The Investigation of Concept Image towards Derivative Representation: A Case Study of Prospective Mathematics Teachers

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Abstract: Derivative concept is one of the essential studies in calculus, which is studied in teaching mathematics. Prospective mathematics teachers who have completed their studies and later become teachers will teach derivative concepts to their students at school. Therefore, knowledge of derivative concepts is vital in transforming knowledge to students. This study aimed to investigate concept images of prospective mathematics teachers on derivative representations. The research design in this study used a qualitative with case study approach. The participants were prospective mathematics teachers at a university in West Java, Indonesia (N=29). The research data was obtained from the test and clinical interview. The findings of this study show that the concept image of all participants on the derivative concept is still limited in function representation. Concerning the meaning of the derivative concept, most participants only view the derivative concept as a tool to solve procedural problems. It concluded that the representation of participants still did not support conceptual understanding of the derivative concept. It is the impact of the teaching design that given. Based on these findings, educators are expected to be able to improve the quality of teaching derivative concepts in the future by using various contexts or representations so that the concept image formed is more comprehensive to support conceptual understanding in learning of derivative concepts.

INTRODUCTION

Over the last few years, research on calculus has developed a lot, not only examining how students understand and thinking processes, but also examining the educator's point of view and effective teaching methods in the construction of knowledge (Thompson, et al., 2008; Park, 2015; Satianov, 2015; Dagan, M., et al., 2018; Villalobos-Camargo, 2021). Among other concepts in Calculus, derivative is considered a difficult concept because of its definition and requires understanding of other concepts such as functions, quotient differences, and limits (Thompson, 1994; Zandieh, 2000; Park, 2013; Figuero & Campuzano, 2013; Arnal-Palacián & Claros-Mellado, 2022). In fact, the derivative concept is a central concept that is very important to study and understand because it is a fundamental tool in various disciplines involving changes and variations in magnitude

(Vrancken & Engler, 2014; Fuentealba, 2019; Moru, 2020). The problem of understanding derivative concepts is still one of the biggest challenges for teaching mathematics at the university level, and a constant concern for higher education institutions (Ferrini-mundy & Graham, 1991; Bressoud, et al., 2015). The findings in previous studies add that the knowledge taught and knowledge learned about derivative concepts still determines the fundamentals, especially in understanding concepts and meanings of derivative concepts (Orton, 1983; Aspinwall, et al., 1997; Bezuidenhout, 1998; Sierpinska, 1992; Zandieh, 2000; Asiala, et al., 1997; Voskoglou, 2017). This finding is in line with the research conducted by Ferrini-Mundy and Graham (1994) which found the phenomenon that students can easily derive a function but cannot relate and interpret the results of its decline in other contexts. Therefore, Zandieh (2000) presented a framework to explore students' understanding of derivative concepts with multiple representations to obtain complete knowledge so that it can be used to solve various kinds of problems. NCTM (2000) suggests that the term representation refers to processes and products that should be viewed as essential elements to support an individual's understanding of mathematical concepts. The derivative concepts can be represented by graphically as the slope of the tangent line to a curve at a point or as the slope of the line a curve seems to approach under magnification, verbally as the instantaneous rate of change, physically as speed or velocity, and symbolically as the limit of the difference quotient (Borji et al., 2018; Huang, 2011; Tokgöz, 2012). Outline of the framework for exploring multiple representations of derivative concepts as presented by Zandieh (2000) is presented in Figure 1.

