Section III:

School-University Partnerships in Mathematics and STEM Education

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Teaching Responsively: Learning from the Pedagogical Reasoning of Experienced Elementary Mathematics Teachers

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Abstract: The participants in this study belonged to a professional development school that embraced the responsibility and challenge of improving students' mathematical thinking. In this study, experienced teachers' pedagogical reasoning was made visible as they analyzed pieces of student written mathematical work in an approximation of practice designed to support professional noticing. Researchers then worked to characterize participants' professional noticing using the lens of responsive teaching. Results indicate that experienced teachers' decisions about how to respond to students' mathematical thinking fall on a continuum and often shift in responsiveness across pieces of student written work. The findings of this study provide guidance for teacher educators who work to develop K-12 educators' responsive teaching practices and have practical implications for the use of approximations of practice to develop responsive teaching practices.

KEYWORDS: Professional noticing, Responsive teaching, Approximations of practice

NAPDS NINE ESSENTIALS ADDRESSED:
Essential 2: A PDS embraces the preparation of educators through clinical practice.

Essential 3: A PDS is a context for continuous professional learning and leading for all participants, guided by need and a spirit and practice of inquiry.

Essential 4: Reflection and Innovation—A PDS makes a shared commitment to reflective practice, responsive innovation, and generative knowledge.

Introduction

Teachers who value teaching practices that use the substance of student thinking as the basis for mathematics instruction tend to enact responsive instruction (Ball, 1993; Dyer & Sherin, 2016; Franke & Kazemi, 2001; Jacobs & Spangler, 2017). Educational scholars have developed many constructs to describe how teachers work to make sense of student thinking and use that thinking to support student learning. These teaching constructs include professional noticing of children's mathematical thinking (Jacobs et al., 2010), cognitively guided instruction (Fennema et al., 1996), formative assessment (Coffey et al., 2011) and teaching responsively (Dyer & Sherin, 2016). While each of these teaching constructs are nuanced in their approach, they all emphasize the belief that teachers should elicit, attend to, and make sense of student thinking to respond in ways that develop mathematical ideas (Kavanagh et al., 2020). Further, each of these teaching constructs involve both observable classroom practices and unobservable teacher reasoning. In this study, we draw on the teaching constructs of professional noticing of children's mathematical thinking and responsive teaching to focus on how experienced teachers engage in pedagogically reasoning as they draw on student thinking to inform their instruction.

To make a teacher's pedagogical reasoning visible, teacher educators are exploring the affordances of practice-based teacher education in which mediated clinical experiences are used to prepare teachers to enact high quality instruction (Ball & Forzani, 2009; Grossman & McDonald, 2008; Kavanagh et al., 2020). One type of practice-based teacher education involves approximations of practice that are designed to simulate components of teaching in a context of reduced complexity (Kavanaugh et al., 2020). The approximation of practice in this study involved teachers bringing pieces of student mathematical written work to participate in a semi-structured interview. The interview was designed to prompt the participants to notice student thinking in each piece of written work, and then share their decisions about how to respond to student thinking, referred hereafter as teacher actions. We also asked the participants to share their pedagogical reason for each teacher action, to help us to determine the purpose for a given action. We then examine each teacher action and related purpose using a responsive teaching lens.

The three participants in this study belonged to a professional development school (PDS) partnership that included twenty-eight public schools and a five-year teacher education program in a College of Education at a research one university. These teachers embraced the responsibility and challenge of PDS essential element three, professional learning and leading (NAPDS, 2021) in which partnerships are formed with an intentional goal of improving student learning in a content or subject area as evidenced by their agreeing to participate in this study. To situate the current study, we review the literature on pedagogical reasoning, responsive teaching, practice-based teacher education with a specific focus on the use of approximations of practice, and professional noticing of children's mathematical thinking.

Literature and Theoretical Framework

Pedagogical Reasoning

Effective mathematics teaching and learning occurs when teachers elicit and makes sense of children's mathematical thinking to make instructional decisions that develop student ideas (Ball, 1993; Dyer & Sherin, 2016; Franke & Kazemi, 2001; Jacobs & Spangler, 2017). This work includes both visible teaching practices, such as how a teacher responds to student thinking, and the invisible cognitive work that involves how a teacher makes sense of student thinking prior to making an instructional decision. This invisible cognitive work is often referred to as pedagogical reasoning. Loughran (2019) described pedagogical reasoning as “the thinking that underpins informed professional practice” (p. 4). According to Loughran et al. (2019) understanding how pedagogical reasoning develops and the way it influences practice is critical for teacher development. In this study, we use the term pedagogical reasoning broadly to describe all the ways our participants reasoned about student thinking. We then characterize how their pedagogical reasoning worked to support responsive teaching.

