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Tracing the trajectory of mathematics teaching across two contrasting educational jurisdictions: A comparison of historical and contemporary influences

Stuart Shaw
Faculty of Education, University of Cambridge, UK: *shawstuartd@gmail.com*
Nicky Rushton
Research Division, Cambridge University Press & Assessment, UK:
nicky.rushton@cambridge.*org*
Dominika Majewska
Research Division, Cambridge University Press & Assessment, UK:
dominka.majewska@cambridge.org

This paper seeks to identify significant trends in mathematics curricula and teaching approaches in two education systems: the United States (a highly decentralised education system) and England (a highly centralised education system), with focus on 16-to-19-year-olds. The paper adopts a two-fold perspective: an historical overview, and comparison of the areas of convergence and divergence across both education systems. The trajectory of mathematical development is expressed through timelines of core concepts and ideas which chronicle the sequence of events and philosophies that have shaped the development of mathematics teaching and learning. By tracing the trajectory of mathematics through history, the paper provides a greater awareness of how different factors influence how mathematics is taught across two disparate educational jurisdictions. The paper affords opportunities to reflect on and draw conclusions about what constitutes meaningful mathematics teaching and curriculum approaches for 21st century learner.

Keywords: England; United States; curriculum; teaching, centralised; decentralised; mathematics education

PREFACE

> I am sure that no subject loses more than mathematics by any attempt to dissociate it from its history (Glaisher, 1890, p. 466).

This statement—appearing as part of the presidential opening address to the British Association for the Advancement of Science—remains as true today as it did in the 19th century. To understand the current state of anything, including mathematics education, it is important to acknowledge how history and other social factors have influenced the current state of the subject. Therefore, study of mathematics education in England and the United States (US) should include key events of historical, social, and scientific significance, as well as growth, change, and recurring underpinnings of disputes in mathematics education. Such issues represent a core set of concepts and ideas that characterise the evolution of mathematics teaching, learning, and assessment across the two jurisdictions.

Many of the issues the US faces today are not new but, rather, “cyclical and seemingly intractable” (Philips, 2015, p. 20). Theoretical debates, mandates, initiatives, reform movements, and standards in recent history still have a strangely familiar ring to them because many have been introduced previously in a slightly altered form and then retracted again (reverberating Glaisher’s comment). Illustrating Philips’ sentiments, Larson and Kanold (2016) describe the history of US mathematics education as “a two-hundred-year pendulum swing between an overemphasis on the rote practice of isolated skills and procedures, and an overemphasis on conceptual understanding, with their respective overreliance on either teacher directed or student-directed instruction” (p. 41).

The history of mathematics education in England has been characterised by some as a loss of freedom. McCourt (2017) describes it as “the story of a country moving from a largely laissez-faire position to a dictatorial one” (para. 1). How could that be? And what might we learn from tracing the paths of mathematics teaching across the two very different educational jurisdictions? These are the kinds of questions that prompted us to write this paper. The comparative account that follows is our attempt to describe and explain how conceptions of teaching and learning have evolved within the field of mathematics education in the US and in England. The central theme of this paper, therefore, is a focus on factors that influence current mathematics teaching in the two jurisdictions, including the teaching of procedural and/or conceptual skills; current instructional approaches to mathematics; best practice for teaching mathematics; influences upon the mathematics curriculum; the influence of textbooks; and, the use of technology in support of mathematics learning.

Note that for the purposes of this article, *current* describes schooling until spring 2020. From that date onwards, the global pandemic affected schooling and caused schools in the US and England to close to some or all students and move to remote schooling for significant periods of time. It is not yet known whether this will have any long-term effects on the way that mathematics is taught in schools.

CURRENT MATHEMATICS TEACHING IN THE US

The way education is organised in US high schools is somewhat different from the way it is organised in other countries (see Richards, 2020). The US has a highly decentralised education system (US Department of Education, 2008) with no national school system. Decentralisation in public education is a term used when administrative and financial decision-making powers are transferred from central Ministries of Education to local governments, communities, and schools (Winkler, 2013). Consequently, schools are able to make their own decisions about many aspects of policy and practice. Ultimately, power to create and administer education policy resides within the individual states (each with its own Department of Education), providing such policy does not violate the provisions of the US Constitution or federal law. It means that there are no national laws prescribing a curriculum for the establishment and recognition of institutions, the governance of institutions, or the recognition of degrees or professions (California State PTA, 2020; Richards, 2020). Significant decisions can also be taken at school district and school board level, although the nature of these may vary depending upon state rules. The recently enacted *Every Student Succeed Act* (*ESSA*, 2015) affords states and school districts the flexibility to develop their own curriculum, instruction, and assessments, which they can shape to better reflect the mathematics students will use. The extent to which states versus districts decide on textbook adoption, for example, is probably an important indicator or driver of how and to what extent state standards are realised locally.

