MODESTUM EURASIA Journal of Mathematics, Science and Technology Education, 2022, 18(6), em2118
ISSN:1305-8223 (online)
OPEN ACCESS Research Paper https://doi.org/10.29333/ejmste/12086

The influence of mathematical knowledge for teaching towards elementary teachers' mathematical self-efficacy

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Received 6 February 2022 ▪ Accepted 7 May 2022

Abstract

This research study investigates how elementary teachers' mathematical knowledge for teaching (MKT) relates to their mathematical self-efficacy (MSE) and whether the MKT can predict the MSE. Researchers administered the MSE scale and the MKT test to 86 in-service elementary mathematics teachers. Two-step regression analyses results indicated that MKT and its proxy measures (teaching experience, training type, and hours) were significant predictors of elementary mathematics teachers' MSE level; however, the MKT was the strongest. Considering these results, the research recommends aligning the MKT and the MSE when building teacher preparation and professional development programs. The implications for teacher preparation and developmental programs are further discussed.

Keywords: teacher preparation program, elementary teachers, mathematical knowledge for teaching, mathematical self-efficacy

INTRODUCTION

As many studies claimed, student achievement is strongly determined by teacher performance, primarily when mathematics is taught to elementary students (e.g., Ball et al., 2005; Carnoy et al., 2012; Taylor & Taylor, 2013; Taylor & Vinjevold, 1999). Therefore, to teach mathematics effectively, teachers need to know what to teach (content knowledge-CK) and how to teach it (pedagogical knowledge-PK). Furthermore, helping students to comprehend abstract mathematical concepts needs from their teacher to amalgamate both domains of knowledge as a result of teaching development and become what is known as pedagogical CK (PCK) (Shulman, 1986) or for mathematics content as mathematical knowledge for teaching (MKT) (Hill et al., 2008a). The MKT is a critical competency for mathematics teachers, and it influences their performance (Hill, 2010); however, the self-efficacy for mathematics teachers is also another factor in teaching effectiveness (Evans, 2013; Oppermann et al., 2016), and it is essential for acquiring the teaching skills (Mannila et al., 2018). Bandura (1997) believes that individuals' possession of knowledge and skills does not alone enable them to face and solve problems. He argued that individuals' feelings and beliefs about their abilities and the extent to which they can use their knowledge and skills about the problems is a significant factor in their behavior while solving the problems they face, especially challenging ones (Bandura, 1997). Self-efficacy determines how individuals confront issues and their effort and perseverance to deal with problems. It also helps define the persistence and determination they will involve when obstacles appear while facing problems (Bandura, 1997). Many studies investigating mathematics teachers' self-efficacy figured that it is dependent on teachers' math background. Teachers' performance is related to their mathematical self-efficacy (MSE) and mathematics teaching efficacy since they affect their teaching practices (e.g., Bates et al., 2011; Livers et al., 2020; Ren & Smith, 2018; Stevens et al., 2013).

This study is interesting to examine the relationship between the MKT and the MSE as two important mathematics teaching capabilities and determine if the MKT can predict the MSE for elementary mathematics teachers in Saudi public schools. Providers of teacher education and professional development programs need to understand the relationship between the MKT and the MSE and if the MKT can predict the MSE before building their programs. We found a gap in the literature to address the relationship between the MKT and the MSE

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Contribution to the literature

- It highlighted the importance of elementary mathematics teachers' MKT for their MSE and how it should be considered a significant factor for teaching effectiveness. It verified that the MKT or its proxy measures could predict the MSE.
- It can help the developer of teacher preparation and professional development programs to improve the confidence of in-service and prospective mathematics teachers in their mathematical capacity through aligning the MKT and the MSE when building those programs.
- It can help future researchers better understand the relationship between the MKT and the MSE and employ other qualitative measures to verify if the MKT can predict elementary teachers' MSE.

and if the MKT can predict the MSE, especially for Saudi elementary mathematics teachers. Therefore, this study investigates how elementary teachers' MKT and MSE are related.

THEORETICAL BACKGROUND

MSE

The MSE is essential in a teacher's career life. It is defined as a teacher's belief in their mathematical ability to effectively teach a specific mathematical subject or task to help students master their learning outcomes (Bandura, 1977; Evans, 2013; Oppermann et al., 2016). Teaching quality can be influenced by the MSE and subsequently the learning quality; when a mathematics teacher has a high level of the MSE in their mathematical knowledge, skills, and teaching abilities, it might motivate their instructional practices (Carney et al., 2016; Evans, 2013; Oppermann et al., 2016; Schreiber & Filo, 2019). Besides, the MSE helps mathematics teachers alleviate their task of recognizing and implementing mathematical content and reform-oriented mathematics instruction (Carney et al., 2016; Oppermann et al., 2016). The MSE of teachers may surpass its impact on instructional practices to teachers' awareness of teaching responsibility and mathematical teaching efficacy (Poling, 2020). However, teachers face challenges while teaching that can inhibit their MSE, and managing their classes, especially for novice teachers (Tunc et al., 2019). More importantly, the level of self-efficacy can be transformed from teachers to learners and become a good predictor of learners' academic achievement (Oppermann et al., 2016). When students gain high self-efficacy from their positive relationships with their teachers, they can develop quality mathematical problem-solving skills (Zhou et al., 2020). Students' MSE is linked to teachers' abilities and capabilities in teaching mathematics (Bagaka's, 2011). However, the relationship between teachers' and students' MSE is not always positive, especially when teachers' MSE is not correlated with their MKT. For example, students with high self-efficacy may benefit less from their interaction with teachers when it is uncombined with quality instructional support (Martin & Rimm-Kaufman, 2015). In addition, when teachers overestimate their mathematical teaching competence, their MSE may hinder the mathematical learning development of their students (Kaskens et al., 2020; Schillinger, 2021). Therefore, the capacity to provide a valid MSE is an essential qualification for future mathematics teachers and is recommended to be tackled in alignment with other teaching competencies in teacher preparation and training programs (Association of Mathematics Teacher Educators, 2017). Teacher education and professional development programs should be designed to develop teachers' MSE through concrete models and ensure that their future teachers will enhance their students' learning and development (Tunc et al., 2019). Other physiological variables such as emotions towards mathematics and teaching are also recommended in teacher preparation programs to improve prospective mathematics teachers' MSE (Ciani et al., 2019). Moreover, the MSE may influence other teaching capabilities, such as knowledge, skills, and professional dispositions.

