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ASSESSMENT OF VISUO-SEMIOTIC SKILLS FOR PRE-SERVICE TEACHERS IN COORDINATE GEOMETRY

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Abstract: Globally, the use of visuo-semiotic models (VSMs) in the mathematics classroom is called for across levels and topics. Literature confirms that visual cognition/literacy in teachers is limited and that their capabilities in visual representation is low. The aim of this study was therefore to explore pre-service teachers' (PSTs) conception of VSMs as a process in doing mathematics in the context of coordinate geometry. A survey with qualitative data was used to collect data from mathematics pre-service teachers taking the geometry module. The PSTs answered open ended questions that comprised coordinate geometry test items. Mnguni's (2014) cognitive process of visualisation was used to analyse visualisation skills portrayed in the test. The study found that different pre-service teachers operate at different levels of visual literacy. The teacher education curriculum should therefore be systematically designed to cater for the skills lacking in the PSTs. It is also recommended that teacher training programmes sharpen their attention on bringing awareness of mathematical visual literacy among PSTs during training.

Keywords: Coordinate geometry, pre-service teachers, visualisation, visual literacy, visual models, Visuo-Semiotic Models.

1. Introduction

The 21st century has seen an explosion in the development of a number of competencies in mathematics education (Arsad et al., 2011). The 21st century competencies incorporates academic communication literacy (Spektor-Levy et al., 2008) and visual literacy (Bottomley et al., 2006). Visual literacy has become one of the most critical skills particularly for the topics that cannot be visualised with physical eyes but through the use of visual manipulatives/models (Mnguni, 2014). According to Mnguni (2014), both learners and teachers need to develop these skills in order to work effectively. As part of this, visuo-semiotic models (VSMs) have become meaningful vehicles for learning science in general and that students have to develop visuo-semiotic skills to enable them to learn effectively (Mnguni, 2014). Similarly, other studies, for example, Larbi and Okyere (2010) and Liggett (2017) attested that, due to the abstract nature of mathematics, visuo-semiotic models are essential to develop conceptual understanding of key mathematics concepts. Visual literacy, as Lee and Lee (2019) postulate, stand in as placeholders of thought and windows into learners' understanding of subject ideas.

To add clarity to the ideas depicted above, some terminology included in the study need to be defined in short before they are elaborated in the subsequent sections. According to Allen (2017), semiotics is the study of signs and symbols, hence, it examines the association between symbols and their roles in how people create meanings. The definition suggests that semiotics is an investigation into how meaning is created and communicated through symbols. There are five semiotic systems that consist of linguistic, visual, audio, gestural and spatial. In this paper, the researcher focuses only on the visual aspects of the concept image. Visuo-semiotics is therefore a sub-domain of semiotics that analyses the way visual images communicate a message (Yi-Ching Su, 2014). Visualisation is another term that is commonly used in the paper and is "based on the production of a semiotic representation of a concept….." (Jones & Tzekaki, 2016). According to Presmeg (1986), visualisation is a process of summoning up a mental image of something, that is, seeing something in the mind, which is facilitated by the use of VSMs. The cognitive process of visualisation covers the internalisation of visual models (IVM), conceptualisation of visual models (CVM) & the externalisation of visual models (EVM) (Mnguni, 2014). The ability to use visuo-semiotic images to enhance

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visualisation skills ultimately results in visual literacy. Visual literacy refers to the ability to understand, produce and use significant images, objects and visible actions.

Stylianou (2010) and Ryan and McCrae (2005) conducted research that revealed that mathematics teachers share many of the same misconceptions as and make errors similar to those made by learners. In their study, Avgerinou and Knight (2005) affirm that mathematics teachers who are visually literate in mathematical context could teach the subject better than those who are not. Visually literate individuals can develop the ability to recognise, interpret and employ the obtrusive syntax and semantics of different visual forms (Felten, 2008).

However, there is little evidence that teachers receive necessary support to meet these demands and little evidence that teacher training programmes adequately prepare pre-service teachers to acquaint them with the skills, and integrate them in instruction successfully (Ball & Cohen, 1999; Stein et al., 2008). Research on visuo-semiotics models has of late focused on students' difficulties using visual models, turning a blind eye on the teachers' conception of these models (Stylianou, 2010). Based on the above ideas, the mathematical VSMs knowledge base that PSTs have, hence determines the level of proficiency in both teaching and learning of mathematical concepts.

Despite evidence of VSMs being integral to the teaching and learning of mathematics, Lee and Lee (2019) asserted that there is limited use or misuse of these models due to teachers' lack of mathematics knowledge. In a different context of mathematics PSTs in South Africa, this study therefore aims to explore the mathematics PSTs' competence in utilising VSMs to provide proficient guidance to the mathematics learners in high schools. Since VSMs are generally context-specific rather than generic, visual literacy is directly interwoven with subject specific knowledge and semiotics (Mnguni, 2019a). Pre-service mathematics teachers' conceptions of using VSMs in the context of coordinate geometry were therefore examined.

Problem statement

Literature documents the misconceptions and difficulties associated with the teaching and learning of mathematics (Larbi & Okyere, 2016; Eshun, 2000; Awanta, 2000). The difficult nature of mathematics maybe due to the way it is taught in schools. In their study, Mnguni (2019a) affirm that the difficult nature of science is due to the fact that teachers are ill prepared to teach science. Similarly, Shahinshah Babar Khan (2012) contends that the existing syllabus of mathematics in Pakistan is tough and teachers do not have command over the subject due to lack of proper training. In all these instances, VSMs were supposedly found to be essential to develop the conceptual understanding of key concepts in mathematics because the use of VSMs can prompt visualisation in the learners' minds (Presmeg, 2016). Schonbörn and Anderson (2009) show that visualisation skills are necessary for learners to comprehend content knowledge presented in external representations (ERs) such as VSMs. Hence, Mnguni et al., (2016) contend that a lack of visual learning is one of the major factors that instigates poor understanding of concepts among learners. Teachers have therefore a task of making mathematics meaningful to learners (Larbi & Okyere, 2016) through the use VSMs in teaching. However, despite the significance of VSMs in teaching mathematics, literature asserts that classroom teachers are still grappling with how to make use of VSMs to teach the subject effectively (Lee & Lee, 2019). In light of this, there is a need to assess the knowledge and skills in using VSMs in a different context of mathematics pre-service teachers in South Africa. In addition, due to the fact that VSMs are subject or content specific (Mnguni, 2014), the study examines the mathematics pre-service teachers' knowledge, ability and competence to select and use appropriate VSMs to enhance understanding of mathematical concepts in the context of coordinate geometry.

Research questions

The following main research question informs the study:

How can VSMs be understood in order to inform the understanding, teaching and studying of mathematics in the context of coordinate geometry?

