Pythagoras - Journal of the Association for Mathematics Education of South Africa
ISSN: (Online) 2223-7895, (Print) 1012-2346

The intersect of early numeracy, vocabulary, executive functions and logical reasoning in Grade R

Authors:
Hanrie S. Bezuidenhout[1]
Elizabeth Henning[1]

Affiliations:
[1]Department of Childhood Education, Faculty of Education, University of Johannesburg, Johannesburg, South Africa

Corresponding author:
Hanrie Bezuidenhout,
hsbezuidenhout@uj.ac.za

Dates:
Received: 16 Aug. 2021
Accepted: 06 May 2022
Published: 15 Sept. 2022

How to cite this article:
Bezuidenhout, H.S., & Henning, E. (2022). The intersect of early numeracy, vocabulary, executive functions and logical reasoning in Grade R. *Pythagoras*, *43*(1), a646. https://doi.org/10.4102/pythagoras.v43i1.646

The current quantitative study, a naturalistic field experiment, was conducted in a public primary school in Soweto, Johannesburg, with the objective to examine how children's achievement on four assessments at the beginning of Grade R, namely their numeracy, their mathematics-specific vocabulary, their executive functions, and their logical reasoning capabilities, predicted their performance on a numeracy assessment at the beginning of Grade 1. A purposive intact group of 59 participants was assessed at the beginning of their Grade R year and again when they entered Grade 1. The results of the study indicate that, apart from existing or prior numeracy knowledge at the beginning of Grade R, mathematics-specific vocabulary was the strongest predictor for numeracy attainment at the beginning of Grade 1. We suggest that early grade teachers consider young children's number concept development as a cognitive, developmental psychology phenomenon and that they help learners build a lexicon of mathematics-specific qualifiers in their teaching with words that represent concepts of, among others, space, position, comparison, inclusion, sequence and magnitude.

Keywords: Early numeracy; mathematics-specific vocabulary; number concept development; executive functions; logical reasoning; early grades pedagogy; Grade R.

Introduction

Grade R children's mathematics-specific vocabulary is a strong indicator of their achievement in Grade 1 numeracy (Purpura & Logan, 2015; Toll & Van Luit, 2014). Grade R numeracy also predicts Grade 1 achievement since mathematics concepts have been shown to develop hierarchically (Bezuidenhout, 2018; Clements & Sarama, 2014; Fritz, Ehlert, & Balzer, 2013). Executive functions (i.e. working memory, inhibitory control and cognitive flexibility) (Cragg, Keeble, Richardson, Roome, & Gilmore, 2017) and logical reasoning (Morsanyi & Szücs, 2015; Nunes et al., 2007) have also been shown to significantly contribute to number concept development. After identifying these four factors as critical contributors for Grade 1 numeracy from the literature, we designed an experiment to investigate predictive correlations between Grade R numeracy, mathematics-specific vocabulary, executive functions and logical reasoning, and numeracy attainment at the beginning of Grade 1.

We were specifically interested in the sample we have chosen because the school from which the sample was purposefully selected follows a custom-designed, dual language of instruction model: the children's home language (isiZulu or Sesotho) is their language of instruction in Grade R but in Grade 1 their tuition changes to a dual language modality when they learn mathematics through the medium of English, with some code-switching in class discussions (Henning, 2012). The sample was selected from a public school in Soweto, Johannesburg.

We propose the argument that the development of four constituents, namely numeracy, mathematics-specific vocabulary, executive functions and logical reasoning, during Grade R collectively prepares children to develop numeracy concepts in Grade 1. Specifically, in a context where the language of instruction in Grade 1 differs from that of Grade R, we view mathematics-specific vocabulary as a particularly important enabler for learning mathematics in Grade 1. Our data confirmed that, apart from Grade R numeracy, mathematics-specific vocabulary is the strongest predictor for Grade 1 numeracy achievement.

The study was undertaken because of the ongoing concern in South Africa about children's consistently weak performance in primary school mathematics (Department of Basic Education, 2014, 2017; Southern- and Eastern Africa Consortium for Monitoring Educational Quality, 2007, 2017; Trends in International Mathematics and Science Study, 2015, 2019). Gustafsson (2019) gives an overview of some of these results. Recognising that young children's poor attainment, and possible ensuing learning difficulties in early mathematics assessments, is a worldwide concern (Chinn, 2015), we acknowledge that it is a specifically serious concern in South Africa. One aspect of this is that in many schools in the country, children learn mathematics through the medium of English, which is not their home language. Some children learn mathematics in a dual language mix in the early grades, which may further compromise their learning (Henning, 2012).

It is not clear how children map their early mathematics concepts onto linguistic representations. Odic, Le Corre and Halberda (2015) investigated such mapping and found that language is used to 'point down' to approximate numbers before approximation does the obverse. We regard this as one of the important findings in recent research, indicating that semantic mapping cues conceptual understanding. This view is also proposed by Spelke (2017), in a proposal that natural language is the source of concepts more than cognitive evolution or culture. The argument that Dowker and Nuerk (2016) propose in this regard is that familiar vocabulary is crucial for forming linguistically named concepts, much as was argued by Vygotsky (1986), who proposed that there is a constant pattern of interaction between the development of concepts (such as number concepts) and the development of language (Kozulin, 1990). Dowker and Nuerk (2016) accentuate not only vocabulary, but also linguistic structures such as syntax, grammar and morphology. We propose that linguistic representation intersects with cognitive modelling. With this view in mind, we argue that young children rely on vocabulary that represents a concept as a 'semiotic mediation' tool (Henning, 2013; Vygotsky, 1986) for learning. As the data of our study suggest, many children struggle to link the English mathematics vocabulary that they encounter in Grade 1, after they had been initiated into the vocabulary of the Sesotho and isiZulu languages of their Grade R classrooms and their homes and communities.

