the second week, PT1 stated that she recognised the errors she had committed in the first week, and thus she distanced herself from her memorisation-based thinking to avoid similar errors. In addition, she stated that her way of thinking changed and she recognized the errors she had committed before. In the third week of the application, the pre-service teacher stated that however hard she tried to distance herself from memorization-based thinking, she still felt its influence. However, she also stated that she used different ways of thinking in the questions, that she handled them differently, and that she started with the interpretation of the questions in the current week, according to the practices performed in the previous week.

3. 2. Findings Obtained for the Pre-service Teacher PT2 for Each Week

Table 4. *Responses given by PT_2 each week*

CODES	1st Week	2nd Week	3rd Week
Habitual action	Doing as known previously		
Understanding	Recognizing, thinking	Making efforts, thinking	Recognizing, thinking
Reflection	Approaching the activity from different perspectives	Approaching the activity from different perspectives, analysing the root of the question, consistency with the root of the question and its solution	Approaching the activity from different perspectives, thinking in different ways
Critical reflection		Necessity of questioning, recognizing the errors previously committed	Necessity of questioning

In the first week of the application, the pre-service teacher PT2 stated that she solved the activity questions as she had known before but that in fact each correct answer is not correct and that she realized that it is necessary to approach questions from different perspectives. Thus, the responses given by the pre-service teacher were coded under the components of habitual action, understanding and reflection. The pre-service teacher stated that she thought about the questions a lot, approached the questions from different perspectives, started to investigate the accuracy of the questions and started inquiries on the questions and answers in the second week. These expressions stated by the pre-service teacher after the completion of the application in the second week were coded as the components of understanding, reflection and critical reflection. From the explanations of PT2 on the application in the third week, it was understood that she thought more about the questions, approached the questions from different perspectives and recognised the errors she had committed before. Accordingly, the explanations of the pre-service teacher were coded as the components of understanding, reflection and critical reflection.

3. 3. Findings Obtained for the Pre-service Teacher PT3 for Each Week

Table 5. *Responses given by PT3 each week*

CODES	1st Week	2nd Week	3rd Week
Habitual action	Memorizing, doing as known previously	Result-oriented thinking	Result-oriented thinking
Understanding	Recognizing, thinking	Recognizing, thinking, making efforts	Recognizing, thinking, making efforts
Reflection			
Critical reflection			

The pre-service teacher PT3 stated that she solved the questions as she had already known before with her memorized knowledge, and that she did not look for any error in the roots of the questions. The pre-service teacher stated that she followed a very simple way of thinking about the questions in the application and that she should have questioned them. In this regard, the responses of the pre-service teacher given to the questions in the first week were coded as the components of habitual action and understanding. The pre-service teacher stated that she conducted result-oriented thinking process to find solutions to the questions in the second week. In addition, she stated that she should have looked at the root of the question and analyzed the consistency between the root of the question and its solution. The pre-service teacher stated that she followed the first solution that came to her mind during the application in the third week. After the completion of the activity, she stated that she should have thought more about the questions and approached the activity from different perspectives.

3. 4. Findings Obtained for the Pre-service Teacher PT4 for Each Week

Table 6. *Responses given by PT4 each week*

CODES	1st Week	2nd Week	3rd Week
Habitual action	Memorizing, doing as known previously		
Understanding	Making efforts	Recognizing, thinking	Recognizing, thinking
Reflection		Approaching the activity from different perspectives, analysing the root of the question, analysing the consistency between the root of the question and its solution	Approaching the activity from different perspectives
Critical reflection		Recognizing the errors previously committed	Necessity of questioning

When the responses given by PT4 to the questions asked after the completion of the application in the first week were examined, it was seen that she used statements indicating that she answered the questions by using memorized knowledge or as she had known previously. In addition, she stated that it is important to look at the root of the question. After the application conducted in the second week, it was observed that the pre-service teacher used expressions indicating that she approached the questions from different perspectives, paid attention to the consistency between the root of the question and its solution, and she recognized the errors she had committed before. Similarly, she included expressions in her explanations indicating the necessity of thinking about the questions, approaching the questions from different perspectives, reaching solutions through an in-depth questioning of the questions.