	Contexts				
	Graphical	Verbal	Paradigmatic Physical	Symbolic	Other
Process-object layer	Slope	Rate	Velocity	Difference Quotient	
Ratio					
Limit					
Function					

Figure 1: Outline of The Framework of Derivative Concept

That framework with multi-representation can be used as a measuring tool for practitioners to review several conditions including; each individual's understanding of the concepts agreed upon by the mathematics community, comparative understanding of concepts between one individual and another, efficiency of teaching strategies based on the introduction of various aspects of a concept, effectiveness of teaching practices with reference to the curriculum used, and evaluation of essential concepts that must be given through a set of teaching materials that have been planned in the curriculum. In line with Zandieh (2000), Giraldo, et all., (2003) provide a view of the derivative concept that is commonly used, in the context of functions and geometry, derivatives for functions $y = f(x)$ at the point a of the domain is described as the slope of the tangent to the graph of the function at point $(a, f(a))$. In addition, the derivative concept is also described as the rate of change of the function $f(x)$ to x. In a physical context, the derivative concept deals with the instantaneous velocity when an object is moving under the action of a constant force. As for the formal concept definition of derivatives $y = f(x)$ defined by Leibniz notation $f'(x) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$ or $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$. The limiting process is consolidated to an instantaneous rate of change, which can then be represented by $\frac{dy}{dx}$. The review is an attempt to organize teaching about derivative concepts to be more comprehensive and meaningful. Zandieh's statement is supported by several studies that focus on mental construction, where the results of these studies confirm that participants show different representations in learning derivative concepts (Borji et al., 2018; Huang, 2011; Tokgöz, 2012; Moru, 2020). According to Duval (2006) and Bressoud (2016), research related to the construction of cognitive structures refers to a review the notion of concept image and concept definition as a theoretical framework for analyzing research findings. This is based on Vinner & Dreyfus (1989) which states that understanding in mathematics more often uses concept images and concept definitions than formal concept definitions as mathematical concepts that are agreed upon and used in the community of mathematicians or scientists. Concept image and concept definitions are two mental entities to explore the extent to which individuals understand a mathematical concept (Vinner & Hershkowitz, 1980; Tall & Vinner, 1981). In principle, concept image and concept definition examine the differences between mathematical concepts as formally defined and the total cognitive representation of individuals associated with these concepts. Concept image is the total cognitive structure associated with the concept that contains all mental images, properties, and processes including words, symbols and pictures related to mathematical concepts. While the concept definition is part of the concept image that relates to individual definitions in the form of words to explain or interpret a concept more specifically (Vinner & Hershkowitz, 1980; Vinner, 1983; EMS, 2014).

In Indonesia, several studies that have been conducted on understanding derivative concepts at the school and university level show that students are accustomed to solving routine problems without being able to provide conceptual meaning based on connections between ideas such as gradients, functions, limits, and continuity (Prihandhika, 2018; Mufidah, 2019; Desfitri, 2016; Destiniar, et al., 2021). This condition has the potential to cause difficulties in the learning process where students are unable to determine solutions to problems that are different from the contexts that have been given previously (Prihandhika, 2020; Nurwahyu & Tinungki, 2020). Tall (1996) said that the obstacles experienced by students in learning can be caused by weak concept images and concept definitions. Edwards & Ward (2004) argues that the formation of a concept image in an individual's cognitive structure can occur because of memorizing a formal concept definition without going through a process of meaning to the concept so that the concept image shown by an individual through a concept definition may not be relevant to the existing formal concept definitions. To minimize potential difficulties to solve the conceptual problem and strengthen the relevance between concept image and formal concept definition in teaching derivative concepts, practitioners should pay special attention to mastery of concepts, plans for essential concepts to be taught, and teaching strategies to be provided (Sbaragli, et all., 2011; Park, 2013). Based on the background and literature study that has been presented, this study aims to investigate the representation of the derivative concept based on the participant's concept image concept. Therefore, the research questions that are the focus of this research are as follows, how are participant's representations in the derivative concept based on concept image?

