



Evaluation of number sense perception of pre-service teachers from different disciplines

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Abstract

This study aims to determine pre-service teachers' perceptions from different disciplines about number sense and reveal the strategies used to serve the components of number sense. The study is designed qualitatively, according to the phenomenology design. The study group consisted of a total of 265 pre-service teachers, 92 from Preschool Education Department (PSED), 88 from Classroom Instruction Education Department (CIED), and 85 from Primary Mathematics Education Department (PME). Three different (F1, F2 and F3) data collection tools were employed in the study. F1 was distributed to 265 pre-service teachers. After analyzing the data collected with F1, interviews were conducted with nine pre-service teachers (3 from each discipline) who employed different solutions using F2. The focus group was conducted with 12 pre-service teachers (4 from each discipline) selected from 24 volunteer pre-service teachers from different disciplines who use different strategies. The data of this session was collected through F3. Descriptive and content analysis were used in the data analysis. According to the findings, pre-service teachers' perceptions about number sense are grouped under four categories: ability, comprehension, number-processing knowledge, and intrinsic situations. The strategies developed depending on pre-service teachers' perception of number sense differed according to discipline.

Keywords: Number sense, interdisciplinary, pre-service teachers, strategy

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1. Introduction

If you have candies in both hands and a little three-year-old girl turns to the hand with more candies, would you think she knows math or wants more candy? If a five-year-old boy says, "I weigh 200 kilograms", would you think this boy does not know math, or do you think he feels too fat? If a 10-year-old boy says that he has to drive for five hours to go to his grandmother, who lives next neighborhood, would you think his knowledge of mathematics is insufficient, or he misses his grandmother a lot? What you would think is up to you; obviously, it may change for everyone, but the thing that does not change is that this leads us, educators, to the number sense.

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Number sense is defined by Howden (1989) as good intuition about numbers and their relationships. NCTM (2000) states that number sense emerges when students develop "flexibility" by dealing with numbers. Number sense develops when students understand the size of numbers, develop several different ways to think about and represent numbers, use a benchmark, and achieve good intuition about the effect of operations on numbers. Olkun (2012), on the other hand, defines number sense as the flexible and fluent use of numbers while solving problems with numerical content. Dede and Şengül (2016) state that people with number sense can rationally interpret situations involving numbers and operations and understand what these situations mean. They also underline that those with a "good" number sense are aware that there is not one but more than one solution to solve problems. Accordingly, they can use various strategies flexibly and produce a suitable solution for the problem.

Different researchers have categorized number sense following different approaches (Greeno, 1991; McIntosh, Reys, & Reys, 1992; Reys et al., 1999; Der CYang & Tsai, 2010). Greeno (1991) classified it under three components: flexible numerical computation, soft numerical computation, and quantitative judgment and inference. McIntosh et al. (1992) described its components as the knowledge of and facility with numbers, knowledge of and facility with operations, and application of knowledge of numbers and operations to computational situations. Reys et al. (1999) expressed number sense in the form of six components. These are; understanding the meaning and size of numbers, understanding and use of equivalent representations of numbers, understanding the meaning and effect of operations, understanding and use of equivalent expressions, flexible computing and counting strategies, and measurement benchmarks. In addition, Yang and Tsai (2010) expressed it as five components: the skills of understanding the basic meaning of numbers, understanding the relative size of numbers, using different representations of numbers, understanding the effect of operations on numbers, and judging whether the result is logical or not.

The concept of number sense, which occupies an important place in teaching, is essential not only for doing mathematics but also for all disciplines where mathematics is used as a tool. Consequently, one of the researched subjects was whether number sense is an innate or acquired feature. For example, Olkun, Fidan, and Özer (2012) pointed out that it is impossible to teach number sense directly, indicating that it is mainly an extension of an innate ability. In addition, like any ability, certain activities can improve children's number sense to a certain extent. They underline that such activities should be chosen according to the child's age and mathematical level. It is vital that the child performs the physical and mental actions in the activities and that the teacher or parent only guides them. Moreover, number sense is a concept that can be learned, taught, and developed (Griffin, 2004). It can be developed using verbal expressions (notifications) in early childhood (Dehaene, 2011), and children's daily life experiences can also enrich it (Tsao, 2004).

