A Mathematical Model to Determine the Optimal Ratio of Researchers of Different Categories for Solving a Scientific Problem in the Military Sphere

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Abstract

In the paper, the authors propose a variant of the mathematical model for justifying the optimal ratio of researchers of different categories to conduct scientific research of the highest possible quality in conditions of limited resources. The discrepancy is formulated between the quality of scientific research and the restriction on financial resources, that is, the problem of resource allocation is solved. The relationship between the quality of scientific research and the number of researchers is proposed to be reflected by the canonical parabola equation. A mathematical model is formulated that reflects the essence of the question under study. The problem is solved using the method of Lagrange multipliers. The results of the study are confirmed by a numerical experiment. Resource constraints have always existed. This is especially true now for the development of the Armed Forces of Ukraine and increasing their combat and mobilisation readiness, which result in the country's defence capability as a whole. Limited funding also takes place in military science. It is very difficult to introduce new full-time positions and divisions. Previously, the number of researchers was justified following regulatory documents when creating scientific institutions and divisions, or by analogy with similar scientific institutions. In other words, the problem was solved empirically or situationally. This scientific study concerns substantiating the number of scientific personnel in conditions of limited resources, taking into account the work that is now performed and will be performed in the future.

Keywords: conditions-constraints, relationships between researchers, mathematical model, quality level, system of equations

1. Introduction

The problem of resource allocation has been solved for a long time. The authors use different approaches and methods. For example, the author considers the problem of multi-stage distribution of enterprise resources in selected areas of activity (Katkova, 2014). The dynamic programming method is used.

Modelling the distribution of resources is proposed by the author as a game of several people (Gornaneva, 2006). The author solves the problems of optimal control over the personnel distribution of combat vehicles in conflict situations (Kononov, 2008). Methods are proposed for solving problems of optimal control over weapon and military equipment distribution using statistical models. The authors allocate resources in a discrete game (Morozov, 2014). The author provides optimal resource allocation and elements of system synthesis (Berzin, 1974; Shuyenkin et al., 1997; Vygodskiy 1962). However, in these papers, the authors mainly use linear models. In this study, the model is nonlinear, so a more advanced apparatus is needed. To solve this problem, the method of Lagrange multipliers is used. The purpose of the paper is to construct a mathematical model to determine the optimal ratio of researchers of different categories to solve a scientific problem (Najafnejhad et al., 2021; Abdolazimi et al., 2021; Farias et al., 2019).

Extreme problems have solutions if there is a conflict situation in their condition. Regarding this study, the conflict situation is as follows. On the one hand, the desire to improve the quality of scientific research leads to the need to increase the percentage of researchers of the highest category (doctors, candidates of Sciences), which can result in non-fulfilment of the condition-restrictions, for example, on the financial component. In other words, the following condition must be met:

$$\mathrm{c}_1 \cdot \mathrm{x}_1 + \mathrm{c}_2 \cdot \mathrm{x}_2 + \mathrm{c}_3 \cdot \mathrm{x}_3 \leq \mathrm{C}_{\mathrm{exp}}, \tag{1}$$

where: C_{exp} funds allocated for the study; X_1 is the number of doctors of Sciences involved; X_2 the number of candidates of Sciences involved; X_3 is the number of specialists involved in a certain profile without an academic degree; c_1 is the remuneration of one doctor of science; c_2 is the remuneration of one candidate of Sciences; c_3 is the remuneration of one specialist of a certain profile without an academic degree (Luchko et al., 2020; Chen et al., 2019; Creff et al., 2020).

It should be noted that the difference in the salary of researchers of different categories indirectly characterises the amount of their contribution to the overall research result. On the other hand, the desire to fulfil a condition-restriction, for example, concerning the allocated finances (1), causes the need to reduce the percentage of researchers of the highest category (Rani et al., 2021; Numada, 2021). This, in turn, can lead to a decrease in the quality of scientific research. Based on this discrepancy, it is necessary to find such an acceptable ratio of researchers of different categories in order to achieve the highest possible quality of scientific research in conditions of limited resources for their implementation. At the same time, the quality of scientific research will be evaluated by the level of ϕ_i, which can be achieved by selecting a certain ratio of researchers of different categories (2):

$$\phi_i = \frac{x_i}{x_0}, \tag{2}$$

where: X_0 is the given number of researchers involved in research, for example, ARW (academic research work), RDW (research and development work), etc.; X_i is the number of researchers of a certain category.

