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Developing Reflective Competence in Preservice Teachers by Analysing Textbook Lessons: The Case of Proportionality

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Abstract: Teachers must often analyze and select the educational resources they consider relevant for their students, being textbooks one of the curricular materials of preferential use. A textbook lesson shows the instructional process planned by the author to promote the learning of a given content by potential students, so it is essential that the teacher is competent to analyze and assess what happens in this process. Reflection on the relevance of a lesson provides didactic-mathematical knowledge to guide the teacher in making decisions about the management of the text. This paper describes the design, implementation, and results of a training action with 45 preservice teachers, oriented to the development of reflective competence through the analysis of the didactic suitability of a lesson on direct proportionality. In the initial evaluations of the lesson made by the preservice teachers, we found some features of didactic suitability indicators in different components. However, the reflections elaborated by the participants are vague or ambiguous. The results of the implementation show an evolution in reflective competence on the part of most of the preservice teachers, who were able to make a detailed assessment by correctly applying suitability criteria, mainly in the cognitive-affective and instructional dimensions. The participants recognise the importance of the training received to guide their reflection on teaching practice, which they need to complement with content-specific didactic-mathematical knowledge to achieve adequate competence in analysing instructional resources.

INTRODUCTION

Various approaches in teacher training propose reflection on teaching practice as a fundamental competence for professional development and the improvement of teaching (Gellert et al., 2013; Ramos-Rodríguez et al., 2017). Developing reflective competence requires adopting conceptual and methodological frameworks that allow us to address this objective, such as the Lesson Study (Fernández and Yoshida, 2004), Professional noticing (Fortuny and Rodríguez, 2012; Llinares, 2012; Mason, 2016) or the competence of didactic analysis in the Ontosemiotic Approach (OSA) to knowledge and mathematical instruction (Godino et al., 2019; Godino et al., 2017). Within the latter framework, the importance of designing and implementing formative actions to promote, among others, the competence of didactic suitability analysis is highlighted. This competence is

aimed at global reflection on teaching practice, its assessment and management for its progressive improvement. The didactic suitability criteria show consensus on what a good mathematics teaching should be like, so they act implicitly as regularities in the discourse of teachers when they have not yet received training on the use of this construct as a guide for their reflection (Breda et al., 2018; Breda et al., 2021; Hummes et al., 2019). However, it is noted that teachers need specific tools and training to direct their attention to the multiple and intertwined factors that affect teaching and learning processes (Seckel and Font, 2020; Sun and van Es, 2015). This has led in recent years to a great deal of research in the field of teacher education using the didactical suitability theory, its components and indicators (Godino, 2013) to organise teachers' systematic reflection on their own or others' practice, develop the competence to evaluate planned or implemented instructional processes, and make informed decisions for improvement (Breda et al., 2018; Burgos et al., 2020; Burgos et al., 2018; Castillo et al., 2021a; Esqué and Breda, 2021; Font et al., 2018; Giacomone et al., 2018; Hummes et al., 2019; Morales-López and Araya-Román, 2020; Pino-Fan et al., 2013).

Teacher guides, textbooks and digital resources are essential tools for teachers that serve as a link between the notions set out in the intended curriculum and the very different and complex world of the classroom (Valverde et al., 2002). Teachers interpret and serve as mediators of the content included in the lessons of the texts they use, so they should have the necessary knowledge and skills to make appropriate use of these resources, considering the needs of their students (Kim, 2007; Lloyd, 2002). Carrying out a critical analysis to support the management of textbooks is a professional teaching task that can be difficult and requires specific training (Beyer and Davis, 2012; Godino et al., 2017; Nicol and Crespo, 2006; Shawer, 2017).

This work is part of a general research focused on the study and development of formative strategies to qualify preservice primary (Castillo and Burgos, 2022) and secondary (Castillo et al., 2021a; 2021b; 2022a) schoolteachers in the critical and constructive analysis of mathematics textbook lessons. The results of these previous interventions showed that participants did not justify their judgements and revealed gaps in didactical-mathematical knowledge that prevented them from adequately interpreting some of the suitability indicators. However, even when the assessment of the didactical suitability indicators was not accurate, the application of the guide for the analysis of textbook lessons helped participants to reflect on the overall suitability of the lesson (Castillo et al., 2021a; Castillo and Burgos, 2022).

In line with these previous experiences, but considering the results obtained for improving the design of the new experimentation cycle, we address the following research question:

> *Does methodology based on the didactical suitability construct help preservice teachers to develop critical and constructive analysis of mathematics textbook lessons?*

To respond to this question, we analyse how reflective competence evolves in preservice primary school teachers, using specifically designed guides based on the facets, components, and criteria of didactical suitability (Godino, 2013), as an instrument to assist their reflection. In this new cycle,

a first phase of initial exploration was envisaged, to detect the participants' prior conceptions about the features they consider adequate in a textbook lesson, and to be able to assess more reliably how the training allows them to develop their reflective competence. In addition, the analysis of didactical suitability is requested across the lesson as a whole. To reinforce reflection and justification by the participants, the instrument is improved so that the assessment of the degree of compliance of the suitability indicators is both qualitative and quantitative. Finally, we analyse in this new intervention whether the formative experience influences preservice teachers' beliefs about what aspects are important in a textbook lesson.

As in these previous experiences, we focus on proportionality for several reasons. Firstly, due to the importance of this content in primary and secondary school curricula. Secondly, because proportionality does not usually receive adequate treatment in school mathematics textbooks at this stage (Ahl, 2016; Burgos, Castillo et al., 2020; Shield and Dole, 2013). Indeed, most textbooks emphasise the rote learning of routines such as the rule of three and avoid arguing about the conditions that allow this procedure to be applied when solving a given problem, which prevents the development of an adequate proportional reasoning (Fernández and Llinares, 2011; Lamon, 2007; Riley, 2010). Thirdly, due to the difficulties in teaching proportionality-related concepts shown by both pre-service and in-service teachers (Ben-Chaim et al., 2012; Berk et al., 2009; Buforn et al., 2018; Van Dooren et al., 2008). These deficiencies can be diagnosed and corrected through reflection on the instructional processes envisaged in textbook lessons, thereby generating teacher learning (Nicol and Crespo, 2006; Remillard and Kim, 2017).

