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EXPLORING INITIAL MATHEMATICAL MODELLING PROCESS USING GEOGEBRA OF FORM FOUR STUDENTS

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ABSTRACT

This study takes a closer look at Form Four students' initial mathematical modelling process when solving mathematical problems using GeoGebra. The modelling process was observed from cognitive perspective to learn the difficulties students faced during the process and the result would give ideas for intervention. Qualitative approach by using multiple case study method was employed. Four participants from Form Four level without prior experience in mathematical modelling were selected. Three different data collection methods were employed and triangulated as well as compared with the idealised modelling cycle to reconstruct individual students' initial mathematical modelling process. Findings from this study showed the importance of situation model and real model with extra-mathematical knowledge (EMK), students partially follow the idealised modelling cycle, novice modellers do display intuitive ideas in displaying their mathematical process and GeoGebra's usefulness was learned.

Keywords: *Mathematical Modelling, Modelling Process, Geogebra*

INTRODUCTION

Currently the global education system is gearing toward 21st Century learning which requires students to possess important skills and knowledge in mathematics that enable them to think independently while taking responsibility for their learning (OECD, 2010). Malaysia is going in the same direction with the introduction of the *Secondary School Standard Curriculum (KSSM)* in 2017 to produce students equipped with 21st century skills. In KSSM four key elements *(process, learning scope, value, and skills)* are considered to develop individuals that are mathematically *fikrah*. The key element highlighted in this study will be mathematical process specifically the *representation* process. When students can recognise the relationship between mathematical concepts and use it to model situations, physical and social problems, they have learned to *represent*. By doing so, they will be able to solve problems easily. This mathematical process focused in KSSM is also known as mathematical modelling.

Malaysian students, however, have not been exposed to mathematical modelling throughout their education. In addition, Malaysian students scored below the OECD average, that is out of 80%, total of 51.8% scored below level 1. According to PISA, students obtaining level 1 and below are known as low performers because they can only answer questions if clear directions are given, and they cannot engage in more complex reasoning to solve real life-based questions. They are unable to interpret the results obtained (OECD, 2013, 2016, 2017,2019).

In this study modelling is considered from the cognitive perspective. The main goal of this perspective is to reconstruct individual student's modelling process and enabling teachers to understand the difficulties students face during mathematical modelling activities (Kaiser & Sriraman, 2006). Although cognitive perspectives on modelling were overtaken by other perspectives such as modelling competencies, it should still be emphasised (Cai et al., 2014) because difficulties students faced during modelling processes (Galbraith & Stillman, 2006) can be identified and these give ideas for teacher's intervention (Blum & Leiss, 2007). In addition, student approach to modelling problems is less known, meaning there is still much to learn from the cognitive perspective on mathematical modelling (Ferri, 2006).

As it is challenging to implement modelling during lessons, additional help would be useful, such as inclusion of technology in exploring mathematical modelling tasks. GeoGebra can be helpful in exploring geometry by minimising the algebra work through using the software (Carreira, Amado, & Canário, 2013) and when given access to Internet it encourages students to face challenging questions especially secondary level students to construct, understand as well as validate a complex model (Villarreal, Esteley, & Mina, 2010).

Some past studies have investigated the student modelling process from the cognitive perspective (e.g., Ferri, 2015; Greefrath & Siller, 2017; Ortiz & Dos Santos, 2011), but we need to be aware that some countries such as Singapore and Canada include mathematical modelling as part of their education curriculum, unlike in Malaysia where students are not exposed to mathematical modelling so there is lack of research in this area. Some of the research done in Malaysia are by Leong (2013); Leong and Tan (2015, 2016); but there is lack of research done on the Malaysian students' mathematical modelling process. Furthermore, although much has been written about the potential of technology in helping mathematical modelling activities, few studies have explored how students use GeoGebra when solving a mathematical modelling question (Carreira et al., 2013; Frassia & Serpe, 2017; Greefrath & Siller, 2017; 2018).

Thus, in this study reconstruction of individual students' initial modelling process will be done. By knowing students' initial modelling process, teachers can help students to deal with difficulties when answering mathematical modelling questions that will allow for independent learning as required by 21st century education. Apart from that, during which step(s) in the modelling process students use GeoGebra and how they choose to use it will also be discovered. Results from this analysis would help teachers build modelling task incorporating digital tools and for intervention during classroom mathematical modelling activities.

The research objective of this study is to describe the Form Four students' initial mathematical modelling process using GeoGebra Version 6.

This study intends to answer the Research Question:

> "How is Form Four students' initial mathematical modelling process when solving mathematical modelling problems using GeoGebra?".