Vygotsky (Kozulin, 1990) proposed that children engage in 'inner conversation' or 'inner speech' to reason logically about concepts. In this sense, there is thus a relation between the development of number concepts, language (also in the form of inner speech) and logical reasoning. The skill of being able to reason logically about properties of numbers and the relations between numbers is required for the development of numeracy skills (Nunes et al., 2007). We add to this argument by proposing that children may need to switch back and forth between their home language inner speech and their interpretive and expressive language in English when they encounter tuition in English. This may increase the cognitive load of the working memory and other executive functions.

The research question of the study is: What are the predictive associations between children's achievement on Grade R[1] assessments of numeracy, mathematics-specific vocabulary, executive functions and logical reasoning, and a numeracy assessment in Grade 1[2]?

1.Assessed in their home language: isiZulu or Sesotho.

2.Assessed in both their home language and in English.

Constituents of early number concept development

Early numeracy

The importance of early development of number concepts has been shown in many studies (Aunio & Räsänen, 2016; Balala, Areepattamannil, & Cairns, 2021; Clements & Sarama, 2014; Desoete, 2015). Research from these and other studies has shown that a variety of early numeracy skills predict overall later mathematics performance. In particular, counting (Sarnecka & Carey, 2008; Wynn, 1990), calculation, number line and magnitude comparison (LeFevre et al., 2010), ordinality (LeFevre et al., 2013), cardinality (Frye, Braisby, Lowe, Maroudas, & Nicholls, 1989; Sarnecka & Wright, 2013) and numeracy-related logic (Aunio & Niemivirta, 2010) have been shown to contribute to mathematical skills and concept development.

Fritz and her colleagues (Fritz, Ricken, Balzer, Willmes, & Leutner, 2012; Fritz et al., 2013) identified five levels of number concept development. While each level describes a specific level of development, comprising the concepts of, first, counting, and then ordinality, cardinality, part-part whole understanding and relationality of numbers, the conceptual levels collectively describe a continuum of number concept development. These levels informed the development of the assessment instrument we used to assess numeracy, namely the MARKO-D SA (mathematics and arithmetic competence diagnostic, as translated from German). This interview-based test was developed in Germany and translated into four South African languages. It has also been validated in South Africa (Henning et al., 2021; Bezuidenhout, Henning, Fitzpatrick, & Ragpot, 2019). The test items were designed to assess the specific concepts assigned to each of the five levels of number concept development.

Around the age of 2, children begin to distinguish number nouns from other parts of speech and realise that these words refer to quantities, although they are not yet able to connect a specific word to the corresponding quantity (Fritz et al. 2013). They learn the order of the 'count list' and use their fingers or objects to link number words to objects which they count in a one-to-one correspondence (level 1: counting). After learning the ordinal properties of numbers (level 2: ordinality), young children develop a sense that each number is not only part of a sequence, but also represents a specific quantity (level 3: cardinality). They learn that the quantity can be decomposed into a specific number of units, for example $5 = 1 + 1 + 1 + 1 + 1$, and that the quantity, or the set, can be broken into smaller sets, such as $5 = 3 + 2$ or $5 = 4 + 1$ (level 4: part-part-whole). This understanding brings about the realisation that numbers have relationships with each other, an important one of which is that the next number on the number line is always one more and that the one before is one less (level 5: relationality). Such combined understanding of the principles of counting, ordinality and cardinality makes it possible for children to determine the relationship between cardinal units.

Language for learning number concepts

Several studies have shown a relationship between early mathematics learning and language (Davidson, Eng, & Barner, 2012; Negen & Sarnecka, 2012; Purpura, Hume, Sims, & Lonigan, 2011; Yang, Dulay, McBride, & Cheung, 2021). An explanation offered by Dowker and Nuerk (2016) for this relationship is that frequent exposure to mathematics-specific vocabulary increases the chance of a child developing an understanding of the conceptual properties of words. Gopnik and Meltzoff (1997) argue that:

> [*A*]aspects of linguistic input can have quite striking effects on conceptual development. Children who hear language relevant to a particular conceptual problem are more likely to solve that problem than children who do not. (pp. 208–209)

Studies have also shown that both the quantity and quality of parents' (Gunderson & Levine, 2011; Levine & Bailargeon, 2016) and teachers' (Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006) 'number talk' influences the development of early number concepts.

One of the most serious challenges in this regard in South Africa is that even if children are exposed to mathematics-specific vocabulary during their early development years, the language of instruction in school may differ from the language in which they have mostly encountered initial mathematics-specific language. Apart from different vocabulary use, variations in semantic structures may also contribute to young children's confusion. Differing semantic structures include the inversion of the order of tens and units in some languages, such as *vyf en twintig* [directly translated as five and twenty] in Afrikaans. Children may also rely on transparency of number nouns themselves, which adds to conceptual clarity in some languages. For instance, *leshome le motso o mong* [eleven] in Sesotho, means 'ten and one' and *leshome le metso e mmedi* [twelve] means 'ten and two'. Although some words can become very long (like the isiZulu word for eight, *isishiyagolombili*), children can no longer apply the rule on which they have relied to learn number words when learning in a different language, like English, where there is less conceptual clarity. In English, 'eleven' and 'twelve' have no real connection to 10. This type of semantic representation may confuse a child.

Susan Carey (2009) introduced the notion of 'input analysers' as the 'evolutionary constructed' mental mechanisms that guarantee 'the relevant (mental) representations [*that*] refer to aspects of the environment that are important to survival' (p. 29). These *metaphorical* input analysers enable humans to represent sensory and perceptual entities in the world mentally. She proposes that for input to be mentally represented, various formats of input are 'filtered' by an innate mental input analyser. Although perceptual (seeing, hearing, tasting, feeling and smelling) and symbolised input (for example, words and notations that represent reality) contribute to the process of conceptual development, input analysers also enable humans to reason about concepts, such as mathematical concepts, on a representational level (Carey, 2009).