Regarding the development of number sense in the national literature, it is not included as the concept of number sense in the Pre-School and Mathematics Syllabus published by the Ministry of National Education [MoNE] in different years 2013 and 2018. However, the curriculum includes many gains involving number and operation knowledge. There are 3 gains for the number sense development of 36-48-month-old children, 3 gains for 48-60-month-old children, and 6 gains for 60-72-month-old children. 29 gains for the number sense for grades 1-4, which serve the gains in other sub-learning areas. For grades 5-8, this number appears as 29 gains, and the sub-learning areas that they serve vary. In addition, the number sense is defined in the Early Childhood Special Education Curriculum (MoNE, 2018) as a "*cognitive development sub-area, which includes the processes related to children's understanding of the numbers and quantity of the beings around them*" (p.9), and the gains of this sub-area are included in the curriculum. Based on definitions, classifications, and importance in the curriculum, number sense, an essential concept for younger age groups and all age groups who can "play with numbers", is also essential in predicting mathematical cases in daily life. In addition to this contextual situation, possessing the concept of number sense is thought to be essential for both students and pre-service teachers, who will teach this concept to the students.

Regarding the related literature, there is a study on pre-service classroom teachers (Altay & Umay, 2011; Şengül, 2013), another one on elementary mathematics teachers (Şengül & Dede, 2014), and one more on elementary-secondary mathematics pre-service teachers (Dede & Şengül, 2016). It can be said that classroom teachers and elementary mathematics teachers have much work to do after preschool teachers provide number sense intuitively. The perception of teachers and pre-service teachers in these disciplines about number sense and the strategies they can develop will guide learning. The problem of this study is to determine the number sense perception of pre-service teachers from different disciplines and to reveal the components of number sense that the strategies they use serve. Revealing these perceptions will contribute to the literature in terms of realizing the existence and developability of the strategies used by pre-service teachers. Based on this context, two sub-problems were generated. These are;

- 1) How do pre-service teachers perceive number sense according to disciplines? Which number sense component do their strategies result from their perception serve
- 2) Which components of number sense do different strategies of pre-service teachers serve according to disciplines? What are the factors affecting the use of different strategies?

2. Method

2.1. Research Design

The study is designed qualitatively, according to the phenomenology design, which focuses on phenomena that we are aware of but do not have an in-depth and detailed understanding (Yıldırım & Şimşek, 2013). Although number sense is not an entirely unknown subject, it is also a subject that cannot be fully comprehended. Aiming to determine pre-service teachers' perceptions about this subject according to disciplines played an essential role in selecting phenomenology design. Include in these subsections the information essential to comprehend and replicate the study. Insufficient detail leaves the reader with questions; too much detail burdens the reader with irrelevant information. Consider using appendices and/or a supplemental website for more detailed information.

2.2. Data Collection Tools

Three different data collection tools were employed in the study. The first is the (F1) form prepared by the researchers to measure pre-service teachers' number sense and consists of twelve questions. The form has been prepared to be used in three disciplines; for this purpose, numbers and mathematical contexts have been kept the same, and only numerical quantities have been altered. In addition, F1 was based on the components of number sense developed by Reys et al. (1999), and these components were included in the solutions. The reason for taking this framework as a basis is that it can be used in all three disciplines by only changing the numerical quantities, which also increases the internal reliability of F1.

The second data collection tool (F2) is used in individual interviews with pre-service teachers who used different strategies to solve questions. This form asks for an explanation of the chosen strategies. It was prepared for the first sub-problem, and it constitutes the basis of the third form.

The third (F3) form is prepared for the interdisciplinary focus group discussion with pre-service teachers. It aims to determine pre-service teachers' perception, how they interpret the strategies, and the factors affecting their use.

Regarding the validity study of the data collection tools, expert opinions regarding their suitability for the study were taken (3 from mathematics teaching, 2 from preschool teaching, and 2 from classroom teaching). Data collection tools were revised accordingly, improving their validity.