METHODOLOGY

This study applied a qualitative approach to explore Form Four students' initial mathematical modelling process when solving mathematical modelling problems using GeoGebra. The multiple case study design is bounded by only looking at Form Four students that have basic skills in using GeoGebra, similar mathematical achievement and are novice modellers whose initial mathematical modelling process by using GeoGebra will be discovered. The case study design is chosen for this study as the findings from the study will resonate with teachers and help them understand the idea as well as the process involved during mathematical modelling activity in an in-depth manner. Knowledge about mathematical modelling process will be learned from individual students' experience while attempting the question. Purposive sampling was done by the researcher because the motive of the study is to discover, understand, and gain in-depth insight (Merriam, 2009).

The modelling problem *Are women faster than men?* is adopted from Hall and Lingefjärd (2017, p. 38). This selection as explained by Hall and Lingefjärd (2017) is because knowledge on various representations such as graphs, tables, algebraic and symbolic expression need to be inculcated in students. Questions on linear models are quite easy to answer but important aspects such as relevance of data or suitability of model choice are often neglected by students. Hence, this question will serve as good mathematical modelling question as students need to go through the entire modelling process to give a valid answer.

Are Women Faster Than Men?

Will women outrun men in the future? How fast do you think women will run in 100 years? In 200 years? Below you can see the results from the Olympic 200-meter gold medalists. Use these data and construct mathematical models for predictions and comparisons with respect to women's and men's records for the 200-meter run in future Olympic Games. Then complete the table with results after 1988. How do these new data change your predictions?

Olympic winners in the 200 Meter Sprint

Year	Men	Time (s)	Women	Time (s)
1988	J. DeLoach, USA	19.75	F. Griffith-Joyner, USA	21.34
1984	C. Lewis, USA	19.80	V. Brisco-Hooks, USA	21.81
1980	P. Mennea, Italy	20.19	B. Wöckel, East Germany	22.03
1976	D. Quarrie, Jamaica	20.23	B. Eckert, East Germany	22.37
1972	V. Borzov, Soweet	20.00	R. Stecher, East Germany	22.40
1968	T. Smith, USA	19.83	I. Szewińska, Poland	22.5
1964	H. Carr, USA	20.3	E. McGuire, USA	23.0
1960	L. Berruti, Italy	20.5	W. Rudolph, USA	24.0
1956	B. Morrow, USA	20.6	B. Cuthbert, Australia	23.4
1952	A. Stanfield, USA	20.7	M. Jackson, Australia	23.7
1948	M. Patton, USA	21.1	F. Blankers-Koen, Holland	24.4
1936	J. Owens, USA	20.7		
1932	E. Tolan, USA	21.1		
1928	P. Williams, USA	21.8		
1924	J. Scholtz, USA	21.6		
1920	A. Woodring, USA	22.0		
1912	R. Craig, USA	21.7		
1908	R. Kerr, Canada	22.6		
1904	A. Hahn, USA	21.6		
1900	W. Tewksbury, USA	22.2		

Data Analysis

As stated by Merriam (2009), there are two stages of analysis for multiple case study which are within case analysis and cross case analysis to build generalisation across cases.

In within case analysis each case will be treated as an individual case and comprehensive understanding will be retrieved from the case. In addition, due to multiple sources of data collection, data triangulation was done during within case analysis. The modelling processes observed were triangulated with the transcribed interviews and collected physical artifacts. It was done for each student. Codes were generated at this stage based on the triangulated data.

The codes formed were categorised based on the modelling cycle by Blum and Leiss (2007). The mathematical modelling activities attempted by students were categorised according to the mathematical modelling cycle (Blum & Leiss, 2007) using definition of the activities from Maaß (2006) and Ferri (2018) definition on extra-mathematical knowledge (EMK) was also included during the category generating stage. As for the categories on how the students used GeoGebra, this was done using Greefrath and Siller (2017). This method of reconstructing mathematical modelling process of students using modelling cycle had been successfully used by Sol, Giménez, and Rosich (2011) and Houston (2007) and since this study focuses on cognitive aspect of the mathematical modelling process the Blum and Leiss (2007) modelling cycle would be a good analytical tool. This cycle has been successfully used by some researchers (e.g., Ferri, 2018; Greefrath & Siller, 2017, 2018; Hankeln, 2020; Ortiz & Dos Santos, 2011).

FINDINGS

The question allows the researcher to observe students' mathematical modelling process by using GeoGebra. The large distribution of data would ensure students rely on GeoGebra but how they would use it could differ. Apart from that, this question also displays how significant it is not to make improbable prediction based on simple trends. Although it seems unusual, this question gives the chance for students to analyse the model they formed and critically validate it instead of coming to a mere conclusion. Even though all the students took different approach to this question, all of them displayed EMK at some point of their modelling process, either in written or verbal manner.

