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Examination of Fractional Number Sense in Eight Graders with High Academic Performance

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Abstract
This study revealed the fractional number sense form associated with daily life and the number sense of 8th-grade students with high academic success. This study aimed to determine fractional number sense which in eighth graders with high academic performance. This study has a qualitative patterned. The sample consisted of 20 students from the middle schools in Pazar, the province of Rize. Participation was voluntary. İnterviews were conducted with participants to address their fractional number sense and strategies. The data were collected using an interview form developed by the researcher. The form consisted of 15 items on number sense components (computational estimation, operation effect, number magnitude, using a benchmark, and equivalent representation) related to daily life. The data were analyzed using descriptive content analysis. As result of this study; Very few participants used number sense. Most participants turned to rule-based solutions to answer the questions. None of the participants was successful in executing number sense. The fact that the items were related to daily life helped us reveal fractional number sense.

Keywords: Number Sense, Mathematics, Secondary Math, Fractions, Daily Life Math

1. Introduction

1.1 Introduce the Problem

Today, what matters is not how much one knows and remembers but how one uses one's knowledge. The World Economic Forum (WEF) (2020) asserts that we must promote creativity to help education keep up with the times. We need people with versatile thinking skills to develop creativity (Aslan, 2001). According to Skemp (1987), the purpose of learning is not to remember but to use and shape knowledge. In order to make sense of information, one must feel it, that is, one must "sense" it. One acquires knowledge by oneself. However, if we want to guide one for a purpose, we must provide one with environments where one can acquire information. We reason, predict, and process information in every step of life. Therefore, we can say that math is everywhere. For example, we use it to calculate a loan interest, credit card statement, inflation rates, tax percentage of a new phone, etc. In some cases, math is precise. However, in some others, we need to sense it, just like an experienced cook sensing how much milk to add to half a kilogram of flour to get a dough as thick as an earlobe. Math is abstract. However, a group of researchers in the USA suggested that math can be sensed (Howden, 1989). Based on students establishing relationships between numbers, interpreting the magnitude of numbers, and discovering shortcuts and methods instead of routine operations, they stated that we could sense numbers and feel them and use them flexibly. Neurologists and educational scientists have defined it differently and sought answers to questions like how to measure it and what its components are. Howden (1989) defines number sense as having a good intuition about numbers and numerical relationships but does not mention its components. Greeno (1991) approaches number sense from a psychological point of view and classifies it as flexible mental computation, computational estimation, quantitative judgment, and inference. McIntosh et al. (1992) emphasize that number sense should be examined in sub-headings and classify them in the most detailed way. Their classification consists of three main headings: (1) knowledge of and facility with numbers, (2) knowledge of and facility with operations, and (3) applying knowledge of and facility with numbers and operations to computational settings. The main headings included 11 subheadings. Sowder and Schappelle (1994) and Markovits and Sowder (1994) collect number sense components in two groups: understanding numbers and rethinking as operations.

1.2 Explore Importance of the Problem

According to them, number sense is based on practicality and flexibility, and it is sufficient to specify it as such. Reys et al. (1992) proceed by grouping the classification made by McIntosh et al. into six components: understanding the meaning of size and numbers, understanding and use of equivalent forms and representation of numbers, understanding the meaning and effect of operations, understanding the use of equivalent expressions and computing and counting strategies, and benchmarks. The National Council of Teachers of Mathematics (NCTM) published a report titled "Curriculum and Assessment Standards for School Mathematics" and explained that students with high number sense were good at understanding the meaning of numbers, developing multiple relationships between numbers, noticing the magnitude of numbers, seeing the effect of operations on numbers, and developing benchmarks (reference) for the measurement of surrounding situations. Based on these definitions, students with high number sense:

1. Know numbers and the relationships between them and use them.
2. Know the operations and the relationships between them and use them.
3. Consider circumstances and use strategies to make calculations.
4. Develop and use strategies.
5. Check data and results and interpret their accuracy.
6. Make operations, measurements, and state expressions using some numbers as benchmarks, and know the equivalent representations of numbers and use them when necessary.
7. Compare numbers based on relative magnitudes, make logic-based predictions instead of intuitive guessing, and use mental or written computation or flexible computation when necessary and perform coherent reasoning.

In light of these features, number sense components were grouped under five headings: equivalent representation, computational estimation, operation effect, number magnitude, and using benchmarks.