Characterizing Responsive Teaching

Responsive teaching is both a teaching stance and a practice that emphasizes the importance of using the substance of student mathematical thinking to guide instructional decisions (Dyer & Sherin, 2016; Hammer et al., 2012; Richards & Robertson, 2015). Importantly, responsive teaching involves instructional moves that work to take-up and pursue student thinking rather attempting to “fix” or “correct” student thinking (Dyer & Sherin, 2016; Richards & Robertson, 2015). Research on responsive teaching include studies that theoretically conceptualize this teaching stance (see Hammer et al., 2012) and studies that identify teacher moves or actions that facilitate responsive teaching (Dyer & Sherin, 2016, Jacobs & Empson, 2016, Lineback, 2015). For example, Dyer and Sherin (2016) identified three teaching actions that result in responsive teaching during classroom discussions that involve: 1) a substantive probe of student ideas; 2) an invitation for student comment; and 3) a teacher uptake of student ideas. In this study, we worked to first determine if teachers understood their students’ mathematical thinking, as this is an important precursor to teaching responsively (Richards & Robertson, 2015). Then, if a teacher demonstrated an understanding of the student mathematical thinking, we asked participants to share how they would respond to this thinking which included both a teacher action and their purpose for that action. We then determine if our participants’ pedagogical reasoning was responsive. For example, consider a teacher deciding to respond to student thinking by asking an open question. This teacher action appears responsive as it could work to pursue student thinking. However, if the teacher explains that the “reason” they asked an open question is for the student to fix a calculation error, the pedagogical reasoning becomes not responsive as it does not pursue or take-up student thinking. According to Yang et al. (2021) even experienced teachers need more deliberate practice to achieve a certain level of proficiency to respond to student thinking in ways that are responsive. To provide spaces that allow teachers to model and discuss their teaching, teacher educators are exploring the affordances of practice-based teacher education.

Practice-Based Teacher Education

Practice-based teacher education is a form of teacher education that uses mediated clinical experiences to prepare teachers to enact high quality instruction (Ball & Forzani, 2009; Grossman & McDonald, 2008; Kavanagh et al., 2020). Although practice-based teacher education was initially developed to be used with prospective and novice teachers, Sztajn et al. (2019) recommend that practice-based teacher education opportunities are beneficial for all teachers regardless of their level of experience or expertise. Practice based teacher education is emerging as an innovative approach to teacher education in that it emphasizes teachers rehearsing (practicing) rather than learning through lecture and discussion (Kavanaugh et al., 2020). One approach to enacting and studying practice-based teacher education involves examining teachers’ engagement in approximations of practice designed to simulate components of teaching in a context of reduced complexity (Kavanaugh et al., 2020).

Approximations of Practice. According to Kavanaugh et al. (2020) approximations of practice should be authentic and involve activities such as video of an actual classroom interaction and/or original artifacts of student work. They should also allow participants the time and space to examine learner thinking and make spontaneous responses (Kavanaugh et al., 2020). According to these researchers, teachers benefit from opportunities to practice their teaching as they engage in repeated cycles of observing, analyzing, and planning in increasingly complex approximations of practice (Grossman et al., 2009; Kavanagh et al., 2020). These characteristics of effective approximations of practice were used to conceptualize the approximation of practice designed for this study. The approximation of practice in this study involved teachers bringing pieces of students' mathematical written work to participate in a semi-structured interview designed to prompt professional noticing (Jacobs et al., 2010). This approximation of practice replicates a space where a teacher, after school is dismissed, sits down with pieces of student written work, to notice students' thinking in those pieces of written work.

Professional Noticing of Children's Mathematical Thinking

Sherin et al. (2011) describes the construct of teacher noticing as involving two main skills: (1) attending to children's strategies and (2) interpreting children's understandings. Jacobs et al. (2010) extended this framework to include a third element, deciding how to respond based on a teacher's understanding of student mathematical thinking. While there are many aspects that a teacher could notice in a classroom, the most critical is to actively notice student thinking (Jacobs et al., 2010). Emphasizing the importance of attending and interpreting student thinking prior to deciding how to respond Jacobs et al. (2010) introduced the construct of professional noticing of children's mathematical thinking, hereafter referred to as professional noticing.