FRAMEWORK

In the following, we present the theoretical-methodological notions of the OSA fundamental to our research: the theory of didactical suitability, which connects descriptive-explanatory didactics with effective intervention in the classroom (Godino, 2013), and the model of categories of mathematics teacher knowledge and competences assumed in this framework (Godino et al., 2017).

Didactical suitability theory

The Theory of Didactical Suitability arises within the framework of OSA, from the need for a theoretical-methodological tool to guide teachers in decision-making in the phases of design, implementation, and evaluation of teaching practice (Godino, 2013). This tool can be applied to examine (partial and global) aspects of the teaching and learning process, for example, to the analysis of curricular programmes, of the planning or implementation of a didactic sequence, or of manuals or textbook lessons, among others.

When the teacher plans an instructional process on a mathematical object (e.g., proportionality) for students at a given educational level (e.g., sixth grade of primary education), he/she must first

delimit what this object represents for mathematical and didactic institutions. To do so, he/she will refer to what experts consider to be the operational and discursive practices inherent to the object whose instruction is being pursued. With all this, the teacher will determine the system of mathematical practices that we designate as the *institutional meaning of reference* of the object (by extension, of the mathematical content). On the other hand, the *intended institutional meaning* is given by the system of practices that are planned on a mathematical object for a certain instructional process. Thus, considering a lesson in a textbook as a potential or planned instructional process about a certain object, the intended institutional meaning is described by means of the sequence of mathematical and didactic practices proposed by the author for the study of the subject in question. Finally, the system of practices that the student manifests in the resolution of the mathematical tasks in which the object is involved determines the *personal meaning* achieved by the student[1].

The didactical suitability of an instructional process is understood as the degree to which this process (or a part of it) meets certain characteristics that allow it to be qualified as optimal or adequate to ensure the adaptation between the personal meanings achieved by students (learning) and the intended or implemented institutional meanings (teaching), taking into account the circumstances and available resources (environment) (Breda et al., 2017; Godino et al., 2016). It involves the coherent and systemic articulation of the six facets or dimensions (Godino et al., 2007): epistemic, ecological, cognitive, affective, interactional, and mediational.

A mathematics instruction process has a higher degree of *epistemic suitability* if the intended or implemented institutional meanings represent well the reference meaning (Breda et al., 2017; Godino, 2013). This requires the presence and interconnection of diverse partial meanings of the corresponding content (Godino et al., 2017) through the inclusion of a representative and well-articulated sample of problem situations; that multiple representations are involved; that the definitions, procedures and propositions fundamental to the topic are clearly and adequately presented; and that the proposed tasks allow students different ways of approaching them and require them to interpret, generalise and justify the solutions, among other aspects.

The degree to which the contents and their development correspond to the curricular guidelines, and how they are related to other disciplinary contents defines its *ecological suitability*.

Cognitive suitability refers to the degree to which the intended (implemented) meanings are in the learners' potential development zone, as well as the proximity of the personal meanings achieved to the intended (implemented) meanings. An adequate degree of cognitive suitability requires that the contents presented in the instructional process have a manageable difficulty for the educational level at which it is aimed, and that the proposed situations respond to different levels of difficulty.

[1] OSA assumes that learning involves the appropriation of the intended institutional meanings by students through participation in the community of practice generated in the classroom.

It is also important to promote the use of different resolution strategies and that students are warned of possible difficulties and errors.

Affective suitability is related to factors that depend on both the institution and the learner. A high degree of affective suitability requires the existence of motivational elements (illustrations, humour, etc.), and the selection of situations responding to learners' interests and allowing them to assess the usefulness of the content. In addition, it involves promoting attitudes of perseverance and responsibility towards mathematics, in particular the flexibility to explore mathematical ideas and alternative methods of problem solving.

Mediational suitability involves the availability and adequacy of material resources, and that the sequencing of content and activities is appropriate (in particular, sufficient time is devoted to the content more difficult to understand). The degree to which the modes of interaction allow for the identification and resolution of conflicts of meaning and foster autonomy in learning defines the *interactional suitability* of the teaching and learning process.

For each of these facets, systems of components and general empirical indicators are developed as a guide for analysis, providing criteria for the progressive improvement of teaching and learning processes (Breda et al., 2018; Godino, 2013). These suitability indicators should be enriched and adapted according to the specific mathematical content to be taught (Breda et al., 2017), but also to the type of instructional process. Thus, in Castillo et al. (2022b), Godino's (2013) system of components and indicators of didactic suitability is reviewed and particularised to develop a Mathematics Textbook Lesson Analysis Guide (TLAG-Mathematics for short) as a resource to lead reflection on the instructional processes planned in textbook lessons. Later, in Castillo et al. (2022c), the TLAG-Mathematics is adapted to the topic of proportionality, which leads to the generation of a new guide (TLAG-proportionality for short) in which explicit indicators are introduced for this content that fundamentally affect the epistemic, cognitive, and instructional (interactional-mediational) facets. These indicators are based on an exhaustive theoretical review of research results and expert judgments assumed by the academic community (Breda et al., 2017) in relation to the teaching and learning of proportionality.

Teacher's didactical-mathematical knowledge and competence model

In the Didactic-Mathematical Knowledge and Competences (DMKC) model proposed by the OSA (Godino et al., 2017), it is considered that the two key competences of mathematics teachers are the *mathematical competence* and the *competence of didactic analysis and intervention*. The latter consists, in essence, of "designing, applying and assessing their own and others' learning sequences, using didactic analysis techniques and quality criteria, to establish cycles of planning, implementation, assessment and proposals for improvement" (Breda et al., 2017, p. 1897). This global competence of analysis and didactic intervention of the mathematics teacher is articulated by means of five sub-competences, associated with conceptual and methodological tools of the OSA: *competence of global meaning analysi*s (identification of types of problem-situations and

practices involved in their resolution); *competence of ontosemiotic analysis of practices* (recognition of the network of objects and processes involved in practices); *competence of management of didactic configurations and trajectories* (identification of the sequence of interaction patterns between teacher, student, content and resources); *competence of normative analysis* (recognition of the network of norms and meta-norms that condition and support the instructional process); *competence of didactical suitability analysis* (assessment of the instructional process and identification of potential improvements). A detailed description of all these sub-competences can be found in Godino et al. (2017).