2.3. Study Group and Data Collection Process

The study group consisted of a total of 265 pre-service teachers, 92 from Preschool Education Department (PSED), 88 from Classroom Instruction Education Department (CIED), and 85 from Primary Mathematics Education Department (PME), attending one of the leading universities in the Mediterranean Region. As shown in Table 1, the

number of males is low among pre-service preschool teachers, but male-female participants are close to each other in the other departments. F1 was distributed to 265 pre-service teachers given in Table 1, and they were asked to answer the questions.

Table 1. Distribution of the study group

| Discipline | Gender | | Overall |
|---------------------------------|--------|------|---------|
| | Female | Male | |
| Pre-school Education | 80 | 12 | 92 |
| Classroom Instruction Education | 55 | 33 | 88 |
| Primary Mathematics Education | 45 | 40 | 85 |
| Overall | 180 | 95 | 265 |

After analyzing the data collected with F1, interviews were conducted with nine pre-service teachers (3 from each discipline) who employed different solutions using F2. These interviews lasted approximately 70-80 minutes. Each pre-service teacher explained why they used the strategies they preferred.

The focus group was conducted with 12 pre-service teachers (4 from each discipline) selected from 24 volunteer pre-service teachers from different disciplines who use different strategies. The data of this session was collected through F3, and the focus group lasted 145 minutes. Pre-service teachers participating in the focus group interview were coded as S1, S2, S3... S12. S1, S2, S3, and S4 for pre-service preschool teachers; S5, S6, S7, and S8 for pre-service classroom teachers; S9, S10, S11, and S12 for pre-service elementary mathematics teachers.

2.4. Data Analysis

Descriptive and content analysis were used in the data analysis. Analysis units were based on the framework developed by Reys et al. (1999) and consisted of 6 components. These are understanding the meaning and size of numbers (UMSN), understanding and use of equivalent representations of numbers (UUERN), understanding the meaning and effect of operations (UME0), understanding and use of equivalent expressions (UUEE), flexible computing and counting strategies (FCCS) and measurement benchmarks (MB).

3. Findings

3.1. Findings of "How do pre-service teachers perceive number sense according to disciplines? Which number sense component do their strategies resulting from their perception serve?"

The perceptions of pre-service teachers about number sense are given in Table 2 according to disciplines.

Table 2. Themes and sub-themes of perceptions of number sense

| | | Frequency (f) | | |
|-------|--|---------------|----|-----|
| | | PST | CT | EMT |
| Theme | Sub-theme | | | |
| 1 | A special ability | 31 | 38 | 9 |
| 2 | Prediction ability | 13 | 21 | 22 |
| 3 | Ability to internalize mathematics | 18 | 25 | 22 |
| 4 | Ability to concretize mathematics | 12 | 19 | 17 |
| 5 | Numerical Ability | 12 | 38 | 45 |
| 6 | Using shortcuts | 3 | 33 | 38 |
| 7 | Understanding mathematics | 34 | 18 | 12 |
| 8 | Understanding the problem | - | - | 55 |
| 9 | Assigning a meaning to the numbers | 10 | 23 | 17 |
| 10 | Proficiency in numbers and operations | - | 25 | 29 |
| 11 | Making sense of numbers | 9 | 12 | 39 |
| 12 | Feeling the numbers | 21 | 34 | 33 |
| 13 | Numbers express the concept of less - more | 41 | 37 | 20 |
| 14 | Relating numbers to each other | 23 | 33 | 33 |
| 15 | Acknowledging numbers | 33 | 13 | 11 |
| 16 | The image created by the numbers in the mind | 35 | 12 | 14 |
| 17 | Enjoying working with numbers | 11 | 33 | 44 |
| 18 | Familiarity with numbers | 18 | 19 | 23 |
| 19 | Uncertainty/Obscurity | 2 | 8 | 2 |
| 20 | Fear of operations and numbers | 1 | 2 | 1 |
| 21 | A spontaneous feeling | 34 | 33 | 12 |

Regarding Table 2, pre-service teachers' perceptions about number sense are grouped under four themes. They are categorized as ability, comprehension, number-processing knowledge, and intrinsic situations. Pre-school and pre-service classroom teachers stated that number sense is "a special ability" with the highest frequency. The other highly mentioned sub-theme is the "numerical ability" under the theme of ability, mainly expressed by pre-service classroom teachers and pre-service elementary mathematics teachers. In addition, pre-service mathematics teachers stated that their numerical ability is low.

