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Measuring pre-service elementary teachers' geometry knowledge for teaching 2-dimensional shapes

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Abstract

This paper reports our efforts to develop a measure of pre-service elementary teachers' geometry knowledge for teaching 2-dimensional (2D) shapes and to evaluate the psychometric properties of this measure. Specifically, the GKT-2D scale was designed to assess pre-service elementary teachers' geometry content knowledge, knowledge of geometry and students, knowledge of geometry and teaching in relation to 2D shapes, based on the van Hiele theory and mathematical knowledge for teaching framework. Using 307 pre-service elementary teachers' responses on the instrument, we examined item- and scale-level reliability and hypothesized factor structure of the instrument. The results suggested that the GKT-2D scale is a reliable and valid measure of the three facets of pre-service elementary teachers' geometry knowledge for teaching 2D shapes, with some limitations further addressed. This paper offers directions for future research in evaluating content-specific knowledge for teaching mathematics to unpack the complex relationship between teacher knowledge, teaching quality, and student learning.

Keywords: geometry knowledge for teaching, pre-service elementary teachers, reliability and validity, van Hiele theory

INTRODUCTION

The quality of instruction that students receive is critical to their learning (Darling-Hammond et al., 2005; Wenglinksy, 2002). In mathematics education, central to such quality instruction is teachers' mathematical knowledge for teaching (MKT) (Ball et al., 2008; Hill et al., 2005). Although there is an assumed relationship between teacher's knowledge, teaching quality, and student learning, we do not know exactly how these are intertwined (Mewborn, 2003). Therefore, understanding and unpacking this relationship is an important research topic in mathematics teacher education as it can provide significant insights into teacher preparation programs and professional development (National Mathematics Advisory Panel, 2008).

Identifying specific kinds of mathematical knowledge teachers need to support student learning and measuring such knowledge reliably and comprehensively would be a good starting point to unravel the complex relationship between teacher knowledge, teaching quality, and student learning. In the field of mathematics teacher education, there have been substantial efforts to conceptualize key mathematical knowledge that shapes the quality of instruction and measure such knowledge (Ball et al., 2008; Chapman, 2007; Ellerton & Clements, 2011; Ferrini-Mundy et al., 2005; Hill et al., 2004; Hill et al., 2008; Herbst & Kosko, 2014; Manizade & Mason, 2011; McCrory et al., 2012; Olanoff et al., 2014; Steel, 2013). Compared to teachers' knowledge for teaching numbers, operations, and algebra (Ball et al., 2008; Chapman, 2007; Ellerton & Clements, 2011; Ferrini-Mundy et al., 2005; Izsák, 2008; McCrory et al., 2012; Newton, 2008; Olanoff et al., 2014; Welder & Simonsen, 2011), investigations into teachers' geometry knowledge is relatively limited and often focused on teaching middle or high school level geometry (Herbst & Kosko, 2014; Herbst et al., 2020;

Components of this study have been presented at the National Council of Teachers of Mathematics (NCTM) Annual Conference on April 2nd, 2019. However, neither this manuscript nor this study has not been submitted for publication elsewhere.

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Contribution to the literature

- This study examined the psychometric properties of the GKT-2D scale using modern test theory techniques.
- The reduced GKT-2D achieved acceptable reliability and validity in measuring three kinds of geometry knowledge for teaching: geometry content knowledge, knowledge of geometry and students, and knowledge of geometry and teaching.
- This study contributes to developing an inclusive, mathematics topic-based, and reliable measure to capture pre-service elementary teachers' content and pedagogical content knowledge.

Manizade & Mason, 2011; Martinovic & Manizade, 2018; Steele, 2013; Zambak & Tyminski, 2017).

Geometry can provide important foundations for learning other areas in mathematics and understanding other subjects (e.g., art, engineering, science, social studies) (National Council of Teachers of Mathematics, 2000). However, U.S. students often experience difficulties in learning basic concepts and problem-solving in geometry (Clements, 2003; Hock et al., 2015). This issue worsens as students transition to middle school, as shown in the Third International Mathematics and Science Study (TIMSS) (Gonzales et al., 2004, 2009; Mullis et al., 2016, 2020; Provasnik et al., 2012). Although U.S. students' overall TIMSS mathematics scores have increased, their geometry scores are still the lowest among the three content domains tested: numbers, geometric shapes and measures, and data display (Mullis et al., 2016, 2020).

One of the reasons for student underachievement in geometry and measurement may be inappropriate teacher preparation for teaching these topics (Bleeker et al., 2013; Browning et al., 2014; Sandt & Nieuwoudt, 2003; Steele, 2013). Thus, more attention needs to be paid to pre-service elementary teachers' geometry knowledge for teaching, considering the critical role of elementary teachers in supporting younger students to build sound foundations of geometry concepts and significantly influencing their geometric reasoning development later in middle school (Carroll, 1998).

There are research studies on pre-service elementary teachers' geometry knowledge for teaching. However, existing studies mainly investigated their geometry content knowledge (Browning et al., 2014; Fujita, 2012; Menon, 1998; Pickreign, 2007; Reinke, 1997). In addition, studies that measured pre-service elementary teachers' both geometry content and pedagogical content knowledge comprehensively and reliably are relatively limited (Livy & Downton, 2018; Martinovic & Manizade, 2018; Robichaux-Davis & Guarino, 2016). Therefore, this study is developed to identify geometry content and pedagogical content knowledge for teaching 2-dimensional (2D) shapes at the elementary level through a literature review and to evaluate the reliability and validity of the GKT-2D scale, an instrument specifically designed to measure such knowledge. The research questions of this study are:

1. Does the GKT-2D reliably and accurately measure pre-service elementary teachers' geometry content knowledge (GCK), knowledge of geometry and students (KGS), and knowledge of geometry and teaching (KGT)?
2. Is the hypothesized factor structure of the GKT-2D adequately supported in a sample of pre-service elementary teachers?