Fraction is one of the subjects that form the basis of math in number sense. However, it has not been addressed in detail. Fractions form the basis of math concepts (percentage, ratio, measurement, etc.) that we encounter in daily life. It is one of the most challenging subjects for students to learn (Yazgan, 2007). Therefore, the more concrete expressive experiences that students have in fractions, the higher their conceptual learning (Olkun & Toluk, 2003). In this sense, teachers who provide learning settings should have enough academic knowledge about the concept of fractions and should also be competent in revealing and interpreting students' math knowledge. It is essential to have a solid conceptual knowledge of fractions in preparing, presenting, and measuring this environment. The fact that teachers can express fractions with appropriate representations for real and concrete situations makes the concept of fractions meaningful and deepens them (Van De Walle et al., 2012). From this perspective, fractions are a component of number sense and are a field that requires detailed examination. Sowder and Schappelle (1989) argue that fractions should be treated separately because they are more complex than other sets of numbers. As a matter of fact, separate definitions have also been made. For example, it is defined as "fraction sense" (Suh, Moyer

& Hae-Ja Heo, 2005; McNamara & Shaughnessy, 2015) or "number sense for fractions" (Carpenter, Fennema & Romberg 1993; Cramer, Behr, Post & Lesh, 1997; Phipps, 2008). However, there is no clear definition. In addition, Carpenter, Fennema, and Romberg (1993) did not classify components for fractions. They define fractional number sense as one's ability to perceive the value of the numerator, denominator, and fraction as a whole, to show them separately, and to express them together rather than revealing their meanings. They exemplified the use of flexible numbers between fractions. McNamara and Shaughnessy (2015) define fractional number sense as knowing fractions' deep and flexible meaning without being tied to a context or question type. Fractions, which occupy a small place in the concept of large numbers compared to the others, contain so much complexity that a fifth-grader asks: Why is the result less than 292 when we multiply 29 and 2/9? (Taber, 2002 p. 67; cited in Van de Walle et al., 2012). If this question were about natural numbers, it would not cause any confusion, but generalizations about natural numbers make it difficult for students to understand fractions. When it is associated with a concrete or real-life situation, students have more conceptual confusion and do not question the information and use it again (Acar, 2010; Kayhan, 2010; Kocaoğlu & Yenilmez, 2010; Şiap & Duru, 2004; Yetkin Özdemir & Kayhan Altay, 2016). In this sense, it is important to determine fractional number sense. The secondary education math (Ministry of National Education, 2018) program pays attention to meaning and estimation skills but does not refer to number sense, which is accepted as a basic practice in learning mathematics by NCTM (1989). Insufficient use of the concept of 'number sense' in the curriculum may cause a slowdown in its improvement. According to some researchers, the level and development of number sense are low because curricula do not address it adequately (Harç, 2010; İymen, 2012; Yang, Li & Lin, 2007). We must first provide students with the right environments and measurement tools to help them sense numbers, which may facilitate the introduction of number sense. Revealing the 'number sense' ability in a subject associated with daily life and examining their transferability to life situations will contribute to the literature, educators, teachers, and preservice teachers. Therefore, this study focused on the fractional sense, which forms a basis of many subjects (decimal numbers, percentage, measurement, ratio, etc.) in math curricula (Streefland, 1991).

1.4 State Hypotheses and Their Correspondence to Research Design

Researchers analyzed the initial knowledge of students in terms of mathematical self-esteem. Based on the description above, the problem formulation in this study is as follows:

1. The main research question is as follows:
2. What is the level of eighth graders' fractional number sense?
3. The sub questions are as follows:
   1. What is the eighth graders' fractional number sense level in terms of components?
4. 2. What number sense components do eighth graders have?

2. Method

This case study adopted a qualitative research design to determine eighth graders' who has high academic performance fractional number sense. Data were collected using a fractional number sense semi-structured interview form developed by the researchers. The data were analyzed using qualitative descriptive analysis. The results are based on the assumption that all participants answered the interview questions candidly and honestly.

2.1 Participant (Subject) Characteristics

The sample consisted of 20 students (11 girls and 9 boys) from the middle schools in Pazar, Rize. All participants had a math grade of 4 or 5 and a TEOG (transition from primary to secondary education) score of 380 to 500 in the spring semester of the 2014-2015 academic year.