The obtained data were divided into sub-themes under the theme of number and process knowledge. Pre-school pre-service teachers mentioned that "numbers express the concept of less – more". Pre-service teachers from other disciplines also expressed this statement and "proficiency in numbers and operations". Moreover, the sub-theme involving "understanding the problem" was not expressed by preschool and pre-service classroom teachers. However, it is one of the most mentioned statements by pre-service elementary mathematics teachers.

In addition, "uncertainty/obscurity" and "fear of operations and numbers" sub-themes were formed; they were expressed by pre-service teachers who do not know the concept of number sense well. Pre-service teachers explained it as "I just feel like, because the word sense was used".

“Familiarity with numbers” from the knowledge of numbers and operations sub-theme meant finding benchmarks. A pre-service teacher explained this statement, which is described as flexible thinking as;

"S6: In fact, if they realize how many more or less from the number, they can solve it. For example, I want them to add 8 and 12. One of them is 2 less than 10; the other is 2 more than 10. If they realize it, they will find that it is 2 times 10. Does it have a name?"

The pre-service teacher described the expression stated in the literature as the benchmark, criterion point, or triangulation but did not mention the conceptual equivalent. In addition, pre-service elementary mathematics teachers used expressions such as "I anchor, I drop anchor, I take reference," and they used these in their question solutions. Moreover, 23 pre-service preschool teachers, 22 pre-service classroom teachers, and 17 pre-service elementary mathematics teachers left the questions unanswered. Another finding is that they could not interpret number sense and answered the questions by rote-based operations. The relationship between pre-service teachers' perceptions about number sense and the components that their strategies served are given in Table 3.

Table 3. Strategies developed based on number sense perceptions

| | | Components that Strategies Serve | | | | | | | | | | | |
|----------------------|-------------------------------------|----------------------------------|-----|-------|-----|------|-----|------|-----|------|-----|-----|-----|
| | | UMSN | | UUERN | | UMEO | | UUEE | | FCCS | | MB | |
| | | 1.S | 2.S | 1.S | 2.S | 1.S | 2.S | 1.S | 2.S | 1.S | 2.S | 1.S | 2.S |
| Themes | Ability | 1/4 | 11 | 13 | 11 | - | - | - | - | 5 | 6 | 6 | 6 |
| | Total 58 | | | | | | | | | | | | |
| | Understanding | 6 | 5 | 4 | 4 | 2 | 7 | 1 | 3 | 7 | 9 | 25 | 22 |
| | Total 105 | | | | | | | | | | | | |
| | Knowledge of numbers and operations | 1/7 | 12 | 11 | 13 | 18 | 14 | 19 | 17 | 11 | 12 | 22 | 27 |
| Total 199 | | | | | | | | | | | | | |
| Intrinsic situations | - | - | - | - | - | - | - | - | - | - | - | - | 2 |
| Total 2 | | | | | | | | | | | | | |

Regarding Table 3, pre-service teachers who perceived number sense as an ability did not use strategies that serve UMEO and UUEE. 58 pre-service teachers answered questions, and they mainly used UMSN and UUERN strategies. The pre-service teachers who perceived number sense as ability are more dominant in preschool teaching, where these strategies are expressed by modeling. Pre-service preschool teachers also expressed the importance of model use.

In addition, pre-service elementary mathematics teachers who have the intuition of making short-cut operations use benchmarks. Five pre-service classroom teachers used this strategy, and the solution of the pre-service classroom teacher (S7) is shown in figure 1.

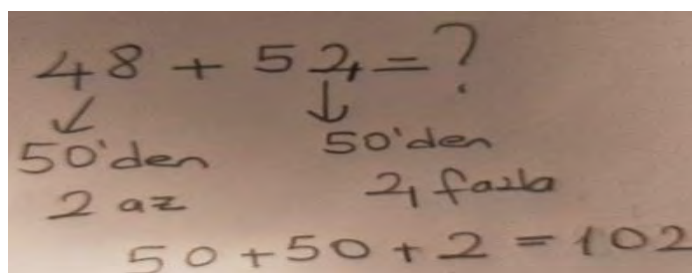


Figure 1. Solution of S7

Regarding Figure 1, 50 was set as the benchmark, and the computation was made accordingly. The majority of pre-service elementary mathematics teachers used this strategy. The solution of Q11, an extended version of this strategy, is shown in Figure 2.