METHOD

In this study, research design used qualitative with case study approach to describing and analyzing the phenomena, attitudes, beliefs, perceptions, and thoughts of participants based on their own life and experiences by holistically and more deeply (Creswell, 2015). The theory of concept image used in this case study to investigate whether participants' understanding of derivative concepts shows the same phenomenon as some previous studies which stated that students' understanding of derivative concepts at the school and university level is still in the context of procedural understanding to solve routine problems without knowing some representations used in understanding derivative concepts (Prihandhika, 2018; Mufidah, 2019; Desfitri, 2016; Destiniar, et al., 2021). The Participants in this study were prospective mathematics teachers attending a bachelor's program (N=29) at a university in West Java, Indonesia. Participants were determined using a purposive technique based on certain criteria (Creswell, 2015). The criteria for participants in this study are students who have obtained a differential calculus course in the first semester of lectures. The research data is generated through the provision of a test in the form of a description of four questions about the concept of derivative with reference to indicators including: 1) Participants can derive a function; 2) Participants can understand the gradient of the tangent line as the first derivative of the function; 3) Participants can understand the derivative as instantaneous speed in the field of physics; 4) Participants can understand the derivative as an gradient of function approximation. The preparation of question indicators refers to the framework for derivative concepts presented by Zandieh (2000). The questions given to participants are as follows:

1. Show how you perform a one-time derivation of a function on $f(x) = x^2 + x - 4$?
2. It is known that the line g intersects the curve f at points A (2,4) and point B (3,9).
   a. Find the slope of the line AB and determine the function of the line g!
   b. Find the equation of the tangent line at point A on the curve f with $f(x) = x^2$
3. If a bullet from a firearm is fired into the air with an angle of elevation θ so that the distance from the origin to the point after t second is $(-t^2 + 12t)$ meter. Then specify:
   a. Average speed over time interval $3.00 \leq t \leq 5.00$ second
   b. Instantaneous speed at t = 4.00 second
   c. The maximum height that the bullet can reach
4. Based on the table below, show the comparison results of $\frac{f(1)-f(x)}{1-x}$ which is the gradient of the secant line with an approximation of 1 to the function $f(x) = x^2$ and determine the derivative $f(x)$ for $x = 1$

x	0,59	0,99	0,999	1,001	1,01	1,1
$\frac{f(1)-f(x)}{1-x}$						

The test results were then confirmed by clinical interviews to find out the participants' thinking and meaning of derivative concepts with a sequence of easy questions to difficult questions (Hunting, 1997). The research data that has been obtained through tests and interviews are then analyzed with three streams of activities carried out simultaneously, including reducing and presenting data, and drawing conclusions (Mayer, 2015). In the process of drawing conclusions, a theoretical framework concept image is used to interpret the meaning of research findings related to the representation of derivative concepts that exist in the cognitive structure of participants (Tall & Vinner, 1981; Vinner, 1983). The research design used facilitates researchers to investigate participants' thought processes more deeply to find out various kinds of meanings and representations based on the concept image and concept definitions possessed by each participant about the derivative concept. The concept image is personal and will continue to grow depending on the knowledge and experience gained by everyone in teaching. Therefore, the concept image between one individual and another individual can show various outputs. In this study, participants' concept images can be seen and analyzed based on concept definitions indicated by mental pictures, procedures, properties which explained in answering questions given by researchers through assignments and clinical interviews (Nurwahyu, 2020), presented in Table 1.

Component of Concept Image	*Descriptions*
Mental Pictures	*All the information imagined by the participants in the cognitive structure based on the experience that has been obtained for further use in explaining the concept and designing procedures to the solving problems given.*
Procedures	*All the syntax chosen by the participants in translating the mental picture to solve the problem.*
Properties	*All axioms, definitions, lemmas, theorems, formulas, or mathematical rules used in the process of explaining concepts and solving problems.*

Table 1: Component of Concept Image

In addition, the research design also supports the interpretation of various phenomena found during the study. However, the various findings that have been obtained cannot be generalized to a wider population scale. This is because prospective mathematics teachers who are participants in the study with prospective mathematics teachers in other places may have different conditions. Therefore, the research carried out is specific and limited and the results only apply to the place that is the subject of the study.