In this view, if children's linguistic input, such as the phonology of a language, its vocabulary and grammatical structure during early development years, is different to the linguistic input at school, it is likely that the input analyser will have difficulty filtering and thus making sense of linguistic input that does not match a child's existing filtering system. In the sample of this study children were exposed to their home language (isiZulu or Sesotho) in the preschool years, while the language of instruction in Grade 1 is English – which includes not only different vocabulary, but also different phonemes, morphemes, syntax and grammar as well as prosody. The tone, pauses, voice inflections and syllabic emphasis of languages differ. Much as Spelke (2017) and Odic et al. (2015) have proposed, our data suggest that there is a link between the fine-grained aspects of language input, such as the vocabulary, and early numeracy.

Executive functions: Manifested in classroom engagement

Increasingly, research findings suggest that the development of mathematics skills also relies on children's executive functions (Cragg & Gilmore, 2014; Prager, Sera, & Carlson, 2016). Some studies show that early numeracy and executive functions are correlated, and others indicate that executive functions are predictors for mathematic conceptual development (Blankson & Blair, 2016; Zaitchik, Iqbal, & Carey, 2014). These authors argue that the three components of executive functions, namely cognitive flexibility, inhibitory control and working memory, collectively influence conceptual development. Tasks such as problem-solving, reasoning and planning make use of a combination of the three executive functions (Diamond, 2013)[3].

Working memory enables children to monitor and code relevant information and revise information by replacing old information with new, appropriate information (Miyake et al., 2000). For instance, when the question "How many more is 5 than 3?" is presented orally or in written language, children must hold all the information of the question in their memory, while calculating the solution. Cognitive flexibility – also referred to as 'shifting' of mental sets, attention switching or task switching – allows one to shift between multiple tasks (Miyake et al., 2000) and to hold focus and refocus attention to relevant tasks (Fitzpatrick, 2014). For example, when a second question, "What is 3 plus 7?" follows the question "How many more is 5 than 3?", children must be able to switch from subtraction to addition. Inhibitory control allows children to deliberately inhibit responses to certain stimuli and choose more appropriate responses (Miyake et al., 2000). For instance, focusing one's attention on a different task or switching between tasks requires inhibition of the automatic inclination to continue with a previous task. Inhibitory control enables children to decide to change activities or to inhibit automated responses.

3.There are researchers who do not share this view, such as Clements, Sarama and Germeroth (2016) who published a review about causal connections between executive functions and mathematics learning.

Fitzpatrick and Pagani (2012) have established a link between children's executive functions and their classroom engagement actions and argue that classroom engagement can be used as 'placeholder measure' for executive functions. Classroom engagement includes tasks such as cooperation with other children, the ability to follow rules and instructions, listen attentively, work neatly and independently and complete work on time. Well-developed cognitive flexibility allows children to disengage and re-engage in classroom tasks, which, in turn, increases participation in classroom activities. Inhibitory skills, such as avoidance, effortful control of behaviour, emotional and social self-regulation, facilitate an increase in learning activities (Fitzpatrick & Pagani, 2012). By means of working memory children can briefly store, maintain and rehearse information, which can increase their engagement in cognitive activities.

Logical reasoning

Handley, Capon, Beveridge, Dennis and Evans (2004) and Morsanyi and Szüs (2015) have established a connection between logical reasoning and mathematical competence. Both mathematical tasks and logical reasoning involve complex cognitive processes which depend on the retrieval and application of normative rules and rely on abstract processes and symbolic representations (Morsanyi & Szüs, 2015). For instance, children should know that if two sets consist of the same number of objects, then the objects in one set are in one-to-one correspondence with those in the other. In terms of counting (level 1 of numeracy assessment in this study), if set A is in one-to-one correspondence with set B, and C is in one-to-one correspondence with A, then sets B and C are equal. In terms of ordinality (level 2 of the numeracy assessment), children can reason about the relations between numbers to compare and quantify values. For instance, if 8 is larger than 7, then it should also be larger than the first six numbers. Or, if 1 is smaller than 2, it should also be smaller than 3 and so forth. In terms of cardinality (level 3 of the numeracy assessment), children can rely on the inversion property of addition and subtraction to reason about quantities. For instance, 1 + 2 = 2 + 1. They may also use logical reasoning to decompose (level 4 of numeracy assessment) numbers when adding or subtracting. For instance, they can reason that 5 + 8 equals 5 + (5 + 3) and thereby know which decomposed values of 8 will assist them best in the addition task. In this study, we used the Revised Culture Fair Test (CFT-R) to operationalise logical reasoning. In this test, children are required to reason about differences and similarities and be able to classify and identify patterns.

Methods

Study design

In this naturalistic field experiment we studied how four competencies that were measured at the beginning of Grade R (i.e. numeracy, mathematics-specific vocabulary, executive functions and logical reasoning) predict numeracy after one year when the children enter Grade 1 (see Figure 1). Studying the naturalistic 'intervention' – namely learning during the Grade R year – allowed the researchers to investigate how these competencies develop in the 'real world' as opposed to a controlled environment.

The hypotheses of the study were:

$H_{1\text{-}0}$: There is no relationship between learners' Grade R numeracy (assessed in their home language) and their achievement in Grade 1 numeracy (assessed in their home language and in English).

H_1: A relationship exists between learners' Grade R numeracy (assessed in their home language) and their achievement in Grade 1 numeracy (assessed in their home language and in English).

$H_{2\text{-}0}$: There is no relationship between learners' Grade R mathematics-specific vocabulary (assessed in their home language) and their achievement in Grade 1 numeracy (assessed in their home language and in English).