THEORETICAL BACKGROUND

The framework of MKT (Ball et al., 2008) and the van Hiele theory (van Hiele, 1984, 1986) constitute the theoretical bases for this study. These two guided us to conceptualize the content and pedagogical content knowledge needed for teaching 2D shapes at the elementary level and helped us develop the GKT-2D scale as a tool for measuring such knowledge.

Mathematical Knowledge for Teaching

Building on Shulman's (1987) taxonomy on content knowledge, pedagogical content knowledge, and curriculum knowledge, Ball et al. (2008) developed the MKT framework, which represents different domains of teacher knowledge that support students' learning of mathematics. The MKT framework consists of two overarching categories, subject matter knowledge and pedagogical content knowledge, each with three knowledge domains. Subject matter knowledge includes:

(1) common content knowledge (CCK) relating to knowing the mathematics content and accurately solving mathematic problems, not unique to mathematics teaching, but critical to plan and carry out instruction,

(2) specialized content knowledge that teachers use to explain, represent, and transform mathematical ideas into a form understandable to students, and

(3) horizon content knowledge regarding teachers' understanding of how different mathematical ideas are related to each other.

The second overarching category, pedagogical content knowledge, includes:

(1) knowledge of content and students (KCS) related to teachers' understanding of students'

conceptions and misconceptions of certain mathematical content,

(2) knowledge of content and teaching (KCT) regarding teachers' understanding of different and affordable instructional methods, and

(3) knowledge of content and curriculum relating to teachers' knowledge of mathematics topics and how these topics are sequenced in the curriculum.

In this study, we focused on three major knowledge domains, CCK, KCS, and KCT, central to the quality of teachers' geometry instruction (Sztajn et al., 2012; van Hiele, 1984). Namely, GCK, KGS, and KGT appropriate to geometry teaching.

The first domain, GCK, relates to teachers' understanding of geometric concepts, properties, and the relationships among the geometry topics they are to teach. The GCK domain in this study is CCK in geometry teaching and fits under the overarching category of subject matter knowledge in the MKT framework defined by Ball et al. (2008). It provides important bases for the other two knowledge domains. The second domain, KGS, relates to teachers' knowledge of students' geometric thought development, including students' way of thinking, common conceptions, and misconceptions about the geometric content they teach. The third domain is KGT regarding teachers' understanding of different methods, materials, and activities that support students' geometry learning. Both KGS and KGT domains are KCS and KCT, respectively, in geometry teaching and fit under the overarching category of the pedagogical content knowledge in the MKT framework. These GCK, KGS, and KGT domains served as key constructs for developing assessment items of the GKT-2D scale in this study.

van Hiele Theory of Geometric Thought Levels

The van Hiele theory of geometric thought levels (van Hiele, 1984, 1986) was another theory that framed this study. It presents several ideas about individual students' geometric thinking development and provides important implications for geometry teaching.

First, it suggests that students' thinking about geometry progresses through five levels: Level 0 (*visualization*), Level 1 (*analysis*), Level 2 (*informal deduction*), Level 3 (*deduction*), and Level 4 (*rigor*). For example, students at Level 0 can recognize geometric shapes by their appearance alone, not by their parts or properties. At Level 1, they begin to see the characteristics of shapes and recognize their properties. At Level 2, students start to understand the interrelationships of properties within shapes (e.g., in a triangle, if two sides are congruent, then the opposite angles to the sides are congruent) and among different shapes (e.g., a square is a rhombus because it has all the properties of a rhombus). Students at Level 3 can see the roles and relationships of axioms, postulates, theorems, and proofs and understand the interaction of necessary and sufficient conditions. And lastly, students at Level 4 (*rigor*) can comprehend and compare mathematical systems with a different set of axioms, such as Euclidean geometry and non-Euclidean geometry (Crowley, 1987).

Second, these levels are discrete, sequential, and hierarchical. Each level has unique languages, symbols, and relations. If a student and a teacher reason at different van Hiele levels, they cannot communicate each other (van Hiele, 1984). When students transition through different levels, they must have mastered lower-level skills to advance to the next level higher (Hoffer, 1981). There are some controversies over van Hiele theory regarding the existence of an earlier level called *pre-recognition* (Clements & Battista, 1992) and students who develop at more than one level simultaneously (Gutierres et al., 1991; Lehrer et al., 1998). However, research generally supports the hierarchical nature of the van Hiele levels (Alex & Mammen, 2016; Ma et al., 2015; Mayberry, 1983) and the behavioral characteristics of students' geometric thought development outlined in the van Hiele theory (Burger & Shaughnessy, 1986).

Third, the progression from one level to the next is more dependent on the instruction received than on the age or maturation of a student, particularly when the instructional activities are appropriate to the student's level of geometric thought (Clement & Battista, 1992; Crowley, 1987; van de Walle et al., 2016). In other words, effective geometry teaching needs to be aligned with the student's level of geometric thought, which can be accelerated by appropriate instructional activities and materials (van de Walle et al., 2016). Therefore, teachers should be able to identify the level at which students are functioning using the trajectories of geometric thought development outlined by the van Hiele's theory (Clements, 2003). Teachers should then offer learning activities that match with students' van Hiele levels of geometric thought and provide opportunities for them to explore and construct geometric ideas, which would promote their advancement to the next higher level (Battista, 2007). Teachers' knowledge of geometric concepts, properties, and relationships play a critical role in identifying students' geometric thought levels and carrying out the instructional activities appropriately (Ball et al., 2008; Fennema & Franke, 1992). For these reasons, we focused on pre-service elementary teachers' GCK, KGS, and KGT in this study as key knowledge domains for teaching 2D shapes.