The analysis and reconstructed initial modelling process using GeoGebra of each participant will be further elaborated as in the following:

Student 1 (S1)

S1 pondered a while after reading the question because he thought the question was unusual as there is limitation to human's ability regardless of gender. At this stage he displayed EMK because the question required prediction of human speed in the far stretched future and he found it strange as he was aware that limitation in human physical ability needs to be considered. He was trying to resonate with his experience to understand the question. So, he formed the situation model by eliciting important information and variables from the question. In addition, he had a unique take on the question as he treated it as a hypothesis that needed to be proven. In his understanding, the question required him to predict if women can outrun men in the future.

Then he structured his understanding by assigning year and time as x and y coordinate to form the real model. He used the spreadsheet in GeoGebra by inserting the real data (year and time) provided in the question as (*x, y*) coordinates, respectively. Then he used 2 variable regressions that formed best fit lines representing men and women. However, S1 appeared dissatisfied and was in deep thought after analysing the best fit line. He even read through the question again. After some time, he made some changes to the spreadsheet; in another column he inserted new values using the formula *200/time* which gave him the value of speed for every runner. The reason for his action was clarified during interview. According to S1, speed would be a better variable compared to time because the focus of the question is on athlete physical ability; thus it would yield a better result if year were plotted against speed. This shows, with the visualisation aid from GeoGebra, he was able to validate his choice of solution method.

With the changes made, he proceeded to 2 variable regressions and formed a growth curve. The curves were labeled *p(x) = men and q(x) = women*. He continued to analyse the curve by finding the intersection point between the curves and listing the predicted speed obtained from the curve till the year 2016. Based on the data, he found mathematical result that in 2040, both genders might have equal speed of 11.30 ms^{-1}. Following that, he googled real data from 1992 -2016 for Olympics' 200-meter gold medalists and listed the data in GeoGebra's spreadsheet. With the newfound real data, he formed new growth curves *g(x) = men and h(x) = women*. Then he realised that there was slight alteration from the first predicted curve (1992-2016) and the curve formed from real data (1992-2016). Hence, he speculated that it would take centuries for women's speed to exceed men.

Although the question was deemed challenging, it intrigued S1 as various fields were included such as Biology, Psychology and Sociology. This made him interpret the mathematical result to real result by looking for justification which were in the form of newly formed assumptions to help him explain the trend achieved. He also acknowledged all the assumptions were not properly documented factors. So, there are no data to be included to improve the model obtained. Therefore, he does believe his answer made sense as it was still a possibility and when more factors are included in the prediction, it could help answer the myth that men are faster than women. After working on his real model, he ended the modelling process by validating the model. He concluded that with lack of data due to insufficient research he would adhere to the current model.

