How are fortuity mathematical movements of mosquitoes represented in children's drawings?

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Abstract: This exploratory and experimental research aims to describe randomness expressed in 5 to 6-year-old children's drawings. This study considers a six-day activity developed in 5 Chilean kindergartens, with a total of 142 participants. The activity on the mosquito's flight considered the corporal movements to generate the idea of randomness in children. The children drew the mosquito flight and described the flight. We qualitatively analyzed and categorized the children's drawings and verbal descriptions, dividing them into static or dynamic representations. A second grouping was established in the dynamic drawings: in one direction and random. The third division considers only random drawings: (1) uncertain walk, (2) possible walk, and (3) casual walk. The results show that children in this stage of development can express in their drawings some of the three basic notions of randomness. The children expressed randomness through random stops, circular paths, and arrows to mark positions or make decision possibilities of the mosquito.

Keywords: *Mathematics education, Basic Notion (BS), Drawings, Random walk, Mathematical intuition*

INTRODUCTION

For decades, researchers proposed the inclusion of probability theory in the mathematics curricula, starting with introducing probability at an early age (Fischbein & Schnarch, 1997; Freudenthal, 1973; Jones, Langrall, Thornton & Mogill, 1997; Kafoussi, 2004; Langrall & Mooney, 2005; Fesakis, Kafousi & Malisiova, 2011). Further research on children's probabilistic reasoning (Clements & Sarama, 2007; Bryant & Nunes, 2012; Batanero, 2015) concluded that randomness plays a vital role in developing children's probabilistic thinking.

The studies carried out in early childhood on probability have consisted mainly of determining when children can respond intuitively or under reasoning about the probability of an event (HodnikČade & Škrbec, 2011). Piaget and Inhelder (1951) observed that children have difficulties differentiating random and deterministic phenomena. For Batanero (2013), these difficulties lie in the irreversibility of randomness; there is no concrete experience reversible for the random, there is no concrete way to reverse a random evolution these phenomena cannot be manipulated to return to the initial state.

Although we agree that randomness does not correspond to reversible experiences, we say that randomness experiences can leave legible traces and could be considered reconstructible instead of reversible. In the same direction, Jones and others (1997) proposed a four-level model, of which the first subjective thinking level takes greater importance to generate probabilistic thinking in early years. Thus, it is necessary to consider activities that seek to strengthen children's experiences related to randomness. We created an enactive task that contributes to the field of probabilistic thinking, this has a focus on a real experience of randomness, and unlike other studies (Varga, 1972; Falk, Falk & Levin, 1980; Fischbein, 1975; Pangue, 2003; Nikiforidou & Pangue, 2010; HodnikČade & Škrbec, 2011) we do not base experience in games related to probability using balls, dice, coins or cards.

The activity designed considered the basic notions (BN) of randomness. BN (vom Hofe, 1995; vom Hofe & Blum, 2016) (*Grundvorstellungen* in German) are mental interpretations of mathematical concepts that mathematically coincide with the concept. BN allows us to understand how young children translate ideas from the real world into the mathematical world. Borovcnik & Kapadia (2018) indicated that probability is a tool for modeling reality and that children work with primary intuitions of chance and probability (Borovcnik, 1992); thus, we have that children represent the random world with primary intuitions but that they must do so with adequate BN or secondary intuitions.

The basic notions of randomness

Randomness arises when an event happens and cannot be previously determined in advance or predicted (Batanero, Green & Romero-Serrano, 1998). The random walk or random movement can be defined as a mathematical model. The type of movement occurs by chance; it has a starting point followed by consequent random steps, where each selected stop does not influence the next possible one. There are random changes of direction besides random stops. This act of stopping and randomly “walking” is repeated until the end of the process, the same as our “aimless walk”. The resulting trajectory is called a “random walk.”

Examples of random walks in the natural realm are mosquito flights (Pearson, 1905) who coined the term “random walk” in his query to the journal Nature about the probability distribution of the distance from the origin of a random flying mosquito (a vector of malaria) after a given lapse of

time. Various types of random walks are of interest, which can offer in several ways in probability theory, such as in Markov Chain or Markov process (Norris, 1998). This is a stochastic model describing a sequence of possible events in which the possibility of each event depends only on the state attained in the previous event (Norris, 1998).