The van Hiele theory and related studies guided us to determine the specific contents of the three knowledge domains for teaching 2D shapes at the elementary level. First, GCK is about teachers' understanding of concepts and properties of 2D shapes and their relationships within a shape and among other shapes presented at each level of the developmental trajectory of students' geometric thought (Sztajn et al., 2012). Second, KGS is about teachers' knowledge of the varied trajectory levels that students progress through from less to more complex geometric thinking. Particularly, it centers around teachers' knowledge and skills to identify students' level of geometric thought as conceptualized by the van Hiele's theory. Third, KGT is about teachers' understanding of various geometric activities that support the progressive development of students' geometric thinking at each level of the van Hiele model. These contents further helped us develop items in the GKT-2D scale as a tool to measure pre-service elementary teachers' geometry knowledge for teaching.

LITERATURE REVIEW

The instruments developed based on the van Hiele theory and MKT framework to assess pre-service elementary teachers' geometry knowledge for teaching comprehensively and reliably were limited. To situate our study, we reviewed the literature for studies on pre-service teachers' geometry knowledge for teaching 2D or 3-dimensional (3D) shapes and the measurements used in these studies. Our review of relevant studies led to several findings.

First, existing studies generally focused on pre-service teachers' GCK with the finding that their overall understanding of geometric concepts is limited and weak (Browning et al., 2014; Fujita, 2012; Menon, 1998; Pickreign, 2007; Reinke, 1997). For example, Fujita (2012) examined pre-service teachers' understanding of quadrilaterals using a questionnaire focused on inclusion relations of quadrilaterals. The study found that most pre-service teachers knew the definition of a parallelogram, but many could not identify rhombi, rectangles, and squares as parallelograms. Pre-service teachers in the study often identified parallelograms only when they were presented as a slanted quadrilateral with one short and one long pairs of opposite sides parallel. Other studies also revealed that pre-service teachers were not able to articulate the meaning of a rectangle and a rhombus (Menon, 1998; Pickreign, 2007).

Second, studies examining pre-service teachers' geometry content understanding mostly assess their van Hiele thinking levels, suggesting that they do not have adequate GCK for the grade level they are expected to teach (Halat, 2008; Knight, 2006; Lin et al., 2011; Marchis, 2012; Mayberry, 1983; Yilmaz & Koparan, 2016). Marchis (2012) examined 36 pre-service elementary teachers' van Hiele level using a set of problems related to 2D and 3D shapes. They found that over 66% of participants were only at van Hiele Level 1 (*analysis*). Specifically, these pre-service elementary teachers could recognize 2D shapes (e.g., parallelograms, rectangles, rhombi). However, they could not correctly provide definitions of these 2D shapes or comprehensively list all of their properties. Regarding 3D shapes (e.g., cubes, square pyramids, tetrahedrons), more than a third could not correctly draw corresponding 2D representations or nets. Halat (2008) analyzed the van Hiele geometry test (VHGT) scores from 125 elementary and 156 secondary pre-service teachers. They found that most pre-service elementary teachers could reason geometrically at or above Level 1 (*analysis*). Many secondary pre-service teachers were only at Level 2 (*informal deduction*). Using 44 elementary and 22 secondary pre-service teachers' VHGT scores, Knight (2006) came to similar findings that their van Hiele levels failed to reach the level that was needed to teach students completing 8th and 12th grade, respectively.

Two studies examined pre-service and in-service teachers' KCS and KCT under van Hiele's theory. Fuys et al. (1988) provided eight pre-service and five in-service teachers with instructional modules based on van Hiele's theory. They found that participants engaged in the modules were able to recognize van Hiele levels of students' thinking and curriculum materials, which is critical for supporting students' geometric thinking development according to the van Hiele theory. Another study (Robichaux-Davis & Guarino, 2016) assessed pre-service elementary teachers' geometry content knowledge, geometry pedagogical content knowledge (GPCK), and spatial reasoning skills at the first three van Hiele levels using four items for each domain. They indicated that the pre-service teachers lacked geometry content and pedagogical content knowledge. They were primarily at Level 0 (*visualization*) and Level 1 (*analysis*). In addition, many of them incorrectly responded to GPCK, which asked them to identify appropriate tools and manipulatives for geometry instruction and assess students' understanding of the inclusion relation among various quadrilaterals.

Third, only a few studies evaluated psychometric properties of the instrument that they used to assess elementary teachers' geometry knowledge for teaching (Esendemir & Bindak, 2019; Mayberry, 1983; Usiskin, 1982). However, these measures assessed teachers' geometry content knowledge only (Mayberry, 1983; Usiskin, 1982) or included a small number of items designed to assess teachers' pedagogical content knowledge (Esendemir & Bindak, 2019). For example, Maybery (1988) conducted several analyses to test the reliability and validity of the 128 questions developed to assess pre-service teachers' geometry content knowledge. Usiskin (1982) also conducted reliability and item analysis for the VHGT instrument, including 25 multiple-choice items designed to identify van Hiele's thinking level, which researchers used to assess pre-service and in-service teachers' geometry content understanding (Halat, 2008; Knight, 2006). More recently, Esendemir and Bindak (2019) examined psychometric properties using the Turkish version of the MKT-geometry (MKT-G) test, developed by the Learning Mathematics for Teaching Project grounded on the work of Hill et al. (2004) (http://www.umich.edu/~lmtweb/history.html). The MKT-G test includes 23 items for measuring pre-service teachers' geometry CCK, four items for assessing SCK, two items for assessing KCT, and one item for assessing KCS. Esendemir and Bindak (2019) analyzed the data collected from 243 elementary mathematics teachers' responses to the MKT-G test via item response theory and tested the reliability and validity of the instrument. Other studies developed to examine teachers' knowledge of student thinking or geometric activities often used a qualitative research design with fewer participants (Fuys et al., 1988; Nason et al., 2012).