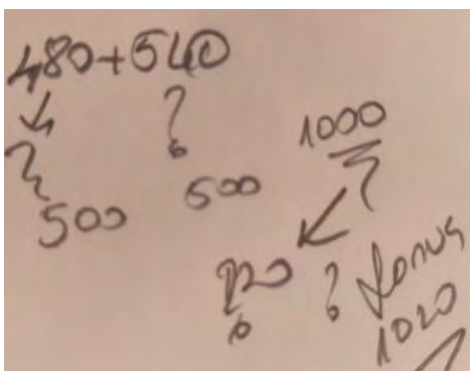


Figure 2. Solution of S11

Regarding Figure 2, 500 was set as the benchmark. Depending on the size of the numerical values, the number of pre-service teachers using the benchmarks was higher than the other strategies in both disciplines. Pre-service classroom teachers used 5, 10,

and 100 as benchmarks, while pre-service elementary mathematics teachers used 50, 250, 500, and 1000. Pre-service preschool teachers were observed to use the phrase "more or less than the specified number" instead of the benchmark.

R: Is it enough to paint the marbles?

S2: Of course, not enough. Actually, when I ask, they cannot make sense of it first?

R: Like how?

S2: I ask, "Is it more than the previous marbles?". They say yes or no.

R: Well, have you ever asked in exercises, for example, how much higher is 7 than 5?

S2: hmmm. I don't know, can they answer?

S4: I asked, they answered, but when I asked how much less than 5, they could not answer.

Only one pre-service teacher expressed this in the focus group, while the other pre-service teachers focused only on the concept of less and more.

3.2. Findings of "Which components of number sense do different strategies of pre-service teachers serve according to disciplines? What are the factors affecting the use of different strategies?" sub-problem

The components of number sense that different strategies used by pre-service teachers serve are given in Table 4 according to disciplines.

Table 4. Components that used strategies serve

| Components that used strategies serve | | | | | | |
|---------------------------------------|------|-------|------|------|------|-----|
| | UMSN | UUERN | UMEO | UUEE | FCCS | MB |
| PSED | 25 | 2 | - | - | 3 | - |
| CIED | 22 | 44 | 20 | 14 | 25 | 58 |
| PMED | 18 | 10 | 21 | 26 | 32 | 51 |
| Total | 65 | 56 | 41 | 40 | 60 | 109 |

According to Table 4, pre-service preschool teachers did not develop strategies that serve UMEO, UUEE, and MB. The strategies developed by this discipline were those that serve UMSN, UUERN, and FCCS.

R: What if we ask for something not included in the sources and the program? For example, for 5, we draw five cars; "how many cars", "five"; we say "paint the five balloons red," but we don't talk about 5 being 1 and 4, 2 and 3?

S3: How so? Isn't that an addition? Shall we start like this?

R: What made you think it was an addition? Noone said plus, equal?

S3: Right

As a result of the dialogues above, pre-service teachers were observed to fail to develop strategies that serve UUERN, and they expressed the reason as "I don't know".

Pre-service classroom teachers used strategies that served each component. Their strategies mainly served MB and UUERN, and the number of strategies serving the other four components is close to each other.

On the other hand, pre-service elementary mathematics teachers mainly developed strategies that serve MB component. The strategy remained the same while the numerical values of the benchmarks in the examples changed compared to other disciplines. In addition, 40 strategies developed by this discipline serve FCCS. A solution example serving FCCS is shown in Figure 3.

Handwritten mathematical solution on grid paper showing the addition of 1840 and 2790. The numbers are written vertically, and arrows indicate the decomposition of 1840 into 1000, 800, and 40, and 2790 into 2000, 700, and 90. The final sum is 4630.

Figure 3. Solution of S12

Regarding Figure 3, S12 can make flexible calculations with numbers. This calculation is related to the FCCS component, and the pre-service teacher explained it as below;

"S12: The question of how it could be solved differently actually led me to this solution. We are used to writing the numbers one after the other and sum up; of course, there is also the grade concern, but when I solved this and other questions, I felt that my mind expanded."