According to vom Hofe & Blum (2016), the BN are related to mathematical content and an actual situation from an individual conceptualization, and they have three main characteristics (vom Hofe, 1995):

- A sense of mathematical concepts comes from the person's connections from known situations, concrete actions, and experiential actions.
- A set of representations that allow the operative action in an imaginative level is correct and objective mental representations.
- A mathematical modeling from an actual situation is a translation from the real world to the world of mathematics and vice versa.

Concerning people's intuitive (primary and secondary) ideas (Borovcnik, 1992), there are 3 BN for randomness:

(1) It can be reduced as uncertainty where it is unknown how to continue or finish. In the event of "moving a glass of water," the glass could fall or not. Nobody previously knew what would happen; we say that the result is unpredictable. Here randomness is considered as a very unusual, unsuspected, and unstructured phenomenon.

(2) It can be reduced as a possibility. In this situation, many results are possible. There is recognition of the possible outcomes; in a race with two classmates, it may be that one of the children ends up; first, one of them falls and cannot finish, both finish at the same time, or both fall. In these possible scenarios, randomness is seen as a one-possibility non-deterministic option. Here the events are random if it can happen, but does not have to occur.

(3) It can be reduced to casual. Here the results do not have a cause that explains the phenomenon—bumping into someone without a previous agreement or meeting someone after a long time. Randomness is seen as a fortuity, where situations or events happen just because fortuity is controlling the results.

Random walk is understood as a path or trail that has a starting point, followed by a randomly selected stop random steps, we would say. Also, there might be no stops. This process is repeated until the end of the walk (Henze, 2013; Soto-Andrade & Reyes Santander, 2011). It is essential to highlight that each selected stop does not influence the next possible one (see Fig. 1).

Fig. 1: Random walk in 2D.

Random walks are the might be movements of a particle (Pemantle, 2007), animals (Smouse & others, 2010), bacteria (Othmer & Stevens, 1997), and insects (Kareiva & Shigesada, 1983). In these examples and figure 1, the next stop or ending point is unknown. There are many possible stops, and it is impossible to predict the next one certainly.

For us, a specific approach to chance, which does not depend only on games of chance, such as dice and card games, can be considered in random movement (Soto-Andrade, Días-Rojas & Reyes-Santander, 2018). In particular, an example of randomness is the movement associated with the mosquito's flight (Pearson, 1905). This phenomenon, very common and close to people, allows us to translate from the real to the concept of mathematical randomness. Many of us go for an "aimless walk" when we want to think or rest; this natural act is related to the mathematical theory of randomness.

Drawings for random walk

At preschools levels, the translation from the real world to concepts must begin with concrete and experiential events. Children can give an intuitive meaning to concepts, resorting to experience to remember them (Saalbach, Grabner, and Stern, 2013). In order to understand how children think, we need a variety of representations inherent and coherent to the children's developmental stage (Reinwand, 2013). According to Hope (2008), drawings are visual representations of our inner images, can support children's thinking, and develop their abilities in perceiving the structures of problems. Thus, children's drawing plays a significant role; these are a powerful means to communicate ideas of learning and thinking.

Nikiforidou and Pange (2010) used drawing as a means of communication between adults and children. This study has shown a decrease in the children's age from 4 to 7 years in which they develop and possess notions in the area of opportunities and probabilistic reasoning. According to Scheid and Ritter (2014), drawings are an essential means of communication in the early stages of development. Drawings are also a medium to explore children's ideas and perspectives, and they

can be interpreted descriptively (Neuss, 2005). Then, it is logical and reasonable that preschool teachers employ drawings to generate scientific knowledge (Chang, 2012).

According to Seidel (2007), in the development of drawing, four age levels are recognized. Level one goes from zero to two years, and at the end of this phase, the child draws scribbles or doodles that represent an apparent aimless movement done with his hands in a space (Seidel, 2007), which could be a random walk. Level two goes from two to seven, where drawings have a self-centered, emotional, and pictorial character; here, everything is perceived as animate and alive, representing what is seen and what is imagined.