In summary, the existing studies primarily examined pre-service elementary teachers' subject matter knowledge for teaching geometry (e.g., CCK or SCK), failing to investigate their pedagogical content knowledge for teaching geometry. Studies that quantitatively measured pre-service elementary teachers' geometry knowledge for teaching on the other important domains in the MKT framework, such as KCS and KCT, were rare. In addition, only a few studies evaluated the psychometric properties of the instrument that they used to assess teachers' geometry knowledge for teaching. And, these instruments were developed to mainly assess teachers' geometry content understanding or included only one or two items to assess teachers' KCS and KCT. These limitations in the existing studies prevent a more inclusive, carefully conceptualized, valid, and reliable capture of pre-service elementary teachers' geometry knowledge for teaching. This study was specifically designed to develop an instrument to measure such knowledge with serious attention to both subject matter knowledge and pedagogical content knowledge for teaching 2D shapes as conceptualized by the MKT framework and van Hiele theory. It evaluates the instrument's item-and scale-level reliability and hypothesized factor structure on a larger sample of pre-service elementary teachers.

METHODS

Participants

Participants of this study were 307 pre-service elementary teachers enrolled in a mathematics methods course at a southwestern university in the U.S. Most of them were White (47.2%) or Hispanic/Latino (36.5%) and females (87.3%) between 18-24 years old (67.8%). This reflects the typical population of U.S. pre-service teachers (Zumwalt & Craig, 2005) and the growing number of Hispanic teachers in the southwest region (Taie et al., 2017).

One of the learning modules in the mathematics methods course addressed geometry. During the 3-week module, participants completed a sequence of instructional activities designed based on the van Hiele theory to help participants increase their

(1) understanding of concepts, properties, and relationships of 2D shapes (i.e., GCK),

(2) abilities to identify students' van Hiele thinking levels based on descriptions of students' reasoning about 2D shapes (i.e., KGS), and

(3) skills to select and implement geometric activities appropriately aligned to students' van Hiele thinking levels (i.e., KGT).

In this study, we focused on van Hiele Levels 0-2 because elementary teachers (EC-6) should be prepared to teach geometry topics at these levels according to the curriculum standards for geometry (e.g., common core state standards, Texas essential knowledge and skills). Specifically, teachers are expected to help students identify and create 2D shapes (Level 0), understand attributes of 2D shapes and sort 2D shapes based on attributes (Level 1), and classify 2D shapes into sets and subsets. For more details on the instructional activities and their underlying design principles, see Yi et al. (2020).

GKT-2D Scale and Data Collection

The GKT-2D scale was originally designed to examine the influence of van Hiele theory based instructional activities on pre-service elementary teachers' geometry knowledge for teaching 2D shapes (i.e., GCK, KGS, and KGT). The first and third authors are mathematics education faculty members in the teacher education program from which participants were drawn. They have been teaching elementary mathematics methods for over eight years. These authors drafted test items for teaching 2D shapes for EC-6 grades aligned with state and national curriculum standards (Council of Chief State School Officers & National Governors Association, 2010; Texas Education Agency, 2013), the van Hiele theory (van Hiele, 1984), and the MKT framework (Ball et al., 2008).

The drafted items were pilot tested via informal interviews with pre-service teachers (n=143) to ensure whether the questions were clearly stated, and the response choices were relevant, comprehensive, and mutually exclusive. Further, their responses on the items were analyzed to identify potentially problematic items based on facility index (FI: % of respondents that answer the item correctly) and discrimination index (DI: the extent to which the item discriminates between respondents in different knowledge levels). For example, items having too low or too high FI (<5% or >95%) and/or weak or negative DI (<30%) were removed or revised (Abellan & Ginovart, 2010). The revision through this pilot testing and item analysis resulted in a revised (current) version of 24 multiple-choice items in three subscales: GCK, KGS, and KGT.

Table 1. Item subscales, concepts tested, and associated van Hiele levels of the GKT-2D instrument

Subscale	Item	Concepts tested	van Hiele level
GCK	1	Properties of squares	1
	2	Properties of rectangles	1
	3	Properties of rhombi	1
	4	Relationships between rectangles and parallelograms	2
	5	Relationships between rhombi and parallelograms	2
	6	Relationships between rhombi and squares	2
	7	Relationships among various quadrilaterals	2
	8	Properties of parallelograms	1
	9	Similarities between isosceles trapezoids and rectangles	2
KGS	10	Student's understanding of rectangles	0, 1
	11	Student's understanding of triangles	0
	12	Student's understanding of rectangles	0
	13	Student's understanding of rectangles and squares	1
	14	Student's understanding of rectangles and squares	2
	15	Student's geometry thinking development	All
	16	Student's geometry learning	All
	17	Student's geometry learning	All
	18	Student's understanding of rhombi and parallelograms	2
KGT	19	Instructional activities for squares	All
	20	Instructional activities for triangles	0
	21	Instructional activities for triangles	1
	22	Instructional activities for rhombi and parallelogram	2
	23	Instructional activities for rectangles	1
	24	Instructional activities for squares and rhombi	2, 0