2.2 Measures and Covariates

First, a literature review was conducted. Studies on number sense, fractional number sense, and number sense components were evaluated. The definitions and questions about the components in earlier studies were examined in terms of common and different aspects. Tests on number sense, interview questions, and interview questions including routine and non-routine problems were evaluated (Jordan, Glutting & Ramineni, 2009; Markovits & Pang, 2007; McIntosh et al., 1992; Reys & Yang, 1998; Singh, 2009; Yang, Li & Lin, 2007; Zanzali & Ghazali,1999). Studies focus on rational numbers, integers, fractions, and the relationships between these numbers, but there is no measurement tool for only fractional number sense. Since fractional number sense includes flexible use of fractions and practical solutions, the dimension of associating with daily life was also taken into consideration. A pool of 30 questions about fractional number sense associated with daily life was developed by modeling the number sense test questions. Textbooks, TIMSS (Trends in International Mathematics and Science Study) math questions, and PISA (Programme for International Student Assessment) math questions were reviewed. However, the questions were about different math topics, and the situations in those questions were not related to the daily life of the sample either. The interview form included the problem situation, real-life pictures suitable for the content of the questions (n=22), three options to help participants express their ideas, and an "other" option to allow them to express their thoughts freely. Experts were consulted to check the validity of the interview questions. Three academics with Ph.D. degrees, one expert in measurement, and eight primary school math teachers, one of whom had a master's degree, checked the interview questions. The experts were asked to rate the questions on a scale of 1 to 5 to indicate whether they represented the number sense components. They were asked to explain their rating and specify which components the questions were associated with. They were also provided with explanation parts to express their views of the wording of the questions. The questions were revised based on expert feedback. Afterward, a pilot study was conducted. Seven questions were removed from the form because the pilot study participants found them too long and tiring and irrelevant to the daily-life situations. Another pilot study was conducted after the necessary revisions were made to the questions.

Table 1: The Characteristics of the Interview Questions

Question	Number Sense	Achievements
1	Equivalent Representation	Making sense of a fraction
2	Equivalent Representation	Displaying on the numerical axis
3	Computational Estimation	Sorting fractions
4	Operation effect	Division
5	Equivalent Representation	Simplification and expansion
6	Number magnitude	Sorting fractions
7	Computational estimation	Displaying on the numerical axis
8	Operation effect	Multiplication
9	Number magnitude	Sorting fractions
10	Using a benchmark	Addition and subtraction
11	Using a benchmark	Making sense of a fraction
12	Computational estimation	Addition and subtraction
13	Number magnitude	Sorting fractions
14	Using a benchmark	Addition and subtraction
15	Operation effect	Making sense of a fraction

2.3 Data Analysis

The data were collected through interviews. Participants' responses to the interview questions were videotaped. One participant did not want to be videotaped. This request was complied with for ethical considerations. The data were analyzed using qualitative methods. All interviews were transcribed. Descriptive analysis, a qualitative analysis approach, was used to group the methods used by the participants in their statements (Yıldırım & Şimşek, 2008). The answers of the participants who used and did not use number sense were analyzed in detail. Participants' statements were collected under two groups: "related to number sense" and "rule-based solution." The statements not related to these two groups were collected under "other" and then grouped as "true" and "false." The data were analyzed using descriptive analysis. Two people (one is an author of this paper, and the other is an experienced researcher who has a Ph.D. in classroom teaching and took courses on qualitative research in data analysis and conducted research based on this method) examined the data and coded them separately. They analyzed the codes together and discussed them until they reached a consensus. The codes were grouped under themes, which were then presented together with quotes.

3. Results

The research data described include the mean, standard deviation, minimum and maximum values. The division of data categories uses the normative reference approach. This section addressed the results regarding the five number sense components:
Participant 9 answered the question regarding the "computational estimation" correctly and used the statement "in infinitely different ways:"
P9: Well, it can be done in a couple of different ways. We can make an addition as much as the amount between those two numbers. There are infinite numbers between two rational numbers. We can add many more numbers.
R: Infinite numbers?
P9: Actually, yes, if you move it little by little, the machine displays different values.
R: What machine?
P9: Considering that a machine adjusts the part to be added there, small amounts can be changed differently.
R: You're calling it a machine?
P3: I just imagined it being a machine.

The comment "if you move it little by little, the machine displays different values" showed that the P9 was able to interpret the question without the need for an operation. She may not have felt the need to show the numbers on the number line. She explained this situation with a pointer next to the expression infinity, which showed that she used number sense through her daily life situation.

The remarks of Participant 12 about the "operation effect" indicated her number sense:
P12: For example, if it were full, I mean, if it were 1 ml, we would need 20 of it.
R: So?
P12: Yes, well, then it's more than 20 because it's less than one whole, I mean the ml of the syringe.
R: What if it's less than that?
P12: Because it's less, I mean, the syringe can hold more than 1 ml, so 20 of them don't make 20 ml.

This response showed that the participant summarized the situation by comparing 3/5 ml to 1 ml. In other words, he realized that the fractional number was bigger than the whole number.

There was no true statement regarding the component of "number magnitude." One of the participants gave a wrong answer to this question:
P6: Well, half a kilo, right? ½?
R: Yes, that's what we call it.
P6: Half of nine is 4.5. So, it's half a kilo. Here, it's a half more, and here it's a half less…It's one and a half more.
R: Which one is close to half?
P6: 4/9, in fact, both are the same, half-half.
R: Why?
P6: Isn't it half from half?
R: Is that so?
P6: I think it is, I think I got this question.
R: Now, you're telling me that 4/9 and 7/15 are equally far from half.
P6: Exactly, I just couldn't express it as you did.