The US has elementary, middle, and high schools. Typically, in elementary schools (kindergarten to grade 5 or 6, ages 5 to 10 or 11 years), students stay in one classroom with one teacher who is certified to teach all the subjects that elementary students learn, including mathematics. The mathematics curriculum is integrated in middle schools (usually grades 6 to 8, ages 12 to 14 years). However, in high schools (grades 9 to 12, ages 14 to18 years) the curriculum is traditionally separated by topic, each usually lasting for the whole school year. There are four high school courses: pre-algebra; Algebra I; Geometry; and Algebra II. Although Algebra I is a high school course, some middle schools offer it in the 8th grade. In many states, all these courses are mandatory for high school graduation as well as for entry to university and other tertiary-level institutions. However, in some places, such as most districts in California, Algebra 2 is not required for high school graduation (Daro & Asturias, 2019).

Although a curriculum is not set at a national level in the US, many states have adopted the Common Core State Standards (CCSS) or have based their own standards upon the CCSS. The CCSS are an educational initiative implemented in 2010 and are sponsored by the National Governor's Association and Council of Chief State School Officers (NGA & CCSO). They comprise a set of academic standards that specify what school students are expected to know and learn in each grade level in mathematics. The CCSS divide the mathematical standards into two sections: the Mathematical Practices that apply to all age groups and the Standards for Mathematical Content that describe what should be taught in each grade.

There are increasing proposals for the phasing out of the "algebra-geometry" pathway in favour of integrated mathematics for all students throughout US high schools (see Jeffrey & Jimenez, 2019). Incentives for more teaching of data science, computer programming, computer-based mathematics are being postulated. Some districts in California, for example, are designing courses that include more 'real-world' mathematics and topics such as financial algebra and mathematical modelling (Johnson, 2021). School classes tend to focus on formulas and procedures despite an emergent chorus of mathematics experts proposing to advance the US mathematics curriculum to ensure it mirrors more closely what learners in higher-performing countries are taught (Larson & Kanold, 2016).

MATHEMATICS TEACHING IN ENGLAND

England has a highly centralised education system (Creese & Isaacs, 2016). Government departments are responsible for many aspects of education, including setting the national curriculum (lists of the content that should be taught in state schools, i.e., non-fee-charging schools, between the reception year at primary school and the end of compulsory schooling in year 11), regulating examinations, and inspecting state schools. Academy schools (non-fee charging schools which receive government funding (DfE, n.d., para 1)) have slightly more freedom than other state schools as they are allowed to set their own curriculum and dates for the school term (Department for Education [DfE], n.d.); however, many academies still follow the national curriculum because national examinations are based on it.

The two main examinations are General Certificate of Secondary Education (GCSEs) taken at age 16 and Advanced (A) levels at age 18. The DfE sets the content and the assessment objectives for these examinations, but individual awarding bodies, such as AQA (formerly the Assessment and Qualifications Alliance) and OCR (Oxford, Cambridge, and RSA Examinations), develop and mark the test papers. There is an accountability culture which depends upon the results from these high-stake tests to judge schools and to evaluate whether educational policies have worked (Creese & Isaacs, 2016). However, in reality, it is difficult to clearly and directly link if results are an outcome of educational policies. Many factors can affect educational outcomes that cannot be controlled for, for example, homework completion and access to additional opportunities.

Although the government sets the national curriculum (the intended curriculum), they do not tell schools how to enact it, or how to organise other aspects of teaching. This means that teachers and schools in England are free to determine what happens within their classrooms. As a result, practice varies by school and teacher.

Kelly, Pratt, Dorf, and Hohmann (2013) and Kelly and Kotthoff (2017) characterised schooling in England as emphasising utility and systems, suggesting that it should be thought of as functionalist. They drew upon evidence from an EACEA/Eurydice study (2011, cited in Kelly et al., 2013) to suggest that placing students into teaching groups according to ability (setting) was widespread in mathematics classrooms, with decisions about placement into sets often being made on the basis of national test results. Their own observations of classrooms in England, reported in the 2013 and 2017 studies, showed that lower sets' teachers coached their students, breaking down knowledge into smaller steps and taking responsibility for what students learnt, giving highly individuated teaching. Higher sets' teachers acted as facilitators and gave students responsibility for their own learning. Students in higher sets were expected to think through things for themselves and make decisions about the mathematics that they used.