Sub-questions

(i) How do PSTs organise, translate, interpret and apply VSMs to model and solve mathematical phenomenon?

(ii) How does the application and interpretation of VSMs by mathematics PSTs inform us about their understanding of teaching mathematics?

2. Literature Review

The concept of visuo-semiotic models (VSMs) in mathematics

Semiotics in mathematics can be representations or manipulatives in teaching and learning mathematics. The semiotic representations incorporate cognitive abilities for understanding using and thinking through visual models (Avgerinou & Quinn Knight, 2005), hence semiotic resources comprise symbolic tools such as diagrams, sketches, graphs, drawings, computer designed visual models, mental images or equations and signs which are used to construct mental models and knowledge (Rundgren & Tibell, 2010; Weliweriya, Sayre & Zollman, 2018). The symbols in semiotics, also called signifiants/signifiers, represent the signified (which is the concept represented by the signifier) and make it publicly understandable (Godino & Batanero, 2003). This description of symbols suggests that a symbol or a signifier on its own is meaningless unless it represents a certain concept. The semiotic processes involved in developing mathematical concepts therefore convey, substitute or yield conceptual meanings (Godino & Batanero, 2003). VSMs, as a sub branch of semiotics, can be referred to as a way visual images communicate a message. VSMs can also be considered as visual prototypical representations that use *discipline-specific semiotics* to represent scientific phenomenon for teaching, learning and research (Mnguni, 2019a). According to Arnersan and Offerdahl (2018), VSMs are used to visually illustrate and represent somewhat unobservable phenomenon and related scientific theories. To clarify VSMs further, Tall (2013, p.16) refers to some *conceptual embodiment* which develop mental images that may *become perfect mental entities*. According to Tall (2013), the term "embodiment" is explained as "giving a body to an abstract idea" (Tall, 2004, p.32). For example, the number line develops in the *embodied world* from a physical line drawn with pencil and ruler to a "perfect" platonic construction that has length but no thickness (Tall, 2008, p.14). This means the physical line, in the embodied environment can be converted to an internal mental abstraction where "length" without the idea of "thickness" is visualised. The physical line then becomes the VSM representing the concept of a number line.

According to Mnguni (2018), the ability to construct and interpret visual information has become an integral skill in the 21st century. In the same vein, Nitz, Ainsworth, Nerdel & Prechtl, 2014) attested that the 21st skills should incorporate the use of VSMs which induce a fertile ground for individuals to develop the ability to interpret, construct, transform and evaluate various visual information. Avgerinou and Ericson (1997) also affirm that visual thinking, visualisation and other related cognitive abilities should be considered as part of visual literacy. Visualisation is defined by Jones and Tzekaki (2016) as the capacity to represent, transform, generate, communicate, document and reflect on visual information. According to Jones and Tzekaki, visualisation is developed from the production of a visual semiotic model representing a concept and that may lead to visual literacy. Mnguni (2019a) averred that visual learning is interwoven with subject-specific knowledge and semiotics, which may vary considerably between disciplines. For example, a learner may be visually literate in mathematics but visually illiterate in chemistry. Alternatively, a student can be visually literate in coordinate geometry and visually illiterate in calculus. The researcher hence explored the pre-service teachers' use of VSMs within the context of coordinate geometry.

Visuo-semiotic models as tools for understanding mathematics

VSMs have been considered by many researchers as integral tools for understanding mathematical ideas (Stylianou, 2010; Presmeg et al., 2016; Mnguni, 2019a). According to Stylianou (2010), it is therefore recommended that students at all levels make use of VSMs/representations to organise, communicate and apply mathematical ideas to solve problems. In a research by Bender and Marrinan (2010), diagrams, drawings and other visual images were considered to have the role of constructing knowledge and interpreting data and equations. When students are able to use VSMs meaningfully in mathematical situations, then the problem or situation becomes more accessible. These models assist students to organise their thinking in a way that will enable them to use different approaches to understand a concept (Fennell & Rowan, 2001). This idea suggests that numbers or solutions that students write are a representation of their thinking. However, students' understanding and ability to use VSMs is normally impacted by the level at which their teachers understand them (VSMs) (Stylianou, 2010).

Cognitive skills that encompass visual literacy in mathematics

This paper describes the cognitive issues involved in teaching and learning mathematics from a semiotic perspective. According to Godino and Batanero (2003), activities in mathematics are characterised by the use of semiotic functions in 4 categories of entities, namely (i) ostensive entities – made up of symbols, tables, graphs, diagrams and notational entities. The entities normally play the role of "representation system" (Godino & Batanero, 2003). The entities hence represent mathematical ideas (ii) extensive entities – are represented by problem-situations, applications and other phenomenological entities that induce mathematical activities. For example, mathematical modelling where real life situations are represented mathematically. (iii) Intensives entities– these cover mathematical generalisations such as mathematical ideas and abstractions (concepts, procedures and theories). (iv) Actuative entities – such as subject's action when faced with tasks. This paper analyses some semiotic processes in the study of coordinate geometry but focussing mainly on the first 3 entities.

Kaput (1991) proposes the need to understand the cognitive processes of VSMs, which are a special form of representations in encoding syntactic and semantic elaborations. Mayer explains that during the process of learning, the physical images enter the cognitive system through the eyes, leading to the construction of mental images within the working memory. The integration of these mental images results in visual thinking skills (Mnguni, 2014; Mayer, 2003). These skills embrace the ability to analyse and assess visual information then apply the knowledge to solve problems. For example, when calculating the volumes of solids of revolution as an application of integration, a learner requires the ability to imagine an object rotating in space, which is a visual information processing skill. According to Mayer's (2003) cognitive theory of multimedia learning, visual learning is in 3 stages, viz, (i) comprehension of visual information which aligns with Mnguni's (2014) internalisation of visual models (IVM). (ii) Processing of this information in cognitive structures which is also equivalent to Mnguni's conceptualisation process of visual models (CVM). (iii) Externalisation of information as visual models (Mnguni's externalisation of visual models (EVM)). The theory means that learning through VSMs embraces the assimilation or absorption of mathematical ideas into the mind (IVM), internal abstraction of ideas (CVM), and projection/ emission of the ideas for use/application (EVM), which can be done through presentations, drawings, writing, etc. Overall, VSMs, according to Mnguni (2014), and Mayer's (2003) theory of multimedia embrace injecting information from the external world into the cognitive structures of the mind, followed by the cognitive processing of this information and ultimately exuding/discharging the information from the mind to the world. Mnguni (2014) summarises the cognitive process of visualisation which is being facilitated by the use of visuo-semiotic models in Figure 1 below:

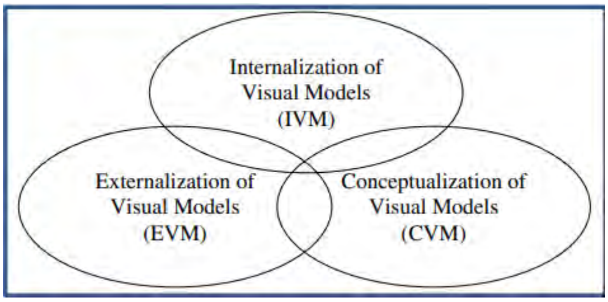


Figure 1: *The overlapping stages of the cognitive process of visualisation* (Mnguni, 2014).