RESULTS

In this section, a description of the participants' answers to each question is presented. A total of 10 participants gave answers related to the derivative concept. The answers are then grouped based on the type of procedures and properties used. Furthermore, the representations used by participants in solving problems categorized using the Zandieh (2000) framework.

Participants' Answer of Questions 1

The first question was asked with the aim of investigating participants' concept image when they were asked to derive a function. The working activity on the first question shows that all participants carry out the process of decreasing function $f(x) = x^2 + x - 4$ use the formula $f'(x) = n.x^{n-1}$ with result $2x + 1$. When confirmed, in the process of decreasing function, all participants imagined two properties in their mental picture, namely the power rule $f'(x) = n.x^{n-1}$ and limit form $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$. However, the limit form is not chosen as a procedure to obtain a solution because it has a complicated solution step when compared to the power rule of the derivative concept. Based on the tendency of the procedure used to derive a function, it was found that the participant's concept image about the form $f'(x) = n.x^{n-1}$ is very dominant. The following is the answer of the participants shown in Figure 2.

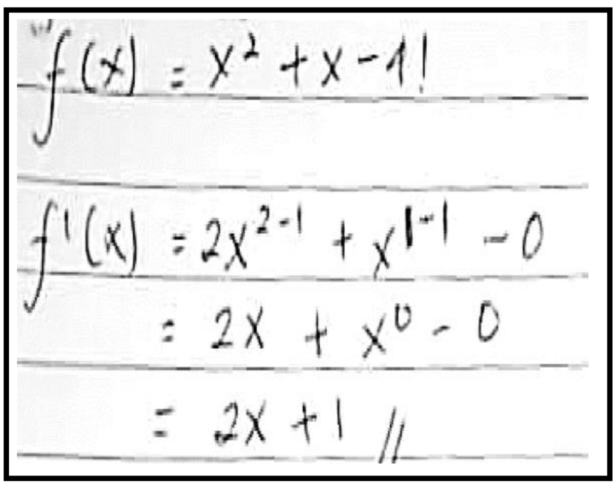


Figure 2: Participants' Answer with $f'(x) = n.x^{n-1}$

Participants' Answer of Questions 2

The second question aims to investigate the participant's concept image about concepts related to derivative concepts, especially those that build geometric representations including the concepts of functions, line equations, gradients, and tangent lines. As many as 43% of participants were able to define procedures to try to solve problems and only 26% of participants were able to carry out procedures to obtain solutions to all the questions given by using properties relevant to the formal concept definition, such as the formula for determining gradients by using $m = \frac{\Delta y}{\Delta x}$ or $m = \frac{y_2 - y_1}{x_2 - x_1}$, the formula to find the equation of a line based on one of the coordinates is to use $y - y_1 = m\,(x - x_1)$ and the equation of a line based on two coordinates by using $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$, and find the equation of the tangent to the curve of $f(x) = x^2$ by using the formula $y - y_1 = m\,(x - x_1)$ for $m = f'(x)$. The following is the answer of the participants shown in Figure 3.