Therefore, we can write (3):

$$x_0 = x_1 + x_2 + x_3, \tag{3}$$

from where:

$$\frac{x_1}{x_0} + \frac{x_2}{x_0} + \frac{x_3}{x_0} = 1. \tag{4}$$

Then expression (1) can be written as (5):

$$c_1 \cdot \frac{x_1}{x_0} + c_2 \cdot \frac{x_2}{x_0} + c_3 \cdot \frac{x_3}{x_0} \leq \frac{C_{exp}}{x_0}, \tag{5}$$

where: $\frac{C_{exp}}{x_0}$ is the average salary of one (any) researcher. Obviously (6):

$$0 \leq \frac{x_i}{x_0} \leq 1. \tag{6}$$

It follows that the ratio $\frac{x_i}{x_0}$ or quantity X_i of the involved researchers of various categories is the main factor that affects the quality of the study. The achieved quality level ϕ_i is the main indicator that characterizes this factor. The possible value of the quality level is within (7):

$$0 \le \phi_i \le 1. \tag{7}$$

2. Materials and Methods

The experience of some studies conducted at the Central Scientific Research Institute of Armament and Military Equipment of the Armed Forces of Ukraine (Shuyenkin et al., 1997). This indicates that the quality (result) of scientific research increases with an increase in the number of researchers involved in a particular category according to the law, which is quite closely reflected by a canonical parabolic equation with the parameter *p* (Vygodskiy, 1962).

So, the relationship between the quality ϕ_i of research and the quantity X_i (or the ratio $\frac{x_i}{x_0}$) of researchers who are involved can be expressed as follows (8):

$$\phi_i^2 = 2p \cdot \frac{x_i}{x_0}, \tag{8}$$

where: $2p$ is the focal chord, or by p = 0,5 (9):

$$\phi_i^2 = \frac{x_i}{x_0}, \tag{9}$$

where: ϕ_i is the level of quality of scientific research that employees of the *i*- th *i*- th ($i = \overline{1,3}$) categories achieve. Dependence (9) is presented in Table 1, and the graph in Figure 1, reflecting the staffing level of researchers.

Table 1. Dependence of the Quality of Research and the Number of Researchers Who Are Involved

	$\frac{x_i}{x_0}$					
	0	0.2	0.4	0.6	0.8	1.0
$\phi_i = \sqrt{\frac{x_i}{x_0}}$	0	0.45	0.63	0.77	0.89	1.0

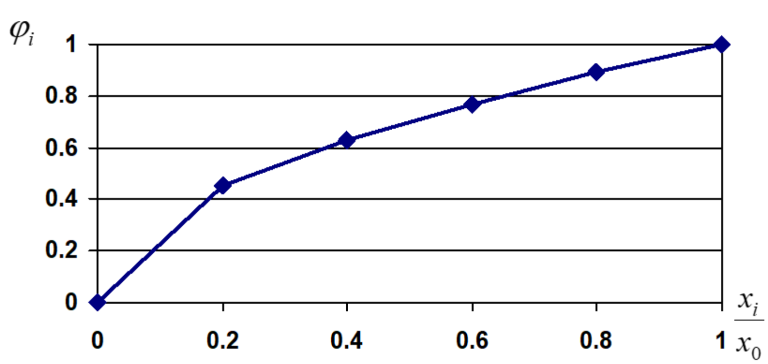

Figure 1. Dependency Graph of the Quality of Research vs the Number of Researchers

As you can see, the quality level value is $\phi_i \to 1$ for $x_i \to x_0$, that is, for $\frac{x_i}{x_0} \to 1$, that is natural and what corresponds to (9). There may be other patterns between ϕ_i and X_i. Then (8) can have a different form. It is obvious that in addition to the number of researchers of a certain category X_i, the contribution α_i of employees in the overall result of scientific research must be taken into account in the expression for the quality level ϕ_i. If we assume that $\alpha_i = \sqrt{2p}$, then expression (8) can have the following form (10) for any quality level:

$$\phi_i = \alpha_i \cdot \sqrt{\frac{x_i}{x_0}}, \qquad 0 \le \alpha_i \le 1. \tag{10}$$