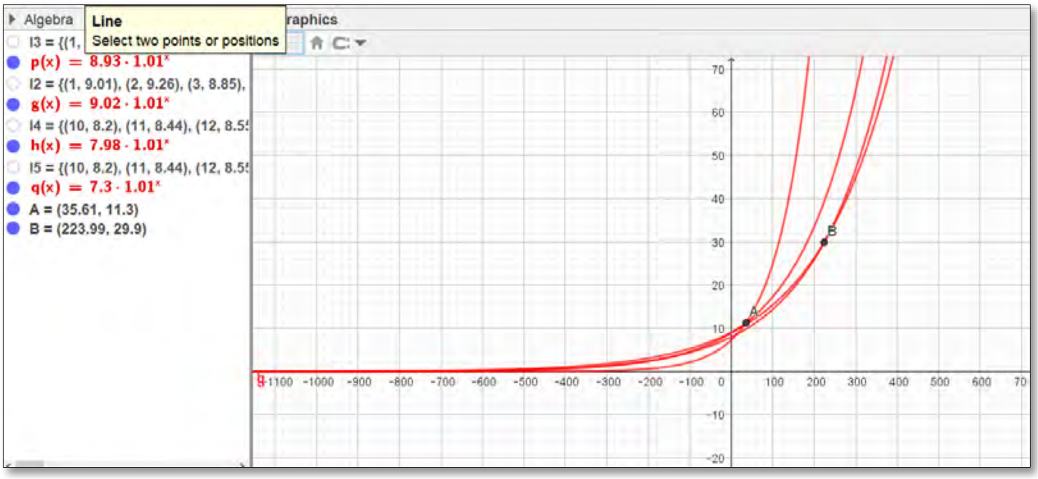

Figure 1. S1's mathematical modelling process

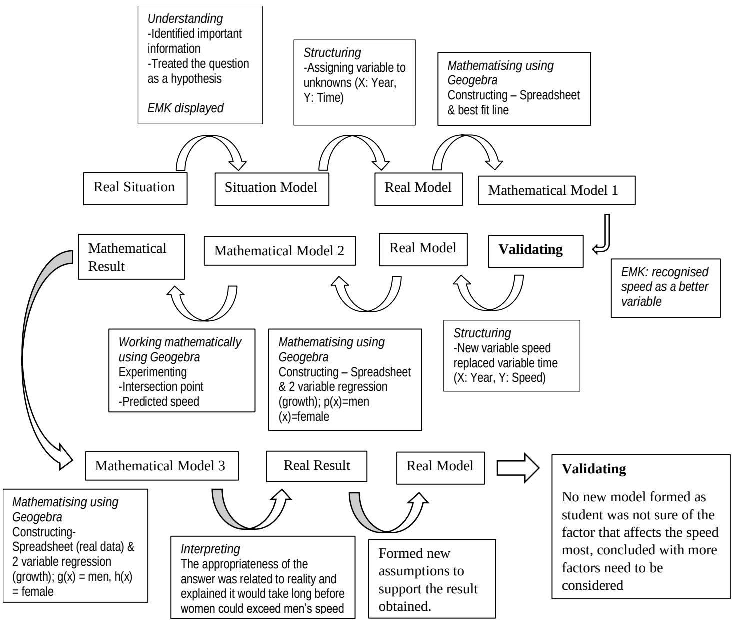

Figure 2. S1 formed growth curve to display the data trend and concluded women will not overtake men

Student 2 (S2)

S2 started by reading the question and she took some time to understand the question. When asked, she explained that the important variable from the question was time, but she tried to understand it by thinking about the relationship between data and reason for the inconsistency pattern of time.

After that, she continued to form the real model by writing assumptions that time recorded by runners will still decrease after 1988 and it will decrease constantly. This assumption was made because she had the idea that she would obtain a straight line. Nevertheless, during interview one of the difficulties she faced during this stage was understood. According to S2, she realised there might be other factors affecting the time obtained but she had no idea on how to incorporate that in the assumption, so she proceeded with the assumptions she wrote but was unsure if it aided her in solving the question. Then, she proceeded with her real model by assigning x as year and y to be time.

Once she assigned the variables, she started using GeoGebra to form a mathematical model by inserting the data into the spreadsheet. However, she was seen struggling a little when she wanted to form the best fit line because she was still exploring the features of GeoGebra. Yet, she managed to find the list feature to group all the coordinates as (year, time obtained by men) and (year, time obtained by women). With that, she successfully formed the best fit line and from the lines obtained, she proceeded to work mathematically for mathematical results by listing the predicted time of year 1992-2016 for men and women, respectively.

Then she tried using internet to find real data for year 1992 till 2016 for Olympic 200-meter gold medalists but she did not use the data that she found in her solution process. This was because the data were on those that broke the record each year, so she was unsure if it would have been useful.

She also faced some strain after obtaining the best fit line because the straight line continued to yield negative values which she felt did not make sense as it would be impossible for time to be negative. She did not proceed to real result as she was unsure on how to interpret the values obtained to reality. Validation occurred during interview, and it was learned that S2 was aware there is limitation to human speed and athletes' condition could affect the running time, but the best fit line continued to predict negative values which she felt did not make sense.