In the model, IVM (the sense organs such as eyes) work with the brain to absorb information from the environment (Mnguni, 2014). CVM is the process where meaning is made and cognitive visual models constructed (Burton, 2004). EVM is the production of external visual models. Hence the study aimed to explore pre-service teachers' conception of VSMs in doing mathematics in order to allow for visualisation of mathematical concepts.

Theoretical Framework:

This study is informed by the cognitive theory of multimedia learning by Mayer (2003). Mayer's multimedia theory states that people learn more deeply from words and images than from words alone (Mayer, p.47). Mayer's (2003) cognitive theory of multimedia learning is related to Mnguni's (2014) theoretical cognitive process of visualization. According to Mayer (2003), when individuals learn from visual models, it is a cognitive process that embraces some mental processes, as described in the cognitive theory of multimedia learning (Figure 1b). The multimedia theory, according to Mayer (2003) is based on the assumption that there are 2 separate (dual) channels of information processing, which are visual and auditory. The auditory channel according to Kirschner et al. (2017) processes sounds that students hear whilst the visual channel processes things that students see. In addition, Mayer foregrounds that meaningful learning requires active processing of information by the learner, and when that happens, learning then becomes an active process of filtering, selecting, organising and integrating information [SOI model], based upon prior knowledge. For the purpose of this study, the paper sharpened its focus on the visual channel.

Figure 1b below depicts the explanation above about Mayer's theory of multimedia learning:

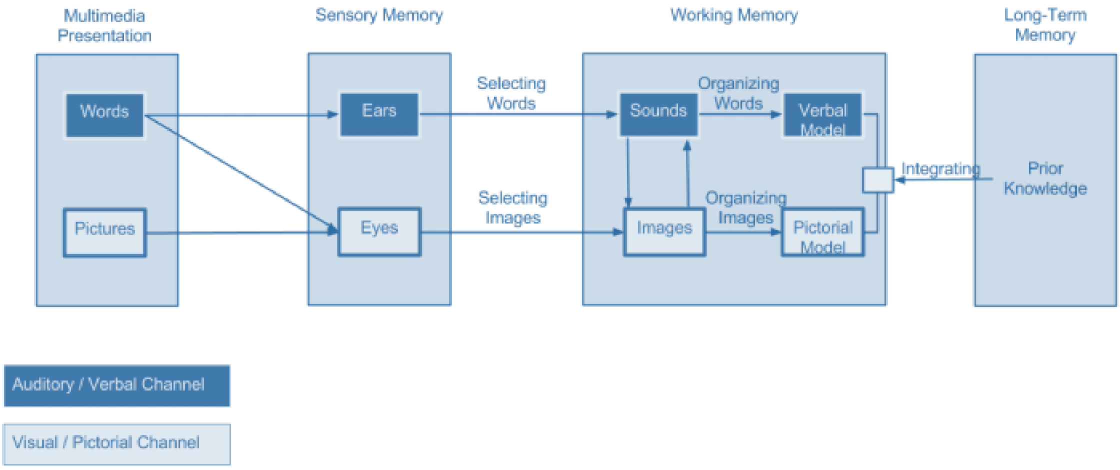


Figure 1b: *Cognitive Theory of Multimedia Learning* (Mayer, 2003)

With reference to Figure 1b above, processing of images begins with the learner's perception of these images (external representations) in the sensory memory. This means the learning process begins with the visual model entering the cognitive system first through the eye which then leads to the construction of a mental image/model in the working memory. The mental models (internal representations) are then integrated to a model ending in long-term memory. This process resonates and overlaps with Mnguni's (2016) stages of visualisation namely; IVM, CVM, EVM.

3. Method

Sampling

Bachelor of Education mathematics pre-service teachers at a university in South Africa were selected to participate in the study. These were secondary school mathematics specialist trainees and the university admission requirements demanded a holder of a National Senior Certificate (NSC) with at least 50% in the language of teaching and learning and at least 50% in Mathematics to take the program. The participants were in their second year of the programme and all of them were taking geometry as a module in addition to other mathematics modules. The 40 participants completed their test instrument electronically. The use of the electronic contact with the pre-service teachers was in line with the social distancing requirements of the Covid-19 pandemic. Teaching practice is a compulsory component in their program, therefore pre-service teachers who had done their teaching practice were considered in this study because the instrument used in the study was based on the pre-service teachers' experiences of teaching.

Design and procedures

This was a qualitative study in which mathematics PSTs completed an open-ended survey with test items on coordinate geometry. The survey was employed to provide a basis for visual semiotic practices of pre-service teachers when teaching coordinate geometry. The instrument (survey) was developed using Mnguni et al.'s (2016) visualisation skills identified for visual literacy. The test items tested both the PSTs knowledge about coordinate geometry and their ability to apply, interpret and use visuo-semiotic models in coordinate geometry. Some of these models (test items) were extracted from text books used by high school learners such as 'The Core Course for A Level Mathematics, Coordinate Geometry: A guide for teachers – years 11 and 12, and others. The survey also provided answers to how PSTs enacted visuo-semiotic models in the teaching and learning of coordinate geometry and the effects thereof to learners' performance.

Analysis

Mnguni's (2014) cognitive process of visualisation was used to analyse visualisation skills required for effective learning in coordinate geometry. The study investigated the 3 factors, namely IVM, CVM, and EVM to test or establish pre-service teachers' understanding of VSMs in coordinate geometry. The items tested the PSTs' skill to interpret VSMs, to make use of VSMs to solve mathematical problems and to reason with models using prior knowledge (Schonborm & Anderson, 2009). These skills include remembering, understanding, analysing and evaluating VSMs used to represent coordinate geometry. For example, Mnguni (2014) submits that after information is internalised, it is ready for conceptualisation or internal abstraction to take place. Incoming visual information is therefore conceptualised by way of interpreting it against prior knowledge. In addition, skills for EVM were probed by exploring PSTs' ability to externalise content knowledge through the use of VSMs, that is to apply, create and synthesise content knowledge. Schonborn and Anderson (2009) suggest that reasoning ability, understanding of concepts of learners and the nature of the mode in which the visual models represent the desired phenomenon are necessary for effective learning with VSMs. As a result, the reasoning ability in solving coordinate geometry test items was investigated as a cognitive function which utilises visuo-semiotic reasoning (Mnguni, 2019a). The researcher explored the knowledge and reasoning ability of PSTs concerning visual literacy in coordinate geometry. Mnguni's theory of cognitive process of visualisation was therefore used to indicate, explore and probe the visuo-semiotic skills utilised in teaching and learning coordinate geometry.

