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Worked Example Mid-Semester Intervention in College Algebra

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Abstract

The enrollment in lower-level mathematics courses has been on the increase in the last decade. It has been estimated that the DFW rate may rise to the level of 50% in courses that can be classified as college algebra. Numerous interventions have been utilized for college algebra and other mathematics classes to help students, including supplemental instruction, peer-led team learning, group learning, flipped classrooms, and integrating technology. In this study, we offered a half-semester course that implemented worked example worksheets, which were used in a previous full-semester intervention, to help students with the college algebra material. Results showed that the thirty students who participated in the worked example pass/fail mid-semester class significantly outperformed the college algebra students who did not participate in the intervention on total points in the course, quizzes, and tests 3 and 4. In addition, participants earned a significantly higher course GPA, attended significantly more regular class lectures, and earned a higher final exam score. Finally, we analyzed the qualitative data to develop five themes about the worked examples and intervention. We conclude that the mid-semester intervention is a good just-in-time mechanism to implement after the semester has started to help students succeed in the class.

Keywords: Worked Examples, College Algebra, Mid-semester Intervention

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“Ejemplos Resueltos” en una Intervención en Mitad del Semestre de Álgebra en la Universidad

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Resumen

Se han utilizado numerosas intervenciones para álgebra universitaria y otras clases de matemáticas para ayudar a los estudiantes, incluida la instrucción complementaria, el aprendizaje en equipo dirigido por compañeros, el aprendizaje en grupo, las aulas invertidas y la integración de tecnología. En este estudio ofrecimos un curso de medio semestre que implementó hojas de trabajo con ejemplos resueltos, que se usaron en una intervención anterior de un semestre completo, para ayudar a los estudiantes con el material de álgebra universitaria. Los resultados mostraron que los treinta estudiantes que participaron superaron significativamente a los estudiantes universitarios de álgebra que no participaron en la intervención. Además, los participantes obtuvieron una nota del curso significativamente más alta, asistieron a conferencias de clase de forma significativamente más regular y obtuvieron una puntuación más alta en el examen final. Finalmente, analizamos los datos cualitativos para desarrollar cinco temas sobre los ejemplos trabajados y la intervención. Concluimos que la intervención de mitad de semestre es un buen mecanismo para implementar después de que haya comenzado el semestre para ayudar a los estudiantes a tener éxito en la clase.

Palabras clave: Ejemplos resueltos, Álgebra universitaria, Intervención de mitad de semestre

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It has been estimated that, each year, fewer than 50% of the 700,000 students enroll in college mathematics courses annually that can be classified as college algebra earn grades of C or higher (Haver, Small, Ellington, Edwards, Kays, Haddock, & Kimball, 2007). College algebra courses lay the foundation for future courses in trigonometry, pre-calculus, calculus, and advanced mathematics courses, in addition to providing the background for statistics, business calculus, and mathematics courses for teachers. However, Haver et al. (2007) state that fewer than 10% of the students entering college algebra each year eventually enroll in calculus. Furthermore, looking at the STEM pipeline more generally, we see that fewer than 40% of students who enter college planning to pursue a degree in science or engineering graduate in six years, with the number dropping to below 25% for underrepresented minorities, according to A Commitment to America's Future: Responding to the Crisis in Mathematics and Science Education (Business-Higher Education Forum, 2005). In the United States, 22% of all college freshman must start college in remedial courses because they do not meet the requirements for entry-level mathematics courses (Evan, Gray, & Olchefske, 2006; Business-Higher Education Forum, 2005).

The high DFW rate in college algebra blocks a considerable number of students (conservatively 300,000 to 350,000 students annually accordingly to Haver et al. (2007)) from matriculating in their desired college career path or at least delays their progress. A variety of studies have implemented different teaching interventions to assist students in their learning of college algebra, such as various types of technology (Alexander, 1993) and flipped classrooms (Overmyer, 2014). In this study, we capitalize on a previous study's use of worked examples (Miller, 2015) to help students learn material in college algebra by implementing a mid-semester worked example intervention that improved students' perceptions about and success in college algebra.

Related Literature

The discipline of cognitive science deals with the mental processes surrounding learning, memory, and problem solving. Worked example research was developed from Sweller's (1988) cognitive load theory. The total load on working memory at any moment in time is referred as the cognitive load. The average person can retain about seven "chunks" of information in his/her working memory and when that limit is exceeded, information is lost.