The solution of a classroom pre-service teacher for the same component is given in Figure 4. A common strategy is used in both solutions; S7 explained the mathematical expression with verbal expressions as follows;

"...I have always loved numbers. So 18 is not just 18 for me; it is the sum of 9, and 9, two less than 20, 3 more than 15. I like separating them into parts because, in this way, I can easily calculate. I think I've been doing this since I was little. For example, I first separated into hundreds and tens in this question, then summed them up; think about it, this is my solution."

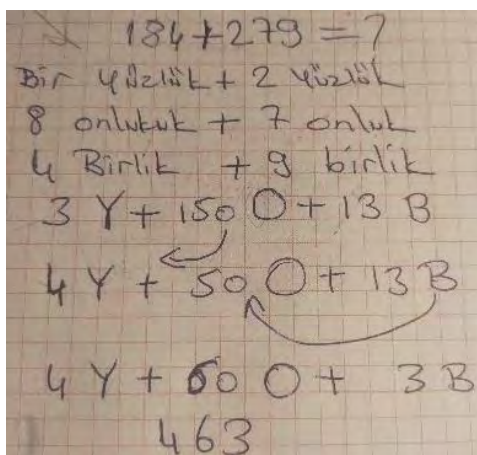


Figure 4. Solution of S7

The components of number sense served by the strategies used by pre-service teachers were determined according to disciplines, and the factors affecting the use of these strategies were determined through focus group interviews. In other words, the reasons for using the number sense components specified in Table 4 are given in Table 5.

Table 5. Factor perception affecting strategy use

| | Factors | Pre-service teachers using different strategies | Total frequency |
|------|--|---|-----------------|
| PSED | Characteristics of the student age group | S1, S2, S4 | 3 |
| | Numbers being already taught by the family | S1, S2, S3, S4 | 4 |
| | The socioeconomic level of the family | S2, S3 | 2 |
| | Students' previous experiences | S3, S1, S2 | 3 |
| | Total | | 12 |
| CIED | The need to learn with fun | S5, S6, S8 | 3 |
| | The ability to make quick operations | S8, S6 | 2 |
| | Lack of test anxiety | S6, S7, S8 | 3 |
| | Using different resources | S7, S5, S6 | 3 |
| | Total | | 11 |
| PME | The characteristics of the curriculum | S9, S10, S11 | 3 |
| | Test system | S9, S10, S11, S12 | 4 |
| | New-generation question types | S9, S10, S11, S12 | 4 |
| | High awareness of families | S10, S11 | 2 |
| | Total | | 13 |

Regarding Table 5, the factors affecting pre-service preschool teachers' strategy use are; the characteristics of the student age group, numbers being already taught by the family, the family's socioeconomic level, and the student's previous experiences. The factor with the highest frequency was "numbers being already taught by the family", which was expressed as a negative situation by the pre-service teachers.

The factors affecting pre-service classroom teachers' strategy preference are; The need to learn with fun, the ability to make quick operations, lack of test anxiety, and using different sources. A negative factor mentioned by pre-service classroom teachers is that students use different sources which have questions promoting rote learning.

The factors affecting the strategies selected by pre-service elementary mathematics teachers are; The characteristics of the curriculum, test system, new-generation question types, and high awareness of families. Only family awareness was evaluated positively, while the other factors were evaluated negatively. Pre-service teachers stated that the strategies that can be used against the anxiety of completing the curriculum, numerous gains in the curriculum, and the anxiety to instruct all of them for the exam would provide convenience in solving new generation questions. These thoughts of the participants were influenced by the teachers in their families and the practice school.

4. Conclusion and Discussion

This study aims to determine pre-service teachers' perceptions from different disciplines about number sense and reveal the strategies used to serve the components of number sense. According to the findings, pre-service teachers' perceptions about number sense are grouped under four categories: ability, comprehension, number-processing knowledge, and intrinsic situations. These categories were expressed as sub-themes. The most mentioned sub-themes are that number sense is necessary for understanding and comprehending the problem. It is a numerical ability, and numbers are perceived as an expression of the concept of less-more. In addition, different sub-themes have been observed for each discipline.