4. Results

The responses of participants in the survey test were not written separately because most of them were repetitive. Each question item of the survey was discussed using some representative excerpts from participants. The excerpts selected represented the views of the majority of the participants on a certain aspect of the study. This means, the excerpts chosen provided an explicit example of the pattern in the study data. The answered questions comprised the use of visual models to represent mathematical concepts that involve geometrical shapes, representation of straight lines on a Cartesian plane, gradient of straight lines, use of mathematical models to solve real life problems and representations of quadratic equations on a graph. Using Mnguni's (2014) stages of visualisation, that is IVM, CVM, EVM, some visualisation skills were identified in the work done by participants.

Internalisation of visuo-semiotic models (IVM)

Below are example questions from the survey that were designed to give information about the PST's IVM:

24. Figure 2 below shows the arrangement of desks in a classroom. Assa, Bernard and Chipo are seated at A(3, 1), B(6, 4) and C(8, 6) respectively.

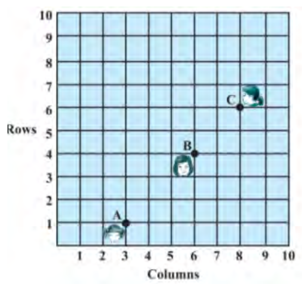


Figure 2: *Question 24*

Do you think Assa, Bernard and Chipo are seated in a line? Give reasons for your answer.

25. In a classroom, 4 friends are seated at the points A, B, C and D as shown in the Figure 3 below.

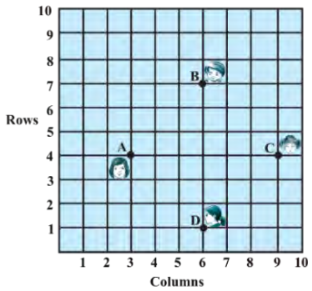
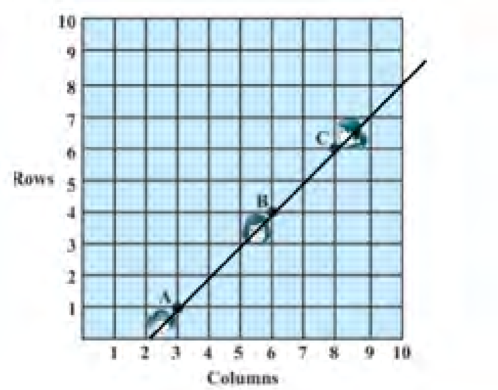


Figure 3: *Question 25*

Linda and Charles walk into the class and after observing for a few minutes Linda asks Charles, "Don't you think ABCD is a square?" Charles disagrees. Find which of them is correct and why?

The skills that embrace the lower and higher-level IVM were identified in the participants' responses. The lower-level IVM stage has visual tasks with minimal cognitive effort to perform (Mnguni, 2014). For example, in item 24, participants were given an image with 3 objects on a Cartesian plane from which they were supposed to determine their position in relation to each other. That means, whether the objects were on a straight line or not, giving reasons for their answers. The PSTs exhibited the skill of ascertaining ideas through observation of images and ability to determine the position of an image. They were able to determine the position of the objects using various approaches. For example, 25% used a straight ruler to join the 3 objects (lower-level skill) and concluded that they were in a straight line. R1 is one of the PSTs who used this approach to answer the question in figure 2 as it is given in the excerpt below:



Yes they are seated in a line because point A, B and C lie on the same line which is indicated when drawing a straight connecting the three points which means they must have the same gradient throughout which suggests that the gradient of segment AB will be equal to the gradient of segment BC

In the same item, 35% of participants made use of algebraic principles to find the gradients of AB, BC, and AC (higher-level skill). One of them is R3, whose work is given below:

Yes,

because by finding the equation of their point(s) we are able to find and locate any point(s) A, B or C, using that equation,

I.e. $m_{AB} = \frac{4-1}{6-3} = 1$ $m_{BC} = \frac{6-4}{8-6} = 1$ **and;** $m_{AC} = \frac{6-1}{8-3} = 1$

But this is not enough for us to confirm that A, B and C are in a line, because that could be three different parallel lines, therefore to finally confirm that Assa, Bernard and Chipo are really seated in a line, we have to use the gradient to determine the equation, i.e.

$(y - y_1) = m(x - x_1)$ **here we can use any point to substitute, I will use A(3;1) as an example**

$(y - 1) = (1)(x - 3)$

$y = x - 2$

Now we can use either the x coordinate or y coordinate of A, B or C to confirm if A, B and C are in line. By using the x coordinate(s) of A, B and C to substitute in the equation we get that;

$y_A = 1$ $y_B = 4$ **and** $y_C = 6$ **therefore Assa, Bernard and Chipo are seated in line.**

In the above response, equality of the gradients indicated that the 3 objects were in a straight line. Out of the 40 participants, 10% made use of distances of segments where they calculated algebraically the distances of AB, BC, AC. The fact that AB + BC = AC led to the conclusion that the 3 points were in a straight line. However, 7.5% calculated the equation of line AB and substituted point C into the equation. This implied that point C was on the extended line AB because it satisfied the equation. Nearly a quarter of participants (22.5%) seemed to have misunderstood the question. For example, R46 presented his response as follows:

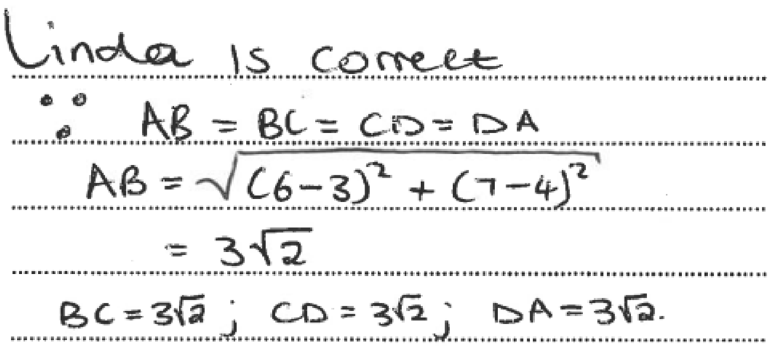
"24. yes, because they are collinear and they are all on the same quadrant"

The question demanded proof that the points were collinear or were in a line, which the PST misconceived. R46 explained that the points were collinear because they were in the same quadrant. This reason suggests that he was not quite privy to the concept of a line despite observing the points on the graph. The visualisation skills to find and ascertain the notion of a line through observation of the images was therefore deficient. Although some other participants proved it algebraically, drawing a line through the three points was going to be a valuable tool that would allow the PSTs to communicate the notion of a straight line accurately. That simple sketch of a line was worth more than the verbal/written explanation from the PSTs.