In other words, an overflow of information in working memory results in cognitive overload. This can be thwarted if one limits the influx of information so that it does not exceed the students' working memory. One way to accomplish this is to transfer information from short-term (or working) memory to long-term memory as information is being processed (or soon after). One needs to minimize the load on working memory for optimum learning to occur, which facilitates changes in long-term memory. One way to reduce the cognitive load on the working memory of a student and build better schema acquisition is through worked examples (Sweller & Owen, 1989; Sweller & Cooper, 1985).

Sweller and Cooper (1985) conducted one of the first studies on worked examples with high school-level algebra students. Through five experiments, they examined the use of worked examples as a substitute for problem solving. Their first experiment found that more experienced students had a better cognitive representation of algebraic equations than less experienced students, as measured by their ability to recall equations and distinguish between perceptually similar equations. Sweller and Cooper (1985) concluded that there was "evidence that expertise in solving algebra manipulation problems is, at least in part, schema based" (p. 67). During this experiment, the students were asked to read and make sure they understood the worked examples. The other experiments alternated worked examples and conventional problems, which increased the motivation of the students to read and understand the worked examples because they needed to use that information to solve the following problems. Throughout the experiments, Sweller and Cooper (1985) also found that worked examples aided in reducing the acquisition time and improved achievement on the test phase of the experiments. However, the students in both the control and experiment groups struggled when presented with problems that varied from those to which they were initially exposed. Sweller and Cooper (1985) concluded, while "worked examples are of assistance to students when faced with similar problems, the advantage does not extend to dissimilar problems" (p. 83).

Zhu and Simon (1987) demonstrated the feasibility and effectiveness of teaching Chinese middle school students' mathematical skills through selected sequences of worked examples. In the first experiment, twenty Chinese middle school students were chosen for the experimental group, half of whom learned using ten worked examples, with the other half working a sequence of ten carefully arranged problems on factoring quadratics. After students spent approximately thirty minutes working examples or completing problems, they took a test. All twenty experimental participants solved the test questions correctly. The experimental group was not compared to a control because their initial goal focused on verifying that "learning by example or by doing was feasible for standard algebra skills and to gain an understanding of the learning process" (p. 144). In subsequent experiments, Zhu and Simon (1987) replicated the results of their first experiment on factoring with another group of students and expanded the research to other tasks devoted to exponents, geometry, and ratios and fractions. When the students were retested after a year or two had passed, Zhu and Simon (1987) found that students retained the material at a very high level and the experimental groups retained material at a slightly higher percentage than the control group that was taught by traditional lecture.

Chi, Bassok, Lewis, Reimann, and Glaser (1989) investigated how ten students studied worked examples on applications of Newton's laws of motion and how this learning transferred. The study was broken into two phases: knowledge acquisition and problem-solving. During the problem-solving phase, students needed to transfer what they had learned from the worked examples to different problems. When compared to less successful students (labelled as "poor" students), the more successful ("good") students (1) verbally generated more self-explanations while studying the worked examples, (2) verbally generated more accurate self-monitoring statements while studying the worked examples, (3) referred to the worked examples less during the problem-solving phase, and (4) reviewed only specific parts of the worked examples when referencing them. When studying worked examples, "good" students generally monitored their own understanding and misunderstanding through self-explanations, while "poor" students did not generate sufficient self-explanations or monitor their learning inaccurately (Chi et al., 1989).

Ward and Sweller (1990) examined the effect of worked examples on high school physics students in Australia in which the worked examples required students to simultaneously attend to multiple sources of information at a time (coined split-attention). They established that students who used worked examples formatted to reduce split-attention achieved test performances superior to those of students who were exposed to worked examples that required split-attention. The worked examples that required split-attention forced a higher cognitive load on students, which allowed less working memory to process the examples. Therefore, worked examples that do not require students to integrate multiple sources of information were deemed to be optimal for learning.