Nentwig-Gesemann (2013) suggests that the qualitative studies that interpret production in children must be appropriate to the participant's age. According to Seidel (2007), a 5-year-old can translate his inner ideas into comprehensible drawings; representations of movement and space can occur at the end of level two, from 2 to 7 years. According to Neuss (2005), through drawing, views of specific scenes, figures, or actions become clear. The drawing is a subjective representation for the researcher and becomes a "symbolic objectivities" in the analysis for articulated content of consciousness communicable, analyzable, and interpretable.

Thus, this study has designed a concrete task in which children move from running, daydreaming, drawing, and speaking about the mosquito flying. The stage of development considered in this study ranges between 5 and 6 years. The analysis of the drawings and oral expressions is done qualitatively (Neuss, 2005), and the BNs are used to categorize this. In the research, the following two questions were posed:

- How do children draw the randomness flight of a mosquito?
- What BN predominate in drawings and verbal expressions about mosquito's flight?

Methodology

Most studies consider an approach to the concept of randomness based on Laplace's rule and in this developmental stage (Bryant et al., 2012). Nevertheless, this is exploratory-experimental and qualitative research. It is researched through activities that do not estimate the probabilities but rather the randomness represented in the drawings.

It is experimental because we intervened in five classes in two kindergartens with an activity that involved the mosquito's flight. It is qualitative research (Miles, Huberman, and Saldaña, 2014) because of the data obtained from the children's drawings and the verbal phrases describing the same (Christensen & James, 2008; Neuss, 2005).

There are three instances in which iconic and verbal data are categorized:

- First categorization: static or dynamic.
- The second categorization is only dynamic: one direction or randomness.
- Third categorization reflects only randomness: (1) uncertain walk; (2) possible walk; (3) casual walk.

One hundred forty-two children participated in this study. The number of children per class was 28, 28, 24, 30, and 32. Children's ages fluctuated from 5 years to 6 years. The latter was not explicitly requested in the instructions, but most children wrote their age next to their names, or the teacher wrote it afterward.

The experimental phase was carried out from September to October in 2017. Furthermore, all the study cases have their corresponding written informed consent from parents and school principals. This document is the permission to implement the activity and to use and preserve children's productions.

The design of the activity was based on Bruner's three phases: Enactive – Iconic - Symbolic (Bruner, 1971). Children do the enactive representation; the iconic phase, in which children represent the random walk; and a symbolic verbal phase. It is complemented by what Neuss (2005) recommends: the questioning of children toward their drawings to obtain a better representation of the same.

The activity was implemented in six regular school days and the different moments were:

Day 1: To provide engagement and motivation, children were presented the short story "The lion and the mosquito" in an adapted Aesop version. This story stimulated questions and comments related to the mosquito. The educator focused the attention on the flight of the mosquito, asking why the lion cannot catch the mosquito

Day 2: The next day, students listened to the music "Flight of the Bumblebee" (1899 – 1900) of Nikolai Rimsky-Korsakov and were asked to imagine with the music the mosquito's flight.

Day 3: In the enactive phase, the teacher asked: How do you think mosquitoes move? Furthermore, inviting students to imagine being mosquitoes and "fly" around the classroom. Children live their experiences in this movement, which has its effect. Furthermore, a child can experience the self as a cause of effects (Zimmer, 2013). This is why the teacher's instruction is to do this activity as freely as possible, allowing children to run in the playground and to fantasize about the idea of being a mosquito, including making insect noises.

Day 4: The iconic phase, the children received an A4 sketchpad and crayons, and they were asked to draw the "mosquito's flight." These productions are analyzed in this study.

Day 5: In the third phase, the symbolic-verbal phase, the children are asked two questions: How many mosquitoes did you draw? Moreover, how is the mosquito's flight? It is expected that the children point out characteristics related to the random walk; for instance, statements like "the mosquito flew like crazy," "it went from here to there," or "it moved from flower to flower." The teacher in charge of the activity writes the answers to the questions next to the drawings, which are categorized again according to the random walk BN and are analyzed in this study.

Day 6: The educator put all the drawings on the wall and, together with the children, look for the best representatives for the following sentences: (1) the flight of this mosquito is uncertain; (2) the flight of this mosquito has more than one chance of way and (3) the flight of this mosquito is crazy, it does not know where it is going.