Overall, the approaches used by participants to solve the problem needed both low-level and high level cognitive effort. Participants were able to observe the images using their eyes, which led to the construction of mental images. The observed images facilitated their conception of ideas regarding straight lines, which resulted in them representing a straight line in different forms, such as distance. They were able to process what they could see in order to understand some spatial relationships between objects (distances of segments). Their ability to determine the position of objects in relation to each other was evidence of the visual-spatial skills that existed in participants' minds. This is a skill that incorporates high level IVM where a relatively high amount of cognitive effort is applied to internalise visual information (Mnguni, 2014)

The internalisation of visual models skill is also reflected by the pre-service teachers' ability to perceive shapes in the study. On item number 25 (Figure 3), participants' ability to recognise, discern, envision and understand the geometrical figure was tested [low-level effort]. However, not all participants could visualise that the 4 points on the plane formed a square. Some stated that the figure was a square but the reasons were not satisfactory. The participants' performance on this question was dismal. The bulk of participants, 43.6% calculated the distance between the points either algebraically or by counting the number of squares between the points only. Using the result, they rushed to conclude that the 4 points formed a square because all the sides are equal. Here is how R8 responded to the question in Figure 3;

R8 responded as follows;


Linda is correct
$\therefore$ AB = BC = CD = DA
$AB = \sqrt{(6-3)^2 + (7-4)^2}$
$= 3\sqrt{2}$
$BC = 3\sqrt{2}$; $CD = 3\sqrt{2}$; $DA = 3\sqrt{2}$.

R4's response is shown below:

Yes, ABCD is a square because the distance from A to B is equal to BC, CD and AD which is 3√2

Both participants never realised that it needed both equality and perpendicularity of sides to prove a square because there are other shapes like a rhombus that also have equal sides but are not squares. More than a quarter of participants (38.5%) had various challenges proving the shape. The responses suggest that pre-service teachers lacked the "ground perception" visualisation skill which according to Mnguni (2014) is the ability to detect attributes of a picture that lie behind objects in the foreground. The ground perception is a skill that is in the internalisation stage of visualisation. My expectation as a researcher was that PSTs could be familiar with the properties of a square to establish that the 4 points formed a square. Only 17.9% ascertained these properties of a square (equality and perpendicularity). They calculated the distances algebraically and used the concept of gradient to affirm perpendicularity of sides. However, their effort to measure the angles at the points of intersection would be adequate evidence of their skill to ascertain facts through observation.

A majority of participants (82.1%) failed to give satisfactory reasons that the geometric shape was a square. Generally, this trend of failing to prove the shape was an indication of a lack of 'internalisation of visuo-semiotic models' skills among participants.

Conceptualisation of visual semiotic models

Below are example questions from the survey that were designed to give information about the PST's CVM:

26. Study the graph below and find out whether the two lines y = x – 1 and 3x + 4y = 24 are perpendicular.

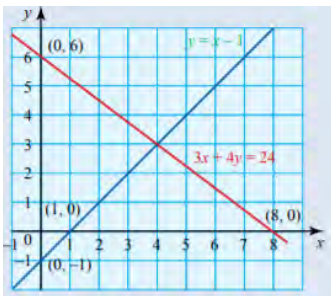


Figure 4: *Question 26*

27. Show whether figure 5a and figure 5b represent the same general equation $\frac{x}{a}+\frac{y}{b}=1$*, where a is the x-intercept and b is the y-intercept .*

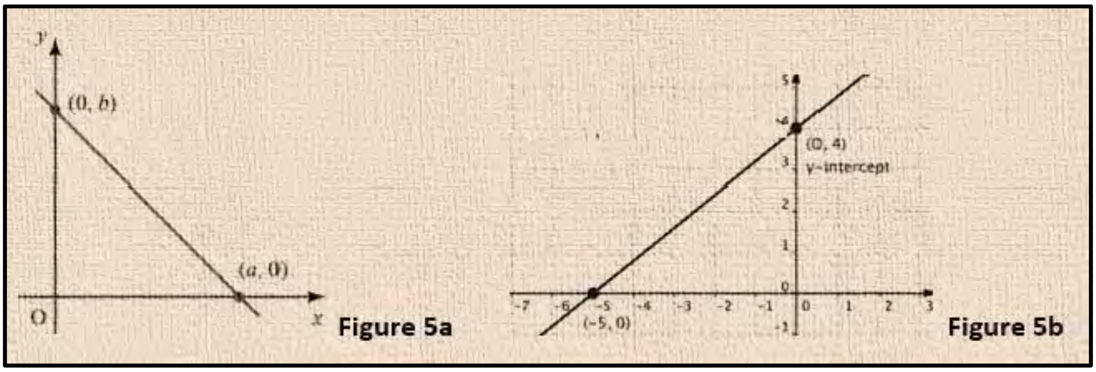


Figure 5: *Question 27*

Figure 6 below shows the curves $y = x^2 - 4x + 1$ and $y = 7 - x^2$.

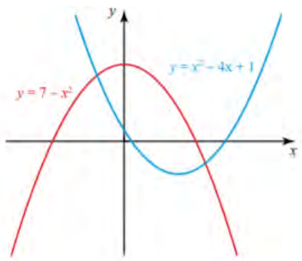


Figure 6: *Questions 32 & 33*

32. In figure 6 above, giving reasons, find the coordinates of the turning points of the two curves
33. In Figure 6 above, state whether the turning points in figure 6 are minimum or maximum points on each of the curves. Justify your answer.

The CVM was tested on items 26, 27, 32 & 33 of the survey. Item 26 explored the ability to assess and carefully examine the given graph (two lines crossing each other) to determine whether the two lines are perpendicular to each other. A majority of participants (87.5%) compared the gradients of the lines to confirm that the two lines were not meeting at 90^0. By mere eyes, the two lines looked perpendicular to each other. The remaining 12.5% just looked at the graph and concluded the lines were perpendicular to each other. For example, R2 presented her answer as follows:

> *They are perpendicular because the two lines are negative reciprocals of each other. They also form right angles*

R2 has an idea that perpendicular lines have gradients that are negative reciprocals of each other. However, this knowledge seemed to be theoretical and memorised for R2 because if she had done the necessary calculations of the gradients correctly, then she would ascertain that the lines were not perpendicular. In addition, observing the two lines, how they were cutting across the Cartesian squares was adequate for R2 to realise that the lines were not meeting at right angles. R2's response suggests that the participant could neither internalise the model nor critically examine it to form the right decision. This response illustrates that the participant (R2) lacked the skill to observe critically and carefully, and make a detailed inquiry into the problem.