This study examined students' perceptions of the worked examples in the mid-semester intervention. Perception is the way a person thinks about or understands something (Merriam-Webster, n.d.). Numerous studies have highlighted the importance of students' perceptions of their learning situations (Jackson & Prosser, 1989; Crawford, Gordon, Nicholas, & Prosser, 1994; Lizzio, Wilson, & Simons, 2002; Trigwell, Hazel, & Prosser, 1996; Struyven, Dochy, & Janssen, 2003). A study conducted by the Midcontinent Research for Education and Learning stated that learning is affected by one's perceptions of what he/she is learning (Marzano & Pickering, 1997). Centra and Gaubatz (2000) cited Koon and Murray (1995) in stating that indicators of student learning "might include student perceptions of their increase in interest in the subject, critical thinking skills, interpersonal outcomes (e.g., cooperative abilities), intrapersonal outcomes (e.g., self- understanding) and other broad course outcomes" (p. 2). Furthermore, Campbell and Mislevy (2012) stated that students' perceptions do matter and have an effect on institutions' retention and attrition.

This study investigated how a mid-semester just-in-time intervention in College Algebra using worked examples helped students to perform better in the course, along with their perceptions and experiences with worked examples. The questions guiding this study were:

1. To what extent does a worked example mid-semester intervention help students perform better in the College Algebra class?
2. What are students' perceptions and experiences with worked examples in College Algebra?
3. What are students' perceptions of how the worked examples contributed to their learning in the College Algebra class?

Methodology

The Course

The study was conducted with students who were enrolled in a large lecture College Algebra course at a large university in the Eastern part of the United

States. The College Algebra course was taught in two large lectures of 200 or more students that met with the instructor in lecture three days each week. A course coordinator worked with the two different instructors to ensure the same structure and format for each section of the course. One day each week, students met in slightly smaller laboratory sections of 120 in which they used technology to work in small groups on lab assignments that integrated information in a variety of forms: graphical, tabular, written, and formulas. Students were placed into College Algebra by either having an adequate ACT/SAT score or by earning a sufficient score on a placement exam. During the course, students completed six online quizzes at home before specific deadlines, four computerized tests during their lab sessions (approximately every four weeks), and eight paper-and-pencil labs. Students were given suggested homework problems, but these assignments were not collected. During the first fifty-minute lab session, students took a retired ACT test to establish a baseline score of background knowledge.

Participants and Setting

The researcher offered a problem session course based on worked examples in which students could enroll at mid-semester, as there were a number of students that was struggling to understand the material in the course, and this was the most effective way to help students. The course lasted eight weeks and was graded on a pass/fail basis. Volunteer participants were expected to attend all problem sessions; students who missed more than one session failed the class. Table 1 shows the demographics of the participants, who were pursuing a variety of majors.

Table 1
Participant Demographics

Race/Ethnicity	% of Participants	Classification	% of Participants
White	75%	Freshman	81.25%
African American	15.63%	Sophomore	12.50%
American Indian	3.13%	Junior	6.25%
Other	6.25%	Senior	0%

The course was designed to last eight weeks for several reasons: (1) it began at mid-semester, which was when many students who were struggling or wanted extra help would seek assistance (students don't usually seek assistant until they start struggling with the material), (2) students are not always motivated at the beginning of the course, since they expect to do well in the class, and (3) students would be motivated to get help after receiving their grades on the first few tests. Thirty students participated in the experiment by enrolling in the class, while the control group consisted of the 460 students who did not participate in the class.

Experiment

Thirty students enrolled in the course and met once each week with the researcher and several undergraduate teaching assistants from mid-semester until the week before they took the Final Exam. During each class, students were given a worked example worksheet and asked to work in groups of between two and four students. Students reviewed an expert's solution of a College Algebra problem, followed by one or two problems that they were asked to solve. The problems that students worked on varied from ones similar to the worked example to a more challenging problem. Students were encouraged to discuss the problems that they found difficult within their group. If they could not resolve the issue, then the group could ask for help from the instructor or one of the undergraduate teaching assistants. When getting assistance, the instructor or teaching assistants solved the problem on a junior legal pad so that all students in the group could see the work.