H_2: A relationship exists between learners' Grade R mathematics-specific vocabulary (assessed in their home language) and their achievement in Grade 1 numeracy (assessed in their home language and in English).

$H_{3\text{-}0}$: There is no relationship between learners' Grade R logical reasoning (assessed in their home language) and their achievement in Grade 1 numeracy (assessed in their home language and in English).

H_3: A relationship exists between learners' Grade R logical reasoning (assessed in their home language) and their achievement in Grade 1 numeracy (assessed in their home language and in English).

$H_{4\text{-}0}$: There is no relationship between learners' Grade R executive functions (classroom engagement) (assessed in their home language) and their achievement in Grade 1 numeracy (assessed in their home language and in English).

H_4: A relationship exists between learners' Grade R executive functions (classroom engagement) (assessed in their home language) and their achievement in Grade 1 numeracy (assessed in their home language and in English).

Participants

At the beginning of their Grade R year, 65 Sesotho and isiZulu speaking children's early numeracy, mathematics-specific vocabulary, executive functions and logical reasoning were assessed in either Sesotho or isiZulu. At the beginning of Grade 1, 67 children's early numeracy was assessed, in both English and in the children's home language. Of this sample, 59 had come from the previous year's Grade R group. The sample (*n* = 59) was thus a purposive intact group. The school was selected because of its dual language of instruction

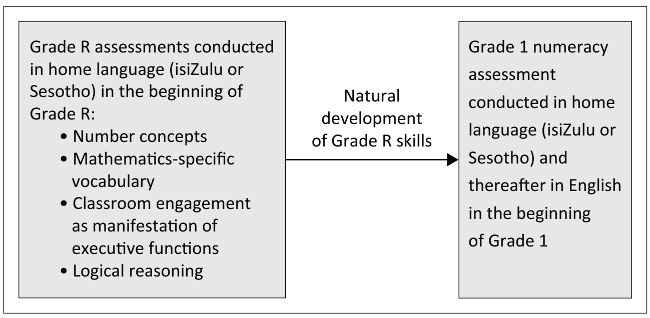

FIGURE 1: Study design.

model. The results of children who were not assessed either in Grade R or Grade 1 (due to being retained in Grade R or Grade 1) were included when data were imputed for a multiple regression analysis. 56% of the sample were male, 48% were isiZulu speaking and 52% Sesotho speaking. The average age of participating children was 6 years and 2 months in Grade R and 7 years and 2 months in Grade 1.

Test instruments

The South African version of the MARKO-D (Bezuidenhout et al., 2019) was used to assess early numeracy. To test knowledge of mathematics-specific vocabulary, we administered the custom-designed Meerkat Maths Language Test (MMLT), which consists of 26 items, assessing numerical language qualifiers (such as more, many, few, just as many), comparative language (such as smaller, taller, same size) and spatial language (such as in between, first, in front of). The MMLT is a five-minute interview-based test in which the items correspond with the concepts assessed in the MARKO-D SA. The children only needed to point to the correct picture, showing their understanding of a mathematics-specific word.

We did not wish to assess executive functions in an unnatural controlled environment (Bezuidenhout, 2018), and rather used a teacher inventory of classroom engagement to describe behavioural manifestations of executive functions. Fitzpatrick and Pagani (2012) and Aunio et al. (2019) argue that a behavioural scale of classroom engagement, administered by the teacher, reflects executive functioning aptly. For example, the successful completion of instructions relies on working memory, shifting between tasks and adapting to routines requires flexibility, and focusing attention on a particular task requires inhibition. The teacher inventory captures teacher ratings of self-regulated learning and productive work behaviour in the form of classroom engagement (Gioia, Isquith, Guy, & Kenworthy, 2012). Cronbach's alpha of 0.83 for the classroom engagement measurement of the current study was lower compared to 0.94 in a Canadian study (Fitzpatrick & Pagani, 2013). The scores correlated with the other instruments used in this study. For the assessment of logical reasoning, subtests 3 (similarities), 4 (complete the rows) and 5 (classification) of the CFT-R (Coppard, 2018) were used.

In terms of ethics, the school's board and parents gave written consent for children to participate in the research. All results were treated as confidential information. Participants could withdraw their consent any time during the project. Ethical clearance was obtained from the Faculty of Education Research Ethics Committee (ethical clearance no. 2017-053), University of Johannesburg.

Ethical considerations

The Board of the School and Ethics Committee of the University of Johannesburg approved this research (ethical clearance no. 2017-053). The authors certify that the study was performed in accordance with the ethical standards. All participants remain anonymous.

Results

An independent samples t-test revealed no significant gender differences on the assessments of early numeracy, mathematics-specific vocabulary, executive functions or logical reasoning. IsiZulu children outperformed the Sesotho children on early numeracy in Grade R (means = 20.55 vs 17.11, $p < 0.01$) and Grade 1 (means = 25.71 vs 21.93, $p < 0.01$), while there was no significant difference when tested in English. There were no significant tester effects.

Table 1 shows that early numeracy scores increased from Grade R to Grade 1 when children were tested in their home language in Grade 1 (means = 18.93 vs 23.86) but decreased when they were tested in English in Grade 1 (means = 18.93 vs 17.73). There was a statistically significant difference between English and home language early numeracy scores in Grade 1 (means = 23.86 vs 17.73).

Table 2 summarises bivariate correlations between predictors and outcome variables. Except for early numeracy in Grade R, mathematics-specific vocabulary had the strongest correlation with Grade 1 early numeracy (tested in English) (0.35, $p < 0.01$). Early numeracy in Grade R (home language) and Grade 1 (home language) showed a correlation of 0.4 ($p < 0.01$). However, Grade R mathematics-specific vocabulary, executive functions and logical reasoning were not significantly correlated with Grade 1 early numeracy scores when tested in their home language. Finally, English and home language early numeracy scores (in Grade 1) had a correlation of 0.28 ($p < 0.05$).