Given (10) the expression for the quality level ϕ_i of research by all participating researchers of various categories can be presented as follows (11):

$$\phi = \sum_{i=1}^{3} \phi_i = \alpha_1 \cdot \sqrt{\frac{x_1}{x_0}} + \alpha_2 \cdot \sqrt{\frac{x_2}{x_0}} + \alpha_3 \cdot \sqrt{\frac{x_3}{x_0}} = \sum_{i=1}^{3} \left(\alpha_i \cdot \sqrt{\frac{x_i}{x_0}}\right), \tag{11}$$

where:α_1 is the amount of contribution to the overall result of involved doctors of Sciences; α_2 is the amount of contribution to the overall result of involved candidates of Sciences; α_3 is the amount of contribution to the overall result of involved specialists. In this case (12):

$$\sum_{i=1}^{3} \alpha_i = 1. \tag{12}$$

To more clearly reflect the fulfilment of conditions (4) and (6), we write the expression (11) in the following form:

$$\phi = \sum_{i=1}^{3} \phi_i = \alpha_1 \cdot \sqrt{\frac{x_1}{x_0}} + \alpha_2 \cdot \sqrt{\frac{x_2}{x_0}} + \alpha_3 \cdot \sqrt{\frac{x_3}{x_0}} =$$

$$\alpha_1 \cdot \sqrt{1 - \frac{x_2}{x_0} - \frac{x_3}{x_0}} + \alpha_2 \cdot \sqrt{1 - \frac{x_1}{x_0} - \frac{x_3}{x_0}} + \alpha_3 \cdot \sqrt{1 - \frac{x_1}{x_0} - \frac{x_2}{x_0}}. \tag{13}$$

You can use this expression to determine the maximum values taking into account (4) and (6), which is reflected in Table 2.

Table 2. Maximum Values of the Quality Level by Limit Values $\frac{x_i}{x_0} = 1$

$0 \le \frac{x_1}{x_0} \le 1$	$0 \le \frac{x_2}{x_0} \le 1$	$0 \le \frac{x_3}{x_0} \le 1$	Maximum value ϕ
1	0	0	$\alpha_1 \cdot \sqrt{\frac{x_1}{x_0}} = \alpha_1$
0	1	0	$\alpha_2 \cdot \sqrt{\frac{x_2}{x_0}} = \alpha_2$
0	0	1	$\alpha_3 \cdot \sqrt{\frac{x_3}{x_0}} = \alpha_3$

3. Results and Discussion

Because the research requires the optimal ratio between researchers of different categories and the maximum possible level of quality ϕ_i from the point of view of mathematical programming, this problem can be formulated as follows: find the optimal value of the ratio $\frac{x_i}{x_0}$, which will ensure that the maximum possible level of research quality is achieved for known X_i, C_{exp}, c_1, c_2, c_3, α_1, α_2, α_3, which can be mathematically written as follows (14):

$$\max \phi = \sum_{i=1}^{3}\left(\alpha_i \cdot \sqrt{\frac{x_i}{x_0}}\right), \quad (14)$$

These conditions reflect the content of the conflict situation discussed above. In addition, the number of constraint conditions is less than the number of variables $\left(\frac{x_1}{x_0}, \frac{x_2}{x_0}, \frac{x_3}{x_0}\right)$. This implies the possibility of solving an extreme problem. It follows from (14) that this problem belongs to the class of nonlinear programming problems. Under the condition of continuity of function (13), this problem can be solved by the method of Lagrange multipliers. Then, as you know, the Lagrange function is written as follows (15):

$$L(x) = \sum_{i=1}^{3}\left(\alpha_i \cdot \sqrt{\frac{x_i}{x_0}}\right) + \lambda_1 \cdot \left[\frac{C_{exp}}{x_0} - \left(c_1 \cdot \frac{x_1}{x_0} + c_2 \cdot \frac{x_2}{x_0} + c_3 \cdot \frac{x_3}{x_0}\right)\right] += +\lambda_2 \cdot \left[1 - \left(\frac{x_1}{x_0} + \frac{x_2}{x_0} + \frac{x_3}{x_0}\right)\right], \quad (15)$$

where: λ_1, λ_2 is the undefined Lagrange multipliers $(\lambda_1; \lambda_2 \geq 0)$. We take the partial derivatives of the Lagrange function for the corresponding variables and equate the obtained results to zero (16):