Compared to our previous version of the GKT-2D scale (Yi et al., 2020), we developed six new items to establish stronger reliability and validity of the instrument. Specifically, three new items were added to assess pre-service teachers' GCK related to the properties of parallelograms (Level 1), the relationship between isosceles trapezoids and rectangles (Level 2), and between parallelogram, rectangles, rhombi, and squares (Level 2). Similarly, we added three items to assess pre-service teachers' knowledge of students' geometric thought development and their knowledge of geometric activities. Each subscale has at least 2-3 items aligned with each van Hiele Level (0, 1, and 2) regarding various 2D shapes addressed in the EC-6 geometry curriculum. Consequently, this newly developed instrument provides better measurements of pre-service elementary teachers' geometry knowledge for teaching across different domains, van Hiele levels, and content compared to the previous version of the instrument. See **Table 1** for the target concepts and van Hiele levels of individual items; sample items for each subscale are provided in **Appendix A**.

First, the GCK subscale has nine items to assess pre-service elementary teachers' content understanding related to the properties of 2D shapes (Level 1) and relationships of properties within a shape and between shapes (Level 2). Our previous studies and pilot tests showed that pre-service elementary teachers were above van Hiele Level 0. Thus, the GCK subscale items targeted to assess pre-service elementary teachers' understanding of geometry content aligned with van Hiele Levels 1 and 2. For example, item 2 asks pre-service elementary teachers to select true statements about rectangles, and item 6 asks them to determine appropriate conclusions about the relationships between squares and rhombi. These two items were used to assess pre-service teachers' GCK at van Hiele Levels 1 and 2, respectively.

Second, the KGS subscale included nine items to measure pre-service elementary teachers' knowledge of students' geometric thinking development and their ability to evaluate students' van Hiele Levels from 0 to 2. For example, item 14 asks pre-service elementary teachers to identify a student's van Hiele thinking level based on a given scenario, "David makes the observation that all squares are rectangles, but not all rectangles are squares. What van Hiele level is David regarding squares and rectangles?" This item was developed to assess pre-service teachers' knowledge of identifying students at van Hiele Level 2.

Lastly, the KGT subscale consisted of six items used to measure pre-service elementary teachers' competency to select instructional activities appropriate to students' geometry thinking levels. For instance, item 23 asks pre-service teachers to select instructional activities aligned with a student's level of geometric thought based on a given scenario, "Mrs. Ball showed a figure (a rectangle with one short and one long pairs of opposite sides) to her class and asked, 'What type of figure is this and how would you describe it to your friend?' Amy answered, 'It is a rectangle, and it has four sides, closed, two long sides, two shorter sides, opposite sides parallel, four right angles…' Which instructional activity would be the most appropriate for Amy's van Hiele level of geometric thinking in relations to rectangles?" This item was particularly used to assess pre-service teachers' knowledge of geometric activities appropriate to students at van Hiele Level 1.

The participants (307 pre-service elementary teachers) completed the GKT-2D scale before and after the geometry module of the mathematics methods course (i.e., pre- and post-test). Their responses were analyzed to examine the scale's internal consistency and hypothesized factor structure.

Data Analysis

Item analysis and confirmatory factor analysis (CFA) were conducted to evaluate the reliability and validity of the GKT-2D scale. The analyses were performed using R (R Core Team, 2019) and M*plus* 8.0 (Muthén & Muthén, 1998-2017), with the pre- and post-test data.

Item analysis

We calculated Cronbach's alpha for the GKT-2D scale and its subscales, GCK, KGS, and KGT (Cronbach, 1951). An alpha value of .70 is considered as indicating acceptable internal consistency reliability (>.90=excellent, >.80=good; George & Mallery, 2003; Nunnally & Bernstein, 1994). Because raw scores are summed over the items to produce scale and subscale scores, this study reported unstandardized alpha values.

In addition to this scale- and subscale-level investigation, corrected item-total correlations were computed to assess reliability at the item level. A small correlation (<.20 or .30) indicates that the item does not measure the same construct as measured by the other items (Everitt & Skrondal, 2010).

Confirmatory factor analysis

CFA was conducted to examine the hypothesized factor structure–i.e., the relationships of the observed variables (GKT-2D items) with the underlying constructs (factors: GCK, KGS, and KGT) as well as the associations among those factors. **Figure 1** depicts the hypothesized factor structure of the GKT-2D scale (initial model). We focused on

(a) the items' loading onto their respective factor and

(b) the factor structure's conformity to the current data (i.e., model goodness-of-fit, factor correlations).

Given that the observed variables were binary (correct/incorrect), the weighted least squares means and variance adjusted (WLSMV) in M*plus* 8.0 (Muthén & Muthén, 1998-2017) was utilized for parameter estimation.