On item 27, figures 5a & b show two different visual representations of the same equation $\frac{x}{a} + \frac{y}{b} = 1$. Participants were required to determine whether the two different graphs depicting straight lines represented the same form of equation $\frac{x}{a} + \frac{y}{b} = 1$. In this question, pre-service teachers were expected to use visualisation skills and knowledge of symbolic language to explain the concept represented by the two graphs. PSTs were expected to analyse and determine that the two lines passing through [(a,0);(0,b)] and

[(0,4);(-5,0)] respectively could be written in the same format $\frac{x}{a}+\frac{y}{b}=1$, where a is the x-intercept and b is the y-intercept, despite differences in their position and direction. This question was also testing participants' understanding of graphical representation where they were expected to be able to imagine or recognise that a line can be represented in different forms, thus, externally expressing a mental visual model (line graph) in a different format (equation). According to Mnguni's (2019a) visualisation skills, this question was testing the skill of "recognition of orientation through mental rotation" in PSTs. Mental rotation skill according to Mnguni (2014) is the ability to perceive various multiple items with different orientation to be the same if orientation is changed or rearranged. Mnguni added that mental rotation is a visualisation skill in the conceptualisation stage of visualisation (Mnguni et al., 2016). The ability to demonstrate the connection between the two line graphs as one and the same form of equation is an indication of the existence of the skill to evaluate concepts. A great number of participants (92.5%) were able to show that figure 5a can be written in the form $\frac{x}{a}+\frac{y}{b}=1$. However, a significant number (62.1%) had challenges showing the relationship or connection between the two line graphs (Figure 5a & b) which were presented in different orientations. The failure to establish this connection between the two line graphs could be a lapse on the PSTs' ability to conceptualise ideas from visual models.

On items 32 and 33 (figure 6), participants had different ways of interpreting the given models (graphs). The items required them to find the coordinates of the turning points of the 2 given curves where numbers were not shown on the Cartesian plane. They were also asked to determine whether the points were minimum or maximum. In each case, they were required to justify their answers. Participants were expected to demonstrate the skill to form a mental image of the coordinates not present in relation to the sketches given. The participants however used various approaches to interpret the sketches to find the coordinates of the turning points of the graphs and determine whether they were maximum or minimum. Generally, the different models used by participants were as follows:

1. Use of calculus to determine the turning points. For example, $\frac{d}{dx}(x^2-4x+1)=0$.

Nearly a quarter (22.5%) of participants used their prior knowledge of differentiation in relation to gradient to find the turning points. The use of prior knowledge concurs with Mnguni's study (2014) that poses that individuals' interpretation of external representations (ER) is dependent on their prior conceptual knowledge that they apply in interpreting the ER.

2. Using the method of completing the square to find the turning points (5%). For example, R20 presented this as follows:

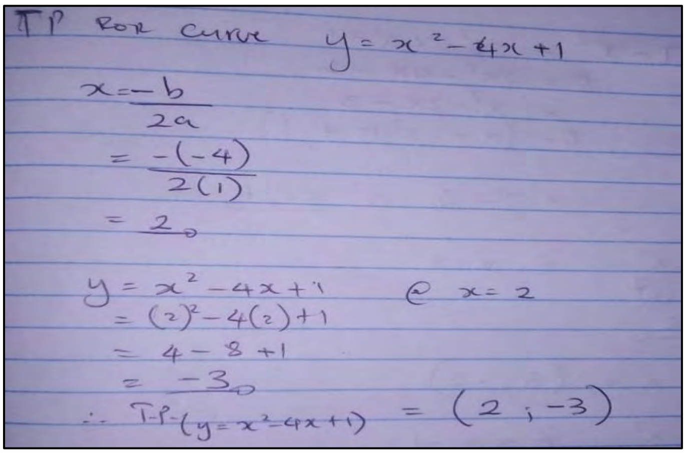
$y = -x^2+7 \Rightarrow T.P = (0;7)$

$y = x^2-4x+1 = (x-2)^2-3 \Rightarrow T.P\ (2;-3)$

3. Some participants used the concept that at the turning point of a quadratic graph,

   $x=\frac{-b}{2a}$ from the general quadratic equation $ax^2+bx+c=0$, (25%).

It was difficult to tell whether participants understood this concept or it was mere memorisation of concepts because most of them could not justify their responses. The researcher expected the participants to explain the origin of $x=\frac{-b}{2a}$ to show conceptualisation of the quadratic graph. 25% of the participants who used this method presented their work as R35 below:


TP for curve $y = x^2 - 4x + 1$

$x = -\frac{b}{2a}$

$= \frac{-(-4)}{2(1)}$

$= 2$

$y = x^2 - 4x + 1$ @ $x = 2$

$= (2)^2 - 4(2) + 1$

$= 4 - 8 + 1$

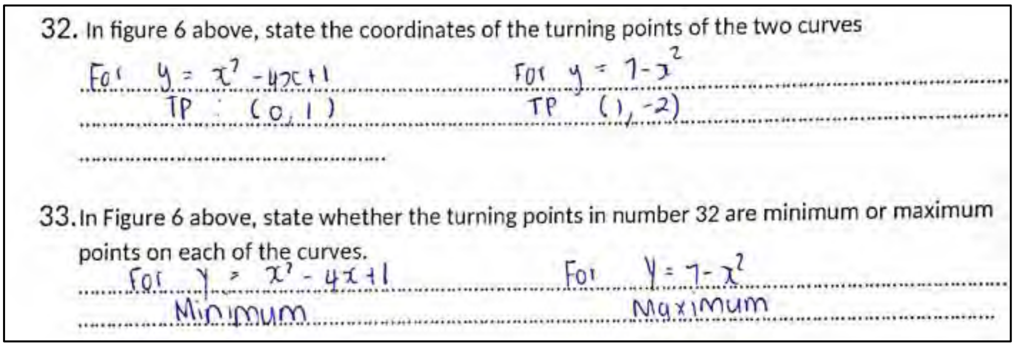
$= -3$

$\therefore$ T.P. $(y = x^2 - 4x + 1) = (2 ; -3)$

The ability to describe the 3 models in their work was a reflection of the skill to interpret visual models and determine algebraically the concept of the "turning point" in relation to the given sketches. However, these models were demonstrated only by a total of 52.5% of the participants, and only a few out of these could justify their answers. Other participants (37.5%) only gave correct answers without showing the method they used. It was therefore difficult to determine how they interpreted the visual model (graphical sketch) to come out with answers. Failure to show procedures and justify the answers allowed the researcher to assume that some of the participants were copying from sources without understanding. In addition, the study was carried out on pre-service teachers who were expected to teach learners for understanding by following all the procedures to the final answer. Unfortunately, the teachers' failure to illustrate how they obtained the answers could be a trend that could extend to their classrooms.