$$\frac{\partial L}{\partial\left(\frac{x_1}{x_0}\right)} = \frac{1}{2} \cdot \frac{\alpha_1}{\sqrt{\frac{x_1}{x_0}}} - \lambda_1 \cdot c_1 - \lambda_2 = 0$$

$$\frac{\partial L}{\partial\left(\frac{x_2}{x_0}\right)} = \frac{1}{2} \cdot \frac{\alpha_2}{\sqrt{\frac{x_2}{x_0}}} - \lambda_1 \cdot c_2 - \lambda_2 = 0,$$

$$\frac{\partial L}{\partial\left(\frac{x_3}{x_0}\right)} = \frac{1}{2} \cdot \frac{\alpha_3}{\sqrt{\frac{x_3}{x_0}}} - \lambda_1 \cdot c_3 - \lambda_2 = 0,$$

$$\frac{\partial L}{\partial \lambda_1} = \frac{C_{exp}}{x_0} - \left(c_1 \cdot \frac{x_1}{x_0} + c_2 \cdot \frac{x_2}{x_0} + c_3 \cdot \frac{x_3}{x_0}\right) = 0,$$

$$\frac{\partial L}{\partial \lambda_2} = 1 - \left(\frac{x_1}{x_0} + \frac{x_2}{x_0} + \frac{x_3}{x_0}\right) = 0. \quad (16)$$

Excluding undefined multipliers λ_1, λ_2 from the first three equations, taking into account that $c_1 > c_2 > c_3$, let's move on to three equations with three unknowns (17):

$$\begin{cases} \alpha_1 \cdot (c_2 - c_3) \cdot \sqrt{\frac{x_2}{x_0} \cdot \frac{x_3}{x_0}} + \alpha_3 \cdot (c_1 - c_2) \cdot \sqrt{\frac{x_1}{x_0} \cdot \frac{x_2}{x_0}} = \alpha_2 \cdot (c_1 - c_3) \cdot \sqrt{\frac{x_1}{x_0} \cdot \frac{x_3}{x_0}} \\ c_1 \cdot \frac{x_1}{x_0} + c_2 \cdot \frac{x_2}{x_0} + c_3 \cdot \frac{x_3}{x_0} = \frac{C_{exp}}{x_0} \\ \frac{x_1}{x_0} + \frac{x_2}{x_0} + \frac{x_3}{x_0} = 1 \end{cases} \quad (17)$$

Let's go directly to the values X_1, X_2, X_3, given that they are integers. Then equations (16), taking into account (1) and (3), are written as (18):

$$\begin{cases} \alpha_1 \cdot (c_2 - c_3) \cdot \sqrt{x_2 \cdot x_3} + \alpha_3 \cdot (c_1 - c_2) \cdot \sqrt{x_1 \cdot x_2} = \alpha_2 \cdot (c_1 - c_3) \cdot \sqrt{x_1 \cdot x_3} \\ c_1 \cdot x_1 + c_2 \cdot x_2 + c_3 \cdot x_3 = C_{exp} \\ x_1 + x_2 + x_3 = x_0 \end{cases} . \quad (18)$$

As can be seen, the systems of equations (17) and (18) are equivalent. They make up the mathematical model of this problem. The next question is to solve the system of equations (18). In this case, it is advisable to do this graphically. To do this, let's first move from the system of three equations to the system of two equations with variables X_2, X_3. Given that (3) $x_1 = x_0 - x_2 - x_3$, the second equation of system (17) is written as follows: $c_1 \cdot x_0 - (c_1 - c_2) \cdot x_2 - (c_1 - c_3) \cdot x_3 = C_{exp}$, or (19):

$$x_2 = \frac{c_1 \cdot x_0 - C_{exp}}{c_1 - c_2} - \frac{c_1 - c_3}{c_1 - c_2} \cdot x_3. \quad (19)$$

This is the equation of a straight line that passes through the 2nd and 4th quadrants of the Cartesian coordinate system. For example, for c_1 = 8000 UAH, c_2 = 6000 UAH, c_3 = 4000 UAH, X_0 = 20, C_{exp} = 100 thousand UAH equation (19) will have the form (20):

$$x_2 = 30 - 2 \cdot x_3. \quad (20)$$

This dependency $x_2 = f(x_3)$ is presented in Table 3 and Figure 2.