Results

The children's productions from days 4 and 5 were associated with an acronym, which corresponds to the class and goes from 1 to 5 with a correlative number for each child. Four productions were eliminated. 4-8 and 5-17 did not have drawings but just one color; 5-17 there was no mosquito; 5-9 was impossible to decipher, and the teacher did not comment on it. The total amount of productions considered was 138.

Iconic results

We divided all productions into the category's static (no randomness) and dynamic in the first categorization. We found 103 drawings that corresponded to static representations. These drawings do not present any path related to the mosquito's movement. This means the child does not consider in his drawing the notion of the mosquito's movement from stop A to B. Table 1 shows the summary of the first and second categorization.

Table 1: Summary of the first and second iconic categorization

	Static	Dynamic one direction	Dynamic randomness
Frequency	103	11	24
Percentage	74.63	7.97	17.40

Thirty-five drawings were dynamics. In the drawings it can be observed the use of semi-continuous lines, dotted lines, arrows, and circles to show the linear path followed by the mosquito, or to indicate the direction followed by the mosquito, as it was represented through the use of arrows. 11 out of 35 showed a linear representation of the mosquito's flight in only one direction, as it is shown in figure 2.

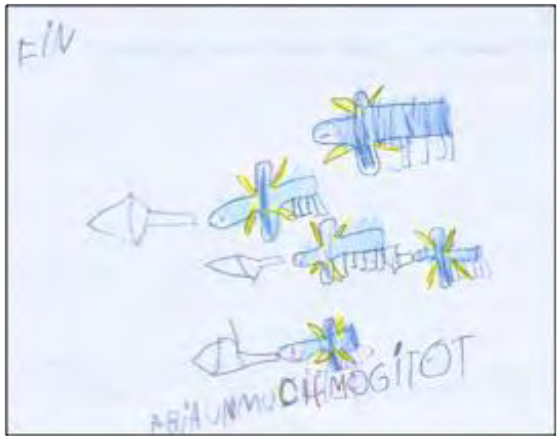

Figure 2: Unidirectional using arrows production 2-19.

24 out of 35 productions show different tracks of the mosquito's flight, no one-directional. Moreover, to show this effect, children drew dots, semi-continuous lines, dotted lines, circular shapes, curve lines, lines over dots to show movement, and the use of arrows to highlight the starting or ending point.

Table 2 shows the resulting 24 children's drawings according to the three BNs of randomness. BN (1) corresponds to the uncertain walk, and it is represented by drawings showing different positions in which the mosquito could have gone through, (2) corresponds to the possible walk, and it is depicted by drawings that point out paths in different directions; and (3) corresponds to the casual walk, where the drawings present a split in the path or closed circular shapes that indicate the mosquito's flight trace, where the mosquito is within that circular shape.

Table 2: Categorization of the random drawings according to BN (1), (2) y (3).

BN	Drawings

(1) Uncertain Walk	1-6, 1-8, 1-10, 1-11, 1-17, 1-25, 1-26, 2-2, 2-26, 4-19
(2) Possible Walk	4-1, 4-4*, 4-12, 4-18, 4-21, 4-22, 4-23, 4-28*, 4-30*, 5-26, 5-30
(3) Casual Walk	4-2, 4-4*, 4-11, 4-12, 4-20, 4-28*, 4-30*

In figure 3, BN (1) of the uncertain walk are shown. Here it is unknown where the mosquito goes and where it will end. In this case, the child drew just dots, and the notion of uncertainty is reflected in the number of unstructured dots. The mosquito was or will be in any of the dots or stops. The impression produced by the drawing reflects quite precisely the randomness of the mosquito's flight; furthermore, the difference in size in the dots gives the idea of closeness or farness, upwards or horizontally.

Figure 3: Uncertain walk production 4-19.

Figure 4 illustrates the use of lines to show the path followed by the mosquito. The dotted line has some variations, and it keeps a direction in the horizontal axis, but it also shows several directions in the vertical axis. In this case, it is possible to associate the image and the BN (2) possible walk since the mosquito's path goes upwards and downwards without following visible rules in the drawing. The mosquito's possibilities are represented in the possibility of going upwards or downwards.

Figure 4: Possible walk production 4-22.