Worked Example Worksheets

The worked example worksheets consisted of worked examples covering the various topics in College Algebra, with each worked example followed by one or two problems for the students to work out. Figure 1 shows two worked examples from different worksheets given to students in different sessions. The worksheets were always given to the students as one sheet (front and back) in a two-column format with headings for all worked examples. Also, each example was labelled with the section in the textbook (Sullivan & Sullivan, 2006), shown in parentheses. This enabled students to easily reference the section from which that problem came. There was a total of approximately 10 to 12 worked examples (expert solutions) and problems on each worksheet. The material on the worksheets reviewed some of the content covered during the previous week's lectures. Due to time constraints, all of the topics from the previous week's lectures were not covered on the worksheets. The worksheets were comprised of problems directly from or derived from the problems in the textbook with no new material being presented. The answers to the problems that students were asked to complete were provided on the bottom of the back page.

Worked-out Example (5.3):
Find the domain, the x and y-intercepts, the vertical asymptote(s), and horizontal or oblique asymptote for the rational function $f(x) = \frac{-4}{(x+1)(x^2-9)}$.

The domain is found by setting the denominator equal zero and solving for x. So $0 = (x+1)(x^2-9) = (x+1)(x-3)(x+3)$. So setting each factor equal to zero we have x+1=0, x – 3 = 0, and x + = 0. Solving these three equations we have x = -1, 3, ar -3. So the domain is all real numbers except -1, 3, and 3.

The y-intercept is found by substituting x = 0 into the function. So $f(0) = \frac{-4}{(0+1)(0^2-9)} = \frac{4}{9}$ and hence the y-intercept is $\left(0, \frac{4}{9}\right)$. The x-intercept is found by setting the function equal to 0, resulting in setting the numerat equal to 0. But the numerator -4 is never 0 and hence there is no x-intercept.

Using the work above there are vertical asymptotes at x = -1, x = 3, and x = -3.

For this rational function, there is a horizontal asympto y = 0.

Problem: Find the domain, x and y intercepts the vertical asymptote(s), and the horizontal or oblique asymptote of the rational function $f(x) = \frac{6x^2-7x-3}{2x^2-7x+6}$.

Worked-out Example (6.8):
The population of a southern city follows the exponential law. If the population doubled in size over an 18-month period and the current population is 10,000, what will the population be 2 years from now?

We start out with a general model $P(t) = P_0e^{kt}$ where P(t) denotes the population after t years, P_0 is the initial population, and k is the growth rate (as a decimal). Since the initial population is 10,000 and t = 1.5, we have $20000 = 10000\,e^{k(1.5)}$, we have

$$2 = e^{k(1.5)}$$
$$\ln 2 = 1.5k$$
$$0.4621 \approx \frac{\ln 2}{1.5} = k$$

Using this k value we have $P(t) = 10{,}000e^{0.4621t}$ and hence $P(2) = 10{,}000\,e^{0.4621(2)} \approx 25{,}198$ people.

Problem: The population of a Midwestern city follows the exponential law. If the population decreased from 900,000 to 800,000 from 2003 to 2005, what will the population be in 2007?

Figure 1. Examples from Several Worked Example Worksheets

Results

Quantitative Data

The performances of the experimental and control groups were compared on several measures: quizzes, tests, Final Exam, total points in the course, grade point average (GPA) in the course, and attendance. The quizzes, tests, and Final Exam were graded out of 100 points, but there were 1000 total points, and the attendance was out of 45 days. To calculate course GPA, the letter grade for each student was quantified by assigning four points for an A, three points for a B, two points for a C, one point for a D, and zero points for an F. The variances for each measure were unequal between the two groups, so a Welch's t-test was used for each comparison. The mid-semester experimental group significantly outperformed the control group on quizzes, Test 3, total points, course GPA, and attendance, using p = 0.05 as the measure of significance. Table 2 shows the results, with the significant differences in bold.

Table 2
Comparison Between Experimental and Control Groups on Various Course Grades

Measure Compared	Experimental Group	Control Group	t-value	Degrees of Freedom	p-value
Quiz Average	87.14	78.46	3.6754	36	**0.008**
Test 1	66.61	67.21	0.1839	30	0.8553
Test 2	72.32	66.46	1.8036	31	0.0810
Test 3	71.96	64.30	2.6897	33	**0.0111**
Test 4	71.79	64.27	2.5341	34	**0.0160**
Final Exam	61.96	57.10	1.9000	37	0.0652
Total Points	788.06	730.81	3.5663	38	**0.001**
Course GPA	2.46	2.06	2.3108	38	**0.0274**
Attendance	41.79	38.19	5.4763	46	**0.001**

The experimental group earned almost 9% higher on the quizzes. The advantage in course GPA was almost half of a letter grade, which corresponds to the 5.7% advantage in total points. The control group outperformed the experimental group on Test 1, but that was before the experimental worked example class began. The experimental group produced a higher average on the other tests and Final Exam, even though the differences were not always significant.

