

## Stress Tolerance in Probabilistic Thinking: A Case Study

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*Abstract: Probabilistic thinking is a structure of thinking characterized by scenarios that allow one to explore reality. Therefore, the characteristic of probabilistic thinking is problem-oriented that will occur in a future full of uncertainty. Nevertheless, few studies examine the students' probabilistic thinking level based on the Stress Tolerance dimensions. Thus, in this study, researchers aim to describe the students' probabilistic thinking level based on the Stress Tolerance dimension in solving probability problems. It is shown that the smallest Stress Tolerance (ST)-Students consider confirming that the first solution is accurate. In contrast, the students with the highest score in ST-dimensions tend to make a simple step in solving the problem. The students' answers to probability problems characterize authentic risk-based decision-making. When we deal with probabilistic situations in everyday life, we all use a series of decision-making in our everyday estimation of probabilities, which sometimes leads to biases. However, the level of probabilistic thinking depends on the stress tolerance of the students. The students with the smallest stress tolerance score tend to get level 4 in probabilistic thinking. In contrast, the students with the highest stress tolerance score tends to reach level 1 in probabilistic thinking.*

### INTRODUCTION

Each student has their motivation to study. Students' motivation has an impact not only on their learning outcomes but also on their mental processes (Kasdhan, 2018). A motivation to study occurs if students are curious about the information or experiences as a motivation to learn something. In a world where people are deluged with information and can attain novel experiences with only a few keyboard clicks, curiosity becomes a potent psychological strength (Kashdan, et al., 2018). Kashdan said that curiosity is about seeking information and experiences for their own sake through self-directed behaviour. Curiosity is a desire to acquire new knowledge and sensory experience that motivates exploratory behaviour (Litman, 2005; Berlyne, 1954, 1960; Loewenstein, 1994). A recent set of studies suggests that being curious about other people's feelings, thoughts, and behaviour is distinct from observing other people surreptitiously to acquire new information (e.g., Litman & Pezzo, 2007; Renner, 2006). Students are required to be able to develop their higher-order thinking skills.

Some researchers defined probabilistic thinking as formal thinking characterized by abstraction, hypothetical, deductive, inductive, and logical thinking (Pfannkuch, et al., 2016; Savard, 2014; Borovcnik & Kapadia, 2014). Pfannkuch characterized probabilistic thinking as how one views

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and thinks about probability, whether from a classical, frequentist, or Bayesian perspective has been presented as crucial to how one engages in probabilistic thinking. Borovnick & Kapadia (2014) described probabilistic thinking as a structure of thinking characterized by scenarios that allow one to explore reality. Therefore, the characteristic of probabilistic thinking is problem-oriented that will occur in a future full of uncertainty.

Jones et al. (1997, 1999) suggested four levels of probabilistic thinking, namely subjective, transitional, informal quantitative, and numerical. Furthermore, Polaki (2002) developed the Jones' probabilistic thinking level in more detail for several subjects or materials in probability theory. In determining the level of probabilistic thinking ability indicators, using the probabilistic thinking rubric as shown in Table 1.

No.	Level	Indicators
1	Level 1 ( <i>Subjective</i> )	1. Students are always bound to a subjective reason
2	Level 2 ( <i>Transitional</i> )	2. Students think naively and often change.
3	Level 3 ( <i>Informal Quantitative</i> )	3. Students can harmonize and quantify their thoughts about the possibilities that will occur.
4	Level 4 ( <i>Numerical</i> )	4. Students can make precise relationships between the sample space and its probabilities and use numerical measurements appropriately to describe the probability of an event.