In the following figure 5, a flower is observed. The mosquito leaves the flower, moves upwards, and then to any of the sides, which corresponds to the BN (2), through the drawing of the possibility to move upwards sideways. The mosquito leaves the flower, goes upwards, and then to any of the sides (left or right), which corresponds to the BN (2) in drawing the possibilities upwards and sideways.

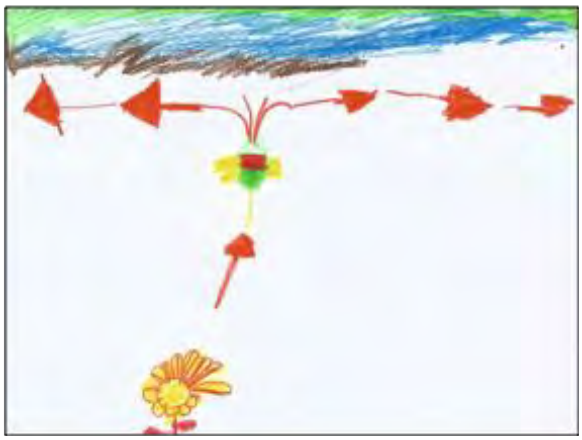

Figure 5: Possible Walk production 4-23.

In figure 6, BN (3) of the random walk are shown. Here is a crazy walk where the mosquito goes and has many positions. In this case, the child lets the pencil take it by fortuity; the simple controls the mosquito's path.

Figure 6: Casual walk production 4-28.

The iconic results show drawings in the three BNs of the random walk. This indicates that children ages 5 to 7 years can draw their mental ideas in relation to the mosquito's flight, and these representations correspond to one or more BNs of the random walk. This result indicates that children in this stage of development have notions that match the mathematical concept of randomness.

Verbal results

Concerning the verbal answers, 8 out of 138 productions did not have an answer. The categorization of the verbal answers related to how the mosquito flies showed that there were 59 out of 130 static answers or unrelated to the question, where the child described that the mosquito flies with its wings, or that it flies like bees, butterflies, birds or planes. It was also considered unrelated when the teacher indicated no answer from the child or when he said he did not know how the mosquito flew. Table 3 is a summary of the first and second categorizations for the verbal answer.

Table 3: Summary of the first and second iconic categorization

	Static	Dynamic one direction	Dynamic randomness
Frequency	62	44	24

Percentage	47.69	33.85	18.46

There were 68 out of 130 answers related to dynamic situations. 44 out of 71 are unidirectional answers. The children mention that the mosquito goes to a specific place: to the right, towards a specific flower within the drawing, towards its home, to the left, downwards, or upwards. These verbal answers correspond to one flight, with one intention and a lineal flight; in some cases, the children mentioned that the mosquito goes straight to its house or straight to see its mother.

Giving only one direction to the flight indicates no stops or random points in which the mosquito might stop. In this group, two children had a unidirectional iconic production (2-19, 4-29), and when they had to give a verbal answer, they indicated that: "they move flying with the right wings" (2-19) and "flies like this (shows the movement with his arms)" (4-29); these answers are considered in the group that does not give information concerning the description of the mosquito's flight. In the case of these two children, it can be said that language is not enough for them to express their ideas about the unidirectional mosquito's flight.

24 out of 68 verbal answers mentioned more than one direction in which the mosquito can go, or they broadly mention the environment of the mosquito's flight. Here, there were answers related to the circle, the action of biting people or going toward flowers (none in particular). Various directions are mentioned, but they are unable to decide one in particular.

Table 4 presents the categorization of the random verbal expression according to the BNs of randomness. The BN (1) uncertain walk corresponds to the expressions in which the mosquito flies anywhere, the BN (2) possible walk corresponds to the expressions in which the mosquito flies from top to bottom, and BN (3) corresponds to expressions in which the mosquito has no apparent causes reasons, motivations to fly.

Table 4: Categorization of random verbal expressions according to BNs (1), (2) (3).

BN	Drawings
(1) Uncertain Walk	2-18*, 4-1*, 4-22*
(2) Possible Walk	1-6*, 1-11, 2-2, 2-6, 2-8, 2-15*, 2-18*, 2-27*, 4-1*, 4-2, 4-3, 4-11, 4-12, 4-20, 4-21, 4-22*, 4-23, 4-30.
(3) Casual Walk	1-6*, 1-16, 1-17, 2-3, 2-15*, 2-26, 2-27*, 4-4, 5-15

The results in table 2 show that the verbal expressions correspond mainly to the BN (2) possible walk. Expressions such as “upwards, downwards, to the left and the right” are more frequent than general expressions like “the mosquito is everywhere” BN (1), or the ones that indicate a random choice, like “the mosquito is biting people.”