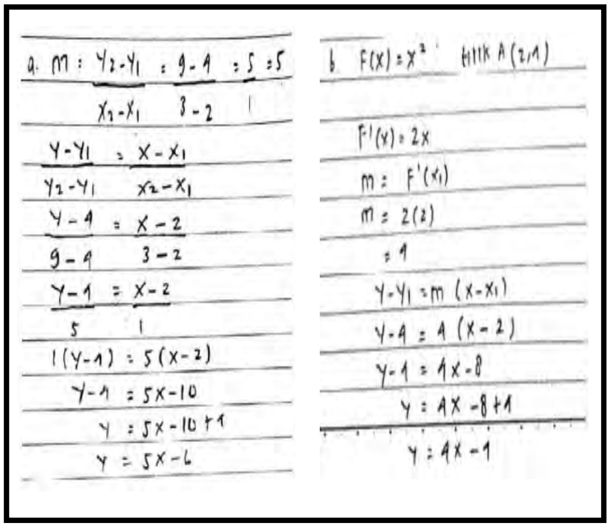
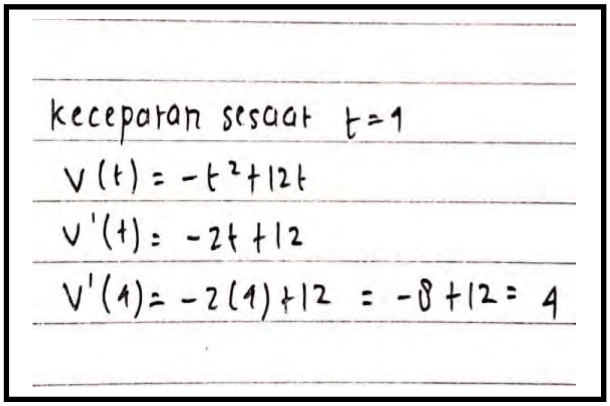


Figure 3: Participants' used slope of tangent

Participants' Answer of Questions 3

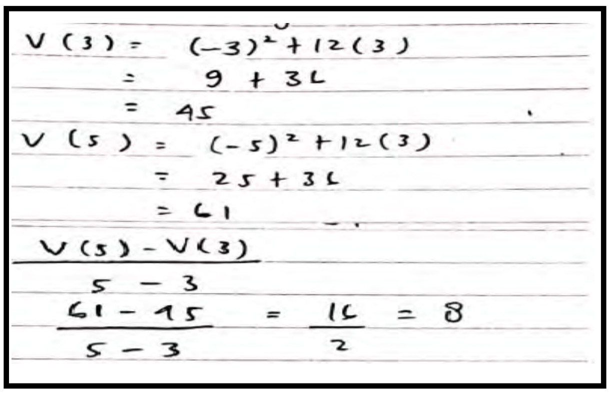
The third question aims to investigate the participant's concept image about the derivative concept of the paradigmatic physical representation, especially velocity. There are three aspects that want to be seen in the third question, namely the understanding and meaning of participants about the average speed at a certain time interval, the instantaneous speed at t seconds, and the maximum height or stationary point on the function $s(t) = -t^2 + 12t$. Regarding the question of average speed, as many as 8% of participants used procedures to solve problems with properties $v = \frac{s}{t}$ or $v = \frac{\Delta s}{\Delta t}$ the formula for finding velocity by comparing changes in distance and time. Furthermore, regarding the question of instantaneous velocity, as many as 13% of participants used a procedure to answer questions with the derivative concept property, namely velocity on $t = 4$. In the last aspect, as many as 20% of participants used properties $s'(t) = 0$ to determine the maximum height or stationary point of an object. Meanwhile, as many as 54% of participants directly substituted the function as a procedure to solve problems without paying attention to the conditions contained in each question.

Based on these findings, it is generally known that the concept image of participants still has a large gap with the formal concept definition so that the percentage of participants who are able to solve derivative concept problems on the paradigmatic physical representation is still below 50%. The following is the answer from the participants shown in the Figure 4, Figure 5, and Figure 6.



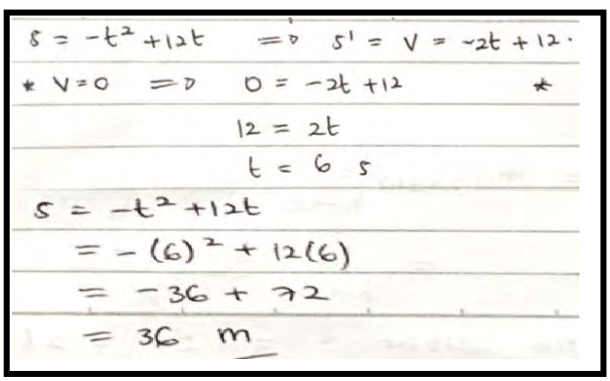
kecepatan sesaat t=1
v(t) = -t²+12t
v'(t): -2t+12
v'(1)= -2(1)+12 = -8+12= 4