The factor loadings of the GKT-2D items were examined for item reliability. An item with an absolute standardized loading less than .30 was considered to

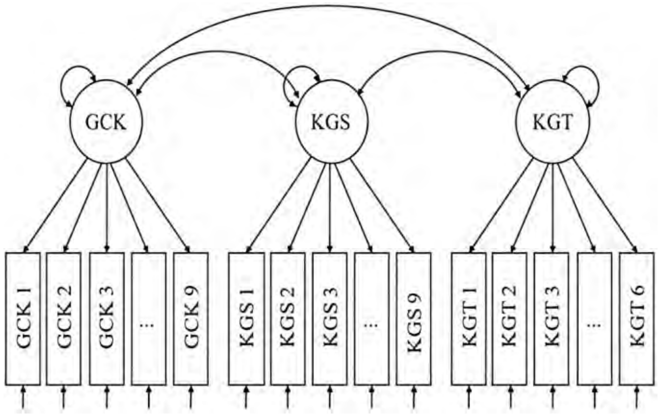

Figure 1. Hypothesized CFA model for the GKT-2D scale

have low reliability because more than 90% variance of the item response was unique and therefore unexplained by the construct (Brown, 2015).

We evaluated model-data fit using both incremental and absolute fit measures: relative model Chi-square (χ^2/df; Wheaton et al., 1977), comparative fit index (CFI; Bentler, 1990; Bentler & Bonett, 1980), and Tucker-Lewis index (TLI; Tucker & Lewis, 1973); and root mean square error of approximation (RMSEA; Steiger & Lind, 1980) and weighted root mean square residual (WRMR; Muthén & Muthén, 1998-2017).

Relative model Chi-square is an index of overall model-data fit and an acceptable χ^2/df value ranges from 2 (Tabachnick & Fidell, 2007) to 5 (Wheaton et al., 1977). CFI and TLI measure how well the hypothesized model fits the data as compared to the null model of uncorrelated variables. A value of CFI and TLI larger than .95 indicates a well-fitting model and a value between .90 and .95 is acceptable (Hu & Bentler, 1999). RMSEA and WRMR consider both the model parsimony (i.e., *df*) and the disparity between observed and estimated covariance matrices of the data. An RMSEA value smaller than .05 (Hu & Bentler, 1999) indicates good fit (<.08=fair, <.10=mediocre; MacCallum et al., 1996). A WRMR value smaller than 1.00 is acceptable (Muthén & Muthén, 1998-2017).

RESULTS

Item and Scale Reliability

To answer our first research question about whether the GKT-2D could reliably and accurately measure pre-service elementary teachers' GCK, KGS, and KGT, we examined Cronbach alpha, item-total correlations, and factor loadings with the data from both pre-test and post-test (**Table 2**).

The Cronbach's alpha values at pre-test and post-test were .41 and .55 for GCK, .27 and .30 for KGS, .17 and .35 for KGT, and .50 and .64 for the overall scale, respectively. The results indicated that the current study

Table 2. Item statistics of GKT-2D (24 items)

Subscale (α, pre/post)	Item	Pre-test				Post-test			
		Item-total correlation	Standardized loading	SE	p	Item-total correlation	Standardized loading	SE	p
GCK	1	.28	.60	.08	<.001	.24	.52	.09	<.001
(.41/.55)	2	.30	.66	.09	<.001	.31	.50	.08	<.001
	3	.18	.40	.10	<.001	.30	.52	.09	<.001
	4	.26	.51	.09	<.001	.28	.51	.08	<.001
	5	.29	.50	.09	<.001	.24	.45	.09	<.001
	6	.14	.32	.10	<.01	.26	.46	.09	<.001
	7	-.00	-.06	.11	.63	.04	.09	.11	.41
	8	.07	.08	.15	.59	.31	.54	.08	<.001
	9	-.02	.10	.10	.31	.26	.49	.08	<.001
KGS	10	.06	.27	.11	.01	-.02	.26	.10	<.01
(.27/.30)	11	.11	.39	.10	<.001	.19	.44	.09	<.001
	12	.09	.40	.11	<.001	.05	.02	.09	.85
	13	.17	.32	.10	<.01	.11	.31	.09	<.001
	14	.19	.56	.10	<.001	.16	.56	.09	<.001
	15	.14	.18	.10	.07	.32	.62	.10	<.001
	16	.19	.36	.12	<.01	.18	.35	.13	<.01
	17	.05	.12	.14	.38	.08	.03	.17	.84
	18	-.10	-.11	.10	.28	.07	.15	.09	.09
KGT	19	.10	.54	.15	<.001	.09	.53	.10	<.001
(.17/.35)	20	.07	.21	.11	.06	.20	.43	.09	<.001
	21	.04	.35	.17	.04	.17	.27	.11	.02
	22	.00	-.08	.14	.54	.11	.15	.12	.22
	23	.19	.16	.11	.14	.15	.41	.10	<.001
	24	.11	.22	.14	.13	.23	.55	.10	<.001

sample did not fully support the (internal consistency) reliability of the original GKT-2D scale.

To locate the sources of the scale-level unreliability, we found that seven items out of 24 had a low corrected item-total correlation (r<.20) and a low standardized factor loading (λ<.30) at both pre-test and post-test. They included one GCK item, four KGS items, and two KGT items. Removing these items improved the reliability of reduced GKT-2D (17 items) at both the scale and item levels. The Cronbach alpha values at pre-test and post-test were .45 and .58 for GCK, .34 and .37 for KCS, .35 and .34 for KGT, and .54 and .67 for the overall scale, respectively. At post-test, the internal consistency reliability of the reduced GKT-2D scale was near "acceptable" (close to .70). Thus, we removed the unreliable seven items and used the data on the remaining 17 items for the subsequent analysis as a data-driven approach to potential modifications in the measure.

Conformity of Reduced GKT-2D Factor Structure with the Current Study Sample

To answer our second research question about whether the hypothesized associations of the GKT-2D items with their target constructs (GCK, KGS, and KGT) are supported in the current sample, we conducted CFA. The CFA results verified the hypothesized factor structure of the reduced 17-item GKT-2D scale. First, all factor loadings were positive as expected, and they were significant at the .05 alpha level with only a few exceptions at pre-test.