Table 1: Level of Probabilistic Thinking (Adopted from Taram, et al., 2019; Polaki, 2002)

However, there are few studies which explore the students' probabilistic thinking level based on the Five-Dimensional Curiosity Scale-Revised (5DCR) curiosity dimensions. The 5DCR distinguishes between experiences of curiosity that differ in emotional valence. The degree to which someone is curious depends on two cognitive judgments. Initially, a person must recognize that an event is exciting and warrants attention. Mysterious, novel, complex, uncertain, and/or ambiguous events tend to elicit interest (e.g., Berylne, 1954, 1960; Silvia, 2008a). Curiosity is initiated if a person notices that an event has novelty potential. A person will only be curious if they also believe they can sufficiently cope with the distress that arises from exploring the novelty potential of a situation (Silvia, 2005, 2008a). If a person believes that a case has novelty and coping potential, a person is said to be curious now (i.e., state curiosity). People who endorse novelty and coping potential with high frequency, intensity, and/or longevity are said to be highly curious (i.e., trait curiosity) (e.g., Silvia, 2008b). From this work on the appraisal components of curiosity, the 5DCR instrument measures a dimension of curiosity referred to as Stress Tolerance—the dispositional tendency to handle the anxiety that arises when confronting the new. Thus, in this study, researchers aim to describe the students' probabilistic thinking level based on the Stress Tolerance dimension in solving probability problems.

## METHOD

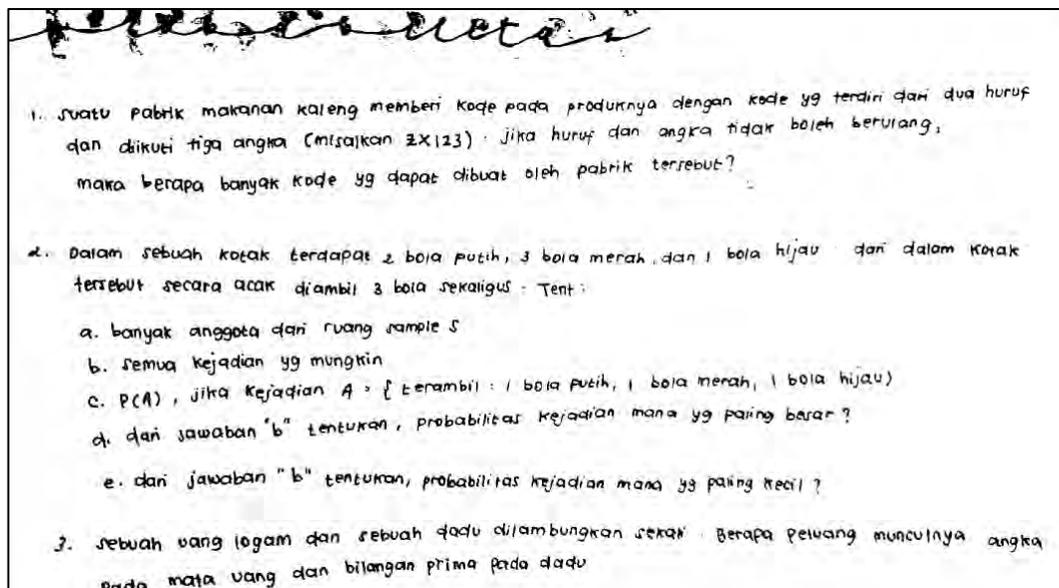
This research was conducted at a university in Yogyakarta, Indonesia. Furthermore, the study was held in May 2022. The number of subjects involved in this study is ten students of 2<sup>nd</sup>-semester students in the mathematics education department. The subject is chosen by purposive sampling based on the dimensions of curiosity. This study uses two instruments: 1) 5DCR Questionnaire of Curiosity and 2) the Probabilistic Thinking Test. The data was analyzed qualitatively. The researchers give the descriptive explanation about the profile of the students' probabilistic thinking based on their Stress Tolerance dimension.

### Stress Tolerance (ST) questionnaire

The Stress Tolerance questionnaire is a part of a five-dimensional curiosity-revised (5DCR) instrument. It is adopted from the Kasdhan, et al. (2018). It consists of 24 questions. There are 4 statements for Stress Tolerance (ST) dimension. The students choose their preferences in the questionnaire from scale 1 (one) to 7 (seven). Scale seven indicates that the statement does not describe him/her at all. In contrast, scale ones mean the statement completely describe his/herself. The scale indicates the degree to which statements accurately describe his/herself.