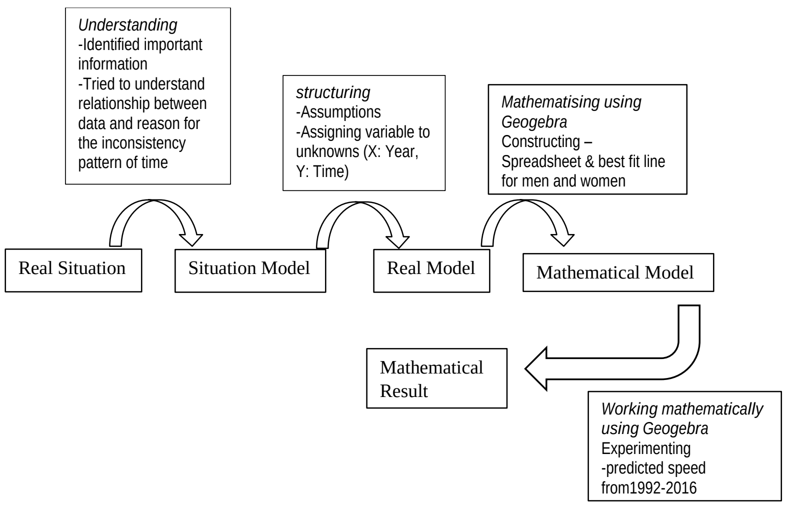

Figure 3. S2's Mathematical Modelling Process for Q

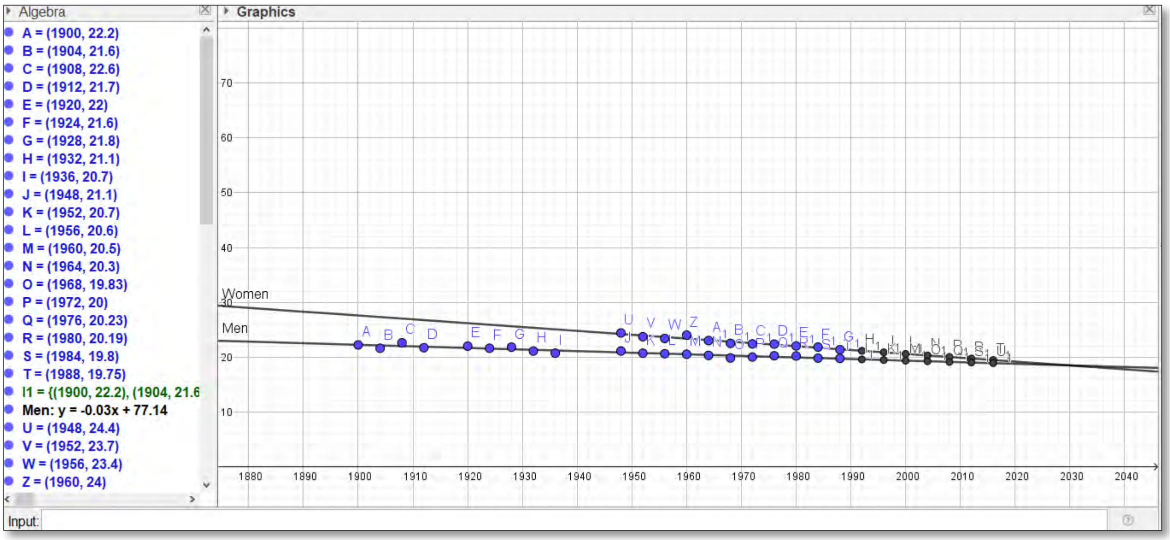

Figure 4. S2 formed best fit line and stopped. She did not produce mathematical result because the negative values obtained did not make sense to her.

Student 3 (S3)

S3 expressed that he did not find the question to be difficult but due to too much information to consider, he found it troublesome. Even so, he was focused during the entire solution process. After reading the question, S3 formed the real model by creating a mental picture of using best fit lines and seeing if they intersect. So, to manifest his mental picture, he assumed the rate at which the results change throughout the years will be approximately constant and he also structured his understanding by assigning variables to unknowns.

Then he used GeoGebra to create his mathematical model. He started by listing points for men and women respectively in the 'spreadsheet'. From the list of points, he used 'best-fit-line' function to obtain best fit lines for the two groups of data. He struggled a bit when he was trying to find the best fit line due to large number of data. Following that, he used 'intersect' feature and he found out that after year 2030, according to the data given and graph plotted, women will outrun men. He then, googled the real data from year 1992-2016 for Olympics' 200-meter gold medalists to be plotted in GeoGebra. After looking at the distribution of the new data, that were far off from the previous predicted line, he decided to try another method. He validated that the mathematical model and result did not depict reality, so he wrote a new assumption to include limitation to human's speed as a factor.

So, he had an idea to form a curve to show the data distribution, but he had no idea on how to do it in GeoGebra, so he ended up making folded line using 'ray' function. With the new real model, he proceeded to form new mathematical model and EMK was displayed at this stage. The new folded line model was a good choice since he was unable to plot a curve. By looking at diverged lines, he found the mathematical result that women might never overtake men. He stopped his modelling process at real result and the validation step only occurred during interview, in which he explained the appropriateness of his answer.

During interview, he expressed that although the question made some sense in reality it was too farfetched to make prediction only by looking at top runners in the world. He felt that average data for both men and women should be considered, and it should be backed by research. Nonetheless, he expressed that GeoGebra was very useful for this question especially in plotting the best fit line.