Figure 4: Participants' used average of velocity



V (3) = (-3)² + 12(3)
= 9 + 36
= 45
V (5) = (-5)² + 12(3)
= 25 + 36
= 61
V(5) - V(3) / 5 - 3
61 - 45 / 5 - 3 = 16 / 2 = 8

Figure 5: Participants used instantaneous



s = -t² + 12t => s' = v = -2t + 12.
* v=0 => 0 = -2t + 12 *
12 = 2t
t = 6 s
s = -t² + 12t
= -(6)² + 12(6)
= -36 + 72
= 36 m

Figure 6: Participants used maximum height

Participants' Answer of Questions 4

The fourth question aims to investigate the participant's concept image about the derivative concept based on the $\frac{\Delta y}{\Delta x}$ approximation of a function. These calculations can help construct an understanding of the derivative concept based on the relationship between the limit concept and the ratio concept. However, based on observations, none of the participants can determine the right procedures based on the properties that can be imagined in the mental picture to solve the given problem. These findings indicate that the verbal representation of the rate of change in studying derivative concepts in class has not been recognized by the participants.

Participants' Response of Clinical Interview

The implementation of clinical interviews aims to re-confirm the participant's concept image through the concept definition presented during the interview, explore the thought process, and examine the participants' meaning of the various representations used in the derivative concept. According to Hunting (1997), clinical interviews can be a tool to observe, assess and interpret individual mathematical behavior. Clinical interviews generally consist of open-ended questions and assignments with the aim of knowing how to respond and the underlying thought processes of individuals (Heng & Sudarshan, 2013).

Clinical interviews were conducted with all participants to investigate the concept image of the derived concept representation. However, the presentation of the interview script in this section is only shown from two participants who represent the tendency and uniqueness of the answers related to the derivative concept. This is due to the limitations of the concept image shown by most of the participants so that researchers have difficulty exploring deeper than the results of clinical interviews conducted. In the interview script, R is the researcher, P1 is the first participant and P2 is the second participant. The following are the results of clinical interviews with P1 and P2 which are shown in dialogue below.

R: can you explain, what is a derivative concept?
P1: derivative concept is a concept to reduce the exponent of a function, for example I have a function $f(x) = x^2$, then the derivative of the function is $2x$.
R: how do you get $2x$?
P1: with using $f'(x) = nx^{n-1}$, if function of $f(x) = x^2$ then the derivative is $f'(x) = 2x^{2-1} = 2x$.
R: did you use the same method to determine the derivative of $f(x) = x^2 + x - 4$?
P1: yes, that's right, I used the same way to answer question 1, the answer is $2x + 1$. Each term in the function is derived, the derivative of x^2 is $2x$, the derivative of x is 1 because $x^0 = 1$. While the derivative of 4 as a constant is 0.
R: is there another way you can think of?
P1: yes, by using the formula $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ but the process is long.
(Clinical interview between R and P1)

R: what is derivative concept in your understanding?

P2: the derivative is concerned with the derivation of the function denoted by $f'(x)$ *or* $\frac{dy}{dx}$.

R: what does this notation mean?

P2: on notation $f(x)$*, the number of accents indicates the number of decreases in the function.* $f'(x)$ *means that the function is derived once,* $f'(x)$ *down twice, and so on. The meaning of notation* $\frac{dy}{dx}$ *is the derivative of y was taken with respect to* x.

R: is there any other notation you know?

P2: no, I only know those two notations.

R: any other ideas you can share about the derivative concept? P2: wait, to determine the derivative of a function using the formula $n.x^{n-1}$ *or* $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$.