Research on the interrelated nature of professional noticing skills often consider attending and interpreting together and explore the relationship of these two skills with the deciding how to respond skill (Fisher et al., 2018; Jacobs et al., 2010; Monson et al., 2018). This research demonstrates that expertise in attending to and interpreting students' mathematical thinking serve as important pre-cursors to deciding how to respond in ways that are considered responsive (Jacobs & Empson, 2016; Richards & Robertson, 2015). Researchers have examined teachers' decisions about how to respond in relation to what teachers have attended to and interpreted about student thinking (see Luna & Selmer, 2021) and identified observable teacher moves that work to take-up and pursue student thinking (Dini et al., 2020; Jacobs & Empson, 2016; Luna & Selmer, 2021). In this study, we also asked the participants to share their pedagogical reasoning so that we could identify their purposes for a teacher action. Therefore, we asked the following question: What teacher actions and purposes for those actions support responsive teaching practices?

Methodology

Study Context

The approximation of practice used in this study (see Figure 1) involves participants examining pieces of student written work created during a mathematics lesson from their own classrooms. Prior to engagement in the approximation of practice, participants (A) teach a typical mathematics lesson, and (B) choose pieces of written work from that teaching event to examine in the practice space. Next, participants engage in a semi-structured interview that included the questions: 1) How would you describe the student work? 2) What does that tell you about student thinking? 3) How would you respond? 4) Why did you make that decision? These questions prompted the participants to (C) engage in professional noticing as they examined their students' work, and then decide how to respond (D-F). Each participant engaged in the approximation of practice four times over a two-month period, hereafter referred to as cycle one, two, three, and four.

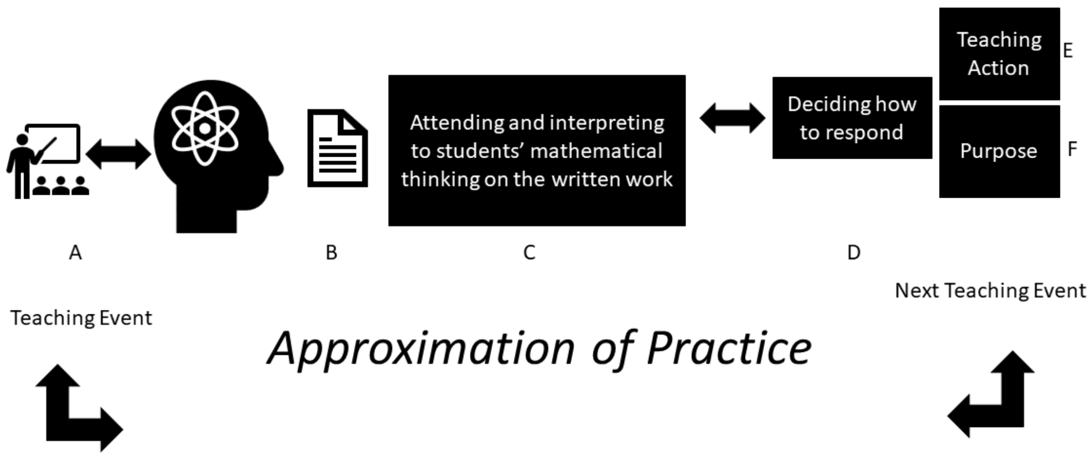

Figure 1. Approximation of Practice

The three participants in this study all worked at Hill Elementary (pseudonym), a public school located in a suburban, ethnically diverse neighborhood in a medium sized city in the Southern Appalachian region of the United States. Over 650 students attend the school. Forty-three percent of students qualify for free or reduced lunch, and 33% of students identified as a minority. At Hill Elementary school, 65% of the students scored at or above the proficient level for mathematics, well above the state and county average. The three participants were active partners in an established PDS partnership with the researchers' university. This study attempts to illuminate essential element four that calls for PDS's to be, "…living laboratories for creating, implementing, refining, and sharing innovative approaches to teaching and learning in efforts to better understand teaching and learning" (NAPDS, 2021, p. 15). Teachers and teacher educators in this PDS partnership engaged in on-going workshops, research projects, and co-teaching, that resulted in multiple collaborative state and national presentations and publications in high quality journals. All partnership projects had an explicit focus on student thinking in the content areas of mathematics and science.

Participants

Recall that even experienced teachers need more deliberate practice to achieve a certain level of proficiency to respond to student thinking in ways that are responsive (Yang, 2021). Therefore, we sought teachers with previous professional noticing experience for participation in the current project to increase the opportunity to observe and capture responsive teaching practices.

The first participant, Ingrid, taught for five years in a fifth-grade classroom. She earned National Board Certification and an Elementary Mathematics Specialist Certification. She was asked to participate in the current study because of her previous involvement in a professional development project that involved teachers videotaping themselves during mathematics instruction and attending bi-weekly meetings during which university researchers and participating teachers analyzed their professional noticing in the video clips.