Iconic and Verbal results

In table 5, we compared the iconic and verbal results. We said that 15 children out of 142 could match their iconic and verbal random representations equally. This means that the child can represent in a drawing the random walk through the use of any BNs, and this representation matches and complements what he verbally indicates.

Table 5: Summary of iconic and verbal results

	Frequency	Percentage
Static drawing/ static verbal	55	39.85
Static drawing/ one direction verbal	32	23.18
Static drawing/ randomness verbal	11	7.97
Static drawing/ no verbal answer	5	3.62
One direction drawing/ static verbal	2	1.44
One direction drawing/ one direction verbal	7	5.07
One direction drawing / randomness verbal	0	0
One direction drawing / no verbal answer	2	1.44
Randomness drawing / static verbal	5	3.62
Randomness drawing / one direction verbal	5	3.62
Randomness drawing / randomness verbal	13	9.47
One direction drawing / no verbal answer	1	0.72
Total	138	100

In relation to the drawings categorized as static, 11 of them (1-16; 1-20; 2-3; 2-6; 2-8; 2-11; 2-15; 2-18; 2-27; 4-3; 5-14) presented also a verbal answer related to the mosquito's flight within the category of random verbal answers. These answers enrich the static drawing, and through them, there is an approach to one of the 3 BNs of the random walk.

For instance, the verbal answer 5-14: “it flies upwards, downwards, to one side and the other” describes that the mosquito goes anywhere, which corresponds to the BN (2) possible walk. There is an indication of the different possibilities the mosquito has in terms of the direction to follow.

Another example of the static drawing and randomness verbal expression corresponds to the categories BN (3) casual walk and BN (2) possible walk. In 2-15: “a mosquito is dancing “cueca” (traditional Chilean dance). It goes for a walk, and the other mosquito dances “cueca” and runs away moving to the sides.” Here there is a connotation of the random walk due to mentioning the word “walk.” Furthermore, the mosquito “runs away,” meaning it goes somewhere without saying why or where and without an apparent cause, which is the BN (3) casual walk. Furthermore, the mosquito’s walk from one side to the other corresponds to the BN (2) possible walk.

A child had a static drawing in a different case, which corresponds to the randomness verbal category, 2-18: “the mosquito walks everywhere, from the bottom to the top and sideways too.” This shows the random walk from the BN (1) uncertain walk, in the phrase “the mosquito walks everywhere” and also in the following statement “... bottom to top and sideways too”, randomness is communicated according to the BN (2) possible walk.

Figure 7 shows the production 4-4, where the mosquito’s flight is presented through continuous lines to indicate the track of the flight. In this case, the lines are not drawn in a linear direction, vertical or horizontal (the straight-line concept as the shortest path); the path done to reach one place to the other corresponds to BN (2). Children draw the mosquito in the center within a circular shape, and there are three paths followed by it. This is the BN (3) casual walk because it does not differentiate the choosing of a path over other; and how it is reached from the edge of the circular shape towards the center of the same, also showing a choice that goes from any point in the edge to the center.

Figure 7: Categories iconic BN (2), (3) and verbal BN (3), production 4-4.

The oral expression that corresponds to the drawing 4-4: “the mosquito first leaves his home and flies from flower to flower and then rests somewhere,” the child indicated that in his drawing there is just one mosquito (there is a bee, a butterfly) and in each action performed by the mosquito, “leaves home,” “goes from flower to flower” and “rests somewhere,” the child draws the flight’s track.

In the drawing, there is clarity in the track left in the action of flying from flower to flower or in the verbal explanation of how the mosquito reaches the center of the circle to rest.

Figure 8 shows the production 1-6, where the iconic was categorized according to the BN (1) uncertain walk, and the verbal was categorized in BN (2) and BN (3). The circular shapes shown in figure 8 indicate the different positions in which the mosquito has gone through; the disposition and size of these circular shapes indicate the uncertainty of the track left by the mosquito in its trajectory, there is a change in the color of the tracks. It can also be observed a rectangular shape and a flower.