(Clinical interview between R and P2)

DISCUSSIONS

Based on the results of clinical interviews with P1 and P2, it was found that the concept image based on the concept definition conveyed was very dominant in verbal representation, especially in the process-object layer of function. P1 and P2 explain that the derivative concept is a concept to derive a function with the formula $f'(x) = n.x^{n-1}$. P1 and P2 also have properties about the formula $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ on their mental picture. However, P2 confirms that it cannot use those properties as a procedure to derive a function. After the researcher reconfirmed the mental pictures of P1 and P2 through follow-up questions about the meaning of the derivative concept, P1 conveyed the concept definition of the derivative concept by using a paradigmatic physical representation of the process-object layer of ratio for the context of velocity. P1 explains that the derivative concept can be used to solve instantaneous velocity problems in physics. Meanwhile, P2 presented a concept definition of derivative concept by using a graphical representation of the process-object layer limit for the slope context. P2 explains that the formula $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ is obtained from the concept of the slope of the tangent line and the formula $f'(x) = n.x^{n-1}$ is obtained based on the elaboration of the previous formula. P2 also conveys some of the notations used in the derivative concept. The following is a representation of P1 and P2 which is shown in Table 2.

Process-object layer	*Representations*				
	Graphical Slope	*Verbal Rate*	*Paradigmatic Physical Velocity*	*Symbolic Difference Quotient*	Other
Ratio			○		
Limit	○	○			
Function		●			

Table 2: Derivative representations of P1 and P2

In Table 2, referring to the framework presented by Zandieh (2000), the black close circle indicates the dominance of the representation that is understood and used by the participants in solving derivative concept problems. The close circle is in the verbal representation which shows that the participant has a mental picture of the formula $f'(x) = n.x^{n-1}$ as the main property to determine procedures for solving various problems in the derivative of function. Meanwhile, the white open circle indicates a conceptually incomplete understanding of the representation described through the concept definition. The open circles are in graphical representations, and paradigmatic physical representations. The open circle shows that participants have a mental picture of the formula $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ as the rate and slope of the tangent line and formula $\lim_{h \to 0} \frac{\Delta y}{\Delta x}$ as a procedure to solve problems on velocity.

Based on the results of the answers to the questions given and the results of clinical interviews, it is known that participants experience epistemological barriers when obtaining derivative concept problems that require an understanding of graphical representations, paradigmatic physical representations, and symbolic representations. The following shows the interplay between concept image and concept definition in Figure 7 to analyze the thought processes that cause difficulties experienced by participants in solving the problems in question 4.

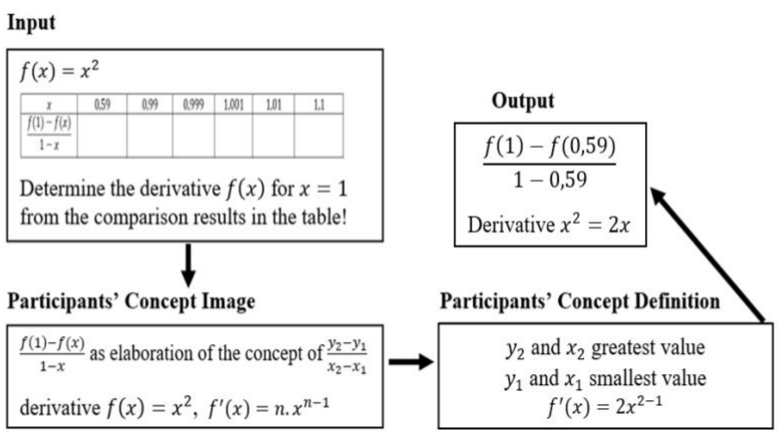


Figure 7: Participants' interplay between concept image and concept definition

Based on the interplay between concept image and concept definition of the participants who tried to answer question 4, it was found that they tried to recall the mental picture of the property difference quotient related to the information contained in the question. However, when asked to determine the derivative of $f(x) = x^2$ for $x = 1$, they again chose the procedure $f'(x) = n.x^{n-1}$ in solving the problem. The procedure is not relevant to the information contained in the question. This procedure was chosen because the participants had difficulty in determining the result $\frac{f(1)-f(x)}{1-x}$ of $f(x) = x^2$ for each x. As a result, the concept definition presented and the output answers given by the participants were not as expected. This may be due to the condition of the participant's concept image regarding verbal representation and symbolic representation that has not been formed in a meaningful way so that the mental picture of $f'(x) = n.x^{n-1}$ appears as a very dominant concept definition in understanding derivative concepts.