The second participant, Kendall, had seven years of teaching experience in a fourth-grade classroom. She also obtained National Board Certification and an Elementary Mathematics Specialist Certification. Kendall was asked to participate because she had been part of a long standing PDS collaboration that involved teaching mathematics and science through a garden-based learning program.

The third participant, Hannah, had eight years of teaching experience in fourth and fifth grade classrooms. Hannah was asked to participate because she led the fourth-grade teachers in implementing the PDS mathematics and science integrated garden-based curriculum. Hannah had then approached university faculty with an idea for a mathematics and science integrated garden-based unit focused on developing student conceptual understandings of area and perimeter. Hannah and university faculty collaboratively developed this unit.

Research Design

We used a single case study design (Yin, 2014) to provide a rich description of the responsiveness of experienced teachers professional noticing while engaged in the approximation of practice. The final data set included 12 interview transcripts and 37 pieces of student written work brought by the participants. We coded the transcripts by identifying and separating out evidence of each professional noticing skill identified as attend, interpret, and decide. We also analyzed each piece of written work to identify the important mathematical elements and created a checklist for each piece of student work. We began our analysis by focusing on the attend and interpret transcript segments.

Data Analysis

Because teachers cannot be responsive to mathematical thinking that they do not understand (Richards & Robertson, 2015), participants had to attend to at least 70% of the mathematical elements identified by the researchers and contained in a checklist created for each piece of written work. To illustrate this coding process, consider a piece of student work brought by Ingrid during her first cycle in the approximation of practice (see Figure 2). The task requires a student to think relationally about the value of a variable that would make the equation true.

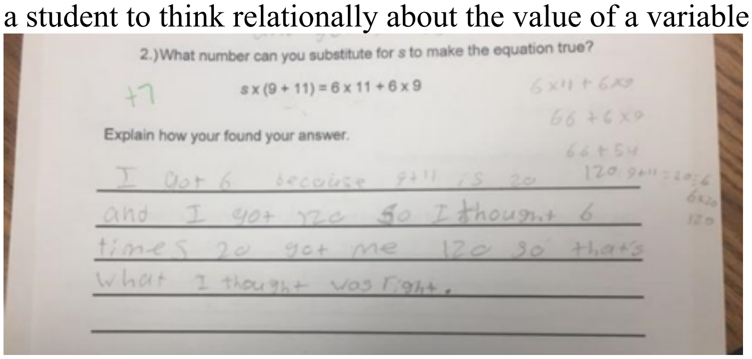

Figure 2. Ingrid's Shared Piece of Student Written Work

Ingrid noticed five out of the five (100%) identified mathematical elements (see Table 1).

Table 1
Mathematical Elements

Mathematical Element	Illustrative Quote
The student calculated 6x11 and 6x9 resulting in answers of 66 and 54	"I can see that he was able to solve the order of operations correctly to come up with the answer of 120"
The student added 66 and 54 together resulting in an answer of 120	"I can see that he was able to solve the order of operations correctly to come up with the answer of 120"
The student added 9+11 resulting in an answer of 20	"He ends up going 6 times 20"
The student multiplied 6x20 resulting in an answer of 120.	"He had 120 on this side and 20 on the other side and So I don't know if he could put together the like he knew 6 would be the right thing"
The student does not share any reasoning	
The student wrote 6 as the answer because 9+11 is 20 and he got 120 (6x20= 120)	"I mean he really shows no work he just said, I thought 6 times 20 would get me 120 and that's what I did and it was right"

This analytical process was implemented for the other 36 remaining pieces written work. Next, we analyzed the transcript sections marked as "decide" to identify the teacher actions and purposes for those actions. These sections contained the participants' responses to the interview questions: How would you respond to this student? Why did you make that decision? Participants often shared more than one decision about how to respond to student thinking noticed in a piece of written work resulting in 59 decide segments.

Teacher Actions and Purposes

To identify teacher actions and purposes we conducted a qualitative content analysis using "theme" as the unit of analysis (Miles & Huberman, 1994). We identified prior studies that examined teachers' actions during mathematics instruction (Herbel-Eisenmann & Breyfogle, 2005; Luna & Selmer, 2021). This analytical process resulted in eight codes for teacher actions (see Table 2). The next step in the analytical process was to identify each teacher action as responsive or not. Teacher actions were considered responsive if they worked to take-up and pursue student thinking rather than fixing student thinking (Dyer & Sherin, 2016; Richards & Robertson, 2015).