In their 2013 study, Kelly et al. described a typical mathematics lesson in England as starting with objectives being set for the lesson. Then content was "taught in small graded steps with differentiated tasks" (p. 561). Lower achievers' tasks had the same structure as the tasks that were demonstrated to the whole class; middle achievers' tasks had slightly less straight-forward solutions and higher achievers were required to draw on previous knowledge to solve their tasks. Teachers assumed that progression in mathematics was a linear process that moved from learning to application, where "application" referred to the application of topics observed in exam questions rather than the relevance to students' lives outside the classroom. Topics were usually taught for a week before the teacher moved on to a new content area, meaning that the emphasis was on mastering the processes required to solve questions and problems rather than developing an understanding of the topic.

HISTORY OF MATHEMATICS TEACHING IN THE US

> Broadly speaking, the education wars of the past century are best understood as a protracted struggle between content and pedagogy. (Klein, 2003, p. 177)

> If it seems we fight the same battles over and over again . . . it is only because we do. (Larson, 2016, p. 8)

The content, tenor, and direction of school mathematics education in the US has been fashioned by decisive episodes, statutory mandates, political initiatives, and rising societal expectations. Significant events since the mid-1950s have included: the inauguration of the "New Mathematics" era, three waves of school reforms (the desegregation movement in 1950s and 1960s; the schools standards movement which started in the1980s and was reinvigorated in 2002 with the introduction of the *No Child Left Behind* (*NCLB*) law; and the school choice movement), the recurrent rise and fall of the "Back to Basics" agenda, the advent of minimal standards competency movement, advancement of the need to emphasise problem-solving (NCTM, 1980), launching of K–12 curriculum standards by the National Council of Teachers of Mathematics (NCTM) (1989 & 2000), the CCSS Initiative resulting in the CCSS for Language Arts and Mathematics for Grades K to 12 (2010), and the recent re-authorisation of the 50-year-old *Elementary and Secondary Education Act* (*ESEA*) leading to the *ESSA 2015*.

The first half century following the founding of the US embodied the emergence of 'the great school mathematics debate' highlighting a pedagogic dilemma around procedural versus conceptual learning: should teachers offer students rules and facts to memorise (procedural) or should teachers give students material to reason about so they can discover and develop understanding of underlying mathematical principles (conceptual)? (Larson & Kanold, 2016). The close of the 19th century witnessed an alignment between progressivism—a movement promoting child-centred education—and the idea that students should be encouraged to be independent and creative thinkers. While progressivism gained traction in the early 20th century, the tension between the teaching of procedural versus conceptual mathematics continued to rumble. For example, the "Crisis-Reform-Reaction" (Fey & Graeber, 2003, p. 521) was characterised by several movements:

- *Excellence in Education*: The movement coincided with the increasing influence of educational psychology; a proliferation of working groups including the School Mathematics Study Group; and public acknowledgement of the inherent value of mathematics for the common good.
- *New Math*: A reflection of the progressive age, with hundreds of new textbooks generated to facilitate quick and radical curriculum changes, though teachers and parents struggled to understand the new-style mathematics.
- *Back to Basics*: Developed in opposition to progressivism, this movement signalled direct instruction and skills practice. Most US states created minimum competency tests in basic skills in the mid-1970s. These were a high school graduation requirement in many states.

Public concerns continued in the 1980s and 1990s over perceptions that students still did not appear to be learning sufficient mathematics (Ravitch, 2000). The cumulative influence of sets of standards (e.g., Curriculum and Evaluation Standards for School Mathematics) and the re-emphasised meaning and role of conceptual understanding engendered the Standards-Based Education Reform initiative. This called for clear, measurable standards for all school students. However, supporters of traditional education considered it unreasonable to expect all students to perform at the same level. By the late 1990s, criticism of the 1988 NCTM Standards began to emerge. These standards did not appear to adequately emphasise procedural skills or place enough emphasis on direct teaching or sufficiently emphasise practice/memorisation (McLeod, 2003). Throughout the 1990s and 2000s, conceptual understanding and sense-making battled procedures, rules, and memorisation for pedagogical primacy. The *NCLB Act* (2001) attempted to provide an equilibrium course for mathematics education. The *NCLB Act* was controversial because it punished schools that did not demonstrate improvement and was subsequently replaced by the *Every Student Succeeds Act* (*ESSA*, 2015-2016). The incoherence of 50 different sets of standards, tests, and passing scores provoked by *NCLB Act* together with exaggerations of student learning on state tests compared to NAEP results (Achieve, 2015) created fertile ground for the concept of the “Common Core” to gain a foothold. The state-led effort to develop the CCSS was launched in 2009. The new mantra was resoundingly clear: “Understanding and procedural skill are equally important” (NGA Center & CCSSO, 2010, p. 4).

Additionally, since the second half of the 20th century, in particular, there has been an unprecedented development and accessibility of progressively sophisticated technology to investigate and deliver mathematics as well as greater accountability for learning via student assessment and teacher evaluation (Reys & Reys, 2014, p. vii).