Pre-service preschool teachers' number sense perceptions are higher in the following sub-themes: Numbers express the concept of less – more, number sense is a special ability, number sense is required to understand mathematics, knowing numbers, the image created by the numbers in mind and a spontaneous feeling. Pre-service classroom teachers' perception is parallel to theirs. They mentioned the following in addition to the sub-themes stated by pre-service preschool teachers: Feeling the numbers, relating numbers to each other, and enjoying working with numbers. On the other hand, pre-service elementary mathematics teachers perceive number sense as necessary for understanding and comprehending the problem, proficiency in numbers and operations, and making sense of numbers. The sub-themes highly mentioned by both classroom and pre-service elementary mathematics teachers are; enjoying working with numbers and numerical ability.

Yang et al. (2009), Dehaene (2011), Wessels (2014), and Naukushu (2016) showed that teachers have a perception of number sense, which is in line with the study. Unlike other studies, this study revealed that some pre-service teachers have a fear of operations and numbers.

Most of the participants did not have a perception of number sense. This rate is approximately 25% for pre-service preschool teachers, 23% for pre-service classroom teachers, and 21% for pre-service elementary mathematics teachers. These participants were observed to fail to produce alternative solutions and mainly use rote-based strategies. In other words, they used a rule-based and procedure-based method. This finding overlaps with the studies of İymen (2012), Şengül and Gülbağcı (2012), Yapıcı (2013), Singh et al. (2019), and Yang and Sianturi (2019; 2021). Moreover, Chen et al. (2013) suggested that in the problem-solving process, teaching rules constantly and refraining from interpreting them stop improving the mathematical perspective and even prevent it. In this study, pre-service teachers with this perception could not develop different strategies.

The strategies developed depending on pre-service teachers' perception of number sense differed according to discipline. Pre-service preschool teachers did not use UMEO and UUEE components. On the other hand, pre-service classroom teachers mainly used strategies serving the MB component, and strategies serving other components were used very little. On the other hand, pre-service elementary mathematics teachers have developed strategies for each component, but their strategies mainly served MB.

Strategies used by pre-service teachers differed according to discipline. Of the 65 strategies serving UMSN, 25 were developed by pre-service preschool teachers, 22 by pre-service classroom teachers, and 18 by pre-service elementary mathematics teachers. Of the strategies serving UUERN, 44 were developed by pre-service classroom teachers and 10 by pre-service elementary mathematics teachers. Of the strategies serving UMEO, 20 were developed by pre-service classroom teachers and 21 by pre-service elementary mathematics teachers. Regarding UUEE, 40 strategies were developed, of which 14 were by pre-service classroom teachers and 26 by pre-service elementary mathematics teachers. Regarding FCCS, 3 strategies were developed by pre-service preschool teachers, 25 pre-service classroom teachers, and 32 pre-service elementary mathematics teachers. For MB, 58 strategies were developed by pre-service classroom teachers and 51 by pre-service elementary mathematics teachers.

Pre-service preschool teachers generally do not use the benchmark, operational fluency, and flexibility strategies. It is primarily due to the characteristics of the student age group, numbers being already taught by the family, the family's socioeconomic level, and the student's previous experiences. The strategies used by pre-service classroom teachers were expressed as learning with fun, the ability to make quick operations, lack of test anxiety, and the differences in the resources used.

Although there is no study on the components served by the developed strategies, there are separate studies involving components, and their results overlap with the study's findings as follows: Understanding the meaning and impact of the operations (UME0) - Purnomo et al., 2014; Yang, Li, and Lin, 2008; Yang and Lin, 2015; flexible computing

and counting strategies (FCCS) - Novita & Herman, 2021, the use of a reference point, that is, MB - CanYetkin Özdemir, 2020. However, no study indicated that pre-service preschool teachers do not use strategies serving MB, UMEO, and FCCS. Moreover, İymen (2012) stated that students had difficulties using benchmarks, which do not overlap with this study. Pre-service classroom and elementary mathematics teachers did not have any difficulties using benchmarks; they just could not name the method.

Pre-service teachers' understanding of number sense, a critical subject, can increase awareness of number sense components, and various strategies can be developed based on this awareness.

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