3. 5. Findings Obtained for the Pre-service Teacher PT5 for Each Week

Table 7. *Responses given by PT_5 each week*

CODES	1st Week	2nd Week	3rd Week
Habitual action	Doing as known previously	Result-oriented thinking, memorizing	
Understanding	Making efforts, Thinking	Recognizing, thinking	Recognizing, thinking
Reflection		Approaching the activity from different perspectives, consistency between the root of the question and its solution, thinking in different ways	Analysing the root of the question, thinking in different ways
Critical reflection		Necessity of questioning	Necessity of questioning

The pre-service teacher PT5 stated that she solved the problems as she had known before and tried to understand the questions in the first week. She stated that she solved the question through trial and error but understood the logic behind the question later in the second week. The pre-service teacher stated that she recognized that the solution is different from what is required in the question in the application conducted in the third week. She also stated that this made her approach the question from different perspectives and realized the necessity of questioning the questions.

3. 6. Findings Obtained for the Pre-service Teacher PT6 for Each Week

Table 8. *Responses given by PT6 each week*

CODES	1st Week	2nd Week	3rd Week
Habitual action	Result-oriented thinking, doing as known previously		
Understanding	Recognizing	Recognizing, thinking, making efforts	Recognizing, thinking
Reflection	Thinking in different ways	Consistency between the root of the question and its solution, thinking in different ways	Thinking in different ways
Critical reflection		Necessity of questioning	Recognizing the errors previously committed

The pre-service teacher PT6 stated that she thought about the question given in the application conducted in the first week in a result-oriented manner and she tried to do it as she had known before. She said that she recognized how she should have approached the question and had recognized that she should have evaluated the questions considering different situations. She stated that considering the previous application, he tried to solve the problems by thinking about them and tried to reach the solution by considering different situations in the second week. After the application, she also stated that there should be a relationship between the question given and its solution; thus, it is necessary to question the given situation. She said that she analysed the questions more deeply, recognized the mistakes she had committed before and tried to answer the questions taking all these into consideration in the third week

3. 7. Findings obtained from face-to-face interviews

Table 9. *Responses given by the pre-service teachers to the interview questions*

Questions	HABITUAL ACTION	UNDERSTANDING	REFLECTION	CRITICAL REFLECTION
1. When you saw the application questions first, how did you think about the questions and their solutions?	"simple thinking" (PT_1, PT_3, PT_6) "focusing directly on the solution"(PT_2,PT_4,PT_6) "not examining the question and root of the question" (PT_1,PT_2,PT_5) "a familiar type of question" (PT_5,PT_6) "at the medium level"(PT_6)	"analysing it in-depth" (PT_1) "realizing that a question covers more than one subject" (PT_3)	"adopting a different approach to questions requiring production" (PT_3) "trying to find solutions in different ways" (PT_6)	"thinking that there might be a error in the question"(PT_2) "another student saw what I did not see"(PT_3) "realizing the benefits of application during the process"(PT_4) "Learning that thinking is necessary" (PT_5) "confusion"(PT_4,PT_6)
2. What kind of change did you observe in your perspective of questions and solutions during the application process?	"not analysing at all at first"(PT_5,PT_6) "focusing deeply on reading questions"(PT_6) "solving the question without checking later" (PT_6)	"staring to analyze more deeply"(PT_1) "adopting a more comprehensive and profound viewpoint" (PT_4) "Learning how to read to understand the root of the question and what it requires" (PT_5)	"starting to adopt different perspectives" (PT_1, PT_3, PT_5) "engaging in mental practices", (PT_3) "starting to adopt a more analytical viewpoint" (PT_3, PT_5)	"solving questions in different ways in the lesson focused on the differential equations" (PT_1) "coming to school as it is a must"(PT_3) "I enjoy attending such lessons" (PT_3) "I experience some problems as I think differently" (PT_3) "adopting a very suspicious point of view"(PT_3,PT_6) "necessity of looking for errors in both the question and its solution" (PT_5, PT_6) "analysing the question" (PT_6)
3. What kind of change did you observe at the end of the application	"focusing only on the solution previously" (PT_4) "they were the subjects we were familiar with"	"Understanding what is in fact asked in erroneous questions" (PT_2) "thinking in the most	"learning how to think simply" (PT_1) "necessity of questioning" (PT_1,PT_5,PT_6)	"converting questions into different questions" (PT_2) "understanding that rote-learning should

arising from participation in the application in your thinking processes?	(PT_3) "Solution-orientedness" (PT_1) "being method-oriented" (PT_5)	detailed manner" (PT_5) "memorization is at the forefront" (PT_6)	"how I can solve the question in a different way " (PT_2, PT_5) "formation of different styles of thinking" (PT_2, PT_3) "questioning"(PT_2,PT_6) "is the question incorrect"(PT_6) "is there a consistency between the question and its solution" (PT_6)	not be drawn on in mathematics" (PT_1) "realizing that students using memorised knowledge solved the problems erroneously" (PT_2) "checking one's own solutions" (PT_4) "questions unknown to me are an advantage for me" (PT_4) "seeing its benefits" (PT_4) "solving questions in a way suitable for the target learner population" (PT_4) "realizing that a more comprehensive perspective is adopted towards some questions" (PT_6) "feeling socially more positive" (PT_3) "each question may a different logic behind" (PT_6) "thinking more deeply and comprehensively" (PT_5)