For nearly two centuries the US educational landscape has been punctuated by recurrent debates over mathematical approaches to teaching and curriculum, which have focused on two key areas:

- What should be the nature of mathematics that students learn: facts, skills, and procedures or concepts and understanding?
- How should students learn mathematics: teacher directed with a focus on memorisation, or student-centred through reasoning and discovery? (Jones & Coxford, 1970, as cited in Larson, 2016, p. 3).

Figure 1 illustrates the prevailing disputations. Commentators may differ over the precise timings of certain episodes, their duration, and what should be included; however, the timelines depicted here show a central set of concepts and philosophies that represent general evolution of thinking.

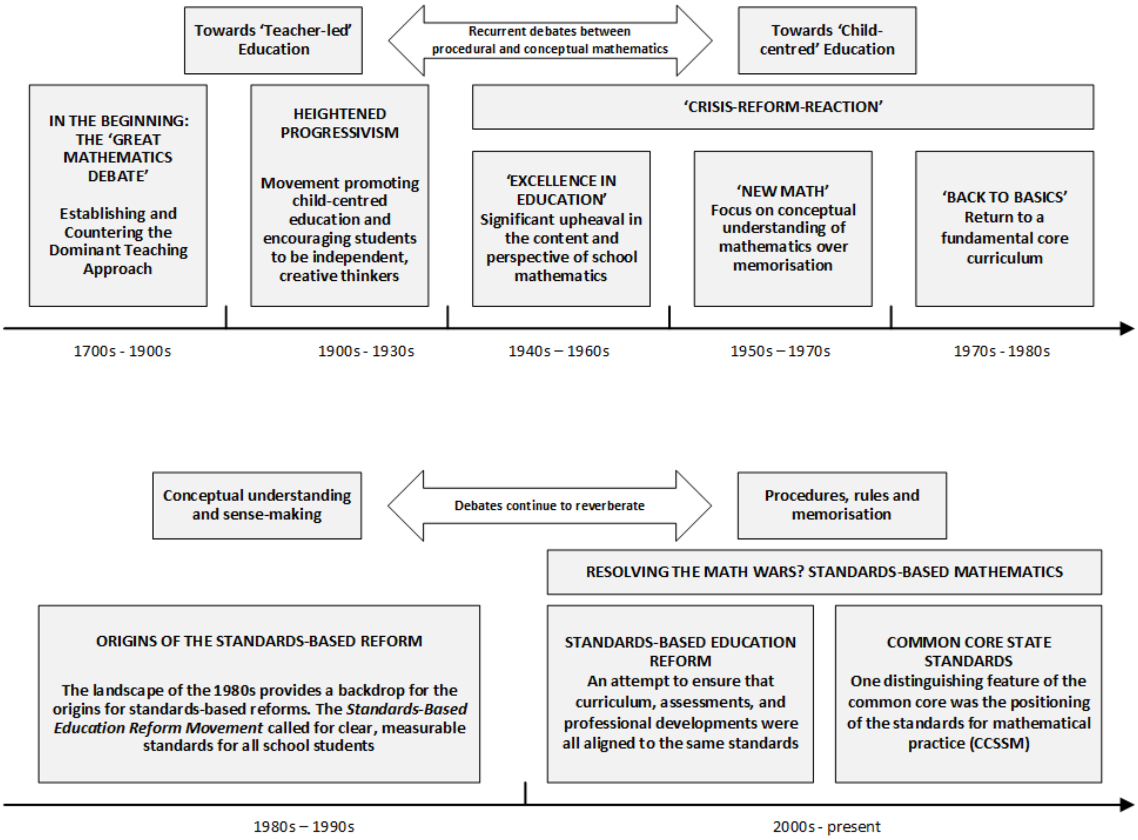

Figure 1: US mathematics timeline

HISTORY OF MATHEMATICS TEACHING IN ENGLAND

> The story of the history of mathematics education in England is . . . the story of a country moving from a largely laissez-faire position to a dictatorial one. (McCourt, 2017, para. 1)

The education system in England has been through a similar amount of change over the same period, although the drivers for reform have been rather different than those in the US. During this time, there has been a shift away from a largely laissez-faire perspective to a more autocratic position. It has moved from a relatively progressive system of teaching which esteemed autonomy and agency in teachers and their students (which remained ostensibly unopposed for a century) to an emphasis on more public education governed by constricted regulation by a select group comprising central government. Some of the significant events since the 1950s include the introduction of O Levels (1951), A Levels (1951), CSE (1965) and GCSE (1988); the new mathematics courses developed from projects in the 1960s, the emergence of the national curriculum in the 1980s and national tests in the 1990s and subsequent amendments to them, and the use of the national numeracy strategy from 1999 to 2009.