Previous works describe the results of interventions with preservice primary (Castillo and Burgos, 2022) and secondary school teachers (Castillo et al., 2021a; 2021b; 2022a), aimed at developing the competence of didactical suitability analysing of the planned instructional process in proportionality textbook lessons by applying the TLAG-proportionality. The results of both interventions showed that participants found it difficult to rigorously assess cognitive, affective (especially in relation to beliefs), interactional and mediational indicators.

METHOD

A methodology characteristic of design research (Cobb et al., 2003) is applied, based on the planning, implementation, and retrospective analysis of an intervention, in a real classroom context. In addition, content analysis (Cohen et al., 2011) is used to examine the response protocols of preservice teachers who took part in the formative experience.

Research context and participants

As part of the research, a first research cycle had been implemented as a pilot test in 2020 with a group of third-year Primary Education students (Castillo and Burgos, 2022). The second cycle (described here) was implemented in 2021 with 45 students also in the third year of the Primary Education degree. In both cases, the training experience takes place within the framework of the course Design and Development of the Primary Mathematics Curriculum (sixth semester), at a Spanish university. In previous courses of the degree, preservice teachers receive specific training on epistemic aspects (mathematical contents that are part of primary education), cognitive (mathematical learning, errors and difficulties), instructional (tasks and activities, materials and resources) and curricular aspects, so that by the time the experience takes place, participants are expected to be able to put into practice the knowledge acquired to design and evaluate didactical units in any subject of primary mathematics. In addition, this course specifically contemplates the use and analysis of textbooks as a resource in the mathematics classroom, as well as the evaluation of teaching and learning processes. Two lessons are given twice a week: one theoretical lesson (two hours long) in a large group and another practical class, in reduced groups, working in teams (of four or five members).

In this paper we analyse the information collected in this second cycle from the observer/researcher's notes and the written responses of the preservice primary school teachers (PPTs onwards) to the assessment task proposed at the end of the course.

Design and implementation

The intervention is organised in four phases that include different didactic resources and moments of individual, group, and final evaluation work.

Initial exploration: What do PPTs value in a textbook lesson?

PPTs are asked, as a voluntary activity prior to the formative session on textbook analysis, to carefully read the lesson "Proportionality and Percentages" by González et al. (2015) and answer the following questions through the Moodle platform of the course:

a) What aspects do you consider most important in a mathematics textbook lesson?
b) What did you think of the lesson you have just read? What positive features would you highlight? What negative characteristics do you observe?

The aim of this diagnostic task is to detect the participants' prior beliefs and conceptions about the features they consider appropriate in a textbook lesson and to involve them in a first reflection on the assessment of these features in a specific unit devoted to proportionality. This will allow us to assess how their competence in didactic reflection is evolving.

Introduction to the didactic analysis of mathematics textbook lessons

The training on the didactic analysis of textbook lessons is developed with the PPTs through two class sessions (each lasting two hours)[2]. In the first session, the textbook lesson analysis is presented as a means of identifying potentially conflicting elements, from the point of view of: (a) the mathematical knowledge students are expected to achieve, (b) the prior knowledge they require to understand the lesson, (c) the instructional process it proposes. Next, a methodology for analysing textbook lessons based on the OSA tools is described.

1. General description of the lesson and division into didactic configurations (elementary units of analysis).
2. Onto semiotic analysis. For each of the didactic configurations into which the lesson is divided, it consists of
    a) detailing the mathematical practices proposed,
    b) identifying the mathematical objects involved in them,
    c) describing the main mathematical processes.

[2] Due to the suspension of face-to-face classes due to the pandemic during the implementation of the experience, these sessions were delivered virtually through the Google Meet platform.

3. Assessment of the didactical suitability of the lesson in the epistemic-ecological, cognitive-affective, and instructional (interactional-mediational) dimensions.

The onto semiotic analysis of a lesson is exemplified using the text "Percentages and proportionality" from Ferrero et al.'s (2015) textbook for the sixth year of primary school. After this first training session, the corresponding practice session of the course (work in teams) is devoted to carrying out the first part of the analysis (general description and onto semiotic analysis of the different configurations) of Gonzalez et al.'s (2015) proportionality lesson for sixth year of primary school.

Didactical suitability as a tool for reflection

In the second formative session, didactical suitability is presented as a global criterion for assessing an intended, planned or implemented instructional process (or part of it), which requires both the analysis of previous practices, objects, and processes and the didactical-mathematical knowledge of that content (in our case, proportionality)[3]. Finally, the TLAG-Mathematics is introduced as an instrument to guide the analysis of textbook lessons, highlighting the need to adapt it to the specific content of the lesson by incorporating didactic-mathematical knowledge on that topic (in our case, proportionality).

Assessment of the didactical suitability analysis competence

After the second training session, the PPTs work individually on the analysis of the didactical suitability of the González et al. (2005) proportionality lesson. It is the same lesson considered in the collaborative work session to carry out the analysis of practices, objects, and processes.

Students are provided with the tables that make up the TLAG-proportionality (adapted from Castillo et al., 2022c) with the components, subcomponents, and indicators of suitability in the following facets: epistemic-ecological (content and adaptation of the lesson to the curricular guidelines), cognitive-affective (learning, attitudes and interests), instructional (material resources, interactions that are promoted in the tasks and their sequencing, etc.). The indicators measure the degree of maximum suitability in each component, so that the lesson will be more suitable with respect to a component to the extent that the corresponding indicators are met in a greater number of configurations.

On this occasion, considering the results of the first research cycle[4], the lesson is considered globally and the suitability in each didactic configuration of the lesson is not analysed.

[3] Given that during the course the PPTs were required to develop a teaching unit on a specific topic (different for each regular work team) as a main learning objective of the course, the teacher used the content of proportionality to exemplify the type of epistemic-ecological, cognitive and instructional analysis that guides the development of the teaching units.

[4] In previous experiences, participants were asked to decompose the text by units of analysis, applying TLAG-proportionality to each of them. The prospective teachers did not relate some configurations to others and found the breakdown to be excessive (Castillo et al., 2021a;2021b).

Furthermore, to organise the assessment of the lesson, two columns were added to the right of the indicators in the TLAG-proportionality tables. In the first column, PPTs must include a numerical assessment of the degree of compliance with the indicator: 0 (the indicator is not met), 1 (it is partially or sometimes met), 2 (it is fully met); in the second column they give the necessary justifications for the score assigned. Then, considering what has been observed through the assessment of the indicators, the PPTs must make a reasoned judgement on the didactical suitability of the lesson (low, medium, or high) in each of the facets.