Qualitative Data

In order to obtain qualitative data, the participants completed a survey at the end of the course. The survey contained general questions about the participants' demographics and open-ended questions about the participants' experiences using and perceptions of the worked example worksheets. Among these questions, students were asked why they started attending the sessions, how the sessions affected their perspectives of College Algebra, and how the participants felt going into tests before and after the sessions. The authors independently coded the responses for each of the questions to develop emerging themes from the data. Important phrases were also identified and categorized. The two authors then met to discuss the themes and highlighted phrases and came to a consensus on the emerging themes discussed in the next five subsections.

Reasons Students Started to Attend the Worked Examples Sessions

Fourteen of the participants attended the worked example sessions because they needed extra help in College Algebra, felt that the sessions would be beneficial, or wanted a better understanding of the material. Participant 1 wrote, "I was starting to get behind on my [College Algebra] assignments. I needed a little extra help, so I started coming to the worked example sessions." Participant 29 started to attend "[f]or further help in [College Algebra]. I have never been very good at math and thought the extra work time would be very beneficial."

Participant 3 had several reasons for going to the worked example sessions:

> To get a better understanding on the material. I didn't do well on my first test and really needed assistance on the assignments. As well I know it's hard for me to do homework because we don't have to turn them in. So, having a class that was pretty much like doing homework for an hour a week really helped me be more independent in my studies.

Ten participants had performed poorly in the past. Participant 22 wrote: "I started to come to these [sessions] because I had failed my first math exam. I thought the sessions would be really helpful and it would make me more prepared for the next exams to bring up my grade." Participant 19 added, "Last semester I took [College Algebra] and I had a very low D when it was time to drop, so I decided to drop. This semester, I decided to do everything possible to get a decent grade in the class, so I decided to start coming."

Impressions of the worked example sessions - before, during, and after

Overall, the participants had mixed thoughts about the sessions before coming. Students thought the sessions would be helpful (11 students), give them practice working problems (5 students), or have an impact on their grades (6 students). Although the students had different expectations, most thought that the sessions would be beneficial in some way. Some also thought that the worked example sessions would be more like a lecture or led by an instructor (4 students), or were unsure about what they would get out of the sessions (2 students). The misinterpretation of the nature of the worked example sessions was not always bad. Participant 10 wrote: "I thought they would be different as in [led] by an instructor, but it turned out we had depended on ourselves to solve the problems[,] which in turn[,] made me better in math."

Before beginning the worked example sessions, nine students experienced difficulty in learning the material and seven felt anxiety. After attending the sessions, 16 commented that they felt comfortable with the material. While not producing a tangible result, comfort (and the confidence that usually accompanies it) can have a powerful impact on one's perspective towards and enjoyment of a class. Participant 10 stated: "Before[,] I relied on the book and class notes[,] which gave me a certain type of question, but after [I had] been going to the worked-out example sessions[,] I learned to think differently when approaching a question and [in] turn[,] made [me] better off in solving questions in my exams." Eight students said that the sessions helped, often mentioning the chance to review and practice the problems. Three students specifically mentioned improved grades as a result of attending the sessions.

Impact on tests

When it came to the tests, the reactions were nearly universally positive. Fifteen students mentioned increased confidence, eight felt more prepared, and six felt better about the material. Eleven students said that the worked example sessions helped or changed their perspectives. Participant 22 responded:

> I felt a lot better going into the next three exams after attending the worked-out examples. I felt a lot more prepared and therefore[,] more confident. This did improve my perspective on the class[;] it wasn't so intimidating anymore. I knew I would improve my scores on the exams to come.

It is obvious that the various benefits that students experienced are related to one another. Feeling more prepared can lead to one feeling more confident and better about the material, which can change one's perspective and attitude about a class.