In the 1950s, O levels and A levels examinations were introduced for 16-year-olds and 18-year-olds respectively. Manipulatives ("objects that can be handled and moved, and are used to develop understanding of a mathematical situation" (Griffiths et al., 2017, p. 3) were increasingly used in teaching and learning and The Association for Teaching Aids in Mathematics (ATAM) was established to produce manipulatives and share good practice in using them.

The 1960s marked a move towards a child-centred view of education (Burghes et al., 2012), culminating in the *Plowden report* (CACE, 1967). Several major mathematics education projects developed innovative curricula, including new topics such as co-ordinate geometry, probability, and statistics. The CSE examination was introduced as an alternative to O level for lower achieving students, leading to recommendations to combine CSE and O level.

In the 1970s and 1980s there was increased government involvement in education and calls for a common curriculum. The government-commissioned *Cockcroft report* (1982) identified six types of mathematics teaching that should be used and made many recommendations about content to be taught. The national curriculum was first published in 1988, listing maths content to be taught by key stage. From 1988, a single examination, GCSEs, replaced O level and CSE.

The 1990s saw yet more government involvement in education. National tests were introduced to monitor students' progress in mathematics at age 7, 11 and 14. The results were used for accountability. There were many changes to the national curriculum and to GCSE examinations. In 1996, the National Numeracy project was launched in primary schools to increase basic skills and raise standards using a prescribed programme of curriculum content for each year group (Brown, 2010). It was used as the basis for the National Numeracy Strategy (Brown, 2010), which was taught in primary schools from 1999.

In the 2000s the numeracy strategy was extended into the first three years of secondary education. At primary school, the numeracy strategy led to mathematics teaching becoming highly standardised. All the numeracy strategies were ended in 2009. There were also many changes to examinations, including the introduction of a non-calculator paper at GCSE and the end of national tests for 14-year-olds.

The 2010s saw further changes to mathematics. Guidance on teaching maths was delivered through new Maths Hubs – a programme which allows mathematics teachers and education professionals to collaborate, with each hub being led locally by an outstanding school or college to support excellent maths practice (NCETM, n.d.c). From 2012, the Maths Mastery approach was emphasised in many UK schools (Boylan et al., 2018). The national curriculum was revised in 2014, but it became optional for some state schools. Several new examinations and tests were launched. A multiplication check was introduced for 9-year-old students. The mathematics National Reference Test assessed changes in performance standards over time. The Core Mathematics qualification allowed students to be examined in and to achieve a qualification in mathematics beyond GCSE but at a lower level than A level. There were also reforms to GCSE content and grading, and A level content.

Unlike the US, almost all these changes have been driven by political agendas and philosophies and, in particular, governmental conceptualisation of what mathematics education should be. The largest changes in mathematics education have often occurred after elections.

Other drivers for reform have included mathematics associations, influential groups of teachers and schools, England's position in the PISA league tables, and improving test results (on national tests, such as GCSEs and A Levels). The last may seem contradictory, but this has occurred when the rise in results is seen as a result of standards having fallen (e.g., Gove, 2012, column 653-655) instead of improvement in the teaching and, therefore, in students' understanding of the subject.

As with the US, mathematics teaching is not identical in all schools, or even across classes within the same school. The balance between active and passive learners varies by schools and the age and ability of students. Higher ability students are given more responsibility for their learning and the teacher is more likely to take on the role of a facilitator, whereas lower ability students are more likely to experience coaching, with the teacher breaking down knowledge into small steps.

There has been an increase in the use of technology in classrooms during this time, particularly for the delivery of mathematics instruction, although students' use of technology remains limited and often only involves calculators (Mullis et al., 2020).

Figure 2 shows the timeline for the evolution of mathematics education in England.