RESULTS

To show the progress achieved in reflective competence, in this section, we confront the pre-training (initial descriptions of the characteristics that a good textbook should meet, and prior assessment of the lesson) and post-training (application of the TLAG-proportionality and reasoned judgement of the didactical suitability) analyses of the lesson. We also checked whether they their beliefs about the characteristics that a "good mathematics textbook lesson" should have, changed after the training experience.

Initial meanings of the characteristics that a good textbook lesson should have

When analysing the PPTs' reflections on the aspects they consider most important in a mathematics textbook lesson, it is possible to identify descriptions that can be associated with indicators of the different components of didactical suitability included in the TLAG-Mathematics. Of the 45 PPTs, 30 made some reference to epistemic-ecological aspects, 37 to cognitive-affective and 29 to instructional characteristics. Table 1 shows the TLAG-Mathematics indicators most frequently referred to in each dimension and component. In the category “other”, we include those references to indicators with a low frequency (less than 4), of which the most common are cited.

Components	Indicator	Freq.
Epistemic-ecological dimension		**40**
Problems	A representative and an articulated sample of problem-situations that allows the contextualization, exercising, amplification, and application of mathematical knowledge	9
Languages	A wide repertoire of representations is used to model mathematical problems, analysing the relevance and potential of each type of representation.	7
Concepts	The fundamental concepts of the subject are presented in a clear and correct way and are adapted to the educational level to which they are addressed.	5
Adaptation to the curriculum	The objectives, contents, their development, and evaluation correspond to the curricular guidelines	7

Intra/Interdisciplinary connections	The contents are related to other intra- and interdisciplinary contents (cross-cutting themes, history of mathematics, …).	5
Others	Procedures are adequately justified, the student is encouraged to justify, content is presented without error	7
	Cognitive-affective dimension	**85**
Individual differences	Expansion and reinforcement activities are included.	10
Increasing difficulty	Situations with different difficulty levels are foreseen.	9
Evaluation	Evaluation and self-evaluation instruments are proposed.	8
Emotions	Tasks and the contents involved are of interest to students. There are motivating elements.	23
Values	The student is encouraged to value the accuracy and usefulness of mathematics in daily and professional life.	14
Others	Necessary previous knowledge is considered, the intended contents are of manageable difficulty, alternative methods of problem solving are encouraged.	21
	Instructional dimension	**39**
Author-student interaction	The author makes an adequate (clear and well-organized) presentation of the topic, emphasizes the key concepts	18
Students interaction	Tasks are proposed that encourage dialogue, communication and debate among students.	5
Sequencing	The content and activity sequencing is adequate, devoting sufficient space to the contents which are more difficult to understand.	6
Others	The use of manipulative materials is promoted, the use of visualisations to introduce concepts, etc. is encouraged.	10

Table 1: Key aspects and frequency (Freq.) highlighted by PPTs in a textbook lesson for each dimension

As it can be seen in Table 1, the cognitive-affective dimension is the one with the highest number of references to indicators.

In the epistemic aspect, the PPTs consider it important that the lesson includes multiple examples and sufficient and varied problems ("sufficient number of activities and focused on the most important", PPT20). They also stress the importance of "clear and simple" language (PPT28) and that the text should be accompanied by " enough and useful" images (PPT20) to "explain the contents" (PPT16), always "related to the exercises" (PPT28). They also mention that the contents should be explained in a clear way ("The content is very important. It is essential that a lesson provides clear explanations so that students can understand it", PPT12). In the ecological aspect, they emphasise the importance of considering the curricular guidelines ("they must be in line with the contents appearing in the primary education curriculum", PPT21) and of considering transversal (and "socio-cultural aspects", PPT29) themes, something with which the PPTs are familiar in the course in which the experience was developed.

In the cognitive-affective aspect, more than half of the appraisals have to do with the interest and usefulness of the contents and tasks proposed to the students, as well as with the inclusion of motivational elements. For example, PPT26 considers it

> relevant that it contains activities related to the daily life and personal environment of the students, so that they can observe the usefulness of mathematics in a real context [...], that the activities and tasks proposed in the textbook are dynamic and attractive for the students, to encourage their motivation.

To a lesser extent, some PPTs appreciate that the lesson includes activities that serve as a review and self-evaluation ("a recap of the lesson as a self-evaluation prior to the exam", PPT3) and that the progression in learning difficulties is considered, advancing "from the simplest to the most complex" (PPT43).

Finally, most of the PPTs' reflections on the instructional aspect mention the attractive design and appropriate presentation of the content, which they tend to interpret as an explanation accompanied by solved examples and illustrative drawings. They also consider the distribution of content to be important ("the order and sequencing of both activities and content, which, for me, should go from the general to the particular", PPT33) and that "activities that encourage cooperative and group work" (PPT28).

Initial analysis of the textbook lesson on proportionality

For each of the dimensions, epistemic-ecological, cognitive-affective, and instructional, we include in Tables 2, 3 and 4, respectively, the types of characteristics and their frequencies when they are valued as positive (PV) or negative (NV) by the PPTs. These are classified by means of the TLAG-proportionality indicators to which they relate, according to the components of each dimension. As before, in the category “other”, we collect those references to indicators with low frequency (less than 4).