TABLE 1: Descriptive statistics for independent, dependent and control variables.

Variable	Possible total score	Mean	Standard deviation	Minimum	Maximum
Dependent variables (Grade 1)					
Early numeracy – English	47	17.73	5.90	7	41
Early numeracy –_home language	47	23.86	5.19	13	37
Independent variables (Grade R)					
Early numeracy	47	18.93	4.60	9	32
Mathematics-specific vocabulary	26	19.41	2.35	13	23
Logical reasoning	45	15.81	5.46	6	30
Independent categorical variable (Grade R)†					

N = 59.

†, Classroom engagement (executive functions): Highly engaged = 67%.

TABLE 2: Correlations between predictor and outcome variables.

Variable	1	2	3	4	5	6
Early numeracy						
Home language Grade 1	-	0.28*	0.4**	0.12	0.14	0.13
English Grade 1		-	0.27**	0.35**	0.28**	0.31**
Grade R predictors						
Home language numeracy Grade R			-	0.2*	0.24*	0.22
Mathematics-specific vocabulary				-	0.27**	0.36**
Logical reasoning					-	0.35**
Classroom engagement (executive functions)						-

Source: Bezuidenhout, H.S. (2018). Diagnostic test for number concept development during early childhood. *South African Journal of Childhood Education, 8*(1), 1–10. https://doi.org/10.4102/sajce.v8i1.584

Note: Correlations involving classroom engagement were conducted using Kendall's tau-B correlation coefficient.

*, $p < 0.05$; **, $p < 0.01$.

To determine associations between each predictor in Grade R and outcome variable in Grade 1, multiple regression analyses were calculated, while controlling for language of instruction. Because the Grade R predictor variables correlated only with the Grade 1 English early numeracy scores and not with the Grade 1 home language early numeracy scores, a regression model was only estimated for the English assessment. Six regressions, which examined associations between each of the predictors while controlling for language of instruction, were run in total on imputed data. In terms of model fit, all the regressions accounted for a significant proportion of the variance in the Grade 1 early numeracy scores.

To examine how each independent variable contributed to the variance in early numeracy, we ran two regression models. Firstly, we examined each predictor separately, controlling for language of instruction (given that the predictors are statistically and theoretically closely related). Of the individual predictors, early numeracy in Grade R was the strongest predictor of Grade 1 early numeracy ($\beta = 0.42$, $p < 0.05$) and mathematics-specific vocabulary the second strongest predictor ($\beta = 0.16$, $p < 0.05$).

In the second regression analysis, while still controlling for home language, Grade R numeracy was omitted as a predictor. In this model we examined both *concurrent* associations (between Grade R mathematics-specific vocabulary, executive functions and logical reasoning, and Grade R numeracy) and *prospective* associations (between Grade R mathematics-specific vocabulary, executive functions and logical reasoning, and Grade 1 numeracy). Mathematics-specific vocabulary, executive functions and logical reasoning significantly contributed to the variance in this second model. Table 3 shows concurrent and prospective associations between Grade R cognitive skills (mathematics-specific vocabulary, executive functions and logical reasoning) and numeracy in Grade R and Grade 1.

Mathematics-specific vocabulary, executive functions and logical reasoning were all significant predictors for early numeracy in Grade R and Grade 1. In terms of concurrent variance, 30% to 47% of the variance in Grade R numeracy (with logical reasoning explaining the most variance and executive functions the least) were explained by concurrent cognitive skills (mathematics-specific vocabulary, executive functions and logical reasoning). In terms of prospective associations, Grade R predicting skills explained 11% to 14% of the variance in Grade 1 numeracy with mathematics-specific vocabulary explaining the most variance and logical reasoning the least.

TABLE 3: Standardised regression coefficients depicting associations between cognitive skills and number concept development for Grade R and Grade 1.

Early numeracy	Grade R	Grade 1
Predictors		
Logical reasoning	0.57***	0.34*
R^2	0.47	0.11
Mathematics-specific vocabulary	0.39**	0.38**
R^2	0.32	0.14
Classroom engagement (executive functions)	0.37**	0.35**
R^2	0.30	0.12

Note: Models are adjusted for children's classroom (isiZulu vs Sesotho).
*, $p < 0.05$. **, $p < 0.01$. ***, $p < 0.001$

In summary, although Grade 1 children's early numeracy scores increased from Grade R to Grade 1, when tested in their home language, the scores decreased significantly when early numeracy was tested in English in Grade 1. There was also a statistically significant difference between the home language and English scores of early numeracy in Grade 1. Apart from Grade R numeracy, mathematics-specific language was the strongest predictor for Grade 1 numeracy, which indicates that children's knowledge of mathematics-specific vocabulary and language of instruction play vital roles in the development of young children's number concepts.

Based on the results presented, we conclude that H_1 was confirmed, while the other three hypotheses were only partly confirmed: relationships between all four independent variables (Grade R numeracy, mathematics-specific vocabulary, logical reasoning and executive functions) and the *English* assessment of numeracy in Grade 1 was confirmed, but in the sample of this study there exists only a relationship between one of the independent variables, namely Grade R numeracy, when numeracy was assessed in the children's *home language* in Grade 1.