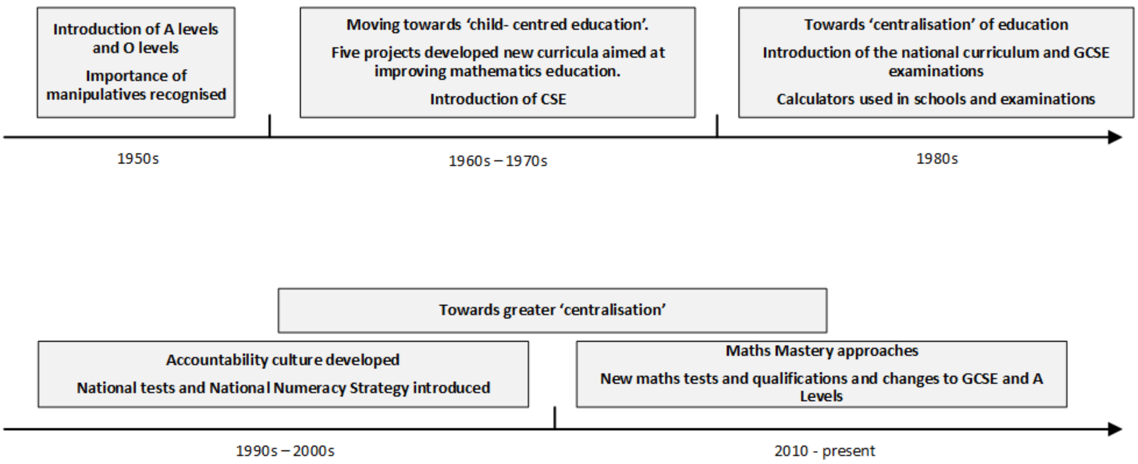

Figure 2: England mathematics education timeline

FACTORS INFLUENCING TEACHING IN THE US AND IN ENGLAND

By contrasting the history of mathematics education in the US and England, we can gain an awareness/understanding of where mathematics education is now and how various factors have influenced how it is taught in both countries. What follows is an attempt to identify and describe some of the salient factors emerging from the comparisons. Nine factors have been identified as worthy of further discussion, though these are by no means the only factors to surface nor are they given in any specific order.

The structure of the mathematics curriculum

There are different ways in which curricula can be structured. Spiral curricula have "an iterative revisiting of topics, subjects or themes throughout the course" (Harden & Stamper, 1999, p. 141). The key features of a spiral curriculum are that *topics are re-visited*, they are in *increasing levels of difficulty*, future learning *is connected to earlier* *learning* and *students' competence increases with each visit of the topic* (Harden & Stamper, 1999). It could be argued that the English national curriculum follows this structure, as topics are re-visited in more detail over time, with the intention of enhancing students' knowledge and understanding of topics. This is different to the US curriculum, as most schools may offer separate topics in each year, and topics may not be re-visited in the same way they are in the English national curriculum. For instance, fractions are taught within grades 3 to 5, but each area within fractions (e.g., recognising fractions relative to a whole or generating equivalent fractions) is only included in the content for a single grade.

The debate between procedural and conceptual skills

The skills-based and concept-based instruction dichotomy has been more influential in the US than England. It does not appear to be as instrumental in driving educational reform in England, although the balance between the two may change because of reforms.

In England, the curriculum content is constructed around content areas and age groups (see DfE, 2013a & b) and emphasises procedural skills rather than conceptual understanding. However, the mastery approach to teaching mathematics helps to redress the balance because it seeks to develop the two alongside each other (NCETM, n.d.a). In the US, mathematical understanding and procedural skills are increasingly considered to be equally important (NGA Center & CCSSO, 2010).

Current instructional approaches to mathematics

There is no one way in which mathematics is taught in English or in US secondary schools because teachers in both countries are free to determine what happens within their classrooms. Therefore, practice varies between them.

Both countries emphasise memory and calculation skills, with an emphasis on *procedural fluency*—the "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately" (NGA Center & CCSSO, 2010, p. 6). An example of its importance in England can be seen in the national curriculum's early emphasis on number facts (e.g., number bonds and multiplication tables) and in its requirement for fluency in written and mental calculations in key stage one and two (5 to 11 years).

Teaching in the US focuses on low-level tasks, emphasising procedures and memorisation, which is similar to the English emphasis on computation skills and quick recall of facts in the key stage 1 curriculum (see DfE, 2013a). In the US, instruction is teacher-led (learners rarely engage in the more challenging tasks), with modest attention to reasoning, problem-solving and the development of meaning (National Research Council, 2012). The widespread mastery approaches now used in England are likely to demonstrate aspects of concepts-based instruction because they emphasise multiple representations of concepts, development of conceptual understanding, and importance of communication and discussion when learning mathematics (NCETM, n.d.a & b, 2018).

Best practice for mathematics teaching

In the US and England, most of the best practices in teaching mathematics listed in the literature (e.g., The Education Alliance, 2006, p. 17) are present in mathematics teaching, although the extent to which they are used will depend on individual schools' approaches and philosophies. Some, such as building new knowledge upon prior knowledge and experience, are likely widespread. Others, such as differentiation, may be used differently in England, where students are set according to ability (Kelly et al., 2013; Kelly & Kotthoff, 2017).

In England, there is a focus on looking to jurisdictions such as Shanghai and Singapore for best practice. The mastery approach to mathematics is currently seen as a way to raise attainment in mathematics (Vignoles et al., 2015). Key to this is its focus upon identifying how mathematics is to be taught, including key and difficult points, breaking it down into small steps, and ensuring that it is taught in a careful sequence (NCETM, 2018) so that students move from being introduced to concepts to having a deep understanding of them (NCETM, n.d.b). There does not appear to be a great deal of overlap between England's mastery approach and the US's National Centre for Educational Achievement's list of mathematics strategies (see NCEA, 2009), as the mastery teaching approach appears to be teacher-led, with opportunities for demonstration, explanation and discussion (NCETM, 2018) whereas the NCEA inquiry-based instruction is led more by students (NCEA, 2009).