The remaining participants (10%) were absolutely lost on the interpretation of the graph in items 32 and 33 as shown below:

R22's response is as follows:


32. In figure 6 above, state the coordinates of the turning points of the two curves

For $y = x^2 - 4x + 1$ For $y = 7 - x^2$

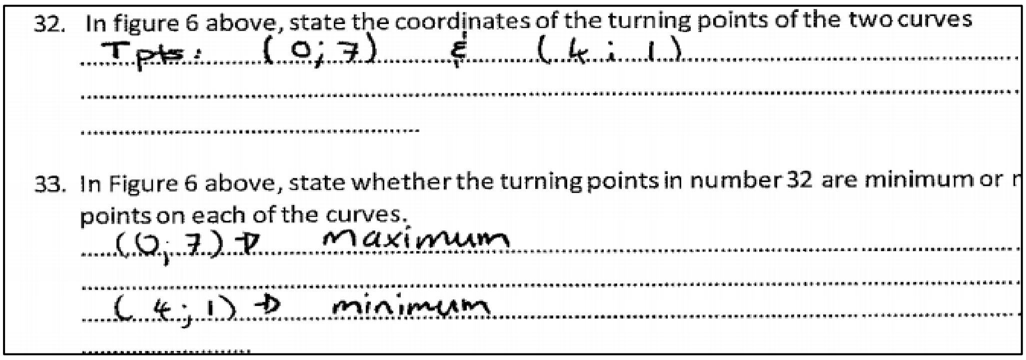
TP : $(0, 1)$ TP $(1, -2)$

33. In Figure 6 above, state whether the turning points in number 32 are minimum or maximum points on each of the curves.

For $y = x^2 - 4x + 1$ For $y = 7 - x^2$

Minimum Maximum

R29 had this to present:


32. In figure 6 above, state the coordinates of the turning points of the two curves

T.pts: $(0; 7)$ & $(4; 1)$

33. In Figure 6 above, state whether the turning points in number 32 are minimum or

points on each of the curves.

$(0; 7) \to$ maximum

$(4; 1) \to$ minimum

From the two responses, R22 and R29 could not even link their answers to the positions of the graphs on the plane. For example, on the graph, the turning point of $y = 7\text{-}x^2$ is on the positive side of the y-axis but R22 is showing this as negative (1;-2). Similarly, R29 is showing the turning point of $y = x^2 - 4x + 1$ as (4;1) but the position of the turning point is on the negative y-axis. Their failure to relate the position of the sketch with the concept of "turning points" (relationship between the signifier and the signified), is an illustration of pre-service teachers' misconceptions about interpreting visual models. The participants were also expected to demonstrate their ability to identify and recognise quadratic graphs, describe their main features and establish by deduction their turning points using symmetry.

Externalisation of visuo-semiotic models (EVM)

Below are example questions from the survey that were designed to give information about the PST's EVM:

Figure 7 below shows the supply and demand of labour for a particular industry in relation to the wage paid per hour. Supply is the number of people willing to work for a particular wage, and this increases as the wage paid increases. Demand is the number of workers that employers are prepared to employ at a particular wage: this is greatest for low wages.

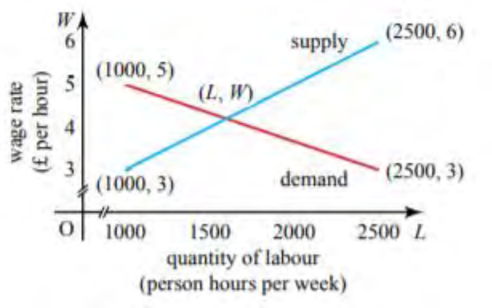


Figure 7: *Questions 29 & 30*

In figure 7 above, show all the necessary steps to find the equations representing the two lines.

29. *In figure 7 above, giving reasons, find the values of L and W at which the market 'clears', i.e. at which supply equals demand.*
30. *In Figure 7, although economists draw the graph this way round, mathematicians/ mathematics educators would plot wage rate on the horizontal axis. Why?*
31. *From Figure 6, find the coordinates of their points of intersection.*

In EVM, *a mental visual model maybe expressed externally in a different format to that in which it occurs in the mind* (Mnguni, 2014, p.6). For example, mental visual models are expressed as external models in different forms such as verbal descriptions, numerical expressions, etc. Skills under this stage were explored in items 28, 29, 30, 31. In these items, the study explored the skills to express, apply or implement some conceptualised knowledge in a new format. So, synthesis and application of visual knowledge are the key aspects in this section. Items 28, 29 and 30 presented a mathematical model (figure 7) starting with a real life situation modelled on a graph. The question was meant to test the pre-service teachers' ability to interpret the graph, analyse and then apply the knowledge to solve real life problems. The ability to solve real life problems becomes externalised knowledge from visually conceptualised models.

From item 29, nearly half of pre-service teachers (45%) managed to find the point of intersection of the line graphs (demand & supply). Another 42.5% had the conception of what needed to be done (equating the two equations) but in the process they made some computational errors which gave them the wrong answers. 12.5% of the participants had correct answers without showing the method used to get the answers. Of interest are PSTs like R5 who is one of the 12.5% who just stated the correct answer but the equations from which the correct answers were derived (item 28) were wrong. This suggests that the answers could have been copied from somewhere without understanding the concepts. This result allowed the researcher to cast aspersions on answers without procedures shown.

In total, 87.5% pre-service teachers at least knew that the two graphs had to be solved simultaneously to find the point of intersection (L,W). However, although some knew that the equations of the 2 lines should be equated, they were using the wrong equations formed in item 28 (40.1%), hence, the answers were incorrect. Nevertheless, the essential point is that they were able to read from the graph to ascertain how (L,W) could be calculated. The ability to demonstrate the skill to solve the problem was evidence of knowledge to externalise ideas from visual models. R18's response is given below to confirm the explanation about the 45% in item 29.

28.

For demand | For Supply
$m = -1/750 \quad c = 19/3$ | $m = 1/500 \quad c = 1$
$y = -\frac{1}{750}x + \frac{19}{3}$ | $y = \frac{1}{500}x + 1$

29.

(L;W) is where demand and Supply are equal

$-\frac{1}{750}x + \frac{19}{3} = \frac{1}{500}x + 1$

$\therefore x = L = 1600$

$W = \frac{1}{500}(1600) + 1$

$= 21/5$

$\therefore L = 1600$

$W = 21/5$

The above interpretation of the graph was done with all the procedures being followed. The candidate showed how all terms of the general equation of a straight line can be found from the given image, that is, the gradient and the constant "c".

However, some participants failed to completely answer the question because they did not justify their answers. They could have justified their answers by, for example, simply sketching lines from the point of intersection to the two axes to describe roughly the required points. Unlike R18's response, R42 responds to the same questions as follows:

R42's Response

Errors made

28.