CONCLUSIONS

The findings of the study showed that the verbal representation in the context of the rate with the process-object layer of function was very dominant in the participants' understanding of the derivative concept. The properties of the formula $f'(x) = n.x^{n-1}$ are so embedded in the mental picture to determine procedural problem-solving procedures. This condition triggers a difficulty for participants in solving other problems, especially on conceptual problems. Based on the results of the answers to the questions given, only a few of the participants were able to determine the completion procedures relevant to the formal concept definition for questions involving graphical representation in the slope context and paradigmatic physical representation in the velocity context. In addition, the results of clinical interviews show that participants' meaning of the derivative concept is still limited to tools to reduce the number of powers of a function. These findings indicate that the participants' concept image regarding multiple representation in the derivative concept has not been fully formed. The use of multiple representations for teaching derivatives in the calculus domain is highly emphasized as a way to develop individual understanding (Orton, 1983; Aspinwall, et al., 1997; Bezuidenhout, 1998; Sierpinska, 1992; Zandieh, 2000; Asiala, et al., 1997; Voskoglou, 2017). According to Zandieh (2000), the concept of derivative can be represented with graphically as the slope of the tangent line to a curve at a point or as the slope of the line a curve seems to approach under magnification, verbally as the instantaneous rate of change, physically as speed or velocity, and symbolically as the limit of the difference quotient.

The dominance of one of the representations in understanding the derivative concept that triggers the difficulties can be caused by teaching that is less comprehensive in explaining a concept being taught. This statement is reinforced by Figueroa & Campuzano (2012) who said that in a first course of calculus, teachers and students generally did not examine the definition of the derivative as a tangent line obtained from the previous equation so that students had difficulties when solving problems involving graphical representations. In addition, Borji & Alamolhodaei (2016) also conveyed research results showing that calculus teaching in their country tends to emphasize symbolic representations and ignore graphic representations. Apparently, other studies from various countries also show a similar phenomenon (Park, 2015; Moru, 2020). The impact of teaching that emphasizes one representation causes most individuals to be highly skilled in solving routine problems, but have difficulty solving cross-context problems that require conceptual understanding, especially about the correlation between the slope of the tangent to the difference quotient boundary, and the application of derivative concepts in various contextual aspects such as velocity in physics and marginal value in economics (Baker, et al., 2000; Moru, 2020).

The findings of this study using qualitative methods based on case study approach with the aim of examining the phenomenon specifically about the concept image and meaning of the participants, where in this study were prospective students of mathematics teachers at one of the universities in Indonesia, showing the same results. in line with previous research. It seems that problems in teaching and understanding about derivative concepts, as previously stated, are still a common phenomenon in many places. Therefore, a review of the formation of the concept image and participants' meaning of the multiple representations used to understand the derivative concept is a strategic step to obtain alternative solutions to problems that are often found in research. According to Tall and Vinner (1981), there are several possible individual responses in the process of forming a concept image, including replacing the concept image that has been formed with a new information or knowledge, maintaining the old concept image, or separately use both concept images to understand the formal concept definition. Based on the results of this study, the complexity of the concept image owned by the individual strongly supports a comprehensive and meaningful understanding of a concept being studied. Therefore, special attention is needed from educators on teaching practices at the school and university level to involve various contexts or representations in the process of transforming derivative concept knowledge in order to form or improve individual concept images for the better in the future.

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