Table 2
Teacher Actions

Teacher Action (The teacher…)	Illustrative Quote	Responsive
…asks the student to elaborate on and/or clarify their thinking	"I would ask him to explain why you would use meters."	Y

… prompts the student to reread the problem situation and consider their related strategy	"I would tell him to re-read it and see what he does."	Y
…asks the student to use a different strategy	"I would encourage her to solve the problem a second way."	Y
…asks the student to work on a new task	"I would give her another one (task)."	Y
…tells, instructs, and/or explains a strategy or concept to a student	"I would go through the procedural steps of how we break this number down."	N
…asks the student a funneling question(s)	"I would ask her, what is the formula for finding volume and ask her, did you follow that formula?"	N
…asks the student to rewrite/recheck their work	"I would tell the student to just slow down and double check the math."	N
…provides the student with test scores	"I would like to show them their test score progression from the beginning of the year until now."	N

This process resulted in four of the eight teacher actions considered responsive to student thinking and included the teacher: 1) asks the student to elaborate on and/or clarify their thinking; 2) prompts the student to reread the problem situation and consider their related strategy; 3) asks the student to use a different strategy; or 4) asks the student to work on a new task. The teacher actions considered not responsive included the teacher: 1) tells, instructs, and/or explains a strategy or concept to a student; 2) asks the student a funneling question; 3) asks the student to rewrite/recheck their work; or 4) provides the student with test scores.

We identified seven purposes for the teacher actions. The first four purpose codes (see Table 3) were identified as responsive as they all worked to take up and pursue student thinking (Dyer & Sherin, 2016; Richards & Robertson, 2015) and included the teacher wants: 1) to test student understanding; 2) to understand additional student thinking; 3) the student to make mathematical connections; and 4) the student to understand a conceptual error. The next three purpose coded were identified as not responsive and included the teacher wants: 1) the student to not have a procedural error, 2) to understand student thinking that is confusing to the teacher; and 3) for students to recognize the importance of persevering in mathematical work. The purpose that involved the teacher wanting to understand student thinking that is confusing was considered not responsive because a teacher cannot be responsive to what they do not understand (Richards & Robertson, 2015). The purpose that involved a teacher wanting a student to not have a procedural error, is clearly focused on fixing student work and is therefore not considered responsive. Finally, while the purpose of the teacher wanting a student to recognize the importance of persevering in mathematical work is important in the creation of a vibrant, mathematical learning environment, it does not involve eliciting and understanding student thinking and was therefore considered not responsive.

Table 3
Purposes for Teacher Actions

Purpose (The teacher wants….)	Illustrative Quote	Responsive
...to test student understanding	"…to make sure she grasped this concept of exactly what kind of division we're doing here."	Y
…to understand additional student thinking	"I would ask him to explain how he figure out the six, because I want to know what he was thinking."	Y
… for the student to make mathematical connections	"I want him to think about the actual relationship of the numbers."	Y
... for the student to understand a conceptual error	"I want to make him look at the bigger picture (the problem context) of how it all fits together."	Y
…to understand student thinking that is confusing to the teacher	"I would do that so I would have a better understanding of what she meant."	N
...for the student to not have a procedural error	"It looks like the reason that these questions were missed was from computation errors to just making sure that he doesn't miss the easy part"	N
...for the student to recognize the importance of persevering in mathematical work	"I just want them to see that their hard work mattered."	N

Decide Sequences

We refer to a teacher action and its related purposes as a decide sequence. There were 59 decide sequences across the 37 pieces of written work. Recall that noticing the important mathematical elements in a piece of written work is an important precursor to teaching responsively (Richards & Robertson, 2015). Therefore a decide sequence would only be considered responsive if the participant had noticed the identified mathematical elements in the piece of student written work. If the participant had noticed the identified mathematical elements and both the teacher action and purpose were considered responsive, the decide sequence was designated as responsive. If either the teacher action or the decide purpose were identified as not responsive, the overall decide sequence was considered as approaching responsiveness. If both the teacher action and purpose were considered not responsive so was the decide sequence. Once the final codes were developed, the researchers coded the remaining data independently then met and discussed differences and modified the codes until 100% consensus across all data points was achieved.

Results

In all, participants brought 37 pieces of written work to examine across the four cycles in the approximation of practice. All three participants noticed all the important mathematical elements in each piece of written work except Kendall who did not notice all the mathematical elements in one piece of student written work. Across the four cycles of professional noticing in the approximation of practice participants posed 59 decide sequences related to the 37 pieces of written work (i.e., teacher action and purpose). Ingrid had the most decide sequences (29) across 19 pieces of written work. Kendall had the second most decide sequences (20) across 10 pieces of written work. Hannah had the least decide sequences (10) across 8 pieces of written work. Next, we present the results for individual participants across the four cycles in the approximation of practice.