Supply: $W = \frac{L}{500} + 1$ demand: $W = -\frac{L}{1750} + \frac{31}{7}$

29.

$\frac{L}{500} + 1 = -\frac{L}{1750} + \frac{31}{7}$

$\frac{9L}{3500} = \frac{27}{7}$

$L = 1333,33$ hr/wk $W = £\,3,67$

For item 28, R42 just listed the equations without showing procedures, one of which is incorrect. It is difficult to imagine how these were formed to determine the participant's understanding of the concept. However, in item 29, the participant is privy to the fact that the two equations should be equated and solved to find the values of L and W where the market clears. The final result is incorrect because the equations used are wrong.

5. Discussion

The aim of this study was to explore pre-service teachers' conception of VSMs as a process in doing mathematics in the context of coordinate geometry. Pre-service mathematics teachers' knowledge, ability and competence to select and use appropriate VSMs to enhance understanding of mathematical concepts in the context of coordinate geometry was therefore examined.

This study found that some mathematics pre-service teachers had challenges with questions that needed higher-order cognitive skills to analyse and assess visual models. The pre-service teachers were failing to respond to questions that needed evaluation, proof and ability to make judgements (which are Bloom's higher level cognitive skills), see items 27 and 33 as examples. It could be for this reason that Jones and Tzekaki (2016) assert that visual cognition in teachers is quite limited. At a higher level, Turner (2010) affirms that visual literacy should espouse the need to interpret a model in situations where relationships are to be identified. A significant number of pre-service teachers (62.1%) had challenges demonstrating the relationship that bound the two line graphs (Figure 5). This is a demonstration of a lack of the higher-order skill to recognise orientation through mental rotation and a lack of the skill to process what one can see in order to understand and visualise the spatial relationship between the two graphs. In addition, participants' inability to prove the square in item 25 could be an indication of visuo-spatial skills deficiency. According to Mnguni (2014), a lack of visuo-semiotic skills implies the inability to process the images they can see in order to ascertain and comprehend the spatial relationships between the objects.

The study also showed that although participants had knowledge of how certain problems should be solved, the skill to externalise their understanding was a challenge for some of them. In a worrisome and quite rampant situation, some participants were just writing answers without explanation or properly showing the procedures they used to attain the answer. The visuo-semiotic procedures required in developing mathematical ideas yield conceptual meanings (Godino & Batanero, 2003) in learners. Failure to explain solutions provides evidence that participants lacked the skill to communicate their ideas. The question items required explanations and in the same vein, their invitation to participate in the study, explicitly spelt out that the study intended to ascertain their skills to provide proficient guidance to mathematics learners in the classroom through the use of visual semiotic models. Dropping answers only raised suspicion in the researcher that there could be a possibility of them regurgitating information without comprehension. Evidence of copying or regurgitating information is reflected by R5's correct answers in item 29 obtained from wrong equations in item 28. This suggests that the answers could have been copied from somewhere without understanding the concepts. According to Turner (2010), once a solution has been found, the problem solver may need to present the solution with an explanation or justification to others. Hence, the PSTs were expected to present their solutions with clearly stated procedures. The skill to communicate involves interpretation that enables formation of a mental model of the situation. According to Stylianou (2010), learners at all levels should be able to use VSMs to communicate and apply mathematical ideas. Failure by pre-service teachers to communicate their mathematical concepts is likely to impact on learners. Pre-service teachers, therefore, have the obligation to make mathematics meaningful to learners (Larbi & Okyere, 2016).

Some participants used the concept $x = -\frac{b}{2a}$ to find the coordinates of the turning points on item 32. However, of concern is that a significant number of participants dared not to explain how $x = -\frac{b}{2a}$ was developed, which compelled the researcher to question the efficacy of the approach used. The use of $x = -\frac{b}{2a}$ without an explanation illustrating its development could be a sign of memorising facts that are not understood. If the pre-service teachers pass on this trend to learners, it could breed challenges in trying to understand the concept of "turning points" of parabolas. This is because the ability to conceptualise VSMs by learners is normally impacted by the level at which teachers understand them (Stylianou, 2010). Failure to explain this is evidence of a lack of externalisation skills or communication skills. Turner (2010) advises against simply following instruction without understanding concepts because this illustrates a low-level cognitive demand.

According to Rundgren & Tibell (2010), external visual models produced by students can be categorised into 3 levels, one of which is the symbolic level. In this level, students produce a visual model that is a qualitative abstraction such as a mathematical model to represent phenomena. For example; the equation of a line or a parabola. Inability to produce equations of the line graphs (item 28) in the study by PSTs exposed a deficit of the skill to express a mental model in a new format, which is the aptitude to synthesise ideas.

The study also found that PST participants exhibited different levels of cognitive effort to interpret visual models of the same task as indicated by the way they presented their responses. For example, in item 24, some PSTs thought of just using a ruler to prove collinearity of the points, which was a mere demonstration of the concept of a line (Bloom's lower level cognitive skill), but some examined the question carefully and made use of distances of segments to calculate algebraically and compare the distances of AB, BC, AC (Bloom's higher level cognitive skill). As a result, Fennell and Rowan (2001) assert that the approaches or representations used by individuals are a reflection of their thinking level. The approaches used by the participants therefore, are likely to determined their level of the ability to use or understand VSMs.

6. Conclusion and recommendations

The current research has shown that mathematics pre-service teachers struggle with tasks that require higher cognitive effort to solve them. In addition, while pre-service teachers may be acquainted with the skill to conceptualise visual models, they can sometimes hardly externalise the concepts. The study also established that participants operated at different levels of visual literacy as reflected by the way they responded to the same questions. The researcher, argue that, because of their differences in visual literacy, pre-service teachers must be treated differently even in the way they are trained. The curriculum should also be designed in a way that fosters visual skills lacking among them. The study results support Mnguni's (2014) findings which posit that, IVM is characterised by low, middle and high levels in relation to the cognitive effort applied to understanding visual concepts. It is therefore essential for teacher educators to identify those cognitive levels at which pre-service teachers operate so that teaching and learning can be systematically organised. This way, appropriate remedial strategies can be put in place to assist pre-service teachers to develop from one stage of visual literacy to the other. In this study, the researcher would want to argue that pre-service teachers and the high school learners share the same challenges and misconceptions about using mathematical processes such as the use of VSMs when teaching and learning mathematics. This inferential assertion emerged from the challenges exhibited by some of the pre-service teachers who wrote a test extracted from high school learners' textbooks. Visualisation in mathematics therefore requires specific training in order to comprehend the entire configuration of relations in coordinate geometry. This conclusion concurs with Ball and Cohen (1999) and Stein et al (2008) who assert that there is little evidence that teacher training programmes prepare pre-service teachers adequately to acquaint them with visuo-semiotic skills and integrate these skills in instruction successfully.

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