MKT

The MKT is a complex construct consisting of broad and deep expertise that qualifies mathematics teachers to deliver high-quality instructions and illustrate why and how mathematics is essential (Hill et al., 2004; Ko & Herbst, 2020). It is conceived as a unidimensional or multidimensional structure (Hill et al., 2004; König & Kramer, 2016), and it might exceed what is included in a mathematics curriculum or coursework. The MKT is assumed to be different from the CK, which alone does not guarantee that teachers will deliver effective teaching (Ko & Herbst, 2020). The MKT is a composite of theoretical and practical knowledge (Gasteiger et al., 2019). The theoretical knowledge describes what a teacher cognitively has, and it is usually tested by the traditional assessments, while the practical one can be noticed in teachers' instructional practices and be observed and evaluated through the alternative assessments (Gasteiger et al., 2019). Most of the theoretical part of the MKT presents to teachers during teacher preparation and professional development programs, whereas they master their practical knowledge while gaining expertise in teaching mathematics (Gasteiger et al., 2019). Ball et al. (2008)

described the MKT in six domains divided into two categories such as subject matter knowledge (SMK) and PCK. The organization and interrelation among these domains are considered differently by researchers who see them as connected or not (Ko & Herbst, 2020; König & Kramer, 2016; Senk et al., 2012). The concept of the MKT varies among researchers according to its jargon, measure, mathematics content topic, and grade level (Ko & Herbst, 2020). For instance, Hill et al. (2004) described teachers' knowledge for teaching elementary mathematics as a multidimensional construct that includes knowledge of various mathematical topics (e.g., number and operations and algebra) and domains (e.g., knowledge of content and knowledge of students and content) (Hill et al., 2004). However, some parts of the MKT are not fully depicted, like the specialized CK (SCK) (Ball et al., 2008; Hill, 2010; Ko & Herbst, 2020) and the knowledge of curriculum (Hill et al., 2008b), although their existence has been confirmed. Also, no differences of impact have been found among the MKT domains (Hill et al., 2005); however, it is possible among the different types of knowledge (SMK that contains: common CK, knowledge at the mathematical horizon, and specialized CK; PCK that contains: knowledge of content and students, knowledge of content and teaching, and knowledge of curriculum) (Hill et al., 2004).

The MKT measures are various due to the nature of the MKT while some tools are qualitative (Hill et al., 2005; Kersting et al., 2012; Ko & Herbst, 2020; König & Kramer, 2016; Schwarz et al., 2008), others are quantitative (Gasteiger et al., 2019; Hill et al., 2004; Mitchell et al., 2014; Senk et al., 2012). Like other types of knowledge, the MKT can be measured directly or indirectly (Kanwar et al., 1990). Examples for direct measures include intellectual and practical tests (e.g., task-based, essay, objective tests, classroom video analysis), and classroom observation instruments (Auletto & Stein, 2020; Gasteiger et al., 2019; Hill et al., 2004; Kersting et al., 2012; König & Kramer, 2016). For indirect measures, self-assessments and questionnaires are utilized in research studies to determine how teachers perceived their MKT (Hill et al., 2005; Mitchell et al., 2014), but their validity can be an issue, particularly with low experienced teachers (Kanwar et al., 1990). Moreover, proxy measures (e.g., math background, teaching experience, teaching degree, competency test results, beliefs about mathematics) are other examples of indirect measures used to assess teachers' MKT. However, researchers argued that these alternative measures are less valid than direct measures and recommended that the MKT measuring tools take into account the content and the context of teaching (Gasteiger et al., 2019; Hill et al., 2005; Schwarz et al., 2008). Ko and Herbst (2020) called for accuracy, distinction, and practicality in the MKT instruments (e.g., two-dimensional model of SMK-G) to measure the MKT through a series of teaching practice questions, and thus will help scholars to meet the complex nature of the MKT. Also, it is recommended that when developing an MKT measure is to include items that measure all MKT domains and demand participants to employ their high-order thinking skills (Hill, 2010; Hill et al., 2008b). Among the qualitative tools, classroom observation tools shall help complete the evaluation picture for teachers' MKT level when they intend to measure the practical element of the MKT found in teacher-student interactions and instructional practices (Auletto & Stein, 2020). Lastly, the validity and reliability of these tools should be tested in factor analytic studies and measure their psychometric quality (Ko & Herbst, 2020; Senk et al., 2012).

The quality of the MKT's impact on mathematics teaching effectiveness is mediated by several important factors related to teachers (characteristics and experiences) or external sources (e.g., school, curriculum, policies). Productive teachers' expertise in mathematics teaching and professional learning and development are frequently reported to positively influence their MKT (Hill et al., 2008b; Ko & Herbst, 2020; König & Kramer, 2016; Mitchell et al., 2014). Also, affirmative teacher beliefs about mathematics (e.g., how students should learn math, how teachers should implement the curriculum, how to connect math content to other domains of knowledge, especially the PK) are claimed as a significant internal factor to advance teachers' MKT (Hill et al., 2008b; Kersting et al., 2012). In addition, teacher characteristics like teaching degree, license level, major, leadership and professional activities, and mathematics background internally impact the quality of the MKT and instruction practices (Hill, 2010; Mitchell et al., 2014; Senk et al., 2012). Some of these influences have external sources like school environment, curriculum materials and regulations, and educational policies (Hill et al., 2008b; Mitchell et al., 2014). Finally, studies indicate a positive relationship for teachers' MKT with student achievement (Hill et al., 2005), and students' learning outcomes, self-efficacy to learn mathematics, and positive relationship with their teachers (Auletto & Stein, 2020) while it helps mathematics teachers to enhance their knowledge and beliefs about their students (Smith et al., 2012).