Participant 6 took the half as the benchmark and made comparisons, which was an important step for the number magnitude. However, he made a mistake by stating that different unit fractions were equally far from the same number. Nevertheless, he could have developed the right strategy if he had not had confusion. The fact that he did not learn the unit fraction caused this mistake.

Participant 11 gave a response about "using benchmark:"
P11: I thought what I drew, and I said it, I imagined it. They are both small, and one is smaller than the other. I don't think I needed to draw it.

14) Which of the following interpretations is correct for a person who has eaten half a sheet of lavash and then a third of the same sheet of lavash? Why?
A) She ate a whole lavash.
B) She ate less than a whole lavash.
C) She ate more than a whole lavash
D) Other

Figure 1: The Drawing of Participant 11 For the Question

If a student refrains from commenting when he/she has not obtained numerical data, he/she may have difficulty interpreting a mathematical question other than numerical data. Consequently, this was the easiest question for all participants. It was also the question the answer of which they based on operations. The participants who regarded making operations as "making explanations" did not make any statement regarding how they related the situation to daily life.

The question regarding "equivalent representation" asked the participants to draw the seven-eighth of a loaf of cornbread and explain it verbally. They were also asked to shade all parts but one. Although all participants gave answers about the number sense, one-third of them gave wrong answers. Almost all participants made representations without making any operations or generalizations. However, the participants who made representations by drawing figures did not pay attention to dividing their figures into equal parts. Below are some examples of the right representations.

Figure 2. The Drawings Regarding the First Question

As can be seen, some participants made sure that they divided their figures into equal parts and stated that they would shade all parts but one. Those who made wrong drawings did not pay attention to dividing their figures into equal parts.

Display 7/8 of the cornbread.

Figure 2: Misrepresentation of the First Question

Each participant was able to turn a fraction into a figure. However, they did not pay attention to dividing their figures into equal parts. Their responses were about number sense, probably because the question was about the representation of a basic fraction. The question asked participants to come up with equivalent drawings, and they all drew figures. However, they did not pay attention to equal representation. The general state of the dataset can be summarized in a table as follows:

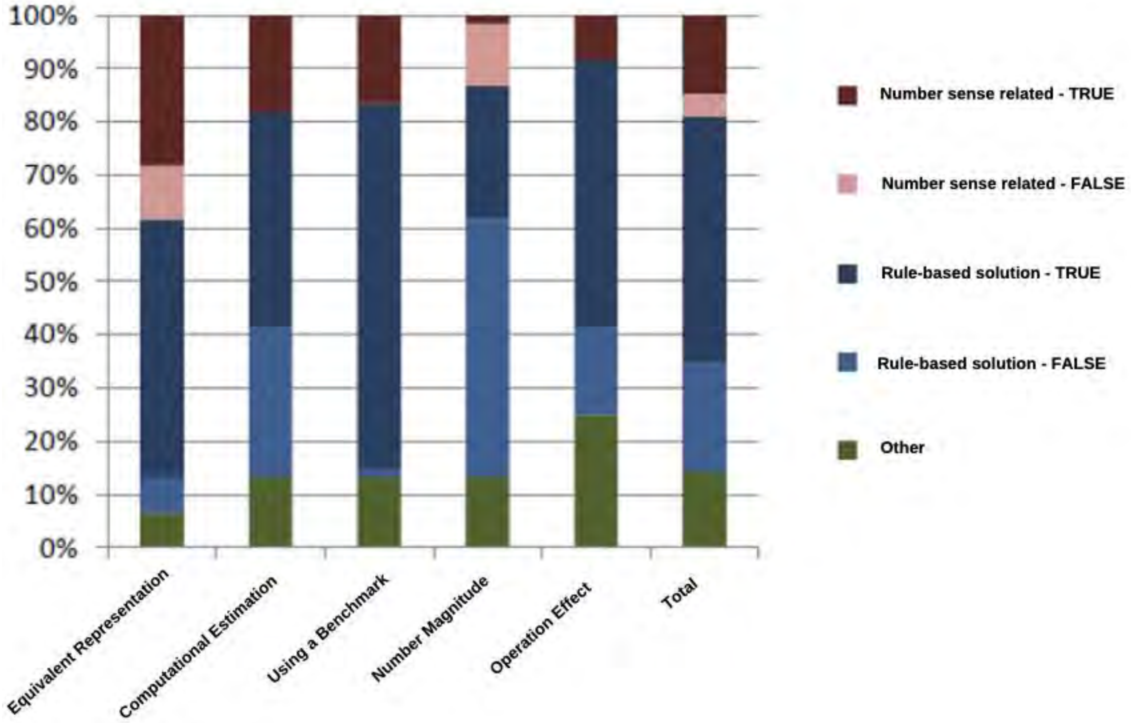

Figure 3: Fractional Number Sense Components and Data State for the Components

The rule-based answers were more than number sense answers. More than half the number sense answers did not end up wrong. However, all rule-based answers ended up wrong. Equivalent representation was the most prominent component. In addition, the rule-based answers were more than the number sense answers. The accuracy of the answers to number sense was higher in rule-based solutions. The computational estimation was the second most prominent number sense component. There were more rule-based answers than number sense answers. All number sense answers were correct, whereas there were more mistakes in rule-based answers. All number sense answers regarding the "using benchmark" component were correct. However, there were some mistakes in rule-based solutions. As for the "number magnitude" component, there were more rule-based answers than number sense answers. In both categories, there were more wrong answers than right ones. As for the “operation effect” component, all number sense answers were correct. However, there were more rule-based answers than number sense answers. These results showed that all number sense answers were correct regarding the "operation effect" component, whereas the rule-based answers led participants to mistakes. Participants answered the “using benchmark” component most correctly, whereas they answered the “number magnitude” the least correctly. They used the most number sense for the “equivalent representation” component, whereas they used the least number sense for the “number magnitude” component. They used the most result-based solution for the “number magnitude” component. In general, there were more rule-based answers than number sense answers. However, the number sense answers were more correct than the rule-based answers. Lastly, almost three in seven participants did not make any explanation.

The graph below shows which number sense components the participants had.

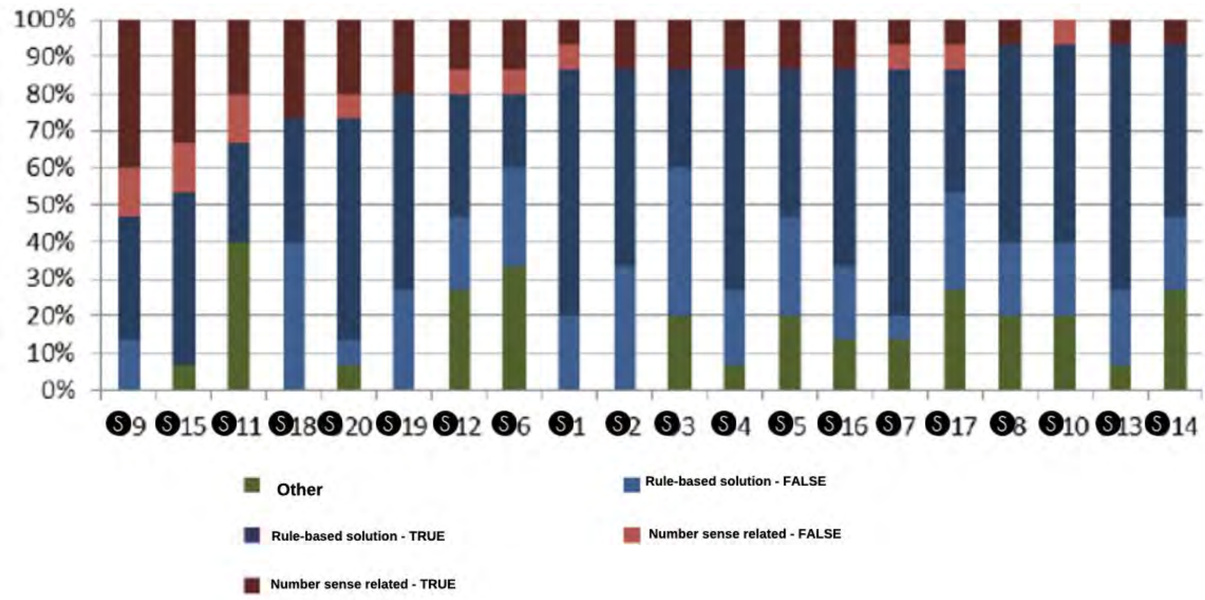

Figure 4: Student Number Sense Component Status

All participants had number sense components. Half the participants used number sense and reached the right solutions. The other students used number sense and reached both right and wrong answers. All participants but two (P15 and P11) used rule-based solutions and gave wrong answers. None of the participants had more wrong number sense answers than right answers. However, two participants had more wrong rule-based answers than right ones. All participants had number sense, albeit little. However, some participants did not have some of the components. The number sense components participants had been summarized in the table below:

Table 2: Distribution of Students by Components

Number of Components	Components	Participant
Five components	Computational estimation, number magnitude, using a benchmark, equivalent representation, and operation effect	P9, P15
Four components	Computational estimation, number magnitude, using a benchmark, and equivalent representation	P20
Three components	Computational estimation, number magnitude, and equivalent representation	P6, P11
	Computational estimation, using a benchmark, and equivalent representation	P18, P19
Two components	Number magnitude and equivalent representation	P7
	Computational estimation and equivalent representation	P3, P5
	Operation effect and equivalent representation	P12, P17
One component	equivalent representation	P1, P2, P4, P8, P10, P13, P14, P16

Two participants had all five components. P20 had four components but did not have the "operation effect" component. P6, P11, P18, and P19 had three components but not the "operation effect" component. However, P12 and P17 had two components, one of which was the "operation effect." Another interesting finding was that participants who had one component only had the "equivalent representation" component.

4. Discussion

This study investigated the fractional number sense components in eighth graders. Participants had low computational estimation. However, their computational estimation level was enough to give the right answers. They preferred rule-based solutions to mental operations when answering the questions. This is consistent with the results of the studies that show that students do not prefer practical solution methods when solving problems (Harç, 2010; Sinnakaudan et al., 2015; Şahin, 2018; Tsau, 2005; Yang et al., 2007). As for the computational estimation, participants had inadequate number sense probably because students' study for exams and focus on reaching the right solutions instead of coming up with practical solutions. In other words, students prepare for exams, and therefore, they consider the solutions that lead to the right results sufficient instead of developing different strategies. These results are consistent with the literature (Bayram, 2013; Harç, 2010; İymen, 2010; Yang, 2002). Research also shows that students have inadequate number sense in terms of computational estimation (Akkaya, 2016; Bayram, 2013; Harç, 2010; İymen, 2010; Yang, 2002).

Participants had a low operation effect. Participants who used number sense got the right answers, whereas those who used rule-based methods did not get the right answers. Participants who used number sense realized the meaning of operations and found solutions considering the situations. The daily life situations had a positive effect on the interpretations. Participants using rule-based methods had both correct and incorrect results. Before commenting on the situation in which operations can occur in numbers, participants thought that the operations should be made in written form. They could not interpret the results of the operations. Participants with sufficient operation effect first tried to understand the situations and then tried to make sense of the operations. However, they could not reveal exact numerical data. Participants with insufficient operation effect tended to obtain numerical data, and they found it sufficient to reach correct results. Participants could not comment on the changes made by the operations on the numbers without numerical data. This can be because learning environments are lacking in inquiry. Soyuk and Yenilmez (2021) state that representation differences form a basis for interpretation in the teaching process. Teachers do not come up with different activities because they find them time-consuming. They prefer different activities to explain topics, but not while solving questions (Çelik, 2015; Işık et al., 2011). Our results are consistent with the literature. Participants used number sense little in terms of the operation effect component. This result is in line with the studies of Lustgarten and Matney (2019) and Harç (2010), who identified the operation effect, which is the lowest number sense component.

Participants had inadequate answers in terms of the number magnitude component. However, the data collection tool had visuals that supported the daily life situation, contributing to the participants' understanding of multiplicities. However, it did not help with strategy development. This result is consistent with the literature. For example, Iymen (2012) found that students lacked an understanding of the number magnitude of exponents. Pesen (2008), on the other hand, determined that students had difficulty understanding the magnitude of fraction numbers. In this study, the lowest component participants had was the number magnitude. This is probably because students have never been to learning environments that promote the number magnitude component. However, Yang, Li, and Lin (2007) reported that students had higher number sense in terms of number magnitude than the other components, which was in contrast to our result.

As for the "using benchmark" component, participants reached solutions through operations but rarely commented on the concept of "half." There were more rule-based solutions than number sense answers. Participants were inadequate in interpreting fraction numbers. This may be because learning environments do not encourage questioning. The difficulties arise from the teaching of the concept (Arslan, 2016; Hoof et al., 2017; İvrendi, 2016; Soylu & Soylu, 2005; Yazgan, 2007; Yılmaz & Yaşa, 2008). Participants regarded the operations to obtain rule-based solutions as explanations and did not interpret the expression. Teachers must have number sense to ensure that students are critical individuals with number sense for interpretation (Yang, 2002). Participants reached the most correct results in this component. However, rule-based solutions were more than expected. As for rule-based correct answers, Yang and Huang (2004) state that reliable and desired answers are not always numerical. They suggest that different measurement tools should be used to assess mathematical abilities.