Discussion

The results of this study show that children find it hard to integrate number concepts that have developed prior to learning English with mathematics vocabulary in English. This finding resonates with Vygotsky's theory of the intersect between pre-linguistic concepts and pre-conceptual language as a starting point for promoting conceptual change and language 'labels' (Kozulin, 1990; Vygotsky, 1986). According to this theory, language and concepts develop concurrently while one supports the other. As children develop vocabulary, they learn the conceptual properties of words and phrases such as *more*, *less*, *in front of*, *bigger*, *just as many* and also the number nouns, in tandem. Spelke (2017) has come to propose number nouns as the origin of numerical cognition. According to Levine and Baillargeon (2016) and Spelke, children align words to constructs they intuitively know or recognise because of words used in their environment. In this sense, language not only supports the development of number concepts, but mathematics-specific vocabulary itself can be seen as input for the development of number concepts and is filtered comfortably by the innate input analyser (Carey, 2009).

The young children in classrooms such as those in our study sample enter a multilingual 'maze' (Henning, 2012) in the first year of formal education, which may negatively influence their opportunity to develop number competency in the building-block years of mathematics progression (Clements & Sarama, 2014). Also, because English is a language with strong local social and economic currency, to which only a few children who come from 'township' areas are exposed prior to formal education, the participants in this study had not yet stored English mathematics vocabulary for fast memory retrieval. In order to make sense of mathematics in a Grade 1 class, children are required to not only align a new English label (word) to an already developed number concept, but also to its equivalent in their home language. This process could easily overload the working memory.

For this reason, we propose that Foundation Phase teachers' pedagogical content knowledge should include a precise understanding of the contribution of all levels of linguistic input (Dowker & Nuerk, 2016) for early numeracy. Further research should focus on ways to include intentional development of mathematics-specific vocabulary in preparation for formal mathematics education.

Our findings also indicated that there is an association between number concepts and other contributors, namely logical reasoning skills and executive functions. For this reason, we also propose that teachers should understand individual differences in young children. These differences include variations in executive functioning, classroom engagement and logical reasoning abilities. Dowker (2008) proposes that teachers need to know which specific skills contribute to number concept development and know how to strengthen each individual child by developing that specific skill.

Knowledge of contributing skills for the development of number concepts would, for instance, enable teachers to explain why the isiZulu speakers in this study outperformed Sesotho speakers on the Grade R and Grade 1 assessments of numeracy in the home language versions of the tests. By including an understanding of the levels of linguistic influences and knowledge of executive functions in their pedagogy, teachers will know that lexical and grammatical composition of number names (LeFevre, 2018), or longer number names, possibly overload the working memory.

In conclusion, the development of early numeracy, mathematics-specific language, executive functioning and logical reasoning during Grade R prepares children for formal education of number concepts in Grade 1. This study's findings emphasise that apart from the development and learning of number concepts in Grade R, mathematics-specific vocabulary is the most important enabler for learning mathematics in Grade 1. Specifically, in the context where the language of learning in Grade 1 differs from the language of learning in Grade R, the importance of developing mathematics-specific vocabulary in dual language mode during Grade R is highlighted. Further research should be conducted to find ways to include the intentional development of mathematics-specific vocabulary before children enter Grade 1 and ensure that they learn the terms in the language of future instruction, with bilingual scaffolding engineered by the teacher and the curriculum.

Acknowledgements

Competing interests

The authors declare that they have no financial or personal relationships that may have inappropriately influenced them in writing this article.

Authors' contributions

H.S.B. and E.H. authors contributed equally to the design and implementation of the research, to the analysis of the results and to the writing of the manuscript.

Funding information

This project was funded by a Global Excellence Stature 4.0 (GES) Postdoctoral Fellowship from the University of Johannesburg.

Data availability

Data is available from the authors on reasonable request.

Disclaimer

The authors declare that the work is their own and that it was written in their own words. All citations from literature are acknowledged in text and referenced. We agree that subject to the ownership of all intellectual property rights in this work, the approved version of this work may be published by the *Pythagoras* journal.

References

Aunio, P., Korhonen, J., Ragpot, L., Törmänen, M., Mononen, R., & Henning, E. (2019). Multi-factorial approach to early numeracy – The effects of cognitive skills, language factors and kindergarten attendance on early numeracy performance of South African first graders. *International Journal of Educational Research, 97*, 65–76. https://doi.org/10.1016/j.ijer.2019.06.011

Aunio, P., & Niemivirta, M. (2010). Predicting children's mathematical performance in grade one by early numeracy. *Learning and Individual Differences, 20*(5), 427–435. https://doi.org/10.1016/j.lindif.2010.06.003

Aunio, P., & Räsänen, P. (2016). Core numerical skills for learning mathematics in children aged five to eight years – A working model for educators. *European Early Childhood Education Research Journal, 24*(5), 684–704. https://doi.org/10.1080/1350293X.2014.996424

Balala, M.M.A., Areepattamannil, S., & Cairns, D. (2021). Investigating the associations of early numeracy activities and skills with mathematics dispositions, engagement, and achievement among fourth graders in the United Arab Emirates. *Large-scale Assessments in Education, 9*(1), 1–21. https://doi.org/10.1186/s40536-021-00106-4

Bezuidenhout, H.S. (2018). Diagnostic test for number concept development during early childhood. *South African Journal of Childhood Education, 8*(1), 1–10. https://doi.org/10.4102/sajce.v8i1.584

Bezuidenhout, H.S. (2019). *Input for young children's number concept development*. Doctoral thesis. Johannesburg: University of Johannesburg.

Bezuidenhout, H., Henning, E., Fitzpatrick, C., & Ragpot, L. (2019). Early mathematics vocabulary and number concept development. *Southern African Assoclafion for Research in Mathemafics, Science and Technology Educafion: Research for inclusive, relevant and equitable quality Mathemafics*, Science and Technology Educafion: Promofing research-based opportunity for all, p. 2.

Blankson, A.N., & Blair, C. (2016). Cognition and classroom quality as predictors of math achievement in the kindergarten year. *Learning and Instruction, 41*, 32–40. https://doi.org/10.1016/j.learninstruc.2015.09.004

Carey, S. (2009). *The origin of concepts*. Oxford: Oxford University Press.