The relationship between professional development and the MKT is usually positive, mainly when it provides hands-on activities and instructional design with micro-teaching; and subsequently improves teachers' MSE (Evans, 2011; Perkins, 2019). Especially large-scale professional development that will equip mathematics teachers with capabilities to enhance their theoretical and practical MKT and, as a result, their teaching effectiveness (Carney et al., 2016; Jacob et al., 2017). There are limitations for teacher preparation programs to educate prospective teachers in some MKT areas noticed in international universities (Schwarz et al., 2008; Senk et al., 2012). Delivering a balanced MKT training program to mathematics teachers will help overcome these shortcomings (Baumert et al., 2010; Smith et al., 2012). Also, teacher preparation and training programs need to raise the bar for teaching qualifications by exceeding the minimum requirement of the mathematical background for teaching certification (Hill et al., 2004; Schwarz et al., 2008), and including specialized CK (SCK) guided with a detailed map and are learning outcome-oriented (Ball et al., 2008; Hill, 2010). These teachers' programs shall address the two main sections of the MKT (SMK & PCK) in all professional training and teacher education program activities and tailor them with teachers' current level of the MKT and experience (Baumert et al., 2010; Hine & Thai, 2019). Differentiated programs that match teachers' needs, school context (Hill et al., 2005; Lee & Santagata, 2020), and curriculum and student knowledge (Copur-Gencturk et al., 2019) are recommended. The empirical part of professional training is essential like the theoretical one, and studies recommended different activities to address this part (Hill et al., 2005; Jacob et al., 2017; Jacobson, 2017; Perkins, 2019). For example, mathematics coaching was claimed to theoretically and practically improve teachers' MKT. Ottmar et al. (2015) recommended the implementation of the responsive classroom (RC) in teachers' professional training programs to meet the behavioral and social needs of teachers and students' interactions and align the MKT with empirical social practices delivered in the classroom (Ottmar et al., 2015).

The Relationship Between the MSE and the MKT

The MSE is considered as one of the teaching capabilities that could be related to and predicted by the MKT (Isiksal-Bostan, 2016; Oppermann et al., 2016; Schreiber & Filo, 2019). However, these two teaching capacities have irregularities in their relationship (Austin, 2013, 2015; Norton, 2019a; Schreiber & Filo, 2019; Thomson et al., 2017). For instance, the MKT positively correlates with and predicts the MSE when mathematics teachers have enough self-sensations for their MSE and MKT, which is more likely to happen with expert teachers (Bjerke & Solomon, 2020; Bray, 2011; Corkin et al., 2015). Researchers claim that when mathematics teachers have a high level of MKT, that will pave the way for high MSE because it eases teachers' cognitive process and allows them to give more attention to the instructional process (Rushton et al., 2016). Other variables that help to validate this relationship are teachers' mathematical degree (Corkin et al., 2016; Ekmekci et al., 2019) and background (Corkin et al., 2015; Stevens et al., 2013), and the mathematical context of the MSE and the MKT scales (Austin, 2015). Therefore, the relationship between the MSE and the MKT could turn to be harmful or disappear when math teachers lack experience or knowledge, have low ability for self-evaluation, or utilize less valid instruments to measure the MSE or the MKT (Austin, 2013, 2015; Francis et al., 2017; Norton, 2019a, 2019b; Stevens et al., 2013; Thomson et al., 2017). Various variables could carry out an effect on the MSE or the MKT together or individually. For example, demographic variables like teachers' age, race, gender, and teaching certification level plus supportive school environment and culture hold a positive impact on both the MSE and the MKT (Brown, 2012; Clark et al., 2014; Corkin et al., 2015; Ren & Smith, 2018), and others like classroom management, communication with students and their parents, textbooks and instructional materials claimed to be mediator factors for novice teachers in their first teaching year (Auletto & Stein, 2020; Isiksal-Bostan, 2016). Expert teachers in mathematics commonly hold high MSE, which is sometimes accompanied by a high level of the MKT (Bjerke & Solomon, 2020; Corkin et al., 2015; Ekmekci et al., 2015). However, novice and expert teachers' actual MKT is not always correlated with their self-perceived expertise in the MKT (Austin, 2013, 2015). One of the reasons that teachers, especially novices, are more likely to give a subjective self-evaluation is because of their limited self-awareness or reluctance to evaluate themselves low (Austin, 2013, 2015; Francis et al., 2017; Norton, 2017; Norton, 2019a). The validity limitations in the MSE and the MKT scales are another reason (Austin, 2015; Norton, 2017). It is logical for novice and prospective math teachers to have limited MKT with low MSE, but it varies depending on their teacher preparation and professional development programs received (Brown, 2012; Campbell et al., 2014; Corkin et al., 2015; Isiksal-Bostan, 2016; Newton et al., 2012; Ren & Smith, 2018; Stevens et al., 2013; Thomson et al., 2017, 2020).