Ingrid

Across four cycles Ingrid shared a total of 19 pieces of written work and had the most decide sequences (29) out of the three participants. Most of her decide sequences (21/29) were considered responsive (72%). Her go-to teacher action involved asking a student to elaborate on and/or clarify their thinking (14 instances) most often for the purpose of testing student understanding (8 instances) followed by the purpose for the student to make mathematical connections (3 instances) and the purpose for the student to understand a conceptual error (3 instances).

Ingrid had 7 out of 29 instances of her decide sequences designated as approaching responsive (25%). Recall that a decide sequence was considered approaching responsive if the either the teacher action or purpose was designated as responsive. Overall, two (of the seven) approaching responsive decide sequences occurred when Ingrid posed teacher actions considered to be not responsive which included asking a student to rewrite/recheck work (1 instance) and the teacher telling, instructing, and/or explaining a strategy or concept to a student (1 instance). Both not responsive teacher actions were for the responsive decide purpose of the teacher wanting the student to make mathematical connections. Ingrid's remaining approaching responsive decide sequences included the responsive teacher actions of asking the student to elaborate on and/or clarify their thinking (4 instances) and asking the student to use a different strategy (1 instance) all paired with not responsive purposes that included the teacher wants the student to not have a procedural error (3 instances) and to understand student thinking that is confusing to the teacher (2 instances). Ingrid had only one not responsive decide sequence that involved asking the student a funneling question for the purpose of the teacher wanting the student to not have a procedural error.

Kendall

Kendall was our only participant to not notice all the mathematical elements in one out of ten of pieces of student written work. This resulted in three decide sequences for that piece of written work being identified as not responsive. Despite this, like Ingrid, most of Kendall's decide sequences (14 out of 20) were considered responsive (70%). For her responsive decide sequences Kendall's most prevalent teacher action was asking a student to work on a new task (10 instances) followed by asking a student to elaborate on and/or clarify their thinking (4 instances). Kendall's most common decide purpose was wanting a student to understand a conceptual error (7 instances) followed by wanting a student to make mathematical connections (3 instances).

Kendall had one approaching responsive decide sequence during her second cycle of professional noticing which involved a not responsive teacher action of asking a student a funneling question for the responsive purpose of the student making mathematical connections. Kendall had five not responsive decide sequences including three decide sequences for which Kendall did not notice the important mathematical elements in the piece of student written work.

The remaining two not responsive decide sequences involved providing a student with past test scores for the purpose of a student recognizing the importance of persevering in the learning of mathematics.

Hannah

Hannah shared the fewest pieces of student work (eight) and had the fewest decide sequences (10) and only two of them (20%) were identified as responsive. Both of Hannah's responsive decide sequences involved the same teacher action of asking a student to elaborate on and/or clarify their thinking but for two different responsive purposes; the teacher wanting to test student understanding and wanting the student to make mathematical connections.

Hannah's had six approaching responsive decide sequences. Five involved the not responsive teacher action of telling, instructing, and/or explaining a strategy or concept to a student for the responsive purpose of wanting to test student understanding. The last approaching responsive decide sequence involved the responsive teacher action of asking the student to use a different strategy for the not responsive purpose of wanting the student to recognize the importance of persevering in mathematical work. Hannah had two not responsive decide sequences. One involved the teacher action of asking the student to rewrite/recheck their work and the other involved asking the student a funneling question; both for the not responsive purpose of wanting the student to not have a procedural error.

Discussion

In this discussion, we identify patterns and changes in patterns for each participant's pedagogical reasoning across the four cycles in the approximation of practice. Our discussion includes the importance of teachers identifying mathematical elements in student work and how teacher actions and purposes inform our understanding of responsive teaching.

Pieces of Student Written Work

Our data indicates that our participants noticed the mathematical elements in self-selected pieces of student written work in all but one instance. This finding is not surprising, as our participants are experienced teachers. However, the one instance in which Kendall did not notice all the mathematical elements reveals something we feel is important. Kendall brought a piece of student work that involved a word problem about building a tower out of different colored blocks (see Figure 3).

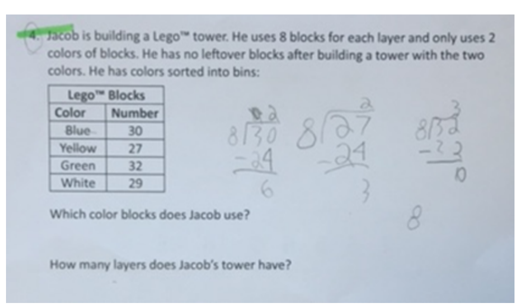

Figure 3. Kendall's Piece of Student Written Work

In the problem, a student named Jacob builds a tower using eight blocks for each layer and only uses two colors of blocks. He has no leftover blocks after building a tower with the two colors. One solution is for a student to add up the number of two-block color combinations and then determine which combination(s) is divisible by eight. A second solution, and the one that the student seems to be utilizing (see Figure 3), is for a student to figure out the number of layers a single color makes, add the remainders for two-color combinations and then determine if any of these sums are divisible by eight.