Table 3. Dependence of the Number of Candidates of Science Involved and the Number of Specialists of a Certain Profile without an Academic Degree in (20)

	X_3								
	0	2	4	6	8	10	12	14	15
X_2	30	26	22	18	14	10	6	2	0

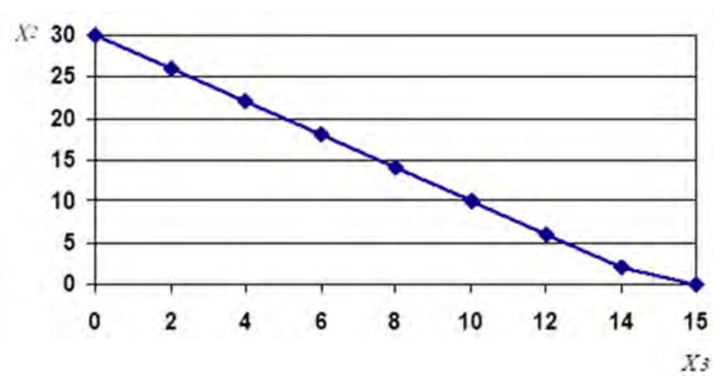

Figure 2. Dependency Graph of the Number of Involved Candidates of Sciences and the Number of Involved Specialists of a Certain Profile without an Academic Degree for (20)

Let us rewrite the first equation of system (18) in the following form (21):

$$\alpha_1 \cdot (c_2 - c_3) \cdot \sqrt{x_2 \cdot x_3} = \sqrt{x_1} \cdot \left[\alpha_2 \cdot (c_1 - c_3) \cdot \sqrt{x_3} - \alpha_3 \cdot (c_1 - c_2) \cdot \sqrt{x_2}\right], \quad (21)$$

from where (22):

$$\sqrt{x_1} = \frac{\alpha_1 \cdot (c_2 - c_3) \cdot \sqrt{x_2 \cdot x_3}}{\alpha_2 \cdot (c_1 - c_3) \cdot \sqrt{x_3} - \alpha_3 \cdot (c_1 - c_2) \cdot \sqrt{x_2}}. \quad (22)$$

When both parts of the equation are squared and taking into account (3), we obtain (23):

$$x_1 = \frac{\alpha_1^2 \cdot (c_2 - c_3)^2 \cdot x_2 \cdot x_3}{\alpha_2^2 \cdot (c_1 - c_3)^2 \cdot x_3 - 2 \cdot \alpha_2 \cdot \alpha_3 \cdot (c_1 - c_3) \cdot (c_1 - c_2) \cdot \sqrt{x_2 \cdot x_3} + \alpha_3^2 \cdot (c_1 - c_2)^2 \cdot x_2}. \quad (23)$$

That is (24):

$$x_1 = x_0 - x_2 - x_3 = \frac{\alpha_1^2 \cdot (c_2 - c_3)^2 \cdot x_2 \cdot x_3}{\alpha_2^2 \cdot (c_1 - c_3)^2 \cdot x_3 - 2 \cdot \alpha_2 \cdot \alpha_3 \cdot (c_1 - c_3) \cdot (c_1 - c_2) \cdot \sqrt{x_2 \cdot x_3} + \alpha_3^2 \cdot (c_1 - c_2)^2 \cdot x_2} \quad (24)$$

Let's move on to a numerical experiment. If, for example, $\alpha_1 = 0{,}5$, $\alpha_2 = 0{,}3$, $\alpha_3 = 0{,}2$, equation (24) will have the form (25):

$$\frac{10^6 \cdot x_2 \cdot x_3}{1{,}44 \cdot 10^6 \cdot x_3 - 0{,}96 \cdot 10^6 \cdot \sqrt{x_2 \cdot x_3} + 0{,}16 \cdot 10^6 \cdot x_2} = 20 - x_2 - x_3, \quad (25)$$

Or (26):

$$\frac{x_2 \cdot x_3}{1{,}44 \cdot x_3 - 0{,}96 \cdot \sqrt{x_2 \cdot x_3} + 0{,}16 \cdot x_2} = 20 - x_2 - x_3. \quad (26)$$

As can be seen from this equation, for $X_3 = 0$ $X_2 = 20$; $X_3 = 20$ X_2 is found from the following equation: $0.16 \cdot x_2^2 + 48.8 \cdot x_2 - 4.3 \cdot x_2 \cdot \sqrt{x_2} = 0$, or: $x_2 \cdot (0.16 \cdot x_2 + 48.8 - 4.3 \cdot \sqrt{x_2}) = 0$. From where it follows that $X_2 = 0$ for: $0.16 \cdot x_2 + 48.8 - 4.3 \cdot \sqrt{x_2} \neq 0$.