Influences on the mathematics curriculum

One of the biggest influences on teaching in the US is the CCSS. All currently available US basal textbook offerings are based upon the CCSS regardless of state standards. Publishers produce a wide range of textbooks, curriculum materials, and resources that local school districts can adopt and that influence the curriculum. However, the contexts in which schools decide upon which curriculum materials are most appropriate varies significantly from place to place (Hudson, et al., 2010).

The introduction of the national curriculum in 1989 had a similar influence on teaching and learning in England. Like the CCSS, it was developed by mathematics educationalists and mathematics academics (Ernest, 1992, as cited in Cooper, 1994). All state schools in England had to follow the national curriculum when it was introduced. Even in today's landscape, when academies are not required to adhere to it, approximately 80% did so in 2014 (DfE, 2014). As with the US, the mathematics curriculum in England is so packed that many teachers do not have the time or space to teach additional topics outside of qualifications, particularly at A Level (Suto et al., 2012).

Comparisons of mathematical practices

The Common Core Standards for Mathematics (CCSSM) enumerates what K to 12 students throughout the US should know in mathematics at the conclusion of each school grade. In England, the national curriculum documents are equivalent for key stages 1 to 3 (approximately ages 5 to 14), and for older students the equivalents are the subject content and assessment objectives issued by the DfE that form the basis for the GCSE and A level specifications.

The CCSS contains eight Mathematical Practices that are common to all grades (NGA Center & CCSSO, 2010, pp. 6–8). There is no equivalent section in the national curriculum or the GCSE content, although some of the content contained within the Practices can be found in these documents (N. Rushton, personal communication, March 30, 2021).

The enacted mathematics curriculum

Decisions as to what makes an appropriate mathematics curriculum will be dependent upon an array of factors, which include beliefs about the nature and purpose of school education (based on experience and data), the respective roles and responsibilities of teachers, and the students themselves (Bernard, 2017). For decades, there has been a movement to "integrate" the US high school curriculum, which has aimed to eliminate courses called "algebra" and "geometry" and advocated teaching some elements from each area in every grade (Will, 2014). Mathematics content is not differentiated in this way in England. Algebra and geometry are two content areas within the curriculum, and content from both areas is taught in all applicable years (algebra from the end of key stage 2, geometry from key stage 1).

The issue of integrated or traditional mathematics courses in the US is closely linked to tracking, where students are separated by academic ability into groups for all subjects or certain subjects within a school. Tracking is one of the predominant organising practices of US public schools. Recently, there has been a strong move against the practice, as it has led to some students taking algebra in 8th grade while others take the same course in 9th or 10th grade. This can limit their options for study in the future (Barrington, 2020).

"Tracking" is not a term that is used in England; instead, students may be *set* (put into subject-specific ability groupings) or, less commonly, *streamed* (put into the same ability group for all subjects) (Education Endowment Foundation, n.d.). A student's set would influence the "tier" that they were entered into for GCSE, which would have similar effects to the tracking used in the US. The higher tier covers additional content, gives access to the top grades, and is usually considered necessary to study mathematics beyond GCSE. However, England differs from the US in that students will be aware of the tier that they are taking and may be aware that the tier affects the pathways that are open to them after GCSE.

The influence of textbooks

Teaching in the US is predominantly structured around textbooks and other commercial schemes (Reys et al., 2004). In most states and districts, the requirements for adopting a curriculum specify the content that must be included. Such adoption requirements apply to elementary and secondary grades through advanced algebra in year 3 of high school. Adoption requirements and criteria are determined by each state regardless of Common Core status. Many use variations of the EdReports (https://edreports.org), criteria which are based on the Common Core. (P. Daro, personal communication, October 5, 2021). The decision as to which curriculum is best for the school is often contingent upon which textbooks best implement the principles, beliefs and values enforced by the state (and which textbooks are available) (Hudson et al., 2010).

This philosophy contrasts somewhat with England, where the national curriculum is influential, and teachers' use of textbooks is so low that Oates (2014) describes an "anti-textbook ethos" (p. 8) within schools. In England, textbooks do not generally set the taught curriculum but comprise one of a range of resources used in the classroom in addition to online materials and self-made resources. Most teachers need to be familiar with the content of the curriculum because the national tests are aligned to it. The structure of the national curriculum also helps teachers to use it, as it sets out what students need to be able to do by a particular school year or key stage.

Supporting mathematics learning through the use of technology

The impact of instructional technology on both students' achievement in mathematics and their attitudes toward mathematics has been known for some time (e.g., Beeland, 2002; Weaver, 2000). However, any beneficial effects are mediated by how technology is integrated into the teaching and learning process.