However, Kendall appears to lack the knowledge needed to recognize the viability of the student's strategy and instead focuses on her preferred strategy; She states, "he should have seen that you have to use two colors, so you are going to have to add the colors together and then divide to see if it is divisible by eight." She then posed three decide sequences. The first of which was, "I would have him really pick apart what the problem is asking because he has forgotten that he needs to be looking at two numbers instead of the one." Imagine this teacher action playing out in a classroom setting. It is highly likely that the student might incorrectly assume that their original strategy was not viable resulting in frustration and confusion. This scenario supports the idea that a teacher's ability to identify the important mathematical elements in each piece of student written work is not just an important but also a necessary precursor to teaching responsively (Richards & Roberts, 2015).

Teacher Actions and Purposes

Our findings suggest that our participants each had a go-to repertoire of teacher actions and related purposes in the 59 decide sequences. All three participants, tended to make similar teacher actions across the four cycles in the approximation of practice, with Ingrid asking a student to elaborate on and/or clarify their thinking (14 out of 29 instances), Kendall asking a student to work on a new task (10 out of 20 instances), and Hannah telling, instructing, and/or explaining a strategy or concept to a student (5 out of 10 instances). While participants did not individually favor a particular decide purpose, as a group they tended to favor the decide purpose of wanting a student to make mathematical connections (23 out of 59 instances) and to test student understanding (14 out of 59 instances), both responsive. Additionally, there was not a one-to-one correspondence between teacher actions and related purposes. These results confirm that participants engagement in the approximation of practice is at times predictable but also allowed the participants to be instructional decision makers in an unscripted space. Teacher educators need to continue to create, facilitate, and study approximations of practice that work to capture and develop teacher professional noticing and responsive teaching to improve mathematics education in various educational settings. These findings also demonstrate the importance of not just eliciting teacher actions but also the often-hidden pedagogical reasoning involving a teacher's purpose for a teacher action.

Responsive Decide Sequences

We used the lens of responsive teaching to identify each decide sequence as responsive, approaching responsive, or not responsive. As participants engaged in the approximation of practice, they posed several decide sequences for each piece of work. Examining these decide sequences for each piece of written work revealed that responsive teaching fell on a continuum as participants would often shift from approaching to responsive for a given piece of student written work. These shifts often occurred through a change in the responsiveness of a teacher action. At other times, these shifts occurred through a change in the responsiveness of a decide purpose. These findings align with other findings that suggest there are various pathways towards teaching responsively and that teaching responsively is not an all or nothing endeavor (Richardson & Robertson, 2015). Our work demonstrates that these shifts can occur through visible teacher actions and/or teacher's often hidden, purpose for a teacher action. We illustrate this phenomenon with an example from Hannah in her first cycle in which she brought a task that involved finding the area and perimeter of two rectilinear shapes (see Figure 4). Hannah expertly analyzed the student thinking as she explained:

> He took it and made it into a full rectangle and used that full rectangle to figure out the missing sides, so this was three and he wrote three over here and then he had five down here because the three and the five equals the eight and then three and four is seven.