### Probabilistic problem test

In this case, the researcher uses an instrument test to determine the students' probabilistic thinking level. The test is an essay. It is containing three (3) questions that have been validated by 2 validators. The students write the answer directly in the answer sheet that is given and did the test for 60 minutes. First question of the test is asking the number of possibilities of a code can be made. Second is asking about the possibility of taking 3 color balls simultaneously from a pocket. Third question of the test asks about the possibility of a head-tail event from tossing a coin and prime number when tossing a die. The probabilistic problems test is shown in Figure 1.



*Translate in English:*

- 1) A food factory gives a code for their products, consisting of 2 letters and 3 numbers (for example, ZX123). If the letters and numbers used were not allowed to be repeated, then how many codes can be made?
- 2) A bag contains 2 white, 3 red, and 1 green ball(s). Three balls are drawn one after the other without replacement. Determine:
  - a) the number of the elements of sample space
  - b) all possible event occurs,
  - c) the probability that the balls are drawn is white, red, and green,
  - d) what is the probability of the most possible event?
  - e) what is the probability of the less possible event?
- 3) A coin and a die are tossed once. What is the probability of getting the head of the coin and the prime number of dice?

Figure 1: Probabilistic Thinking Test

## Interview

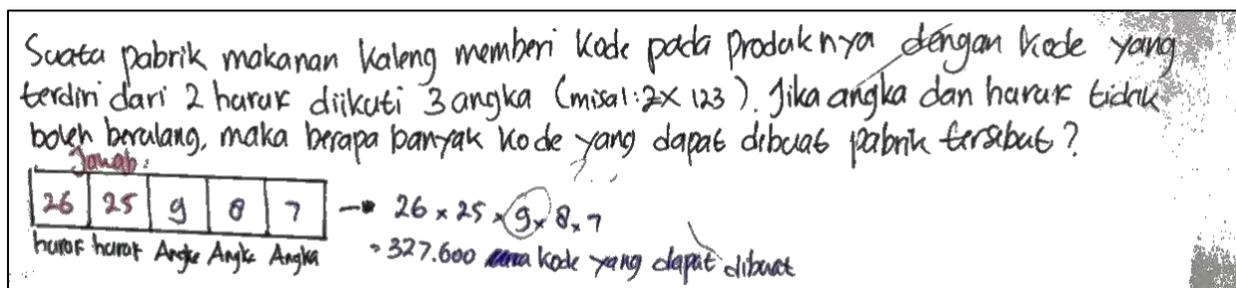
Researchers interviewed the students to confirm their answers. The researchers ask the students what, why and how the step of the probabilistic thinking level. This interview is conducted after the researchers give the test of probabilistic thinking.

## RESULTS AND DISCUSSION

In each dimension of curiosity, the researchers choose one student who represents their work in solving probabilistic problems based on the Stress Tolerance dimensions.

### The smallest vs the highest ST score of student probabilistic thinking

The student with a minor in ST dimension score writes down the number of possibilities of a code that can be made by combining the alphabetic and numerical code. It is shown in Figure 2 that students write the solution to the first problem in detail.



Suatu pabrik makanan kaleng memberi kode pada produknya dengan kode yang terdiri dari 2 huruf diikuti 3 angka (misal: 2x123). Jika angka dan huruf tidak boleh berulang, maka berapa banyak kode yang dapat dibuat pabrik tersebut?

Jawab:

26	25	9	8	7
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huruf huruf    Angka    Angka    Angka

$\rightarrow 26 \times 25 \times 9 \times 8 \times 7$   
 $= 327.600$  maka kode yang dapat dibuat

Translate in english:

A food factory gives code for their products that consist of 2 letters and 3 numbers (for example ZX123). If the letters and numbers used were not allowed to be repeated, then how many code that can be made?

Answer:

26	25	9	8	7
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Letters      Numbers

$$26 \times 25 \times 9 \times 8 \times 7 = 327.600 \text{ codes}$$

Figure 2: The smallest ST-Student's solution in number 1

When the researchers interviewed the student about the solution, the student could explain his idea about how he got the answer. He can distinguish the combination of the letters and the numbers for the product's code. Like the solution in number 2, the subject made precise relationships about the sample space and its probabilities and appropriately used numerical measurements to describe the probability of an event (see in Figure 3).