3. Conclusion

In the current study, it was intended to reveal how the reflective thinking skills of the pre-service elementary education mathematics teachers change as a result of involvement in error-based activities. To this end, a three-week application was conducted by using error-based activities. Each week, three error-based questions were asked and these questions were discussed with the pre-service teachers. The pre-service teachers were asked two questions about the application conducted each week and they were asked to answer these questions in their diaries after the application. Face-to-face interviews were held with the pre-service teachers in order to learn their thoughts on this application in depth. In accordance with the determined theoretical framework, the statements of the pre-service teachers were coded and evaluated according to the components of reflective thinking.

All of the pre-service teachers responded to the questions in the first week of the application in line with the understanding dimension of the reflective thinking. In the first application conducted in the first week, all the pre-service teachers tended to conduct the activities on the basis of their prior knowledge and skills. All the pre-service teachers adopted rote-learning approaches to the questions by focusing on the result rather than the process. In the second and third applications, from their responses to the questions and what they wrote in their dairies in the second and third weeks, it was concluded that they more focused on the process. On the other hand, they stated that although they tried to get themselves distanced from memorized knowledge-based and result-oriented thinking, they could not completely get rid of this type of thinking. In addition, in the interviews conducted with the pre-service teachers, they stated that trying to do the activities on the basis of memorized knowledge did not make them feel good and that this was an obstacle to deep and reflective thinking. In their study, Haghverdi et al., (2012) concluded that students answered the questions by focusing on the result rather than the process and thus they did not see the errors in the questions.

The pre-service teachers' performance in doing activities and the efforts they made showed that they read, understood and tried to solve the activities. Thus, it can be argued that all the pre-service teachers were successful in the understanding dimension of the reflective thinking. In addition, in order for the pre-service teachers to understand the activities, they had to recognize some important points in the questions presented during the applications. In the first application, the pre-service teachers experienced some problems in recognizing the important points in the questions. In the subsequent applications, the pre-service teachers started to recognize the important points and errors in the questions and to reflect on these errors. In the interviews conducted with the pre-service teachers, it was also revealed that they started to analyse the questions in the subsequent applications more deeply and to conduct more detailed and comprehensive investigations on the questions. Similarly, in some of the other studies focused on error-based activities, it has been revealed that however hard they tried, elementary education math teachers and pre-service teachers experienced difficulties in understanding the questions in the first applications (Gedik and Konyalıoğlu, 2014; Özkaya and Konyalıoğlu, 2019). However, the teachers and pre-service teachers were determined to have recognized the erroneous questions given to them more easily and tried to solve the questions by better understanding them after the first applications.

When the data obtained from the pre-service teachers were examined, it was found that many expressions stated by them could be included within the reflection dimension of reflective thinking. In the first week of the application, the pre-service teachers were not familiar with the activities. However, after their first experience, they started to think differently. The pre-service teachers' reflection-based responses were started to be observed after this experience. From the responses of the pre-service teachers, it was understood that they were familiar with some questions and solutions in the activities. Yet, it is clear that after the first application, they adopted different approaches to the activities in the subsequent applications. In the subsequent applications, the pre-service teachers tried to look at the error-based questions from different perspectives and to think about them differently. Soncini et al. (2020) stated that teaching strategies related to handling errors had a positive effect on students' adopting different ways of thinking. As for the analysis of the root of the question addressed under the reflection component, the pre-service teachers faced two different situations. One of them is about the lack of information in the root of the question and the other is about the error in the root of the question. Although there were few pre-service teachers who examined the root of the question in the first week, all the pre-service teachers tried to find errors in the questions in the second and third weeks of the application. In this way, they began to see the deficiencies and errors in the root of the question. Moreover, they also stated in the face-to-face interviews that they performed the question analysis method they learned in the application in their other lessons. In general, the pre-service teachers began to see deficiencies and errors in the root of the question. However, in the first week of the application, they could not notice the consistency between the root of the question and its solution on their own. After the pre-service teachers evaluated the activities with the researcher, they noticed the consistencies and inconsistencies between the root of the question and the solution. During the three-week application process, the perspectives of the pre-service teachers began to change and they started to consider thinking in different ways, analysing the root of the question, question-solution integrity and consistency. At the end of the application, the thinking styles of the pre-service teachers changed proportional to their efforts to cope with errors. Janet (2017) stated that encouraging instead of preventing errors in learning would contribute positively to students' analysis of questions and solutions.