Chinn, S. (2015). 'The Routledge international handbook of dyscalculia and mathematical learning difficulties', In S. Chinn (ed.), *The Routledge international Handbook of Dyscalculia and Mathematical Learning Difficulties* (p. 1–17). New York, NY: Routledge.

Clements, D.H., & Sarama, J. (2014). *Learning and teaching early math: The learning trajectories approach*. New York: Routledge.

Coppard, S.L.M. (2018). *Assessing the culture fairness of an intelligence test by adjusting the test times and pictorial examples: A pilot study with grade 2 learners in four Johannesburg schools*. Masters dissertation. Johannesburg: University of Johannesburg.

Cragg, L., & Gilmore, C. (2014). Skills underlying mathematics: The role of executive function in the development of mathematics proficiency. *Trends in Neuroscience and Education, 3*(2), 63–68. https://doi.org/10.1016/j.tine.2013.12.001

Cragg, L., Keeble, S., Richardson, S., Roome, H.E., & Gilmore, C., 2017, 'Direct and indirect influences of executive functions on mathematics achievement', *Cognition, 162*, 12–26. https://doi.org/10.1016/j.cognition.2017.01.014

Davidson, K., Eng, K., & Barner, D. (2012). Does learning to count involve a semantic induction? *Cognition, 123*(1), 162–173. https://doi.org/10.1016/j.cognition.2011.12.013

Department of Basic Education, 2014, *The Annual National Assessment of 2014: Diagnostic report foundation phase mathematics and home language*. Pretoria: Author.

Department of Basic Education, 2017, *The SACMEQ IV project in South Africa: A study of the conditions of schooling and the quality of education*. Pretoria. Retrieved from http://www.education.gov.za

Desoete, A. (2015). Predictive indicators for mathematical learning disabilities/ dyscalculia in kindergarten children. In S. Chinn (ed.), *The Routledge international handbook of dyscalculia and mathematical learning difficulties* (p. 90). New York: Routledge.

Diamond, A. (2013). Executive functions. *Annual Review of Psychology, 64*, 135–168. https://doi.org/10.1146/annurev-psych-113011-143750

Dowker, A. (2008). Individual differences in numerical abilities in pre-schoolers. *Developmental Science, 11*(5), 650–654. https://doi.org/10.1111/j.1467-7687.2008.00713.x

Dowker, A., & Nuerk, H.C. (2016). Linguistic influences on mathematics. *Frontiers in Psychology, 7*, 1035. https://doi.org/10.3389/fpsyg.2016.01035

Fitzpatrick, C. (2014). Bridging the gap between advantaged and disadvantaged children: Why should we be concerned with executive functions in the South African context? *South African Journal of Childhood Education, 4*(1), 156–166. https://doi.org/10.4102/sajce.v4i1.66

Fitzpatrick, C., & Pagani, L.S. (2012). Toddler working memory skills predict kindergarten school readiness. *Intelligence, 40*(2), 205–212. https://doi.org/10.1016/j.intell.2011.11.007

Fitzpatrick, C., & Pagani, L.S. (2013). Task-oriented kindergarten behavior pays off in later childhood. *Journal of Developmental & Behavioral Pediatrics, 34*(2), 94–101. https://doi.org/10.1097/DBP.0b013e31827a3779

Fritz, A., Ehlert, A., & Balzer, L. (2013). Development of mathematical concepts as basis for an elaborated mathematical understanding. *South African Journal of Childhood Education, 3*(1), 38–67. https://doi.org/10.4102/sajce.v3i1.31

Frye, D., Braisby, N., Lowe, J., Maroudas, C., & Nicholls, J. (1989). Young children's understanding of counting and cardinality. *Child Development* 60(5), 1158–1171. https://doi.org/10.2307/1130790

Gioia, G.A., Isquith, P.K., Guy, S.C., & Kenworthy, L. (2012). *Behavior rating inventory of executive function.* Retrieved from http://www4.parinc.com/Products/Product.aspx?ProductID=BRIEF

Gopnik, A., & Meltzoff, A.N. (1997). *Words, thoughts and theories.* Cambridge, MA: MIT Press.

Gunderson, E.A., & Levine, S.C. (2011). Some types of parent number talk count more than others: relations between parents' input and children's cardinal-number knowledge. *Developmental Science,* 14(5), 1021–1032. https://doi.org/10.1111/j.1467-7687.2011.01050.x

Gustafsson, M. (2019). *TIMSS, SACMEQ and PIRLS*. Retrieved from http://www.myemissions.co.za/Holistic%20view%202019%2001%2022.pdf

Handley, S.J., Capon, A., Beveridge, M., Dennis, I., & Evans, J.S.B. (2004). Working memory, inhibitory control and the development of children's reasoning. *Thinking & Reasoning, 10*(2), 175–195. https://doi.org/10.1080/13546780442000051

Henning, E. (2012). Learning concepts, language, and literacy in hybrid linguistic codes: The multilingual maze of urban grade 1 classrooms in South Africa. *Perspectives in Education, 30*(4),70–79.

Henning, E. (2013). Teachers' understanding of mathematical cognition in childhood: Towards a shift in pedagogical content knowledge? *Perspectives in Education, 31*(3), 139–154.

Henning, E., Balzer, L., Ehlert, A., & Fritz, A. (2021). Development of an instrument to assess early number concept development in four South African languages. *South African Journal of Education, 41*(4).

Klibanoff, R.S., Levine, S.C., Huttenlocher, J., Vasilyeva, M., & Hedges, L.V. (2006). Preschool children's mathematical knowledge: The effect of teacher "math talk." *Developmental Psychology, 42*(1), 59. https://doi.org/10.1037/0012-1649.42.1.59

Kozulin, A. (1990). *Vygotsky's psychology: A biography of ideas*. Cambridge, MA: Harvard University Press.