Jawab: \* Bola putih : P, Bola merah : M, bola hijau : H

1. Ruang Sampel :  $\{M_1M_2M_3, M_1M_2P_1, M_1M_2P_2, M_1M_2H_1, M_1M_3P_1, M_1M_3P_2, M_1M_3H_1, M_2M_3P_1, M_2M_3P_2, M_2M_3H_1, M_1P_1P_2, M_1P_1H_1, M_2P_1P_2, M_2P_1H_1, M_3P_1P_2, M_3P_1H_1, P_1P_2H_1\}$   
= 17 ruang sampel.

2. kejadian yang mungkin;  $M_1M_2M_3; M_1M_2P_1; M_1M_2P_2; M_1M_2H_1; M_1M_3P_1; M_1M_3P_2; M_1M_3H_1; M_2M_3P_1; M_2M_3P_2; M_2M_3H_1; M_1P_1P_2; M_1P_1H_1; M_2P_1P_2; M_2P_1H_1; M_3P_1P_2; M_3P_1H_1; P_1P_2H_1;$

3.  $P(A) = \{P, M, H\} = 3/17$

4. paling besar :  $\{M, M, P\} = 7/17$

5. paling kecil :  $\{P, P, H\} = 1/17$

Translate in English

Answer: White ball (P), Red ball (M), Green ball (H)

1. Sample Space =  $\{M_1M_2M_3, M_1M_2P_1, M_1M_2P_2, M_1M_2H_1, M_1M_3P_1, M_1M_3P_2, M_1M_3H_1, M_2M_3P_1, M_2M_3P_2, M_2M_3H_1, M_1P_1P_2, M_1P_1H_1, M_2P_1P_2, M_2P_1H_1, M_3P_1P_2, M_3P_1H_1, P_1P_2H_1\}$   
= 17
2. Possible event =  $M_1M_2M_3, M_1M_2P_1, M_1M_2P_2, M_1M_2H_1, M_1M_3P_1, M_1M_3P_2, M_1M_3H_1, M_2M_3P_1, M_2M_3P_2, M_2M_3H_1, M_1P_1P_2, M_1P_1H_1, M_2P_1P_2, M_2P_1H_1, M_3P_1P_2, M_3P_1H_1, P_1P_2H_1$
3.  $P(A) = \{P, M, H\} = \frac{3}{17}$
4. The biggest =  $\{M, M, P\} = \frac{7}{17}$
5. The smallest =  $\{P, P, H\} = \frac{1}{17}$

Figure 3: The smallest ST-Student's solution in number 2

The smallest score of ST-student made solution in number 3 of the probabilistic test showed that there was not only one solution derived to determine the probability of the event in tossing a coin and a die. The student gives two alternatives to find the possibility of event head-tail and prime number in tossing a coin and a dice consecutively. The first solution of the smallest Stress Tolerance students is illustrated in Figure 4.

3. Sebuah uang logam dan sebuah dadu dilambungkan sekali. Berapa peluang munculnya angka pada mata uang dan bilangan prima pada dadu

Jawab: -  $P(\text{angka pada mata uang}) = S = \{A, G\}$  \*  $A = \text{angka}, G = \text{gambar}$

$= 2$

$P(A) = \frac{1}{2}$

- Prima: 2, 3, 5 = 3  $\rightarrow S = \{1, 2, 3, 4, 5, 6\}$

$= 6$

$P(\text{prima}) = \frac{3}{6} = \frac{1}{2}$

\* Peluang munculnya angka pada uang dan bilangan prima pada dadu:  $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ , atau  $\frac{3}{12}$

Translate in English:

3. A coin and a die are tossed once. what is the probability of getting a head of the coin and prime number of a die?

Answer:

-  $P(\text{tail of a coin}): S = \{A, G\}$  |  $A: \text{Head}, G: \text{Tail}$

$= 2$

$$P(A) = \frac{1}{2}$$

- Prime number = 2, 3, 5 = 3  $\rightarrow S = \{1, 2, 3, 4, 5, 6\}$

$= 6$

$$P(\text{Prime}) = \frac{3}{6} = \frac{1}{2}$$

Figure 4: The smallest ST-Student's solution for Number 3 (First Solution)

Furthermore, the ST student gives two alternative solutions to solving problem number 3. The second answer shows that he considers confirming that the previous solution is accurate. The student wrote the sample space of the event by mentioning the elements of it, one by one, as a set of the probable events (See in Figure 5). In the last of the second solution, the student counts the number of elements of the sample space and then determines the probability of the event. As a result, the level of probabilistic thinking of the subject is categorized as level 4.