According to the critical reflection component, the pre-service teachers were expected to evaluate themselves in terms of their own thinking and activities. Changing perspectives, changing fixed ideas, thinking to find different ways of approaching activities, and recognizing errors that have been believed to be correct are the issues addressed within this component. In this connection, the pre-service teachers started to question what they did during the activities. The pre-service teachers' thoughts on their performance started to change after the first application. Necessity of questioning is one of these changes. It was seen in the interviews that the pre-service teachers understood the importance of questioning. In the second and third weeks of the application, it was observed that the pre-service teachers conducted thinking processes on the basis of questioning, so that they started to analyse the questions, read the activity more carefully, and reflect more on the activity. In addition, it

is obvious from the statements of the pre-service teachers that they started to be aware of the errors they had made before. The pre-service teachers started to be aware of the errors related to the questions and change them after the first application. In studies on learning from errors, it was observed that students began to question the questions more and approached the questions and solutions with a critical perspective (Booth et al., 2016).

In general, in the current study, it was seen that the pre-service teachers constructed new knowledge after evaluating their previous experiences with error-based activities. In this three-week study, according to the findings obtained from face-to-face interviews, it was found that the pre-service teachers read the questions superficially, adopted result-oriented approaches and did not feel the need for conducting an in-depth analysis of the questions at first. During the application process, they were observed to have started to read the questions more carefully and to question the questions to understand them better. In terms of reflection, the pre-service teachers sought more than one solution to the questions. In addition, the pre-service teachers' acts of focusing on correcting their mistakes, trying to overcome their deficiencies and evaluating the thoughts of their peers can be considered within the context of critical reflection. It is seen that this three-week error-based activity application contributed to the development of the reflective thinking skills of the pre-service teachers. Sultana et al. (2020) emphasized that different applications should be integrated into undergraduate courses to develop students' reflective thinking. In addition, many researchers have stated that errors develop higher-order thinking skills (Dweck, 2012; Moser et al., 2011; Melis 2004) .

On the other hand, Borasi (1994) stated that thinking processes during the detection and verification of errors should be interpreted in order for learning to be meaningful. In order for a student to achieve meaningful learning, existing knowledge must be uncovered and, accordingly, thinking processes must be uncovered. This is important in determining the biases of students and building new knowledge on the existing knowledge (Bransford et al., 2000). In this respect, the pre-service teachers understood the role of errors in having access to thinking processes in the current study. As a result, these pre-service teachers will be able to access the mathematical thinking processes of their future students through their errors. Making students think is usually done by getting them to interpret their answers (Jacobs et al., 2010). Interactive thinking of both pre-service teachers and students about errors reveals their understanding of mathematics. In the literature, it has been well established that errors and misconceptions are important for both students and teachers in learning mathematics (Ingram et al., 2015). If teachers think of errors as a powerful tool that reveals students' "learning difficulties" (Borasi, 1987, p. 2), they can also reveal students' thoughts on mathematics (Ingram et al., 2015). In the current study, the pre-service teachers have been able to see the importance of errors in mathematics before they start teaching students in their future professional career. Therefore, it is important to carry out such studies in order to change the opinions of pre-service teachers on the basis of errors and to make them think more broadly before becoming an active teacher. Melis (2004) stated that when making a mistake, the thoughts of the person were expressed verbally and an alternative solution was sought. This happens naturally when trying to understand math mistakes that are an essential part of learning. It also shows the positive effect of mistakes on reflective thinking (Melis, 2004). Similarly, in the current study, the pre-service teachers changed their reading strategies, for example they tried to read in detail and to find different ways to solve questions. In the current study, the pre-service teachers developed especially in terms of reflective thinking skills, activities and questioning their solutions. Erdoğan (2020) also emphasized that practices that would provide positive feedback should be included in developing students' inquiry and reflective thinking skills.

The lack of sufficient research on this subject in the literature suggests that the subject should be examined among pre-service teachers, students and teachers. Future research may become more effective if it is carried out for a longer period and with more pre-service teachers. In addition, it is thought that the importance of the issue will become more apparent when in-service teachers and students are involved in such studies.

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