Researchers argue that the MKT and MSE are attainable through teachers' preparation and professional development programs (Campbell et al., 2014; Corkin et al., 2015; Isiksal-Bostan, 2016; Newton et al., 2012; Stevens et al., 2013; Thomson et al., 2017, 2020). Mathematical background for teachers could play an essential role in improving teachers' level of the MSE and the MKT after professional development programs or teacher education courses (Corkin et al., 2015; Stevens et al., 2013). Also, the quality of improvement increases when the content of professional training is more direct and intensive toward the MKT (Corkin et al., 2015, 2016; Ren & Smith, 2018; Stevens et al., 2013; Thomson et al., 2020).

Teaching mathematics effectively would become achievable when teachers have an acceptable level of the MSE and the MKT together (Aksu & Kul, 2019; Austin, 2015; Rushton et al., 2016). Researches on the correlation between the MKT and the MSE suggest that it is linked to in-service and prospective teachers' teaching effectiveness (Oppermann et al., 2016; Schreiber & Filo, 2019). Others identify both constructs as crucial factors

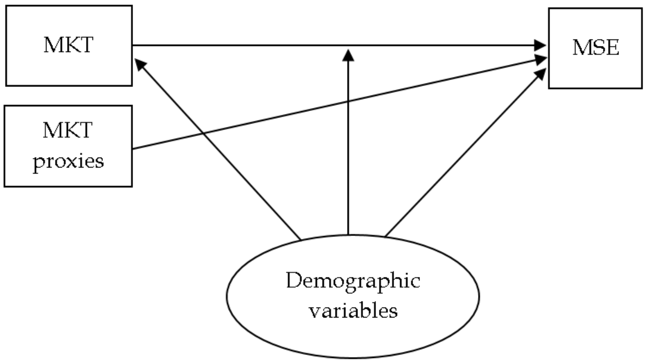

Figure 1. Theoretical & conceptual framework of the study

toward improving students' achievement by improving teachers' theoretical and practical skills to manage students' errors during class discussion (Bray, 2011). Also, teachers' ability to validate their MSE and MKT increases the quality of assessing teachers' effectiveness (Austin, 2015). Teachers' ability to understand mathematical content in students' practices becomes strong with the MSE and the MKT, and thus their instructional quality and practices, and problem-solving skills (Yun & Ah, 2016). Moreover, they can predict mathematics teachers' knowledge in technology, pedagogy, and content (TPACK) and help teachers identify significant math events, interpret them meaningfully, and connect them with instructional practices (Ekmekci et al., 2019). In conclusion, there is still a need to understand both constructs, the MKT and the MSE, and whether MKT is a predictor of the MSE. The theoretical and conceptual framework of the study are shown in **Figure 1**.

The current study addresses the following research questions:

1. How confident are elementary mathematics teachers in their MSE, and are their MSE rates different among their demographic variables?
2. What is the level of the MKT of elementary mathematics teachers according to professional organizations' standards (National Center for Assessment, NCTM, CAEP, and AMTE), and are their MKT scores different among their demographic variables?
3. Is the MKT a predictor of elementary mathematics teachers' self-efficacy after controlling for demographic variables (e.g., age, gender, years of teaching experience, teaching level, a teaching degree, etc.)?
4. How different are elementary mathematics teachers who rank high in their MKT and MSE than those who rank low in these teaching capabilities?
5. Is there a correlation between proxy measures for the MKT (teaching experience, teaching degree, type of degree, appraisal rating, training type, and hours) and the MSE?

METHOD

Our study is quantitative research focuses on the influence of MKT towards elementary teachers' MSE; therefore, it used the correlative descriptive method to explore the relation between variables and then identify the direction of this relationship, as well as using survey to collect data. The population for this study was all elementary mathematics teachers in Saudi public schools and according to the Saudi ministry of education, there are about 20,261 elementary mathematics teachers in Saudi public schools (9,687 male and 10,574 female). This study used the voluntary response sampling to collect respondents. After delivering the MSE measure and the MKT test to 796 elementary mathematics teachers in Saudi Arabia public schools, 86 voluntarily completed both research tools. The majority of respondents were young (67% under 40 years old), which corresponded with the national average teacher age of 38 (OECD, 2020); thus, their years of teaching experience are less than 11 years. Male nearly five times the number of female mathematics teachers among respondents, and a large percentage of them teach upper elementary grades (4-6) with a bachelor's degree in mathematics education. Respondents also were asked about their professional growth and appraisal and reported that over 80% of them received (excellent) in their recent annual rating, and they received professional training mostly in *PK* (OECD, 2020). Lastly, they have high self-esteem in mathematics, with over 91% of them evaluating themselves above the good level.

Respondents received their invitation to participate in our study via e-mail and text messages through their school districts. QuestionPro, an online survey platform, was utilized to build and distribute our study's instruments, and then the IRB approval was obtained from Imam Abdulrahman Bin Faisal University. The purpose of the research and how to participate have been clarified for elementary teachers before starting. Researchers developed their own MSE measure and the MKT test for elementary mathematics teachers in Saudi Arabia public schools.

MATERIALS AND DESIGN

MSE Measure

The MSE measure assesses the confidence level of elementary mathematics teachers in their specialized mathematics knowledge, skills, and teaching to link that later to their MKT. Researchers considered various high-inference instruments like the *mathematics teaching efficacy beliefs instrument* (MTEBI) (Enochs et al., 2000; Swars et al., 2009), and new ones like the *mathematics experiences and conceptions surveys* (MECS) (Jong et al., 2015). We found some lack of validity (Austin, 2013, 2015; Thomson et al., 2020; Xenofontos & Andrews, 2020) and reliability (Kieftenbeld et al., 2011), and a state of a discrepancy between these measures and the purpose of this study (Austin, 2013, 2015). Therefore, this measure has been developed based on multiple standards from national and international professional organizations (National Center for Assessment, NCTM, CAEP, and AMTE), whose standards are implemented directly or indirectly in mathematics teacher education programs in Saudi Arabia where the study took place.