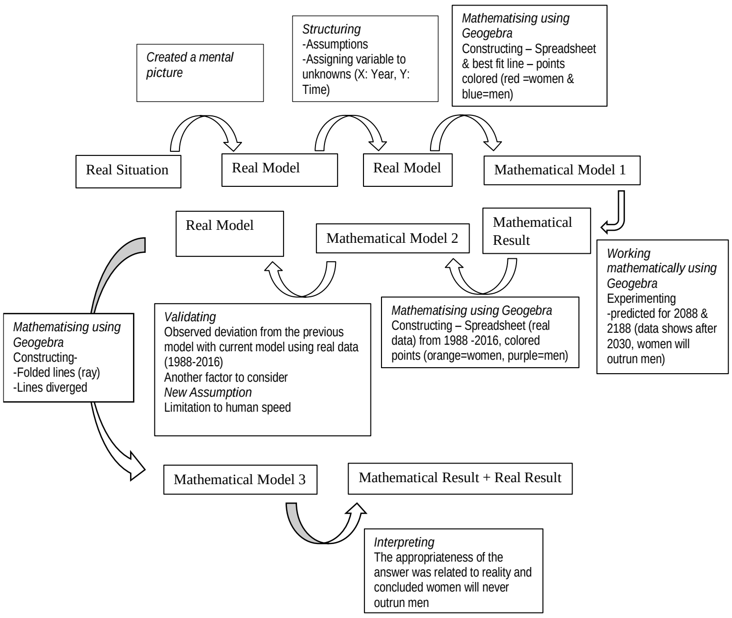

Figure 5. S3's Mathematical Modelling Process for Q2

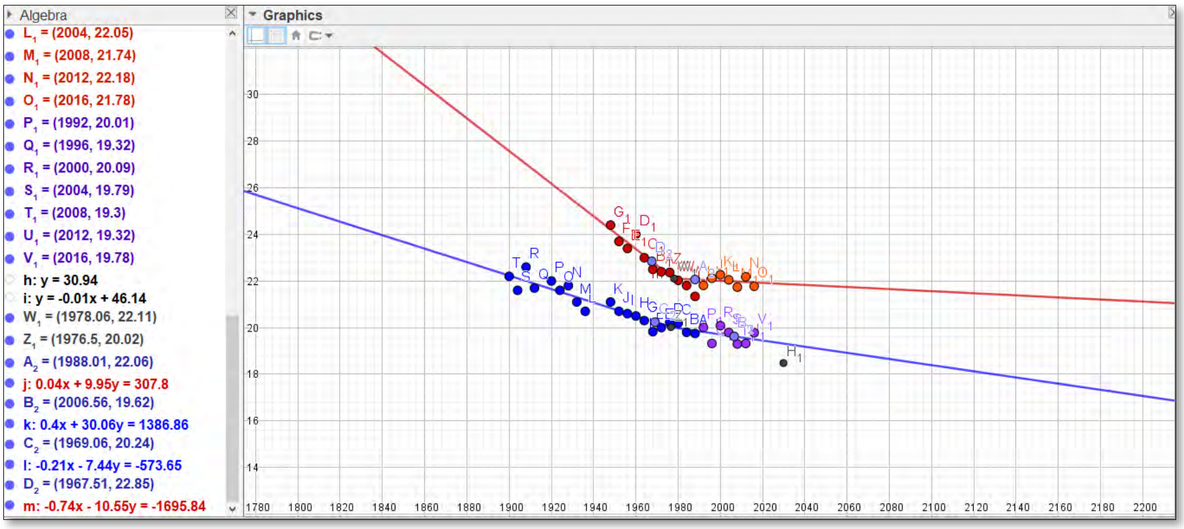

Figure 6. S3 Formed Folded Lines to Display the Data Distribution and Concluded Women Will Not Overtake Men as The Lines Diverged

Student 4 (S4)

After reading the question, S4 formed a real model after reading the question by having the strategy to structure the information to points and create best fit lines to observe the data distribution for both men and women. So, he started structuring his understanding by identifying important information and assigned *x*-axis to be the year and *y*-axis to be the time taken. Following that, he used GeoGebra and started by keying in data from year 1900-1988 into the 'spreadsheet'. Then, he started plotting the points and created a list of points.

Only during interview, the researcher learned that S4 had difficulty using the spreadsheet, but he overcame the difficulty by doing some research online and after learning to use it, he expressed that GeoGebra was very useful because the spreadsheet made creating list of points and constructing best fit line easy with just a simple command in the 'input bar'. Once the points were plotted, he constructed best fit line for men and women; from the line he tried to answer the question. He found intersection for year 2088 and 2188 to find running time for men as well as women. He found the mathematical result and made conclusion based on the model that women will outrun men in the future.

After that, he continued to search online for data from year 1992 till 2016 for Olympic 200-meter gold medalists. Then, all of a sudden, he cleared the entire model he formed and created new best fit lines for men and women incorporating the newfound data. Based on the best fit lines formed from real data for the period 1900-2016, he found that the time predicted for year 2088 and 2188 were longer with the new mathematical model.

It could be seen that his conclusion was solely based on the straight lines he formed. Although, he did not make changes to the result obtained, he was aware that the result he got was inconclusive for future prediction because via interview he expressed that in his method, he did not consider human speed limitation as a factor. Thus, leading him to predict women's running time to be 15.49 seconds in year ??? and women would outrun men in future. This shows he did not consider the shortest time obtained so far in reality for female is 21.34 seconds which shows the mathematical model he obtained could be tweaked further. The method he chose was straightforward as can be seen in the assumption made that with decreasing record time would help him to form the straight line. In addition, it is the usual way tested in examination whereby students are usually not required to relate their answer to reality to see if it makes sense in real life.