Component	Indicator	PV	NV
Problems	A representative and articulated sample of tasks is presented to contextualise and apply proportionality and percentages.	12	8
	Mental arithmetic situations involving proportional reasoning are proposed.	9	-
Languages	The language level is appropriate for the students.	8	-
	Different types of representation (graphical, symbolic, tables of values, manipulative, etc.) are used to model proportionality situations.	7	-
Concepts	The fundamental concepts of proportionality for the corresponding educational level are presented clearly and correctly.	-	6
Others		6	6
Total		42	20

Table 2: Aspects positively or negatively valued by the PPTs in the epistemic-ecological dimension

Of the 45 PPTs, 24 indicate positively, and 17 negatively, some feature in the epistemic-ecological dimension. The PPTs positively value the great variety of exercises in each configuration of the unit, the inclusion of mental arithmetic activities (something they had not reflected on in the first task), and the appropriateness of the language and especially the use of graphs and the proportionality table as a means of representation ("explains the relationship between two magnitudes through a proportionality table, a resource that I find easy to apply with the students", PPT5). In a lesser extent (category "other" in Table 2, PV), they refer to the fact that students are encouraged to create problems ("they are asked to invent and formulate problems themselves", PPT12) or to the alignment of the objectives and contents with the curriculum. Besides, the PPTs highlight as shortcomings in the lesson aspects related to the problem component (also the most positively valued in this facet). Most of them consider that "too many problems" are proposed or that these are not appropriate or are decontextualised. They also find deficiencies in the definition of fundamental concepts. For example, PPT25 states "as a negative aspect, I would highlight that not all the concepts that can help the student to understand the content, for example, proportional magnitudes, are present". A minority (category "other" in Table 2, NV) mention that procedures are not presented correctly or that various types of reasoning are not used in the justification of the statements.

Component	Indicator	PV	NV
Previous knowledge	The necessary previous knowledge (fractions and their equivalence, magnitudes and their measurement) is covered.	-	4
	The intended contents are of manageable difficulty.	7	4
Individual differences	Expansion and reinforcement activities are included.	20	-
Increasing dificulty	Situations with different difficulty levels are foreseen (with integers and non-integers, divisibility relations between quantities, ...).	7	-
	Warns of possible conceptual or procedural errors: illusion of linearity, assuming necessary conditions as sufficient, using erroneous additive strategies.	9	-
Evaluation	Evaluation and self-evaluation instruments are proposed.	9	-
Emotions	Tasks and the contents involved are of interest to students. There are motivating elements.	23	6
Values	The student is encouraged to value the accuracy and usefulness of mathematics in daily and professional life.	13	4
Others		6	9
Total		94	27

Table 3: Aspects positively or negatively valued by the PPTs in the cognitive-affective dimension

In the cognitive-affective dimension, 43 out of the 45 PPTs indicated some positive and 21 some negative characteristics. In the first case, they highlight as potentialities of the lesson those qualities that they considered desirable in the previous task, i.e., the presence of reinforcement (through the final review) and extension activities (in the "test yourself"), and that the proposed situations include elements to capture the students' interest ("use of a character as a common thread

to motivate students", PPT19; "workshops and challenges appear to motivate students", PPT20). In addition, they think that including problems related to shopping and sales can show the usefulness of proportionality in everyday situations. They also point out that the "keep in mind" labels included in the lesson can help students with their possible difficulties. Infrequently (category "other" in Table 3, PV) they rate positively that students are supported in learning ("help students in doing the activities", PPT39) or that practical or realistic work is encouraged.

When they express limitations in this facet, they also focus on the degree of interest of the tasks for the students ("the situations are out of the context of the student", PPT26; "problems not very realistic for the context of an 11-year-old child", PPT42) or their usefulness ("not practical exercises, not applicable to the students' daily life", PPT38). Although not frequent, some more specific reflections on the learning of the mathematical content covered in the lesson are observed. For example, in relation to prior knowledge, some PPTs consider that there is a lack of an adequate relationship with fractions or that the conversion of units in magnitudes should be remembered. They also mention that the degree of difficulty of the content on percentages or scales is not adequate ("the content which, from my point of view, can cause the greatest difficulty in learning is the calculation of the percentage of a quantity and scales", PPT23).

Component	Indicator	VP	VN
Author-student interaction	The author makes an adequate (clear and well-organized) presentation of the topic, emphasizes the key concepts	14	7
Students' interaction	Tasks are proposed that encourage dialogue, communication, and debate among students.	4	7
Sequencing	The content and activity sequencing is adequate, devoting sufficient space to the contents which are more difficult to understand.	7	-
Material resources	The use of manipulative (Scali meter, pantograph, proportion compass) and informatics (Geogebra, Excel, ...) materials is encouraged.	4	4
Others		-	6
Total		29	24

Table 4: Aspects positively or negatively valued by the PPTs in the instructional dimension

Of the 45 PPTs, 24 indicated some positive and 18 negative characteristics in relation to the instructional dimension. As when they specified the characteristics, they considered essential in a textbook lesson, the clarity and appropriateness of the presentation of the lesson was again the most prominent positive aspect ("clear and simple explanations", PPT12; "presents an explanation with examples in each section", PPT18). The relevance of the sequencing also stands out ("the contents are structured in an appropriate way, going from the simplest to the most complex", PPT29). Regarding the weaknesses of the lesson, the PPTs also focus their attention on the interactive aspects, where in this case they highlight that the explanation is insufficient or could be improved, and that the fundamental concepts are not included ("the theory and its explanation are insufficient", PPT30; "key concepts are not explained", PPT15). They also consider the degree of interaction between students to be inadequate, mentioning, for example, that an expository

method is followed which constrains collaborative work ("It did not seem to me to be a lesson far removed from traditional mathematics teaching", PPT17).

By comparing the results in Tables 1, 2, 3 and 4, we observe a greater number of indications of what is considered appropriate or inappropriate in the planned teaching process when a specific textbook lesson is proposed as a specific means of reflection. It is interesting to note that all those characteristics that they identify as positive or negative in the cognitive-affective or instructional dimensions, had been pointed out as qualities that a mathematics textbook lesson should have in the previous question. However, in the assessment of the proportionality lesson, we recognise some new attributes in the epistemic-ecological aspect. In addition, in general, fewer characteristics are indicated negatively than those indicated positively.

Didactic suitability analysis of the lesson by means of TLAG-proportionality

The a priori analysis of the lesson according to the different facets, components and indicators of didactic suitability by means of TLAG-proportionality was carried out independently by the researchers and an external expert collaborator. The analyses were then compared in order to reach a consensus on an overall assessment. As a result, the didactic suitability was rated as low in each of the dimensions: epistemic-ecological (score 15 out of 62, 17 out of 31 indicators rated 0), cognitive-affective (score 7 out of 24, 6 out of 12 indicators rated 0), and instructional (score 3 out of 18, 6 out of 9 indicators rated 0). However, as can be seen from Table 5, the majority of the PPTs considered the degree of suitability to be medium to high in the epistemic-ecological and cognitive-affective dimensions (73.34% and 51.11% respectively). The assessment was better in the instructional facet, where 53.33% considered the suitability degree as low. To a lesser extent, 28.89% of the PPTs correctly rated cognitive-affective suitability as weak.