After reviewing and mapping all four groups of standards, researchers found three constructs (mathematics knowledge, mathematics skills, mathematics teaching) that overlap among those professional standards and are consistent across the literature as mathematics teaching qualifications. The mathematics knowledge construct has five mathematics content standards (number and operations, algebra, geometry, measurement, and statistics and probability). The mathematics skills have six process standards (reasoning and proof, methods and techniques of mathematics learning and teaching, problem-solving, communication, connections, and representation).

The mathematics teaching construct has mathematical teaching tasks that implement mathematics knowledge and skills domains into teaching. The relative weight of each construct was calculated before writing items for each construct. As a result, mathematics knowledge, skills, and teaching constructs included relatively 19, 9, and 24 items. The mathematics teaching construct has more items than the other two because it represents the six subdomains of MKT (Hill et al., 2008a). A concise and neutral statement starts each construct, asking elementary mathematics teachers about their confidence level of correctly performing the following mathematics knowledge, skill, and teaching task.

Respondents rate their MSE on a 4-point Likert scale with categories labeled “1=not confident at all”, “2=not confident slightly”, “3=slightly confident”, and “4=absolutely confident”. In addition, demographic information collected by this instrument includes gender, age, years of teaching experience, teaching level, a teaching degree, type of degree, appraisal rating, training types and hours, and their self-evaluation in mathematics. Researchers defined instruments’ terminologies in the consent form in addition to the description and procedure of how to fill out the measure.

MKT Test

To assess the MKT for elementary teachers, researchers built a test based on the five mathematics content standards (number and operations, algebra, geometry, measurement, and statistics and probability) and six process standards (reasoning and proof, methods and techniques of mathematics learning and teaching, problem solving, communication, connections, and representation) noted earlier and then aligned it with the Saudi elementary mathematics curriculum. This instrument was developed based on the same standards utilized for building the MSE measure, which will help to compare elementary teachers’ MSE level and their MKT level. We decided to develop a new MKT test that would parallel with the MSE measure to increase the validity of our results (Austin, 2013, 2015). Moreover, we reviewed high-quality MKT tests nationally and internationally for validity and reliability (e.g., Education and Training Evaluation Commission, 2020, Ball et al., 2008) to use them as references for this tool.

40 questions were written for the first draft of this test, covering three cognitive levels (remembering, applying, thinking) and eleven standards. These three cognitive levels correspond to the revised taxonomy of Bloom (Krathwohl, 2002) as follows: remembering (remember), applying (understand, apply, analyze), thinking (evaluate, create). After reviewing by mathematics education experts (two professors and two associate professors), the final version of the MKT test had 33 questions and its distribution across standards and cogitative levels presented in **Table 1** and **Table 2**.

Table 1. MKT questions according to mathematics content and process standards

Standards	1st draft		Final draft	
	NoQ	%	NoQ	%
Number & operations	6	15	5	15.15
Algebra	7	17.5	6	18.18
Geometry	6	15	5	15.15
Measurement	4	10	3	9.09
Statistics & probability	4	10	3	9.09
Reasoning & proof	2	5	2	6.06
Methods & techniques of mathematics learning & teaching	2	5	2	6.06
Problem solving	3	7.5	2	6.06
Communication	2	5	1	3.03
Connections	2	5	2	6.06
Representation	2	5	2	6.06
Total	40	100	33	100

Note. NoQ: Number of questions

Table 2. Cognitive levels of MKT questions

Levels	1st draft		Final draft	
	NoQ	%	NoQ	%
Remembering	18	45	5	15
Applying	15	37.5	21	64
Thinking	7	17.5	7	21
Total	40	100	33	100

Note. NoQ: Number of questions

According to the elementary mathematics teacher standards and their relative weight, the number of questions for each standard and cognitive level is different. For the content and face validity purposes, the total number of questions was minimized to 33 and that will help also to reduce the duration of the test and the participation dropout rate.

Multiple-choice questions were used to write the MKT test with four options (correct answer and three distractors) and a timer of 50 minutes to finish the test implemented. Online respondents were made to electronically return the MKT test immediately after completing the MSE measure, and each one was given an ID number in addition to an IP address to match their responses in both instruments. The theoretical maximum score in the MKT test is 33 points.

Validity and Reliability

In a pilot study with a sample of 29 elementary mathematics teachers (17 male and 12 female) collected from the population of the study holding the same characteristics by the convenience method of sampling, we measured the construct and content validity for both instruments. Feedback collected from the pilot study helped to improve the instrument's quality. Furthermore, four mathematics teacher educators (two professors and two associate professors) who were either involved in developing or implementing the national elementary mathematics teacher standards for teacher education programs across Saudi Arabia Universities examined the content validity of both instruments. They evaluated how much these measures were aligned with the objectives and research questions. Their comments and feedback about both tools were considered when reviewing and revising.

The reliability of the MKT test was verified by using split-half method and was acceptable (Spearman-Brown coefficient=0.81). Also, the KR20 value computed and was 0.73. The coefficients of ease, difficulty, and discrimination were calculated. The values of the coefficients of ease and difficulty ranged between 0.862 and 0.242, and the values of the discrimination coefficients ranged between 0.25 and 0.88. The reliability of the MSE measure was verified by using the Cronbach's alpha method and was acceptable (α=0.93).

The discriminatory validity of the MKT test and the MSE measure was calculated. First, the MKT scores and the MSE rates of the pilot sample arranged in a descending order, then using Mann Whitney test (U) we compared the highest 27% to the lowest 27% which belong to eight respondents in each level. The result showed statistically significant differences at the level of ($p<.01$) between mean ranks of the low and high MKT scores and MSE rates. This means that the MKT test and the MSE measure could distinguish respondents in terms of their MKT and MSE.