Ruang Sampelnya:  $S = \{A_1, A_2, A_3, A_4, A_5, A_6, G_1, G_2, G_3, G_4, G_5, G_6\}$

$= 12$

$P(A, \text{Primo}) = \frac{3}{12}$

$= \frac{1}{4}$

$$\begin{aligned} \text{Sample space: } S &= \{A_1, A_2, A_3, A_4, A_5, A_6, G_1, G_2, G_3, G_4, G_5, G_6\} \\ &= 12 \\ P(A, \text{prime}) &= \frac{3}{12} = \frac{1}{4} \end{aligned}$$

Figure 5: The smallest ST-Student's solution in number 3 (Second Solution)

Besides, another student with the next-to-the smallest score of ST-dimension was writing the solution in detail. She wrote the diagram to ensure the coin has two possibilities (head and tail) events (see in Figure 6). Both are bound to a subjective reason, think naively and often change, harmonize, and quantify their thoughts about the possibilities that will occur, make precise relationships about the sample space and its probabilities, and use numerical measurements appropriately to describe the probability of an event.

Jawab:

Sebuah uang logam  $\begin{cases} \text{angka} \\ \text{gambar} \end{cases}$

$$\begin{aligned} n(A) &= 1 \\ n(S) &= 2 \end{aligned} \quad P = \frac{n(A)}{n(S)} = \frac{1}{2} \text{ peluang angka}$$

Sebuah dadu (1,2,3,4,5,6)

$$\begin{aligned} n(A) &= 3 (2,3,5) \\ n(S) &= 6 \end{aligned} \quad P = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

Peluang munculnya angka pada mata uang dan bilangan prima pada dadu

$$P(A) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Answer:

A coin  $\begin{cases} \text{Head} \\ \text{Tail} \end{cases}$

$$\begin{aligned} n(A) &= 1 \\ n(S) &= 2 \end{aligned} \quad P = \frac{n(A)}{n(S)} = \frac{1}{2} \text{ tail possibilities}$$

A die (1,2,3,4,5,6)

$$\begin{aligned} n(A) &= 3 (2,3,5) \\ n(S) &= 6 \end{aligned} \quad P = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

Probability of head of a coin and prime number of a die:

$$P(A) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Figure 6: Another smallest score of Stress Tolerance-Student's solution in number 3

In contrast, the students with the highest score in ST dimensions tend to make a simple steps in solving the problem. She wrote the simplest way to find the solution to the problem. For example, the key to the number 3 was not noted on the answer sheet. She claimed that the problem was complex for her. She does not know how to start and how to resolve it. It can be seen in the solution of number 2 that she crosses out the first answer. The first solution is shown in Figure 7.

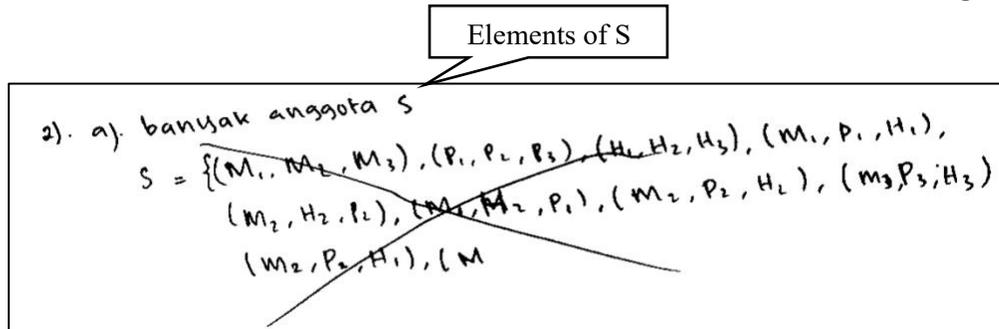


Figure 7: The highest ST-Student's solution in number 2 (first solution) - First attempt

However, the revised answer only mentioned the sample space without determining the possibility of taking three balls simultaneously. The highest ST student wrote the number of the sample space but not the elements of it (see in Figure 8). After the interview, the subject only mentioned that she did not worry about the result. In other words, she does not know the number of possibilities of the event. As a result, the level of probabilistic thinking of the subject is categorized as level 1.