Many US mathematics classrooms do not use calculators at all while others use them in judicious ways (Usiskin, 2012). In England, calculator usage is also contentious, and has been for many years. There is a concern that students will not be proficient in using mental and written methods if they have access to calculators (e.g., DfE & Gibb, 2011). For this reason, the national curriculum states that calculators should not be introduced until the end of key stage 2 (7 to 11-year-olds) when students' mental and written arithmetic should be secure. However, teachers are instructed to use their judgement, and some may allow younger students to use calculators in lessons for specific tasks.

Nowadays, of course, the basic, limited-function technology of the early calculator has been replaced by increasingly more complex, affordable, and readily available calculators, dynamic tools and computer algebra systems. Many have argued that technology should not be intended for use in isolation away from other aspects of mathematics teaching, but rather as a mechanism for supporting mathematical practice and the kinds of problems encountered within a CCSSM environment (Larson & Kanold, 2016, p. 83).

The benefits of using technology are the same for both countries, but in England the opportunities for this type of integration would appear to be more limited than in the US. Graphical calculators and interactive whiteboards are used widely, but computers are not available in many classrooms (Mullis et al., 2020). Computers tend to be used occasionally rather than regularly and may only be demonstrated rather than being available to students (Mullis et al., 2020). This limits students' access to the benefits of technology. Possibly for this reason, in England, the term *digital divide* tends to refer to differences in students' access to technology in the home rather than the classroom (see Coleman, 2021). The lack of availability of technology within mathematics classrooms needs to be addressed and once it is, teachers would need to be trained in how to make effective use of it in their teaching (Gamage & Tanwar, 2017; INNOVA, 2016).

CONCLUDING THOUGHTS

By mapping out the course of mathematics education throughout history in the US and England, we have shown how different factors have influenced the current landscape of mathematics education in both countries, for example, the role of government in

England and research-based reform movements in the US. Our comparison also reveals several features that separate mathematics education in the US and England:

- By differentiating students by mathematical ability: "tracking" in the US and putting into sets in England. While on the face of it, this may appear to be a similarity, the key difference is that compulsory GCSE mathematics exams are "tiered", meaning that students in England are often aware of the implications of setting (e.g., limitations of future or higher-level study), whereas the lack of compulsory mathematics exams in the US means that students may not be as aware of the implications of tracking.
- By the reliance on textbooks which is significant in the US and minimal in England.
- By calculator and computer usage, which yields a "mixed" picture, but perhaps there is a greater reliance on computers in the US.

The trajectory of school mathematics education in the highly decentralised US has been fashioned by pivotal events, statutory mandates, political initiatives and rising societal expectations. The highly centralised education system in England has been through a similar amount of change over the same period, although the drivers for reform have been rather different than those in the US. The history of mathematics education in England is a story of influential reports, teacher-led initiatives and, for the last 40 years, government-led interventions.

The recurrent tension between conceptual understanding and sense making on the one hand, and procedures, skills, rules, facts, and memorisation on the other, continues to reverberate in the US. Though there is a developing consensus that values traditional mathematical learning goals while broadening the definition of mathematical literacy to meet the needs of 21st century learners, perhaps the most favourable position for the "pendulum" to assume is halfway between the two mathematical concepts (Larson & Kanold, 2016). Indeed, this is now the preferred stance of NGA and CCSSO (2010) who contend that "mathematical understanding and procedural skill are equally important" (p. 4). In England, the national curriculum content emphasises procedural skills rather than conceptual understanding. However, the mastery approach to teaching mathematics helps to redress the balance, as it seeks to develop the two alongside each other. In addition, the countries' curricular structures may be another key difference, with England employing more of a spiral curriculum and the US employing a system where topics are taught for a year without being re-visited in the future, as discussed above.

Across both jurisdictions, two dominant trends seem to be wending their way through mathematics classrooms: more applications and more active (as opposed to passive) learning. Both trends seem to be continuing despite the pandemic's disruption of in-class teaching throughout the two countries.

Two questions continue to pepper the mathematical landscape of each education system:

1. What should be the "essence" of mathematics taught? (Ginsburg, 1996)

2. How should students be taught and how should they learn mathematics?

What this comparison demonstrates is that there is a requirement to emphasise that the current objective is not altogether different from that of the past: know *how* (procedural skill), know *why* (conceptual understanding), and know *when* (application) (Larson, 2016). This has become increasingly the case where 21st century mathematical competences demand deeper learning if they are to be transferable, that is, if learners are to apply what has been learnt in one context to another, less familiar context (see, e.g., National Research Council, 2012).

Declaration and conflict of interest

The authors declare no known competing financial interest or personal relationships that would influence the work within this paper. The research and article have passed through an internal review and have been approved for external submission.

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