Data Analysis

This study investigates whether and how elementary teachers' MKT and MSE are related in a descriptive correlational research design. Quantitative data were collected from a voluntary response sample of 86 elementary mathematics teachers in Saudi public schools by the MSE measure and the MKT test. The descriptive analysis, the independent t-test and ANOVA were employed to answer the first and second questions, while the Pearson correlation coefficients, ANOVA, and two-step hierarchical multiple regression analyses were conducted to address question three. We used the independent t-test to answer the fourth question and the Pearson correlation coefficients, ANOVA, and multiple regression for the fifth question. The missing data was only in the demographic variables and was less than 3%.

FINDINGS

The research questions organize the results section.

Elementary Mathematics Teachers' MSE

Results showed a high level of elementary teachers' confidence in their MSE (M=3.22, SD=0.46), which was above the theoretical mean of 2.5. Elementary mathematics teachers in our study were more aplomb about their capacities in mathematical teaching (M=3.28, SD=0.41), than in mathematical knowledge (M=3.22, SD=0.55) and mathematical skills (M=3.16, SD=0.54). The MSE rates were not significantly different among teachers' demographic variables except for age (young and old), teaching experience (novice and expert), mathematical knowledge training (yes and no), and PK training (more or less than 20 hours). Subsequently, elementary mathematics teachers who received mathematical knowledge training (M=3.47, SD=0.37) on average were more confident in their MSE than those with no training (M=3.15, SD=0.46). Also, respondents with higher age, teaching experience, and PK training hours are more likely to have positive MSE on Tukey's HSD and Fisher's LSD as post-hoc tests.

Elementary Mathematics Teachers' MKT

The MKT test shows that Saudi elementary mathematics teachers have a medium level of MKT (M=21.43, SD=3.92, Min=12, and Max=31). The total percentage of the right answer is 65% which is higher than the passing score in the Saudi professional licensing test for teachers (Education and Training Evaluation Commission, 2020) but lower than what research studies recommended of 70% or higher for qualified mathematics teachers (AlSalouli, 2016; Ben Motreb & AlSalouli, 2015). Teachers' scores in the MKT subscales were also close to medium. On the subscales, respondents on average had 67% right answer (M=14.77, SD=3.01, Min=9, and Max=21) in the mathematical knowledge (22 items), 57% right answer (M=5.12, SD=1.65, Min=1, and Max=8) in the mathematical skills (9 items), and 78% right answer (M=1.55, SD=0.57, Min=0, and Max=2) in the mathematical teaching (two items). Although they have on average exceeded the recommended level of 70% right answer as mentioned above in four standards areas (number and operations,

Table 3. The means & standard deviations of teachers' scores in MKT according to content and process standards (n=86)

Standards	TD	Min.	Max.	Mean	SD	%
Number & operations	5	1	5	4.33	0.91	0.87
Algebra	6	0	6	3.65	1.56	0.61
Geometry	5	1	5	3.24	1.07	0.65
Measurement	3	0	3	2.28	0.76	0.76
Statistics & probability	3	0	3	1.27	0.87	0.42
Reasoning & proof	2	0	2	0.71	0.73	0.36
Methods & techniques of mathematics learning & teaching	2	0	2	1.55	0.57	0.78
Problem solving	2	0	2	1.35	0.68	0.68
Communication	1	0	1	0.70	0.46	0.70
Connections	2	0	2	0.83	0.69	0.42
Representation	2	0	2	1.53	0.63	0.77
Total	33	12	31	21.43	3.92	0.65

Note. TD: Total degree; SD: Standard deviation

Table 4. The means & standard deviations of teachers' scores in MKT according to cognitive levels (n=86)

Cognitive levels	TD	Min.	Max.	Mean	SD	%
Remembering	5	0	5	3.42	0.93	0.68
Applying	21	8	20	13.32	2.86	0.63
Thinking	7	1	7	4.69	1.32	0.67
Total	33	12	31	21.43	3.92	0.65

Note. TD: Total degree; SD: Standard deviation

Table 5. The means, standard deviations, & inter-correlations for MSE & predictor variables (n=86)

Variable	M	SD	1	2	3	4
MSE	3.22	.46	.50**	.36**	.28**	.64**
Predictor variable						
Teaching experience	2.34	.88	-	.33**	.10	.24*
PK training hours	2.14	.77		-	-.02	.15
MK training hours	.21	.41			-	.49**
MKT	21.43	3.92				-

Note. M: Mean; SD: Standard deviation; *p<.05; **p<.01

measurement, methods and techniques of math learning and teaching, representation), **Table 3** demonstrates the results, but their percentage of right answer in the three cognitive levels were all below 70% (as shown in **Table 4**). The MKT scores were not different among teachers' demographic variables except for age (young and old), teaching experience (novice and expert), and mathematical knowledge training (yes and no) but with a slight difference on Tukey's HSD and Fisher's LSD as post-hoc tests.

The Relationship Between the MSE and the MKT

To approach our third research question, a two-step hierarchical regression analysis was conducted to evaluate the prediction of elementary mathematics teachers' self-efficacy from their MKT after controlling for their demographic variables. Assumptions were checked first, the MSE rates are assumed to be normally distributed and linearly related to the MKT, and no outliers found after checking the minimum and the maximum values of standardized residual (within -3 to 3). Also, the P-P plot for all models suggested that the assumption of normality of the residuals has been met as well as the variance of the residuals found to be constant after examining the plot of standardized residuals vs standardized predicted values which suggested that the homoscedasticity assumption has been met.

