Mulling Changes to Math Instruction

A framework proposed in California seeks to boost achievement by increasing the engagement of all students.

Jo Boaler and Jennifer Langer-Osuna

Despite California's status as the world's fifth largest economy and a hub of technology innovation, its students' math achievement ranks among the lowest in the United States, which itself ranks 37th in the world on the Program for International Student Assessment (PISA). Even before the pandemic slowed learning for many, only 39 percent of California students demonstrated proficiency on the state math assessment. During the pandemic, the proportion of math-proficient students fell to one-third.[1] At the same time, the STEM workforce faces shortages and a lack of diversity,[2] which call into question the longstanding practice of filtering the majority of students out of advanced pathways in math, often from a young age.

To address these problems, a committee of California math educators proposed changing the approach to teaching math based on what research has revealed about what will improve math achievement and engagement. In 2021, California started the process of revising the framework that guides math instruction.

The committee recognized the increasing need to prepare students to navigate 21st century workforce demands with skills in problem solving, reasoning, and data analysis. Yet the form of instruction that dominates math classrooms nationwide has students working through narrow questions and repeating methods shown to them. This instruction has led to widespread math anxiety and low achievement. The teaching of math as a disconnected set of procedures turns off even high-achieving students, as it so frequently offers students no access to meaning or deep understanding.

The committee met throughout 2021, drawing on research on effective teaching as well as practical wisdom from classroom educators. We were part of a writing team who distilled the committee's recommendations into a proposed framework for California math instruction. It was released for a 60-day period of public review and comment in March and will be considered by the state board for adoption later this year. For the benefit of other state boards who might be exploring revitalizing math education in their states, we share the evidence that underlies four of the recommendations in the proposed framework: open high-level pathways to more students, teach to big ideas and connections, teach through collaboration and discussion, and encourage data literacy.

Open High-Level Pathways to More Students

The proposed framework offers options for providing high-level opportunities to many more students so that they might be better prepared to qualify for STEM jobs in California. The framework proposes keeping high-level pathways open to more students for a longer time while also enabling exceptional students to move through courses at a faster pace.

One of the problems that districts face in keeping math pathways open for as many students as possible is that high schools typically have more prerequisite classes in front of the highest level courses—calculus or statistics—than there are years in high school. Thus students must complete algebra in middle school to enable them to reach the highest levels. The proposed framework acknowledges that middle schools need to offer algebra as an option in eighth grade, at least until high schools change, but recommends that tracking decisions are not made before then. Some California districts put students on different pathways in fourth grade, and many put students on different pathways in sixth grade, the beginning of middle school, using data from elementary school.

Such approaches fail to reflect the fact that all students can grow and learn. Instead, setting students on tracked math pathways in elementary school reflects a widespread belief in a pervasive myth that relatively few students have a "math brain" capable of understanding advanced math concepts.

Design problems have plagued the research on tracking. For example, when students are put into different classes and taught different content, high achievers are able to score at higher levels by design. Some studies have overcome this problem by monitoring student achievement and course taking over years, examining the achievement of students in tracked and detracked cohorts.

In one such study in New York City, middle school students were placed into regular or advanced classes for the first three years of the study. In the last three years of the study, all students took the advanced content and worked in the same groups. The researchers followed six cohorts of students through to the end of high school. They found that the students who worked in heterogeneous groups took more advanced math in high school, enjoyed math more, and passed the state test a year earlier than students who had been taught in tracks.[3] Further, the advantages accrued across the achievement spectrum. Other studies of initiatives to detrack middle school classrooms that compare student achievement with and without tracks have shown similarly promising results.[4]

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Despite these promising results from detracking, districts remain caught in a system where the only way students can reach high-level courses in high school is to compress important middle school content to fit prerequisite courses into the math sequence. One of the recommendations of the new framework is that this progression be reviewed and that high schools reduce the number of courses needed so that all students can have access to high levels and learn the intended content of middle school.

Teach to Big Ideas and Connections

Math comprises important ideas and connections. Curriculum standards and textbooks tend to divide up math into smaller topics, which has led students to believe that it is disconnected and procedural. In a review of the research, the National Research Council, concluded that

> [s]uperficial coverage of all topics in a subject area must be replaced with in-depth coverage of fewer topics that allows key concepts in the discipline to be understood. The goal of coverage need not be abandoned entirely, of course. But there must be a sufficient number of cases of in-depth study to allow students to grasp the defining concepts in specific domains within a discipline.[5]

The proposed framework in California shares a set of big ideas in math, organized by grade level and content standards. These ideas were first set out in the California Digital Learning Integration and Standards Guidance initiative, which was released in May 2021.[6] The selection of a few big ideas allows for the teaching the most important topics more deeply and coherently and also allows teachers and students to work on collaborative problem solving. This approach builds on research that has shown that teachers who organize content around big ideas are more successful (see box).[7]

The proposed framework includes many examples of big-idea tasks from across preK-12 to illustrate how they can foster understanding of multiple math standards. Students benefit from viewing math as a vibrant, interconnected, relevant, and creative set of ideas. As educators create opportunities for students to engage with and thrive in math through teaching to big ideas, they value the different ways questions and problems can be approached and learned, and many more students view themselves as belonging to the mathematics community.[8] Such an approach prepares more students to think mathematically in their everyday lives and helps society develop more students interested in and excited by science, technology, engineering, arts, and mathematics pathways.