As for the equivalent representation component, almost all participants could make representations without any operation. However, Pesen (2010) showed that third graders were still drawing mistakes and that those mistakes persisted until eighth grade. Participants preferred talking to drawing to reveal what they knew. This result coincides with Çelik (2015), who argues that some students and teachers are too lazy to draw models. Although participants encountered a situation related to daily life, they first turned to operations and explained the results of their operations. However, the answer with the highest number sense, which is still not sufficient for interpretation, was obtained in the "equivalent representation" component. In this component, most participants turned to rule-based solutions. This tendency has also attracted the attention of Ekenstam (1977). This component is where the answers related to number sense are observed the most. İymen (2012) also found students more successful in the equivalent representation component related to exponential numbers. However, the use of number sense is still low. The rule-based answers were more than the number sense answers. The presentation of environments associated with daily life contributed to the number sense answers. This result coincides with Yazgan (2007), who maintains that real-life situations contribute to students' questioning. Yazgan (2007) adds that equivalent representation means revealing the relationships between the skills when they are needed and that people who cannot perceive these cannot find practical solutions to daily-life problems. The fact that equivalent expressions were not put forward more may have led students to operations.

Participants had low fractional number sense. High academic performance did not reveal a situation related to the existence of fractional number sense. While some participants were able to show the fractional number sense in each component, some could not. In this sense, incomplete concept knowledge might have caused those participants to show the number sense representation incompletely. The main reason for the challenges of conceptual learning in fraction teaching is that teachers move on to numerical notation and operations without making students grasp the important conceptual elements of fractions (Van De Walle, 2012). The amount represented by the fraction is related to the reference whole, the division into equal parts in fractions, fraction comparisons, inability to determine the unit in compound fractions, incorrect sums of fractions, and the fact that they are not related to the effect of multiplication and division by fractions on numbers cause the knowledge of fractions to be superficial. Therefore, since it is difficult to understand and deepen the concept of fractions, it becomes challenging to use them flexibly and practically (Van De Walle, 2012). This can explain why students cannot reveal their fractional number sense because they have not yet mastered and deepened that knowledge. Students do not have difficulty understanding and applying fractional information, but they cannot use number sense because they are not encouraged to rearrange and interpret it. Yang (2005) also states that organizing in-class activities to do and deepen math contributes to the development of number sense.

In conclusion, each student has number sense in terms of at least one component (equivalent representation). However, this is insufficient. Classroom activities help students deepen conceptual learning and use number sense. Each material and method used by teachers affects number sense. It is helpful to use components to define number sense, but each contains superficial and intense questions. Components also have levels. While the equivalent representation is adequate to express superficial number sense, the operation effect requires intensive number sense.

Students with high academic performance have low number sense. Therefore, researchers should look into the effectiveness of education programs in the development of fractional number sense. Insufficient number sense is associated with learning environments. Therefore, teachers should be provided with in-service training to help them develop and evaluate fractional number sense to achieve more effective results. Further studies should be conducted to examine the relationships between number sense components for the leveling result obtained for the definition of number sense or fractional number sense to help students understand the concept better and what they need to pay attention to in classroom activities.

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References

Baumeister, R. F., Campbell, J. D., Krueger, J. I., & Vohs, K. D. (2003). Does High Self-Esteem Cause Better Performance, Interpersonal Success, Happiness, or Healthier Lifestyles? Psychological Science in the Public Interest: *A Journal of the American Psychological Society*, 4(1), 1–44. https://doi.org/10.1111/1529-1006.01431

Berne, P. H., & Savary, L. M. (1996). Building self-esteem in children (New expanded ed). Crossroad Pub. Co.

Blazar, D., & Kraft, M. A. (2017). Teacher and Teaching Effects on Students' Attitudes and Behaviors. *Educational Evaluation and Policy Analysis*, 39(1), 146–170. https://doi.org/10.3102/0162373716670260

Bringula, R. P., Basa, R. S., Cruz, C. D., & Rodrigo, M. M. T. (2015). Effects of Prior Knowledge in Mathematics on Learner-Interface Interactions in a Learning-by-Teaching Intelligent Tutoring System: *Journal of Educational Computing Research*. https://doi.org/10.1177/0735633115622213

Chong, S., & Cheah, H. (2009). A Values, Skills and Knowledge Framework for Initial Teacher Preparation Programmes. *Australian Journal of Teacher Education,* 34(3). https://doi.org/10.14221/ajte.2009v34n3.1

Gal, I., & Ginsburg, L. (1994). The Role of Beliefs and Attitudes in Learning Statistics: Towards an Assessment Framework. *Journal of Statistics Education,* 2(2), 3. https://doi.org/10.1080/10691898.1994.11910471

Garfield, J., & Ben-Zvi, D. (2007). How Students Learn Statistics Revisited: A Current Review of Research on Teaching and Learning Statistics: How Students Learn Statistics Revisited. *International Statistical Review,* 75(3), 372–396. https://doi.org/10.1111/j.1751-5823.2007.00029.x

Guerriero, S., & Guerriero, S. (n.d.). Teachers' Pedagogical Knowledge and the Teaching Profession. 7.