Five demographic variables (gender, teaching level, teaching degree, type of degree, appraisal rating) turned out to be insignificantly correlated with the MSE (criterion variable). Therefore, we removed insignificant variables and the rest of the demographic variables (age, teaching experience, PK training hours, mathematical knowledge training hours) were entered together in one block, and a case of multi-collinearity was found. Age and teaching experience were significantly correlated (r=0.88); thus, the age variable was removed from the model since teaching experience is theoretically more related to the MSE than age to avoid any negative impact on the regression analysis.

When the three demographic variables (teaching experience (exper), PK training hours (pkt), mathematical knowledge training hours (ckt)) were entered and each VIF value is well below 10, tolerance scores are above 0.2, and the Durbin-Watson=1.561, they significantly predicted the MSE, $F(3,82)=14.89$, $p<.001$, adjusted $R^2=.33$. Means and standard deviations for predictors and criterion variables are presented in **Table 5**.

Table 6. The beta weights and significance values

Variable	B	SEB	β	R^2	Δ R^2
Block 1				.35	.35
Teaching experience	.213	.05	.41**		
PK training hours	.135	.06	.23*		
MK training hours	.275	.10	.25*		
Constant	2.38	.14			
Block 2				.57	.22
Teaching experience	.168	.04	.32**		
PK training hours	.099	.05	.17*		
MK training hours	-.023	.10	-.02		
MKT	.065	.01	.55**		
Constant	1.23	.21			

Note. *p<.05; **p<.001

Table 7. t-Tests of high & low MKT groups

Variable	Group	M	SD	n	t (df)	p	η^2
MKT	12-17	2.66	0.2	14	8.61(49)	0.00	0.72
	25-31	3.52	0.33	17			

Note. M: Mean; SD: Standard deviation

Table 8. t-Tests of high & low MSE groups

Variable	Group	M	SD	n	t (df)	p	η^2
MSE	2.28-2.76	17.88	3.72	17	5.56(29)	0.00	0.52
	3.68-3.97	24.5	2.58	14			

Note. M: Mean; SD: Standard deviation

However, as indicated by the R^2, only 33% of the variance in the MSE could be predicted by entering the teacher's teaching experience and pedagogical and mathematical knowledge training hours in the equation. When the MKT variable was added, it significantly improved the prediction, $F(1, 81)=40.60$, $p<.001$, R^2 change=.22. The entire group of variables significantly predicted the MSE, $F(4, 81)=26.71$, $p<.001$, adjusted $R^2=.55$, and this is a large effect (Cohen, 2013).

The beta weights and significance values presented in **Table 6** indicate which variables contribute most to predicting the MSE when teaching experience, PK training hours, and mathematical knowledge training hours are entered together as predictors. With this combination of predictors, the MKT test score has the highest beta (β=.55) and significantly predicts the MSE.

Respondents' predicted MSE level is: Z predicted MSE=.32Zexper+.17Zpkt-.02Zckt+.55Zmkt.

High and Low MKT & MSE

After identifying the direction of the relationship between the MSE and the MKT, the data were re-examined to identify the difference between high and low teachers in their MKT and MSE. These groups were derived by splitting the teachers' scores for each measure into three groups (M+1SD≤High, M−1SD<Middle<M+1SD, Low≤M−1SD). Normality is assumed since the MKT scores and the MSE rates follow a normal distribution in each subgroup. Also, homogeneity of variances has been assumed and both groups have equal variances on the MKT and the MSE since both sample sizes are roughly equal and Levene's test is significant p<.001. Next, an independent-samples t-test was conducted to compare differences on average between high and low groups. The test was significant for the MSE variable, $t(49)=8.61$, $p=0.00$, $\eta^2=0.72$, and the MKT variable, $t(29)=5.56$, $p=0.00$, $\eta^2=0.52$, (**Table 7** and **Table 8**). As measured by eta squared (η^2), the effect size values for both variables indicated a moderate to a large difference between the high and low groups.

The Relationship Between Proxy Measures of MKT and MSE

A multiple regression analysis was conducted to evaluate how the proxy measures of the MKT predicted the MSE level. Out of six typical teacher knowledge proxies, there were only three significant predictors (teaching experience (exper), PK training hours (pkt), mathematical knowledge training hours (ckt)) (Darling-Hammond, 2002; Darling-Hammond et al., 2005; Nye et al., 2004; Strong, 2011; Stronge & Hindman, 2006; Wenglinsky, 2002) while the criterion variable was the MSE. The linear combination of the three teacher's knowledge proxy measures was significantly related to the MSE level, $F(3, 82)=14.89$, $p<.001$, with an R^2 of .35. The sample multiple correlation coefficient was .59, indicating that approximately 35% of the variance of the MSE level in the sample can be accounted for by the linear combination of the three proxy measures (**Table 6**). Respondents' predicted MSE level is: Z predicted MSE=.41Zexper+.23Zpkt+.25Zckt.

DISCUSSION AND CONCLUSION

This study investigates the relationship between the MKT and the MSE and how valid elementary math teachers' MKT can predict their MSE. Elementary mathematics teachers showed high confidence in their mathematics knowledge, skills, and teaching practices. Elementary mathematics teachers tend to over-report their self-assurance (Norton, 2017). Age, teaching experience, and hours of training in mathematical knowledge and PK filtered their level of the MSE. Therefore, older, expert, and highly trained elementary mathematics teachers are expected to be more confident about their MSE. They rated themselves higher in mathematical teaching than in mathematical knowledge and skills, which supports the idea that teachers are more confident in their practical knowledge than theoretical ones (Gasteiger et al., 2019).