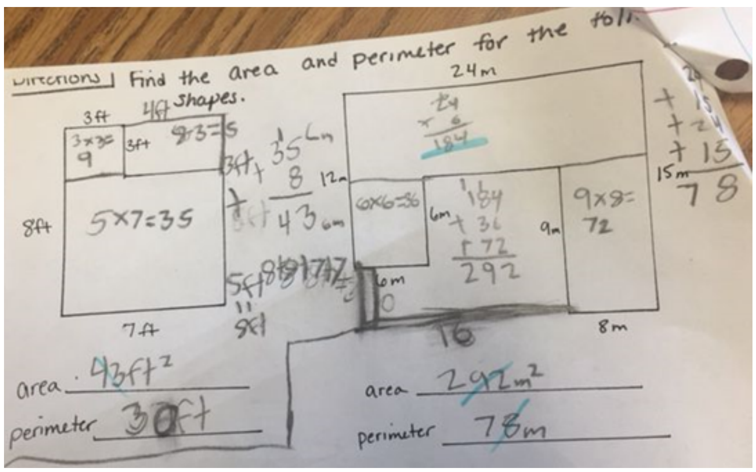

Figure 4. Hannah's Piece of Student Written Work

Hannah noticed that although her student had composed the rectilinear shape into a large rectangle, he didn't use this composition to find an answer. She also noticed that while he did his calculations (e.g., five times seven and three times nine) correctly; he, "incorrectly wrote 35 and eight (instead of nine)." She then states, "It is just a computation error or writing the wrong number and I would ask him to slow down and double check his math he did to find an answer." This decision about how to respond was coded as the teacher asks the student to rewrite/recheck their work for the purpose of the student fixing a procedural error. This decide sequence was identified as not responsive. She then shifted her noticing to the student's work determining the perimeter of the first shape stating:

> The student added up all the sides to get the perimeter. This worked for the first shape to find the right answer…. I can almost imagine he is pushing the sides out to find the perimeter, but it doesn't show if he understands they are adding up each side of the rectilinear figure and not the perimeter of a big rectangle.

Hannah then applies what she noticed about how the student found the perimeter in the first shape to his work on the second shape,

> …he does the same thing for the second figure by taking 24, 15, 24, 15 because it works for the first figure, but he is not taking into account that these are not the

> same on both sides and that this is a whole different shape (interpreting why the student did not find the correct perimeter of the second shape).

Hannah then shares a second teacher action:

> I would tell him it works here (finding the perimeter for the first figure) because this side is really up here so there are no extra sides that I am counting…but when I am looking here this face and this face together equal your bottom, but nothing equals this section there is nothing to pull over and then you're not accounting for these spaces on the inside.

She went on to state, "I want the student to connect how their method worked for the first shape and how that relates to the second shape." This decision about how to respond was coded as the teacher tells, instructs, and/or explains a strategy or concept to a student so the student makes mathematical connections. This decide sequence was identified as approaching responsive.

We find this example interesting for several reasons. First, Hannah engaged in noticing complex student thinking. She recognized that the student understood the conceptual idea of finding area and was merely making computational errors. Yet, pedagogically, she first focused on the student fixing these procedural errors. One could argue that fixing these errors is important, or not, but using the lens of responsiveness, a teacher action and/or related purpose focused on fixing, rather than pursuing student thinking are considered not responsive. Hannah then connected the student's strategy for finding the perimeter of the first figure to his work in finding the perimeter for the second figure and posed a not responsive teacher action coded as tells, instructs, and/or explains a strategy or concept to a student for the responsive purpose of the wanting the student to make mathematical connections.

In this example, Hannah shifts from a not responsive (i.e. fixing a procedural error) to a responsive (i.e. making mathematical connections) decide purpose. Imagine if Hanna made one more shift, from a not responsive (i.e. tell, instruct, explain) to a responsive (e.g. ask student to elaborate on and/or clarify thinking) teacher action. We suspect that all teaching professionals often experience shifts in responsiveness. In this example, a teacher educator might simply ask Hannah, or any teacher engaged in the practice space, to consider a shift in her teacher actions towards an action that creates a space for the student to make, rather than being told, these mathematical connections. In this slight shift, a teacher might become responsive to student thinking.

Conclusion

In this study, we examined teachers' pedagogical reasoning in an approximation of practice with experienced teachers. The participants were prompted to professionally notice their students' mathematical thinking in pieces of student written work. The participants noticing of important mathematical elements, teacher actions and purposes were analyzed using the lens of responsive teaching. We encourage teacher educators to use approximations of practice with experienced and prospective teachers to enhance student centered teaching (Grossman et al., 2009; Kavanagh et al., 2020). Our results inform professional development in several ways. First, a teacher needs to be able to notice their students' mathematical thinking to teach responsively. Some teachers may need exposure to learning opportunities (e.g., case studies, video analysis, practice spaces) to develop this component of their teaching practice. Second, a teacher's ability to engage in responsive teaching involves shifts in both teaching actions and purposes for those actions. A teacher educator could facilitate these shifts in teacher actions through carefully crafted questions, such as, "What does your student understand?" How can you use that understanding to help them think about (mathematical concept)? If on the other hand, a prospective or practicing teacher often poses purposes for a given teacher action that does not work to pursue student thinking, teacher educators could provide learning opportunities that include case studies of teachers who demonstrate a responsive teaching stance. Importantly, the teachers in this study, embraced the responsibility and challenge of PDS partnerships to engage in continuous learning (NAPDS, 2021) and we believe that this established relationship between PDS teachers and researchers helped to create the space for implementing an approximation of practice that allowed teachers to reveal and have their teaching practice examined with an intentional goal of improving student learning (Dresden et al., 2014).

Author Bios

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