Figure 8: Categories iconic BN (1) and verbal BN (2), (3), production 1-6.

Concerning the verbal part in the production 1-6, the child indicated two positions and one ending point 1-6: “the mosquito flew towards a house, then towards one side and downwards to bite people.” BN (2) is present when he says that the mosquito went to one side and downwards because this BN is associated with the path’s possibilities. When he says that the mosquito bites people, it could be associated with the BN (3), in terms of the casual of the election of people it could bite.

These results show that 35 children resort to different ways to express the mosquito’s flight. Concerning the iconic expressions categorized as random, different ways to describe the

mosquito's flight were found. Among these, random stops can be distinguished, which describe the different positions of the mosquito. In these expressions, it can be affirmed that the union of those stops could have complemented the notion of random walk.

Another type of pictorial expression that could be observed was the random path; in this case, the children drew curve lines representing the mosquito's track or footprint. Arrows to continue the flight or mark a position were also observed, but arrows to show the decision-making process was unexpected. This decision-making process was characterized by either deciding a path or showing a split in the path, which presents the mosquito's possibility of deciding between two tracks.

CONCLUSIONS

This study shows that the drawings of children younger than 7-year-old show basic notions of the random walk. They are expressed with points, lines, arrows, colors, and verbal expressions. This means that the concept of randomness can be built on over the years, based on children's experiences and observations of their environment. It can be said that when 5-year-old children participate in a concrete activity, they can activate certain notions related to randomness, and these notions can be expressed through drawings or verbally. These results match Nikiforidou and Pange's (2010) proposal of diminishing the age from 7 to 5 in which children possess and develop basic notions related to randomness.

The observation of the mosquito's flight, imagine that they can fly and draw this flight, allowed the children to describe what they understand as random in a close and contextualized way. In this case, some children recognize the determinist, expressed with lines and an accurate direction drawn by a line and an object to be addressed. Some children recognize randomness by identifying several possibilities for the flight in random points or curves indicating a non-directional flight.

It surprises the number of children who represent the mosquito's flight statically and whose verbal expressions coincide with some of the BNs of the random walk. Concerning the static representations, these can be understood within the levels of development; here, children understand a drawing as a picture. Hence, an alternative would be asking them to draw this situation again after some time has passed, after the dynamism of flight has been developed.

Amazingly, children use different expressions to communicate their notions about randomness, especially arrows to indicate possible positions or decision-making. According to Seidel (2007), children are very talented at drawing and have excellent graphomotor skills, so this type of activity complements their thoughts and perception and helps them develop the language. The same author adds that these children develop advanced comprehension of signs, presenting unusually expressive creations through pictorial and linguistic metaphors (Luci-Arriagada & Reyes-Santander, 2016; 2017).

Then, this study presents the relation between the development of drawings in preschool years and the capacity to express random BNs. Gender differences were not considered in this study. According to Brosat and Tötemeyer (2007), girls' drawing skills overcome boys up until they are 8 or 9-year-olds. A possible study in this area could be related to observing the performance of boys and girls from the random BNs to the BNs of probabilities.

Our study shows that it is possible to find in drawings and phrases the random BNs; additionally, we believe that how research is done should be coherent with the stage of development of the participants. This is the first study that relates drawings to mathematical concepts. It would be interesting to apply the same activity and compare it with other groups of children to see if the results remain or other cultural factors (Farsani 2020, 2021; Breda et al., 2021; Krause & Farsani, 2021) that allow children to express their ideas better through drawings.

We agree with Langrall and Mooney (2005) and Greer (2001) on the need to perform long and focused experiences to enable children to build on existing intuitions and construct new ones. This study shows that preschool children have BNs of the mathematical concept of randomness based on perceptive configurations through an enactive and adequate activity. The study also shows that the children come very close to the notions about randomness as a random walk like a walk that has possibilities and a casual walk.

Practical implications:

Given the results of this paper, it is possible to think about how a potential curricular proposal may encapsulate and draw upon young learners' mathematical intuition and creativity in mathematical activities. Of course, further cross-cultural studies are encouraged as the findings of this paper emerge only from one Latin-American country.

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