Habrat, A. (2018). The Role of Self-Esteem in Foreign Language Learning and Teaching. Springer.

Hailikari, T., Katajavuori, N., & Lindblom-Ylanne, S. (2008). The Relevance of Prior Knowledge in Learning and Instructional Design. *American Journal of Pharmaceutical Education,* 72(5). https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2630138/

Islami, R. E., Sari, I. J., Sjaifuddin, S., Nurtanto, M., Ramli, M., & Siregar, A. (2019). An Assessment of Pre-service Biology Teachers on Student Worksheets Based on Scientific Literacy. *Journal of Physics: Conference Series*, 1155, 012068. https://doi.org/10.1088/1742-6596/1155/1/012068

Kaplan, J. J., Gabrosek, J. G., Curtiss, P., & Malone, C. (2014). Investigating Student Understanding of Histograms. *Journal of Statistics Education*, 22(2), 4. https://doi.org/10.1080/10691898.2014.11889701

Kholik, N., & Nainggolan, F. (2017). The Correlation Between Students' Self- Esteem And Their Writing Ability At The Second Grade Of Sma N 1 Prıngsewu. 10.

Lee, C., & Meletiou-Mavrotheris, M. (2003). Some Difficulties of Learning Histograms in Introductory Statistics. 8.

Nurtanto, M., Sofyan, H., Fawaid, M., & Rabiman, R. (2019). Problem-Based Learning (PBL) in Industry 4.0: Improving Learning Quality through Character-Based Literacy Learning and Life Career Skill (LL-LCS). *Universal Journal of Educational Research,* 7(11), 2487–2494. https://doi.org/10.13189/ujer.2019.071128

Pamungkas, A. S., Setiani, Y., & Pujiastuti, H. (2017). Peranan Pengetahuan Awal dan Self Esteem Matematis Terhadap Kemampuan Berpikir Logis Mahasiswa. Kreano, Jurnal Matematika Kreatif-Inovatif, 8(1), 61–68. https://doi.org/10.15294/kreano.v8i1.7866

Rahmatina, D., & Zaid, N. M. (2019). students' mısconceptıons ın ınterpretıng the mean of the data presented ın a bar graph. *International Journal of Insights for Mathematics Teaching* (IJOIMT), 2(1), 57–74.

Ramirez, C., Schau, C., & Emm, E. (2012). The Importance Of Attitudes In Statistics Educatıon5. 15.

Satriani, I. (2014). Correlation Between Students' Self Esteem And. 2, 6.

Schield, M. (1999). Statistical literacy: Thinking critically about statistics. 8.

Schleicher, A., Organisation for Economic Co-operation and Development, & Programme for International Student Assessment (Eds.). (1999). Measuring student knowledge and skills: A new framework for assessment. Organisation for Economic Co-operation and Development.

Sharma, D. A., Chevidikunnan, M. F., Khan, F. R., & Gaowgzeh, R. A. (2016). Effectiveness of knowledge of result and knowledge of performance in the learning of a skilled motor activity by healthy young adults. *Journal of Physical Therapy Science,* 28(5), 1482–1486. https://doi.org/10.1589/jpts.28.1482

Thompson, R. A., & Zamboanga, B. L. (2004). Academic Aptitude and Prior Knowledge as Predictors of Student Achievement in Introduction to Psychology. *Journal of Educational Psychology,* 96(4), 778–784. https://doi.org/10.1037/0022-0663.96.4.778

Wallman, K. K. (1993). Enhancing Statistical Literacy: Enriching Our Society. *Journal of the American Statistical Association,* 88(421), 1–8. JSTOR. https://doi.org/10.2307/2290686

Yang, J. C., & Quadir, B. (2018). Effects of Prior Knowledge on Learning Performance and Anxiety in an English Learning Online Role-Playing Game. *Journal of Educational Technology & Society,* 21(3), 174–185. JSTOR.

Zuffiano, A., Alessandri, G., Gerbino, M., Kanacri, B. P. L., Di Giunta, L., Milioni, M., & Caprara, G. V. (2013). Academic Achievement: The Unique Contribution of Self-Efficacy Beliefs in Self-Regulated Learning beyond Intelligence, Personality Traits, and Self-Esteem. Learning and Individual Differences, 23, 158–162. https://doi.org/10.1016/j.lindif.2012.07.010