Suitability degree	Epistemic-Ecological	Cognitive-affective	Instructional
Low	2	13	24
Low to medium	10	9	4
Medium	26	17	16
Medium to high	4	4	0
High	3	2	1

Table 5. Frequencies in the assessment of the didactic suitability by dimensions (n=45)

The judgements used by PPTs to justify their allocation are of two types.

a) Quantitative. Based on the number of indicators in each facet rated 0, 1 or 2 points, or their average. For example, PPT12 rates instructional appropriateness as low indicating:

   Most of the indicators have a rating of 0, many others have a 1 and there are no indicators rated with 2 points, therefore, we can say that the instructional suitability is low.

b) Descriptive. Description of the positive or negative aspects of the lesson, understood as the degree of compliance with the indicators. For example, PPT34 assigns a low suitability in the instructional aspect, due to:

   A large part of the proposed tasks are very mechanical, and manipulative materials that would allow for more meaningful learning are not used. In some of the didactic

configurations, I think that more time should be devoted to them, such as the one dedicated to proportional magnitudes. It is appropriate to spend enough time on this before moving on to work on the rule of three. In addition, it should be noted that there are few exercises that encourage collaboration between classmates and group reflection. Nor do we find that the teaching unit works in an interdisciplinary way, it does not relate to any other specific topic.

Although there is no significant difference between the justifications of one type and those of another (47.41% quantitative, 52.59% descriptive), we do observe most of the descriptive type in the evaluations as low (correct) or low to medium (partially correct) in the three dimensions.

Those PPTs that incorrectly assessed the didactic suitability of the lesson in the epistemic-ecological dimension did not identify significant deficiencies in relation to the mathematical content. Thus, a high percentage of PPTs rated positively (2 points) some indicators which in the a priori analysis we identified as having no compliance (0 points) and which represent serious shortcomings in the planned teaching process on proportionality. Among these, we highlight the following:

- Adequate representations are used to distinguish multiplicative relationships within and between quantities. A total of 42.22 % of the PPTs gave this indicator a score of 2 points. These either simply said that "the indicator is fully met" or considered only the presence of pictorial or graphical representations ("drawings and graphs are used in magnitude problems", PPT28), without assessing whether these representations allow distinguishing between both types of relationships, as recommended for an adequate development of proportional reasoning (Fernández and Llinares, 2011; Shield and Dole, 2013).
- Fundamental procedures (arithmetic strategies, reduction to unity, rule of three, calculation of percentages) are presented clearly and correctly. As before, 42.22 % of the PPTs scored 2 points. In this case, when justifying their grading, they mentioned that these procedures have a specific place in the lesson ("a specific section is dedicated to the reduction to unity and the rule of three", PPT35) or that the student is taught the procedure to follow to solve the proposed problems ("the explanations make it clear which procedures to follow in carrying them out", PPT16).
- Proportionality appears to be related to rational numbers. In total 55.56% of the participants gave this indicator a score of 2 points. Most of them did not justify it and when they did, they based their assessment on the inclusion of fractions in the rule of three procedure, percentages and the scale ("percentages, the rule of three and scales are closely linked to rational numbers", PPT44).
- The relationship of proportionality with magnitudes is made explicit (the numerical values involved in proportionality situations correspond to measures of quantities of magnitudes). Up to 62.22% of the PPTs rated this indicator with 2 points, mainly due to the presence of

> different magnitudes in the examples and activities proposed ("the relationship between different magnitudes is presented: euros and tickets, km and time...", PPT5).

Those PPTs who did rate the adequacy in this facet as correct or partially correct (i.e. low or low to medium, respectively), recognised the above indicators as partially or not at all fulfilled, and identified some other shortcomings in the lesson. For example, they noted that the lesson does not state the fundamental properties of the proportionality relation and the procedures are not justified ("we do not find arguments by the author, justifications are omitted", PPT5). They also found that the application situations are not very diverse and representative of the contents (the majority of missing value), do not make explicit the multiplicative relationship in proportional situations, do not allow distinguishing multiplicative comparisons from additive ones, and mainly involve internal ratios ("scarcity of situations involving the use of external ratios", PPT8) i.e. between different values of the same magnitude and not external ones, between values of different magnitudes. Furthermore, they recognise that the representations do not allow them to identify and distinguish the multiplicative relationships that are established within and between proportional quantities. Finally, they also point out that the proportional nature of percentages and scales is not established ("percentages are included, but despite this, there is no section or explanation in the book that relates them to proportionality", PPT29).

From a cognitive-affective point of view, the lesson does not take individual differences into account and does not encourage flexibility and creativity in the resolution on the part of the pupils. Thus, the research team scored 0 points for the corresponding indicators in the TLAG-proportionality. However, 51.11% of the PPTs consider that the lesson takes individual differences into account "through the 'Unit review' where there are activities to revise and reinforce the contents" (PPT32) and the presence of two procedures (reduction to the unit, rule of three) to solve different problems, scoring 2 points for both indicators in the TLAG-proportionality. However, the activities proposed in the "Unit review" or "Test yourself" sections are not considered as reinforcement or extension, and no other strategies (progressive construction, additive, multiplicative, etc.) than those described by the PPTs are used.

Those participants who rated the lesson adequately in this dimension identify shortcomings in relation to prior knowledge, attention to individual differences, development of flexible thinking and proposal of appropriate evaluation or self-evaluation instruments. For example, PPT5 indicates:

> At the cognitive-affective domain, the flexibility to explore mathematical ideas and alternative methods of problem solving is not encouraged. In terms of the difficulty of the content presented, it is manageable for the educational level at which it is aimed, but the use of different strategies and situations with different levels of difficulty is not promoted. Nor is prior knowledge considered.

Finally, in the instructional aspect, we observe that the presentation is not entirely adequate. Indeed, concepts (constant of proportionality, direct proportionality relationship) and properties (symmetry of the proportionality relationship, additive and homogeneous character of the linear function) essential to the subject are omitted, various argumentative resources are not used, the

student is not given the chance to take the responsibility of the study, and the sequencing does not follow the didactic recommendations on the progression in the teaching of proportionality (Shield and Dole, 2013; Streefland, 1985). More than half of the PPTs recognised most of these shortcomings in the lesson (leading them to rate instructional appropriateness as low). However, 33.33% of the PPTs considered the sequencing of content to be fully satisfactory (2 points) ("the sequencing is adequate, the apparently more difficult content has more exemplification", PPT41), regardless of the relevance of progressing from additive to multiplicative thinking, delaying the introduction of the rule of three until proficiency in other less formal procedures was acquired (Shield and Dole, 2013).