As shown in **Table 3**, especially at post-test, all items had a standardized factor loading greater than .30. The only exception was item 13, but its loading was close to the cut-point. In general, the GCK items produced higher loadings and thus greater measurability of the construct (mean=.45, median=.51) compared to the KGS items (mean=.43, median=.41) and KGT items (mean=.39, median=.41).

Second, the CFA model with 17 items demonstrated an adequate fit at both pre-test and post-test. For the pre-test data, the χ^2/df value (1.18) was smaller than the lower cut-point (2). The CFI (.89) and TLI (.87) values were very close to the cut-point of .90. The WRMR value (.90) was smaller than the cut-point of 1. The RMSEA value (.03) and the upper value of its 90% confidence interval (CI=.00-.04) were less than .05, indicating a close fit of the hypothesized model. Similarly, all values of the fit measures were satisfactory in the post-test–χ^2/df=1.31, CFI=.91, TLI=.90, WRMR=.94, and RMSEA=.03 [.02; .05].

Third, all factor correlations were significant and indicated positive associations among the GCK, KGS, and KGT constructs as hypothesized. We found that the pattern of the factor correlations was somewhat different before and after the geometry module instruction. In the pre-test, the association between the KGS and KGT constructs (ψ=.90) was the strongest, followed by the

Table 3. Factor loadings of reduced GKT-2D (17 items)

Subscale	Item	Pre-test			Post-test		
		Standardized loading	SE	p	Standardized loading	SE	p
GCK	1	.58	.08	<.001	.56	.09	<.001
	2	.64	.09	<.001	.55	.08	<.001
	3	.38	.10	<.001	.51	.09	<.001
	4	.45	.09	<.001	.48	.08	<.001
	5	.53	.09	<.001	.44	.08	<.001
	6	.36	.10	<.001	.45	.09	<.001
	8	.14	.15	.36	.51	.08	<.001
	9	.03	.10	.76	.51	.08	<.001
KGS	11	.40	.10	<.001	.50	.09	<.001
	13	.35	.10	<.01	.28	.09	<.01
	14	.60	.10	<.001	.62	.09	<.001
	15	.21	.10	<.05	.65	.09	<.001
	16	.32	.13	<.05	.41	.13	<.01
KGT	19	.60	.17	<.001	.55	.10	<.001
	20	.20	.11	.06	.46	.09	<.001
	23	.21	.11	<.05	.36	.11	<.001
	24	.19	.14	.16	.57	.11	<.001

Table 4. Factor correlations among three constructs of reduced GKT-2D

Construct	Pre-test			Post-test		
	1	2	3	1	2	3
1. GCK	.45			.58		
2. KGS	.65***	.34		.77***	.37	
3. KGT	.66**	.90**	.20	.66***	.89***	.34

Note. Cronbach's alpha for subscale is presented on the diagonal; *p< .05, **p< .01, ***p< .001

associations between the GCK and KGT constructs (.66) and between the GCK and KGS constructs (ψ=.65) that were similar in strength. In the post-test, the association between the KGS and KGT constructs (ψ=.89) was still the strongest association. However, the association between the GCK and KGS constructs (ψ=.77) considerably increased after the instruction, making it stronger than the association between the GCK and KGT constructs (ψ=.66) (**Table 4**).

In summary, the CFA model was supported with the current study sample at each measurement time point (pre-test and post-test), suggesting that the reduced GKT-2D' factor structure adequately represents GCK, KGS, and KGT in pre-service elementary teachers.

DISCUSSION

This study has some limitations. First, it ended up with a small number of test items (17), which could underestimate the reliability and validity of reduced GKT-2D. Second, since our participants were all from one teacher education program, our findings may not be generalized to different samples and contexts. Third, it mainly focused on the construct validity, the extent to which the measure can accurately represent the constructs of GCK, KGS, and KGT items, no other kinds of validity (e.g., predictive validity). Despite its limitations, it does contribute to several understandings relevant to examining the reliability and validity of the GKT-2D scale, which is developed to measure pre-service elementary teachers' geometry knowledge for teaching 2D shapes.

First, this study contributes to the development of an inclusive, conceptualized, mathematics topic-based, and reliable measure to capture pre-service elementary teachers' both content and pedagogical content knowledge central to effective geometry teaching and student geometry learning. In developing this measure, we focused on three domains of geometry knowledge for teaching 2D shapes, including GCK, KGS, and KGT.

Developing reliable measures of teachers' mathematical knowledge for teaching has been attempted on the topics of numbers, operations, and algebra at the elementary level (Hill et al., 2004) and geometry at the middle or high school levels (Brakoniecki et al., 2016; Herbst & Kosko, 2014). Such an effort is rare in quantitatively measuring pre-service elementary teachers' geometry knowledge in the existing literature, especially for both content and pedagogical content knowledge for teaching 2D shapes. In contrast, as shown in our review of relevant literature, the direct measure of teachers' geometry knowledge for teaching 2D shapes was often focused on teachers' geometry content understanding (Halat, 2008; Knight; 2006; Mayberry, 1983) or without validated tools (Fujita, 2012; Marchis; 2012; Pickreign, 2007; Reinke, 1997).