Thoughts about perspective and confidence

When asked about the impact of the worked example sessions on the students' perspectives of the College Algebra class, fourteen students felt that the sessions helped. Seven were more explicit in mentioning the extra practice or improved understanding, and four cited improved confidence. The number of students who mentioned improved confidence (both for this question and others) was prescient. Another question on the survey specifically asked how the worked example sessions affected the students' confidence when taking tests. Twenty-four of the 30 students responded that their confidence improved, while the other six students were neutral. Therefore, the worked example sessions resulted in no negative results in terms of improving confidence.

When asked to surmise how they would have performed in the College Algebra class if they had not attended the worked example sessions, 17 felt that they would have done worse in the class, compared to 11 who said that they would have had similar results. Again, no students opined that they would have fared better by not attending the sessions. It is understandable and expected that some students would feel as though they did not benefit much from the sessions. These 11 students may have had strong backgrounds in math, but still attended the worked example sessions. Alternatively, some of these students could have just been supremely confident in their abilities.

The worked example sessions were still beneficial to the students, even if it was not measured in their grades or overall performance. Participant 12 wrote: "I think I could do just as well, but I would have to put a lot more time into studying without the worked-out example sessions." Participant 22 had similar thoughts: "I think I would still be doing well in the class, but I think I would have to put a lot more time and effort into my studying if the worked-out sessions weren't available." Participant 21 was more expansive:

> If the sessions were not available, I feel I would be doing about the same in the course. As mentioned for the first exam[,] I received a 95[,] but I put forth a lot of hard work. I would have continued this effort throughout the semester[,] but attending the worked-out example sessions was less stressful. I do feel that the sessions were still a benefit to me.

Strengths and weaknesses of the worked example sessions

The most common strengths of the worked examples sessions mentioned by students were the worked examples themselves (12 students) and the opportunity to ask questions to the helpers (17 students). Similar traits were also indicated: the chance to practice more problems (3 students) and the opportunity to work in groups (2 students). Taken together, the chance to work on problems and get help (from the dedicated helpers or from other students) were deemed to be the most advantageous aspects of the worked example sessions.

No matter how good a program is, it will still have aspects that can be improved upon. Six students thought that clarity was a problem, but they were not always clear about where they encountered those issues: written explanations on the worked example worksheets or verbal explanations from the helpers. Another six students were discouraged by topics on the worksheets that were not covered in the class yet, so pacing and coordination were issues. Two students stated that there were too many distractions in the class, but that is somewhat unavoidable when students work in groups and helpers offer assistance.

One of the weaknesses could also be considered in a positive light. Five students lamented the fact that there were not more sessions, wished that they were longer in duration, or would have liked them to have started earlier in the semester. Regardless of the specific request, these students all echoed Oliver Twist: "Please, sir, I want some more." That speaks to how much the students felt that they benefited from the extra practice.

Despite the aspects of the class that the students did not like, 28 of the 30 students said that they would recommend the class to another student, and the other two students did not answer the question. Therefore, none of the students would not recommend the worked example sessions. The strengths of the recommendations varied a bit, though. Participant 13 wrote, "I would definitely recommend it;" whereas, Participant 14 responded, "I would tell him to take it if he is struggling." Participant 29 saw the added benefit of the worked example sessions counting as an actual pass-fail class: "I would recommend a friend to take the course. It is an extra credit hour and is very beneficial in helping you practice math problems." Participant 18 was very pragmatic: "Definitely take it; no harm was done, only pros. [I]t is what you make it." The strongest recommendation came from Participant 17, even if it was a bit unrealistic: "I would say don't sign up for the actual class[,] just go to the sessions [if needing] help."

Discussion

The worked examples appeared to benefit the participants, in both their grades and their perspectives of the class. It should be noted that for the first test, neither the experimental group nor control group participated in the worked examples sessions, as they did not start until mid- semester. However, there was a trial period four weeks before the worked examples course started, between the first and second tests. All students in the College Algebra course were allowed to come to these sessions, which were conducted in the exact same way as the actual worked examples course. Thus, there were some individuals in the control group who participated in at least one worked examples session and there are individuals in the experimental group who did not get introduced to them until the formal sessions started at mid-semester.

The students from the experimental group attended significantly more classes than the students from the control group. In the survey, many of the participants mentioned they realized the necessity of being in class. They found that attending class and the worked example sessions were very beneficial, often resulting in being able to better understand the material. Thus, this could have impacted their attendance in a positive way.