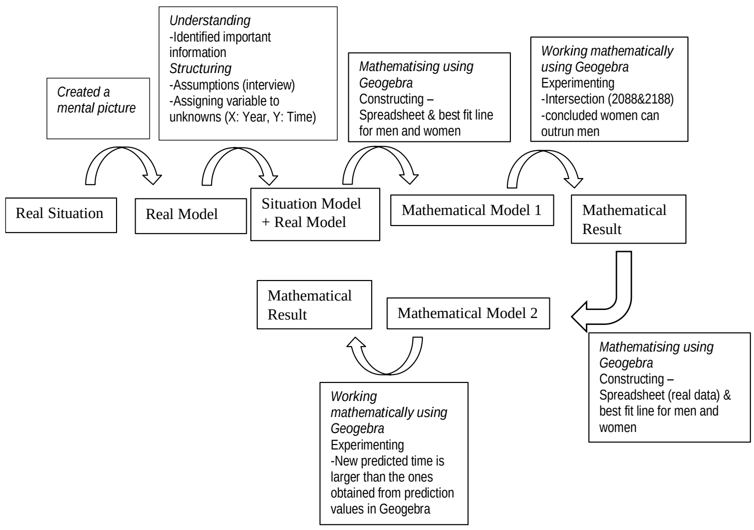

Figure 7. S4's Mathematical Modelling Process for Q2

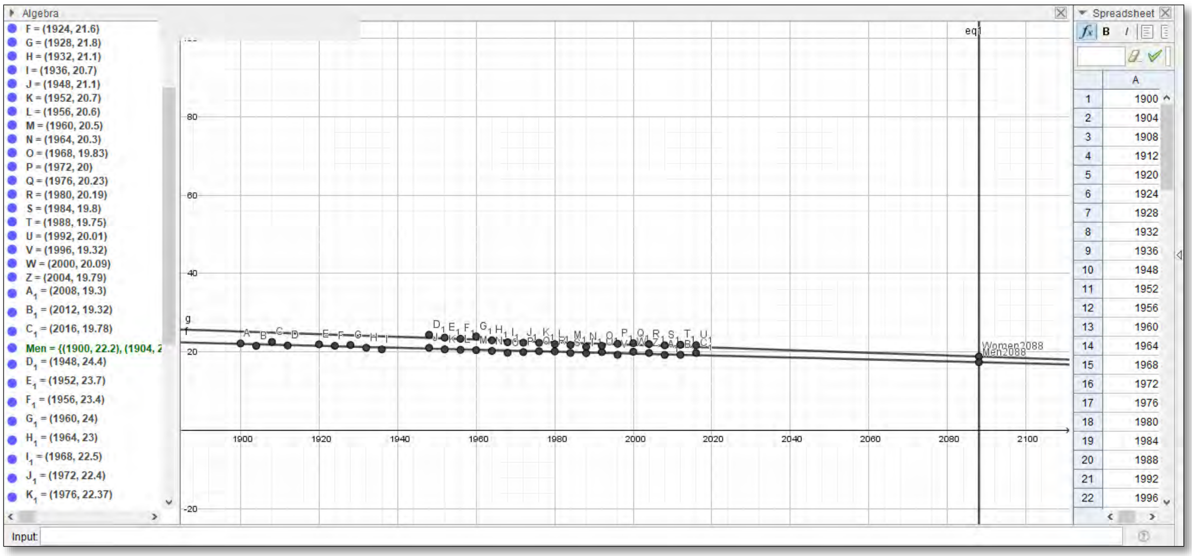

Figure 8. S4 Formed Best Fit Lines and Concluded Women Would Overtake Men as Displayed by The Model. He Did Not Try to Interpret It to Real Result

DISCUSSION

The novice modellers in this study tend to create a mental picture prior to the solving process and it is in the form of strategy they plan to embark with their solving process. Their experience and understanding contribute to forming their mental picture. This was also observed by Hankeln (2020); the German students in her study tend to look at the real-world situation outwardly and swiftly try to look for mathematical representation. Apart from that, it was also observed that situation model and real model occur almost simultaneously as they pursued the questions. This is in line with Schaap et al. (2011); they chose to use the term mathematising as situation model and real model are not always distinguishable as the activities such as understanding, structuring, and simplifying can be interrelated.

It was obvious that only with a proper situation model and real model formation will students be able to proceed with the modelling process successfully and create an adequate mathematical model. The drawback is that creating a suitable situation model and real model can be challenging to novice modellers. This was observed when S2's mathematical modelling process was reconstructed. In the initial stage, she seemed to display understanding of the question, but she was unable to mathematise the real situation. She could not see the connection between the real situation given in the question and the mathematics in it. Similarly, it was reported by Ortiz and Dos Santos (2011), that although students might understand the question by re-reading it and identifying the important information, sometimes they still are unable to make progress in the modelling process. In addition, S2 also tried to fit her flawed understanding of the situation into a symbolic representation that hindered her from moving past mathematical results. This was also seen in Hankeln (2020), when one of the participants did not try to establish better understanding of the situation but quickly tried to convert it to a mathematical problem.