Second, this study shows that the reduced GKT-2D scale reasonably measures three different kinds of geometry knowledge for teaching 2D shapes: GCK, KGS, and KGT. This finding supports the existence of the three kinds of geometry knowledge for teaching conceptualized by the van Hiele theory and relevant research studies (Ball et al., 2008; Sztajn et al., 2012; van Hiele, 1984). The fitted CFA model produced a good model fit and adequate loadings of the items onto the construct designed to measure. However, the reliability of the reduced GKT-2D scale was not fully supported by the current study sample based on the Cronbach's alpha value, which could have been because the number of items was small. So, further work is needed to focus on developing additional items and testing their reliabilities.

Third, this study demonstrates that strong associations exist between KGS and KGT, between GCK and KGS, and between GCK and KGT constructs both at pre-test and post-test levels. As shown in the finding of this study, the correlation values between the items of KGS and KGT, between the items of GCK and KGS, and between those of GCK and KGT constructs are all positive and significant. This result resonates with the findings from the Hill et al. (2004)'s study that teachers' common and specialized content knowledge is associated with their KCS for teaching numbers and operations, patterns, functions, and algebra topics. However, the factor correlation between the KGS and KGT subscales was higher than optimal, alarming an issue for discriminating these two kinds of knowledge (Ball et al., 2008). Most of the items in the KGT subscale were designed to measure pre-service teachers' knowledge of geometry instruction appropriate to students' level of geometric thought. To select instructional activities accurately, pre-service teachers should understand students' geometric thinking levels, which, we understand, led to a high association between KGS and KGT subscales. Thus, further work for refining the items is needed to improve discriminant validity, especially for the KGS and KGT subscales.

Although further work is needed to overcome the weaknesses of the reduced GKT-2D instrument, this study greatly contributes to building a knowledge base to quantitatively measure pre-service elementary teachers' content and pedagogical content knowledge for teaching geometry. Future research needs to investigate whether and to what extent pre-service elementary teachers use the three kinds of geometry knowledge for teaching (i.e., GCK, KGS, and KGT) in the EC-6 classroom setting and how their knowledge level in each domain shapes the quality of their geometry teaching. Such research will extend our understanding of the entire link between teacher knowledge, teaching quality, and student learning in geometry.

CONCLUSION

In conclusion, this study suggests that the reduced GKT-2D scale reasonably measures three kinds of geometry knowledge for teaching 2D shapes: GCK, KGS, and KGT needed for quality geometry teaching at the elementary level. Although the current study sample did not fully support the internal consistency reliability of the reduced GKT-2D scale, overall, the reduced GKT-2D has a great potential to be used to measure teachers' both content (i.e., GCK) and pedagogical content knowledge (i.e., KGS and KGT) for teaching 2D shapes conceptualized by the MKT framework and van Hiele theory.

This study also revealed strong and positive associations between any two kinds of geometry knowledge for teaching 2D shapes (i.e., GCK and KGS, GCK and KGT, KGS and KGT). Consequently, it implies that mathematics teacher educators actively build on the associations between these knowledge domains in their curriculum and teaching design so that pre-service teachers can develop necessary mathematical knowledge for teaching. It further indicates that by carefully designing pre- and post-test to assess teachers' mathematics knowledge for teaching with a focus on its specific domains, teachers' development of such knowledge in the context of their exposure to the instructional activities can be reliably captured. Equipped with such measures, mathematics teacher educators will effectively understand the significant relationships between teacher knowledge, teaching practice, and student learning and use this understanding to guide their effort to improve mathematics teacher preparations (Charalambous & Pitta-Pantazi, 2015). Thus, it is important to extend the efforts to measure teacher geometry knowledge for teaching 2D shapes to other geometry and mathematical topics that pre-service elementary teachers are expected to teach.

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APPENDIX A

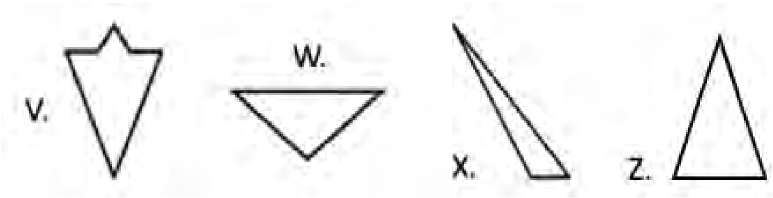
Sample Items

Item 5. Using the statement below, which conclusions can you make? (**GCK**)

- Statement 1: Shape A is a parallelogram
- Statement 2: Shape A is a rhombus

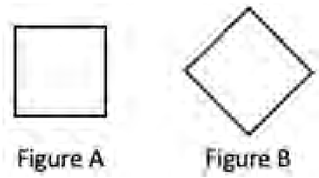
A. Statement 1 and 2 cannot both be true.
B. If statement 1 is true, then statement 2 is true.
C. If statement 2 is true, then statement 1 is true.
D. If statement 1 is false, then statement 2 is true.
E. Statement 1 and 2 cannot both be false.

Item 11. Suppose that Carol is at van Hiele Level 0 with her understanding of triangles. Which of the shape below might Carol says is NOT a triangle? (**KGS**)



A. Shape V B. Shape W C. Shape X D. Shape Z E. All of the above

Item 24. Mrs. Lee showed **Figure A** to her 2nd grade student, Jacob, and he said, "It is a square." When Mrs. Lee turned the square 45 degrees (**Figure B**), Jacob said, "Now it is a rhombus because it looks like a diamond."



All of the following activities would be the most appropriate for Jacob's level of geometric thought, EXCEPT for? (**KGT**)

A. Manipulating and constructing geometric shapes
B. Identifying a shape in a simple drawing
C. Comparing shapes according to their characterizing properties
D. Identifying a shape in a variety of orientations
E. Describing geometric shapes using standard and non-standard language

Note. Items are available upon request.