Reanalysis of the characteristics that a good textbook lesson should have. The case of proportionality.

To find out whether the intervention motivates a change in the beliefs, we asked PPTs if they had changed their opinion about the lesson after analysing it by means of the TLAG-proportionality. Of the 45 participants, two did not respond to the question and 16 maintained their initial opinion about the lesson. If this was positive (6 PPTs), they consider it possible to use the material with improvements that they recognise more clearly after the analysis of the didactical suitability. If this was negative (10 PPTs), the participants justify that they initially observed that it was not entirely suitable, although they have been able to carry out a more thorough review. For example, PPT8 states:

> I have now recognised many more shortcomings that I had not noticed before. [...] Not knowing how to do an analysis correctly, I did not identify as many conflicts as I do now, because at that time I only identified the lack of clear explanations that would facilitate the acquisition of the content and the completion of the activities and problems.

A total of 27 PPTs claim to have changed their opinion and consider themselves to have a more critical view of the lesson, which they justify by identifying a greater number of conflicts (17 PPTs) or relevant aspects to consider in a lesson that they had not appreciated before (10 PPTs). In this case, the participants elaborate a detailed report on what are these important features or conflicts that they had overlooked in their initial assessment, referring to the indicators or the evaluation of the didactical suitability in the different facets. For example, PPT21 states:

> My opinion has changed because, in the first analysis I made I thought it was a fairly complete and detailed lesson but, now I think that several elements can be improved (the indicators I have rated with one and include those I have rated with zero).

Similarly, PPT45 comments:

> It has changed a lot due to the fact that I have been able to carry out a more complete and thorough analysis which has allowed me to locate the weaknesses and strengths of the book lesson, as well as to establish guidelines to counteract those shortcomings since I can locate the conflicts in a more detailed way and focusing on specific aspects.

In addition, to assess how the training received influenced the PPTs' beliefs about what aspects are important in a textbook lesson, we finally asked them to distinguish the three indicators they considered most important for the assessment of the didactical suitability of a textbook lesson, in each of the dimensions[5]. Table 6 shows the three most frequently selected indicators in each of the facets.

Component	Indicator	Freq
Epistemic-ecological dimension		
Problems	A diverse and representative sample of tasks (missing value, comparison, tabular, etc.) is used to contextualise and apply the contents of proportionality and percentages.	15
Languages	The language level is appropriate for the students	17
Concepts	The fundamental concepts for the corresponding educational level are presented clearly and correctly: direct proportionality between magnitudes, covariance of quantities, invariance of ratio, constant of proportionality	17
Cognitive-affective dimension		
Previous knowledge	Previous knowledge of fractions, equivalence of fractions and measurement of magnitudes, necessary according to the educational level, is considered	16
Individual differences	Access, achievement and support for all learners is promoted, e.g. by encouraging the use of a variety of correct strategies (progressive building, additive, multiplicative, etc.).	17
Emotions	Logical reasoning, original ideas or useful, practical or realistic work are encouraged.	15
Instructional dimension		
Students interaction	Tasks are proposed that encourage dialogue, communication and debate between students in which different solutions are explained, justified and questioned, using mathematical arguments	26
Autonomy	There are opportunities for students to take responsibility for the study: they explore examples to investigate and conjecture; they use a variety of tools to solve problems and communicate them.	28
Sequencing	The content and activity sequencing is adequate, devoting sufficient space to the contents which are more difficult to understand.	26

Table 6: Indicators considered most important in each facet (n=45)

Comparing Table 6 with Table 1, which summarised the key features that the PPTs included in response to the question "What aspects do you consider most important in a mathematics textbook lesson?", we observe that:

[5] It should be noted that the number of indicators in the epistemic-ecological dimension is much higher than in the cognitive-affective and instructional dimensions.

a) In the epistemic-ecological facet, the importance given to the material including a diverse and representative sample of tasks that allow the content to be contextualised (in this case, that of proportionality and percentages), and that the fundamental concepts appear clearly and correctly (increasing the number of PPTs that indicate this as a fundamental feature of the lesson) is maintained. The third indicator selected as essential to consider the lesson as suitable in this facet relates to the relevance of the language for the educational level. In the previous assessment, more attention was paid to clarity and richness of expression and representation.
b) In the cognitive-affective dimension, the value given by the PPTs to emotions (fostering creativity and practical and useful work on the part of the pupils) is maintained, this being the aspect most highly valued in the initial task. The new indicators selected as most important incorporate prior knowledge and guaranteeing access to all students through flexibility in the use of correct strategies, which had barely been mentioned in the initial reflection.
c) In the instructional facet, which was the least referred to in the initial task (above all they mentioned the importance of clarity in the presentation and organisation of the lesson), the relevance given to interaction between students and sequencing of the contents is maintained, but the PPTs choose as the third indicator the one related to the autonomy component.

The PPTs have selected as fundamental some indicators of the TLAG-proportionality that they had previously assessed incorrectly in the lesson. Therefore, they could judge a textbook lesson as adequate based on these features that they consider to be a priority, when in fact it has shortcomings that they would not consider in the management of the resource. This raises the need to reinforce their didactic-mathematical knowledge in these aspects. In particular, on the cognitive facet, ensuring access for all learners requires knowledge and flexibility in the use of a variety of correct strategies. However, more than half of the PPTs showed in their assessment, a limited vision of the possible ways of dealing with proportionality situations-problems and the need to progress in this type of strategies so that students achieve adequate proportional reasoning. In the instructional aspect, one third of the PPTs considered the sequencing of content to be adequate, focusing their assessment only on the greater exemplification of the most difficult content and without reflection on the relevance of progressing from the intuitive to the formal, delaying the introduction of the rule of three (Streefland, 1985). Also, almost a quarter of the participants felt that the lesson includes moments for students to be autonomous in their learning. In this sense, it is necessary for PPTs to understand that to promote autonomy it is not enough to pose problems for students to solve (as these PPTs considered); it involves incorporating situations that allow students to be spontaneous, dynamic and participatory (Santaolalla, 2014), developing the intellectual capacity to face real problems and new situations (Rey and Penalva, 2002).