The participants completed a lot of work during the sessions. It was their decision to attend their lecture and complete the worked examples worksheets. The students asked questions at their own discretion. Assistants circulated around the classroom answering questions and helping students who were having difficulties, but the students needed to be engaged in the sessions. With this, it is important to note the differences in performance between the two groups. Although not all aspects of the data collected showed significant differences, the experimental group outperformed the control group on every measure after Test 1. Starting the sessions earlier may have produced more drastic results.

The students who attended the worked example sessions repeatedly mentioned how much they felt the sessions helped them in College Algebra. Primarily, they appreciated the opportunity to get some extra practice and ask questions, which led to them being more comfortable with the material and resulted in better performance on assessments. A few also mentioned that it took them less time to learn the material, which corresponds with the findings of Sweller and Cooper (1985). Confidence was also a recurring theme in the survey responses, even when the question did not ask about it. The self-fulfilling prophecy states that one's expectations often culminate in actions that result in confirming those expectations. When students lack confidence and feel that math (or anything, for that matter) is too difficult, the resulting pessimistic attitude leads to the expected failure. While an optimistic perspective will not always result in success, a positive outlook can eliminate the debilitating effects of a negative view. Henry Ford summarized it well: "Whether you think you can, or think you can't, you're right."

Confidence can be a powerful tool. When one has self-confidence in an endeavour, difficult tasks no longer seem to be insurmountable. The belief that success is possible can result in a greater work ethic. After all, there is no point even trying to complete an impossible task. Conversely, having some measure of success can lead to a snowball effect: success leads to the expectation of subsequent success, which encourages hard work, which culminates in gaining a deeper understanding of the material, which results in more success. Like a snowball that picks up speed, momentum, and mass as it rolls down a hill, an initial success can begin the process that results in future success. With the success and increased confidence comes enjoyment, which makes the work feel like less of a task, which can result in doing the work more often. However, it is described or examined, this success-confidence-growth cycle can quickly morph into a self-perpetuating process.

Implications for Teaching

Introductory math courses are often taught in large lecture halls that are less than personable, which makes it difficult for students to become engaged in the class. This detachment often results in malaise. Factor in the many opportunities for distractions and it is understandable why many students receive a D or F, or withdraw from the course. Students do not always have access to tutoring or recitation sessions in a particular course because some universities or departments are not able to offer such resources. Worked examples are a teaching technique that requires few outside resources (including personnel, time, and funding), but could have a great impact on the success of students.

By implementing worked example sessions, students are given an extra day to practice the material and better understand what was covered during the lecture. It also provides students with an opportunity to ask questions they have about the content. Students are typically not as likely to ask questions during lecture, but they may be more willing to seek help in a more one-on-one atmosphere, especially one that invites questions. The sessions can be thought of as large-scale, ultra-efficient office hours: a way for instructors to help a lot of students outside of the class in a way that could not be done during normal office hours.

A primary resource for worked example sessions are the assistants who help the instructor answer questions. These teaching assistants need to prepare for the sessions by reviewing the worksheets, making sure they understand the expert solutions, and knowing how to approach the problems in multiple ways so that they can assist students who might need additional help. Being a teaching assistant will also be a valuable experience, particularly for students who are studying math, engineering, and education, since one can learn a lot about both teaching and math by explaining a concept to someone else. Rather than offering monetary compensation, course credit can be used as an incentive for those who volunteer to help, reducing the strain on department budgets. Therefore, this can be accomplished with limited resources which is another important component of these worked example sessions. That is, departments, instructors, and teaching assistants will not need to expend great amounts of effort to produce positive outcomes for the students. Since worked examples helped the students who participated in them, it would be very beneficial to implement them in introductory math courses.

Limitations

In this study, we did not have the opportunity to have a larger experimental group to compare to the control group. With a larger group, we could have produced more robust data. Also, we did not have the opportunity to have question-and-answer sessions for the control group, so an alternative treatment was not provided. The students from the control group were also not surveyed to gather their impressions of the course, so their opinions were not obtained, and could not be compared to those from the experimental group. Prior studies advise that students be given the worked examples and then be asked to complete a similar problem without referencing the example. However, this was not possible in this study because the participants had the worked example worksheets in their possession.

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