LeFevre, J. (2018, November 17). Three connections between language and math and their educational implications. Paper presented at Art & Science of Math Education Conference, University of Western Ontario, London: Canada.

LeFevre, J.A., Fast, L., Skwarchuk, S.L., Smith-Chant, B.L., Bisanz, J., Kamawar, D., & Penner-Wilger, M. (2010). Pathways to mathematics: Longitudinal predictors of performance. *Child Development, 81*(6), 1753–1767. https://doi.org/10.1111/j.1467-8624.2010.01508.x

LeFevre, J.A., Jimenez Lira, C., Sowinski, C., Cankaya, O., Kamawar, D., & Skwarchuk, S.L. (2013). Charting the role of the number line in mathematical development. *Frontiers in Psychology, 4*, 641. https://doi.org/10.3389/fpsyg.2013.00641

Levine, S.C., & Baillargeon, R. (2016). Different faces of language in numerical development. In: D. Barner & A.S. Baron (eds.), *Core Knowledge and Conceptual Change* (p. 127). Oxford: Oxford University Press. https://doi.org/10.1093/acprof:oso/9780190467630.003.0008

Miyake, A., Friedman, N.P., Emerson, M.J., Witzki, A.H., Howerter, A., & Wager, T.D. (2000). The unity and diversity of executive functions and their contributions to complex "frontal lobe" tasks: A latent variable analysis. *Cognitive Psychology, 41*(1), 49–100. https://doi.org/10.1006/cogp.1999.0734

Morsanyi, K., & Szücs, D. (2015). 'The link between mathematics and logical reasoning: Implications for research and education', In S. Chinn (ed.), *The Routledge international Handbook of Dyscalculia and Mathematical Learning Difficulties*, New York, NY: Routledge.

Negen, J., & Sarnecka, B.W. (2012). Number-concept acquisition and general vocabulary development. *Child Development, 83*(6), 2019–2027. https://doi.org/10.1111/j.1467-8624.2012.01815.x

Nunes, T., Bryant, P., Evans, D., Bell, D., Gardner, S., Gardner, A., & Carraher, J. (2007). The contribution of logical reasoning to the learning of mathematics in primary school. *British Journal of Developmental Psychology, 25*(1), 147–166. https://doi.org/10.1348/026151006X153127

Odic, D., Le Corre, M., & Halberda, J. (2015). Children's mapping between number words and the approximate number system. *Cognition, 138*, 102–121. https://doi.org/10.1016/j.cognition.2015.01.008

Prager, E.O., Sera, M.D., & Carlson, S.M. (2016). Executive function and magnitude skills in preschool children. *Journal of Experimental Child Psychology, 147*, 126–139. https://doi.org/10.1016/j.jecp.2016.01.002

Purpura, D.J., Hume, L.E., Sims, D.M., & Lonigan, C.J. (2011). Early literacy and early numeracy: The value of including early literacy skills in the prediction of numeracy development. *Journal of Experimental Child Psychology, 110*(4), 647–658. https://doi.org/10.1016/j.jecp.2011.07.004

Purpura, D.J., & Logan, J.A. (2015). The nonlinear relations of the approximate number system and mathematical language to early mathematics development. *Developmental Psychology, 51*(12), 1717. https://doi.org/10.1037/dev0000055

Sarnecka, B.W., & Carey, S. (2008). How counting represents number: What children must learn and when they learn it. *Cognition, 108*(3), 662–674. https://doi.org/10.1016/j.cognition.2008.05.007

Sarnecka, B.W., & Wright, C.E. (2013). The idea of an exact number: Children's understanding of cardinality and equinumerosity. *Cognitive Science, 37*(8), 1493–1506. https://doi.org/10.1111/cogs.12043

Southern and Eastern Africa Consortium for Monitoring Educational Quality. (2007). *The SACMEQ Project in South Africa: A study of the conditions of schooling and the quality of education.* Retrieved from http://www.sacmeq.org/reports

Southern- and Eastern Africa Consortium for Monitoring Educational Quality. (2017). *The SACMEQ IV Project in South Africa: A study of the condition of schooling and the quality of education*. Retrieved from http://www.sacmeq.org/reports

Spelke, E.S. (2017). Core knowledge, language, and number. *Language Learning and Development, 13*(2), 147–170. https://doi.org/10.1080/15475441.2016.1263572

Toll, S.W., & Van Luit, J.E. (2014). The developmental relationship between language and low early numeracy skills throughout kindergarten. *Exceptional Children, 81*(1), 64–78.

Trends in International Mathematics and Science Study (TIMSS). (2015). Retrieved from http://timssandpirls.bc.edu/

Trends in International Mathematics and Science Study (TIMSS). (2019). Retrieved from http://timssandpirls.bc.edu/

Vygotsky, L.S. (1986). *Thought and language* (rev. ed.). Cambridge, MA: Massachusetts Institute of Technology.

Wynn, K. (1990). Children's understanding of counting. *Cognition, 36*(2), 155–193. https://doi.org/10.1016/0010-0277(90)90003-3

Yang, X., Dulay, K.M., McBride, C., & Cheung, S.K. (2021). How do phonological awareness, rapid automatized naming, and vocabulary contribute to early numeracy and print knowledge of Filipino children? *Journal of Experimental Child Psychology, 209*, 105179. https://doi.org/10.1016/j.jecp.2021.105179

Zaitchik, D., Iqbal, Y., & Carey, S. (2014). The effect of executive function on biological reasoning in young children: An individual differences study. *Child Development, 85*(1), 160–175. https://doi.org/10.1111/cdev.12145