This dependency $x_2 = f(x_3)$ is presented in Table 4 and Figure 3.

Table 4. Dependence of the Number of Involved Candidates of Sciences and the Number of Specialists of a Certain Profile without an Academic Degree in (26)

	X_3								
	0	2	4	6	8	10	12	15	20
X_2	20	4.68	5.56	5.59	5.22	4.63	3.89	2.59	0

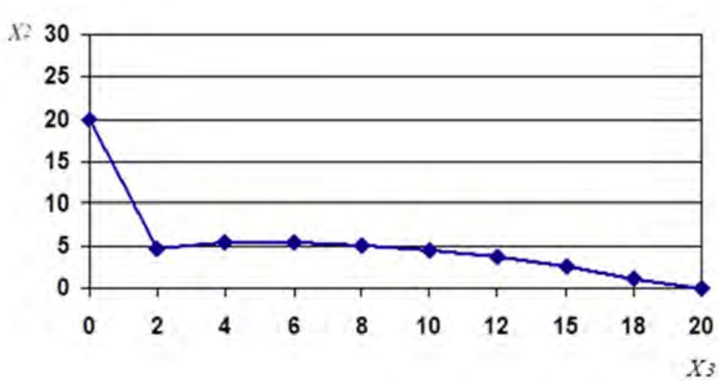

Figure 3. Dependency Graph of the Number of Attracted Candidates of Sciences vs the Number of Involved Specialists of a Certain Profile without an Academic Degree for (22)

If you combine the graphs in Figure 2 and Figure. 3, then we obtain the intersection point, which will be the solution of the system of equations (18). This is shown in Figure 4.

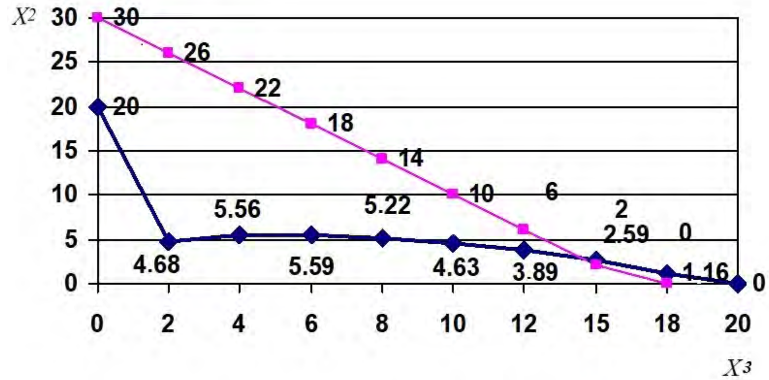

Figure 4. Dependency Graph of the Number of Involved Candidates of Science vs the Number of Specialists of a Certain Profile without an Academic Degree for (20) and (22)

Intersection of curves in Figure 4 indicates the compatibility of equations (24) and (26). As a consequence, you can determine the coordinates of the resulting intersection point: $x_2 \approx 3; x_3 \approx 15$. According to (3) $X_1 = 2$. Thus, for this example, the ratio $\frac{x_i}{x_0}$, which affects the quality of scientific research will be equal to (27):

$$\frac{x_1}{x_0} = 0.1;\ \frac{x_2}{x_0} = 0.15;\ \frac{x_3}{x_0} = 0.75. \quad (27)$$

That is for $\alpha_1 = 0.5,\ \alpha_2 = 0.3,\ \alpha_3 = 0.2$ the objective function will be $\max \phi = 0.245$.

The procedure described above, namely solving the system of equations, will be easier if you use, for example, the Mathcad application. To make sure that at a point with coordinates $X_1 = 2$; $X_2 = 3$; $X_3 = 15$ objective function (14) will reach its maximum, it is necessary to determine the sign of the second derivative of this function. If this sign is negative at a given point, then indeed the objective function reaches its maximum.