CONCLUSIONS

In this paper we have described the design, implementation, and results of a training intervention with preservice primary school teachers, aimed at promoting their reflective competence by analysing the didactical suitability of textbook lessons. Since, without a guide, teachers tend to make descriptive and not very analytical analyses of curricular materials (Nicol and Crespo, 2006), the usefulness of the TLAG-proportionality as a tool to guide and promote reflection on the degree of didactic suitability of a textbook lesson on this subject, is shown.

The results of the experience show the evolution of the participants' competence in didactical suitability analysis. Initially, the PPTs made evaluations of the lesson in which, although they implicitly use didactical suitability criteria, their narratives are superficial and limited to a few aspects that they consider essential. In the final evaluations, critical and detailed judgements can be observed that contemplate a greater number of components of the different facets which appear interconnected in the instructional processes. Thus, in their opinions PPTs explicitly mention didactical suitability indicators, which are correctly assessed by most of the participants in the cognitive-affective and instructional dimensions. Furthermore, although only 12 PPTs rated epistemic-ecological appropriateness adequately, they manage to recognise the importance of argumentation, and are more specific in aspects central to the content of proportionality, for example, by correctly assessing the need to precisely establish the multiplicative relationship between proportional magnitudes or to differentiate between additive and multiplicative comparisons.

In previous experiences (Castillo et al., 2021a), the results reflected inconsistencies between the reasoned judgements about the didactical suitability of the lesson and the ratings of the indicators in the TLAG-proportionality. In this new intervention, all participants have referred to the TLAG-proportionality indicators in their reasoned judgement of suitability (either quantitatively or descriptively) and their judgement is consistent with the indicators' ratings in the instrument. This improvement in the results may be because, on this occasion, the participants applied the guide in a general way to the lesson and were asked to justify the scores given to the TLAG-proportionality indicators, which allowed them to reflect in a more detailed way on their decision.

On the other hand, we observed a significant change in participants' motivation with the TLAG-proportionality utility compared to previous experiences (Castillo et al., 2021a). This improvement may be explained by the fact that in this intervention participants have had the opportunity to find the meaning of the suitability criteria (Schwarz et al., 2008) through the pre-training task of initial exploration. As suggested by Schwarz et al. (2008) the use of analytical tools to help teachers to have a critical stance must have accessible meaning for teachers, otherwise they may acquire a rejecting attitude towards the task or qualify it as too difficult and impractical for their profession.

The results of the intervention show that the analysis of textbook lessons, using didactical suitability as a resource, is a training activity that preservice teachers find useful and interesting. According to the participants, the TLAG-proportionality helps them to select the critical aspects to consider in the analysis of a textbook lesson:

> These [TLAG-proportionality] tables are a guide to know the quality of the textbook in question. Otherwise [without the guides] you would not look at details that are relevant for students' learning or, on the contrary, you would look at details that were not very important (PPT9).

> The tables [of the TLAG-proportionality] lead us to go deeper into the most important aspects that must be considered in a textbook lesson, so that it works correctly, as this is the main objective. Not having these tables with the indicators makes it difficult to analyse such a lesson, as there will be aspects that are too important to be left out (PPT38).

However, the results also warn about the importance of prospective teachers consolidating didactical-mathematical knowledge and competences needed to become a reflective education professional. Indeed, the application of the TLAG-proportionality requires the implementation of a series of mathematical and didactic knowledge, which may not be part of the users' previous training. Its systemic structure, by means of descriptors associated with the different components of the various dimensions, allows the teacher trainer to identify difficulties in the interpretation and assessment of specific indicators, and to make decisions on those aspects that need to be reinforced in future training interventions.

The training action described in this article is complemented by action decisions on how the teacher should use the lesson, as well as the proposed resolution of identified conflicts. However, due to space limitations and that the interest of this paper is to show the progress in reflective competence, confronting the pre- and post-training analysis of the lesson, these results will be shown in a forthcoming paper.

We agree with Remillard and Kim (2017) that the type of analysis conducted is especially important for primary school teachers, who typically do not have a broad background in mathematics and are therefore more inclined to rely on curricular resources:

> At the beginning I thought that almost everything would be fine, that there would be no conflict because if there were many conflicts it would not be a book that pupils use at school, etc. [...] One look at the book and it might seem like the best book for your students, but when you analyse the book, it might have more conflicts than you would have imagined. That happened to me (PPT11).

It is common for teachers to consider textbooks as a guarantee of quality and that they can replace the teacher, even "other teachers... have so little confidence in their personal training that they do not dare to make any changes in textbooks or omit anything even if it seems reasonable" (Santaolalla, 2014, p.194). Hence the importance of prospective teachers gaining confidence in their own training, and of having sufficient didactical-mathematical knowledge to be able to detect deficiencies in textbooks and other educational resources. In this respect, we observed that the participants in our experience felt more secure in carrying out these tasks when they had a tool to guide their analysis, acquiring a more critical view of the quality of the text and modifying their initial opinion of the lesson, if necessary.

With a view to future research, it would be desirable to carry out further interventions in which to analyse, over a longer training period, whether reinforcing progressively the didactic-mathematical knowledge supporting the indicators of suitability helps prospective teachers to progress in their reflective competence and how it does so. On the other hand, in this manuscript we have been able to corroborate that prospective teachers are influenced by their convictions when analysing texts (Shawer, 2017). Thus, most of the participants in our experience considered that a good book should have a variety of tasks, but fewer recognised the important role of argumentation or communication of ideas. We believe that analysis guides like the TLAG-proportionality for such content, or the TLAG-Mathematics in general, can help to identify the beliefs that prospective teachers have about teaching and learning mathematics. Given that it is difficult to assess the change in beliefs without analysing how these are manifested in their teaching practice (Lloyd, 2002), it would be interesting, along the lines proposed by Yang and Liu (2019), to infer the beliefs of prospective teachers in the analysis of lessons by contrasting them with a more in-depth study applying a specific questionnaire. It would be also relevant to observe whether with a continuous training, participants manage to develop and materialise rich beliefs about the role of these resources in the teaching of mathematics.

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