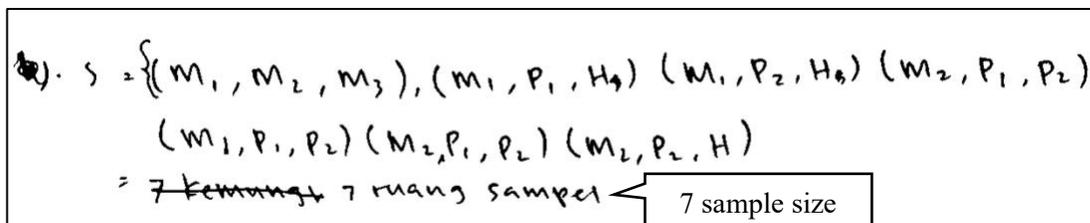


Figure 8. The highest ST-Student's solution in number 2 (revised solution) – Second attempt

### Stress Tolerance in Probabilistic Thinking

Stress Tolerance was found to have the most vital links to dispositional mindfulness, work engagement (vigor, dedication, and absorption), and low levels of work burnout, as well as being moderately linked to work-related curiosity and a willingness to defy social norms and express opposing viewpoints. In addition, stress tolerance had the most vital inverse relationship with negative emotionality and positive relationships with extraversion and conscientiousness, respect and trust, psychological needs satisfaction, and the humility to separate intellect and ego (Kasdhan, et al., 2018). However, the relation between stress tolerance and probabilistic thinking was on the structure of thought characterized by scenarios that allow one to explore reality. Mysterious, novel, complex, uncertain, and/or ambiguous events tend to elicit interest (e.g., Berylne, 1954, 1960; Silvia, 2008a). If a person notices that an event has novelty potential, Stress Tolerance occurs. The

students will only be stressed if they also believe they can sufficiently cope with the distress that arises from exploring the novelty potential of a situation (Silvia, 2005, 2008a).

All probabilistic analysis is based on the idea that (suitably trained and intelligent) people can at least recognize good probabilistic arguments presented by someone else or discovered or thought of by themselves, but not necessarily generate good assessments. The fact that there was correspondence about the gambles – and occasionally some disputes about them – indicated that people do not automatically assess probabilities in the same way or accurately (e.g., corresponding to relative frequencies or making good gambling choices). Their understanding of mathematical ideas must be developed via experience and exposure to the challenges of solving various issues (Aguilar & Telese, 2018).

Unlike the typical research finding, which sets out to identify underlying the level of probabilistic thinking, the researchers note that this research is founded upon 'clinical' methods where the problem to which subjects answer in probability problems characterize authentic risk-based in decision making. The basic concept is that we all use a series of decision-making in our everyday estimation (explicit or implicit) of probabilities, and this decision-making sometimes leads to biases. Just as associational thinking serves us well in many contexts, so does decision-making. When we deal with probabilistic situations in everyday life, we can often 'muddle through,' but occasionally, not appreciating the comparative nature of valid probabilistic thinking can lead to judgmental disasters. These systematic deviations may be linked to thinking in terms of associations, whereas excellent probabilistic judgment always necessitates comparative thinking.

## CONCLUSION

The smallest Stress Tolerance-Student considers confirming that the first solution is accurate. In contrast, the students with the highest score in ST dimensions tend to make a simple step in solving the problem. The student's answers to probability problems characterize authentic risk based on decision-making. The students with the smallest stress tolerance score tends to get level 4 in probabilistic thinking. In contrast, the students with the highest stress tolerance score tends to reach level 1 in probabilistic thinking.

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