4. Conclusion

The paper substantiates the optimal ratio of researchers of different categories to solve a scientific problem using a mathematical model. The results obtained adequately reflect the research process. To solve the scientific problem for this example, it is necessary to involve two doctors of Sciences, three candidates of Sciences and fifteen specialists of a certain profile without an academic degree. That is, the optimal ratio of researchers in the given conditions is found.

A law that is quite closely reflected by the canonical parabolic equation expresses the dependency of the quality of research on the number or ratio of involved researchers of different categories. It is possible that other laws can be used. For example, using the tangent chord method, you can try to find more accurately the intersection point of graphs depending on the error. The results obtained using a mathematical model to justify the optimal ratio of researchers of different categories to conduct scientific research of the highest possible quality in conditions of limited resources can be compared with the results of the tangent chord method. This will be the subject of further research.

References

Abdolazimi, O., Shishebori, D., Goodarzian, F., Ghasemi, P., & Appolloni, A. (2021). Designing a new mathematical model based on ABC analysis for inventory control problem: *A real case study. RAIRO – Operations Research*, *55*(4), 2309-2335. https://doi.org/10.1051/ro/2021104

Berzin, E. A. (1974). *Optimal resource allocation and system synthesis elements.* Moscow, USSR: Sovietskoe Radio.

Chen, Y.-K., Chiu, F.-R., & Chang, Y.-C. (2019). Implementing green supply chain management for online pharmacies through a vadd inventory model. *International Journal of Environmental Research and Public Health*, *16*(22), article number 4454. https://doi.org/10.3390/ijerph16224454

Creff, S., Noir, J. L., Lenormand, E., & Madelénat, S. (2020). Towards facilities for modeling and synthesis of architectures for resource allocation problem in systems engineering. *ACM International Conference Proceeding Series,* F164267-A, 211-216. https://doi.org/10.1145/3382025.3414963

Farias, K., Hadj-Hamou, K., & Yugma, C. (2019). Mathematical formulations for a two-echelon inventory routing problem. *IFAC-PapersOnLine*, *52*(13), 1996-2001. https://doi.org/10.1016/j.ifacol.2019.11.496

Gornaneva, O. I. (2006). Modeling resource allocation as game in normal form. *Natural Sciences*, *2*(2), 16-22.

Katkova, T. I. (2014). Multi-stage distribution of enterprise assets by strategic areas of activity. *Bulletin of National Technical University «Kharkiv Polytechnic Institute»*, *35*(1078), 113-120.

Kononov, V. B. (2008). Methods for solving the problems of optimal control of the distribution of the composition of military equipment in conflict situations described by statistical models. *Bulletin of Kharkov University of Air Force named after I. Kozhedub*, *2*(69), 68-71.

Luchko, M. R., Lukanovska, I. R., & Ratynskyi, V. (2020). Modelling inventory management: Separate issues for construction and application. *International Journal of Production Management and Engineering*, *7*(2), 117-124. https://doi.org/10.4995/ijpme.2019.11435

Morozov, V. V., & Shalbuzov, K. D. (2014). On the solution of a discrete resource allocation game. *Bulletin of Moscow University*, *152*(2), 10-16. https://doi.org/10.3103/S027864191402006X

Najafnejhad, E., Tavassoli Roodsari, M., Sepahrom, S., & Jenabzadeh, M. (2021). A mathematical inventory model for a single-vendor multi-retailer supply chain based on the vendor management inventory policy. *International Journal of Systems Assurance Engineering and Management*, *12*(3), 579-586. https://doi.org/10.1007/s13198-021-01120-z

Numada, M. (2021). Development of matching modeling for human resource allocation of shelter management by the set theory. *Journal of Disaster Research*, *16*(4), 719-732. https://doi.org/10.20965/jdr.2021.p0719

Rani, P., & Mahapatra, G. S. (2021). Entropy based enhanced particle swarm optimization on multi-objective software reliability modelling for optimal testing resources allocation. *Software Testing Verification and Reliability*, *31*(6), article number e1765. https://doi.org/10.1002/stvr.1765

Shuyenkin, V. A., Donchenko, V. S., Konstantinov, S. N., & Shapirovskiy, V. U. (1997). *Mathematical models of inventory management*. Kyiv, Ukraine: OOO International Financial Agency.

Vygodskiy, M. Y. (1962). *Handbook of higher mathematics*. Moscow, USSR: State Publishing House of Physics and Mathematics.

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