Teach through Collaboration and Discussion

When students enter these careers, they will almost certainly need to collaborate with others, connecting ideas and perspectives as they solve complex problems. In fact, PISA now assesses collaborative problem solving internationally, and the upcoming National Assessment of Educational Progress in math will include collaboration as a practice. The proposed framework we helped write highlights an approach in which students work together, learning to reason and critique each other's reasoning. Math problems can also be used to build students' awareness and understanding of important problems

Box 1. Developing Teachers' Capacity: A District Example

To realize the ambitious vision for math instruction set forth in the framework, teachers will need support to develop what for many will be new teaching practices. Drawing from a research-practice partnership between a research university and a K-8 school district on the West Coast, we offer an example of a professional development approach that was focused on collaborative learning in the elementary grades. The district serves primarily Latinx and Pacific Islander students, with a majority designated as English learners.

Historically, the district has emphasized literacy but chose to emphasize math for the first time with the hiring of a new district math coach for elementary teachers. The coach worked with a small cohort of classroom teachers meant to serve as site-based instructional leads. Their work focused on developing teacher curiosity about student mathematical thinking; using open tasks that invite student mathematics thinking; eliciting, interpreting, and responding to student thinking in whole-class and small-group discussions; and making sense of student work from an asset frame. Instructional leads were supported to develop their own practice but also to share their work with interested colleagues at their school sites.

With the district superindendent's support, the district coach has autonomy sufficient to engage cohorts of teachers in slow, deep, responsive work. The hope is that the development of instructional-lead cohorts helps scale the work into each elementary school, such that instructional leads work in collaboration with the district coach. By beginning with teachers who opt in to professional learning communities, curiosity and excitement builds, and real change in instructional practice grows over time.

facing California, such as water shortages, fires, and climate change.

When students work together, their solutions tend to be more sophisticated and they tend to learn more.[9] Math communication and collaboration can simultaneously raise student achievement and work against inequities.[10] In a meta-analysis of research on cooperative math learning from prekindergarten through the university level, researchers Gulfer Capar and Kamuran Tarim found a mean effect size on student grades of 0.59,[11] indicating that this teaching method influences student achievement more strongly than traditional methods.

Discussions also offer students opportunities to explain their mathematical thinking, make sense of others' reasoning, and jointly develop flexibility with numbers, which serves as the basis for number sense. A key component of successful collaborative learning is student agency within the classroom to use their own ideas and resources to make sense of and solve problems.[12] It marks a significant departure from the typical passive engagement of students in traditional lecture-style classrooms.

Despite the evidence that collaboration benefits students, instructors often struggle to implement an active learning environment that centers on collaboration.[13] This struggle highlights that simply placing students in groups does not necessarily result in effective collaboration.[14] In some cases, social status and other factors can hinder it.[15] Complex instruction (CI), for example, is one pedagogical approach to collaborative learning suggested by the framework. CI centers on students working on "groupworthy" tasks in small groups, valuing multiple perspectives, and broadening opportunities for contribution and success.[16] Studies of CI teaching in high school math classrooms have shown an increase in student achievement,[17] an increase in students' appreciation for each others' ideas,[18] and more effective collaborative work.[19]

Encourage Data Literacy

The content currently taught in high school math courses was set out in the 1800s and has not changed since. Yet mathematics has changed considerably, particularly as regards

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data science. All students need to be able to interpret data and consider its source, purpose, and meaning.[20] If schools do not help students develop data literacy, they will be left vulnerable to misinformation, often shared through social media, and lack the foundations of important understanding. Teachers of all grades can help students though data investigations, engaging in "data talks," and generally infusing data inquiry into different lessons.

The University of California and California State University systems communicated to high schools that they value students who have followed a data science–statistics pathway as highly as those who have followed a calculus pathway. Stanford has also updated its admissions statement to include a data science possibility. Thus the framework sets out a high school course option in data science that students can take after they have taken integrated 1 and 2 or algebra and geometry. The course would be an excellent prelude to an AP statistics course. Students could also choose to take data science and calculus courses.

The availability of data science as a possible high school course reflects the broad nature of mathematics and the need to offer high school students more choice. Students who are intending to major in STEM in college will still need to take courses that enable them to start college with a calculus course, but many students will be better served by a data science–statistics pathway in their high school years.

Conclusion

Change is needed in California. Given the state of math achievement, it is hard to defend the status quo. The framework sets out an approach that allows students to problem solve, reason, collaborate, investigate, and connect ideas while also delaying tracking decisions so that more students can pursue math pathways. It takes an approach that the most successful teachers have been using for many years and would scale it to teachers and schools across the state. Its success will depend on teacher learning opportunities provided over the next few years, as well as the opportunities to educate counselors, school leaders, and parents. If time for learning is provided, a different mathematical future is a real possibility for the students of California.■

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cont'd on page 43

cont'd from page 11...The Impact of COVID-19...

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cont'd from page 17...The Urgent Need for Tailored Math Instruction

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cont'd from page 31...10 Lessons Learned...

while offering innovative opportunities to get students ready for life beyond the standards.

Bonus: The state-level working of the system is so far removed from the general classroom that it is incumbent on state leaders to make extra effort to communicate and network with practitioners. ■

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