In addition, mathematical modelling questions lack relevant data that makes them challenging but with extra-mathematical knowledge (EMK) students could tackle the question. For example, EMK helped S3 during validation which enabled him to create a new assumption that led him to form a more refined mathematical model. The influence of EMK was clearly explained by Ferri (2018), as mathematical modelling problems require everyday knowledge to link mathematics with the real situation posed in the question. So, EMK makes the process interesting for students as it gives them a chance to increase their mathematical competency such as estimating and measuring. Hence, the usefulness of mathematics becomes obvious.

As for this study, after reconstruction of students' initial mathematical modelling process, it can be said that students only partially followed the idealised modelling cycle. The obvious difference is after understanding the question, some students approached the question by identifying the important information and further structuring it to form the real model as outlined by Blum and Leiss (2007), but some students started off by creating a mental picture in the form of strategy to solution process which means the real model is formed immediately after understanding the real situation as encountered in Hankeln (2020) where students try to mathematise the situation quickly. Eventually, from the real model students do proceed to the mathematical model. Then they work mathematically to obtain mathematical results and surprisingly most of them interpret it to real results by relating it with their assumptions and real situation. The next phase according to Blum and Leiss (2007) is validation; however, in this study, it was the phase least attempted by students.

Two types of validation obtained in this study can be explained by referring to Ferri (2006). Intuitive validation occurs when students might feel the answer is incorrect, but no explanation could be given as the question might not be in their range of experience. This was observed in S2 as she unconsciously tried to validate the answer obtained as she felt the mathematical model obtained was not right, but she did not know how to explain it. The next type is knowledge-based validation as displayed by S1 and S3; this type of validation depends on extra-mathematical knowledge which allows the student to make a more conscious reflection in relating the results to the real situation. According to Ferri (2006), learners tend to make inner-mathematical validation as for them it means the answer and the working obtained is correct. They do not associate the result to reality which makes mathematical modelling beneficial to be included in the learning process. Therefore, to help students validate, it would be effective to provide the modelling cycle as a guidance tool for their action in answering modelling question (Blum & Leiss, 2007).

During the interview all the students explained the additional factor or changes that could be made to improve their model but none of them tried to further refine it with the ideas they had in mind. This could be because they are accustomed with the practices in Malaysian schools that require only one correct answer, and no additional steps are needed to improve the solution. Thus, most of them end their mathematical modelling process once a mathematical result or real result was obtained because they are accustomed not to question the appropriateness of their answer (Ferri, 2018).

It was also discovered that although participants were novice modellers, they do have intuitive ideas on displaying their mathematical modelling process. This finding is supported by Ortiz and Dos Santos (2011), whose participants were also students that did not experience the structure of mathematical modelling, but they attempted the questions posed in a creative way almost like the classic modelling behaviour described by Blum and Leiss (2007). Besides, in the Malaysian context Leong and Tan (2015) conducted a study to assess students' mathematical modelling competencies and they revealed that beginner modellers were able to solve modelling problems at different levels of competence. Despite being newly exposed to mathematical modelling questions, the majority of participants did try to interpret the mathematical result to real result.

As for this study, higher order thinking skills were also required in the mathematising stage when students need to choose an appropriate choice for the solution path especially as they need to include GeoGebra. Similar idea is shared by Brown and Edwards (2011) as the authors believe higher order thinking is important for student success in modelling and application tasks in secondary school mathematics. According to the authors, students display higher order thinking when they need to make a suitable decision about a solution method, make connections between representations and verify their conjectures.

In this study, GeoGebra was mainly used during mathematical model formation and calculation of result. The visual mathematical representation helped students in validating their mathematical model. In addition, technology as partner explained by Galbraith et al. (2003) was prominent in this study as students creatively used GeoGebra in their modelling process. Moreover, in this study including GeoGebra did benefit students who saved time by avoiding trivial calculations and manipulations. With GeoGebra, visualisation became more apparent, and this helped them to further analyse the model formed. Despite the positive exploration students did using GeoGebra, one of the setbacks they faced was in attempting to convert their mathematical language to the software's language and vice versa. Some students faced difficulty progressing from mathematical model to computer model, terms used by Siller and Greefrath (2010). All the participants had good mathematical competency, yet some struggled to manifest their mental picture with inclusion of GeoGebra as they experienced at least one technical difficulty related to the software.

CONCLUSION

Since in 2021 mathematical modelling is part of the Malaysian secondary mathematics syllabus, teachers need exposure to this concept especially on how to evaluate students' modelling process. Hence, by using identical approach to this study, adapting a modelling cycle would enable teachers to envision the specific phases and actions students undergo for appropriate intervention to occur (Shahbari & Tabach, 2019). Subsequently, the modelling cycle would be a strategic instrument for mathematical modelling (Ferri, 2011).

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