On the contrary, elementary mathematics teachers' MKT was medium-level and showed their highest score in the number and operation section. Respondents might have old mathematical knowledge or forget part of this knowledge since the number and operation are dominant in the elementary mathematics curriculum. Age, teaching experience, and hours of training in mathematical knowledge slightly explain the differences among teachers' MKT scores. This difference between the MSE and the MKT results could be explained by the Dunning-Kruger effect (Kruger & Dunning, 1999). First, our respondents received a high annual rating that might negatively contribute to their high level of the MSE and mislead the interpretation of teacher competence and practice, mainly when regular teacher appraisal is not used for improving teacher knowledge, skills, or teaching practices (OECD, 2020). The second factor, the self-assessment is not a common practice in the Saudi professional development system; thus, our respondents might lack awareness of its purpose and process of how to do it and how to use it for their professional development, which could lessen its outcomes. In addition, the possibility of impacting the annual appraisal rating could cause teachers to assess themselves subjectively.

However, as hypothesized, elementary mathematics teachers' MKT predicted their MSE level after controlling other variables in the regression analysis: the higher their score in the MKT, the more confident they were in their MSE. In other words, mathematics teachers need to acquire a legitimate level of the MKT to become confident enough in their specialized knowledge and abilities, and this should be considered when building teachers' preparation and professional development programs. This result is consistent with previous findings by several studies (e.g., Isiksal-Bostan, 2016; Oppermann et al., 2016; Ren & Smith, 2018; Schreiber & Filo, 2019) that show a relationship between both mathematics teaching capabilities and support the claim that their MKT can predict their MSE, and it should be considered as a prerequisite for mathematics teacher effectiveness. In addition, the result of this multiple regression analysis is consistent with that found by Thomson et al. (2017), who argued that the mathematics CK alone did not predict preservice mathematics teachers' efficacy beliefs; however, the PCK did.

Moreover, this relationship between the MKT and the MSE is filtered through three teachers' demographic variables: teaching experience, training type, and training hours. This finding is evidenced in other studies that found the relationship between the MKT and the MSE is more robust for experts (Bjerke & Solomon, 2020; Bray, 2011; Corkin et al., 2015) and trained teachers (Corkin et al., 2015, 2016; Ren & Smith, 2018; Stevens et al., 2013; Thomson et al., 2020). Finally, it should be noted that the result for our fourth question showed that the difference between the high and low groups in the MSE and the MKT ranges from moderate to large, and even the lowest average scores in the MKT were still located on the upper part of the MSE scale. This finding can be explained by teachers' lack of objectivity in self-evaluation (extreme self-assessment), and teachers' skills for self-evaluation are weak (conceptual and practical) because of the lack of training for it. Therefore, the lack of experience and knowledge, as well as the inability to self-evaluation, negatively affect the measurement of the relationship between the MSE and the MKT (Austin, 2013, 2015; Francis et al., 2017; Norton, 2019a, 2019b; Stevens et al., 2013; Thomson et al., 2017)

Implications and Limitations

This study came to address the need mainly in the Saudi education system to determine whether the MKT can predict MSE. The results highlighted the importance of elementary mathematics teachers' MKT for their MSE and how this relationship should be considered as a significant factor for teaching effectiveness (Bandura, 1997). The positive and robust relationship between the MKT and the MSE indicates that teaching competencies (e.g., knowledge, skills, and attitudes (Tigelaar et al., 2004)) could interact and thus predict one from the other. Moreover, this relationship is significant while the MSE is at the center of teachers' capacities and is highly related to teaching effectiveness (Michos et al., 2022).

Accordingly, this linkage between the MKT and the MSE entails important implications for research, teaching practice and preparation, and professional development programs for elementary mathematics teachers. Teacher educators, researchers, and policymakers can benefit from the findings of our study in understanding how elementary teachers' MKT relates to their MSE. Developers of teacher preparation and professional development programs can utilize our study's findings to improve the self-assuring of the mathematical capacity of in-service and prospective mathematics teachers through aligning knowledge, skills, and attitudes when building those programs. The balance between teachers' competencies and different domains of knowledge (six domains of the MKT) is essential to address when building teacher education and development programs, especially when it is evident, as in our study, that they are correlated with each other. Findings also showed that self-assessment practices are not supported by other professional growth strategies (Ross & Bruce, 2007). Therefore, elementary mathematics teachers might overestimate their mathematical abilities while not receiving the efficient level of preparation or professional development in the MKT. Researchers recommend training teachers on self-evaluation skills to elevate their validity level for measuring teaching effectiveness.

One of the limitations is the small sample size, not randomly selected, and all respondents were elementary and public-school teachers. Their perspectives and responses about the MSE and the MKT could differ from other in-service teachers in other grade levels and private or international schools. However, we tried to overcome this limitation and reach the highest number of elementary mathematics teachers by sending the study instruments in two distribution methods online and paper formats via the Saudi ministry of education channels. Also, the current study's MSE measure and the MKT test are domain-specific, directed to the elementary mathematics teachers' standards in Saudi Arabia, and limited to those constructs included in them but we considered other international standards (NCTM, CAEP, and AMTE) as well and aligned our instruments to them. In addition, our respondents might have used some help from curriculum and internet sources while providing their responses to the measure or the test, which could lessen the validity of our results. Therefore, we included clear instructions in the consent form and at the beginning of each instrument and asked teachers to not use any external resources while completing the MSE measure of the MKT test and limited the time of response to 50 minutes.

Future research should consider utilizing qualitative tools for collecting data to understand better the MKT and the MSE and the relationship between them. Furthermore, it is important to know how the relationship between the MSE and the MKT relates to mathematics teaching effectiveness, therefore, researchers are planning to investigate it in the following research. Also, we suggest examining how other factors like motivation and attitudes toward teaching mathematics, and beliefs about mathematics teaching interact with MKT and compare that with MSE factor.

Author contributions: All authors have sufficiently contributed to the study, and agreed with the results and conclusions.

Funding: No funding source is reported for this study.

Declaration of interest: No conflict of interest is declared by authors.

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