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Effects of Language on Children's Understanding of Mathematics: Implications for Teacher Education

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ABSTRACT

Teacher educators have a moral and civic obligation to examine ways in which language and mathematics are connected and supported in teaching and learning mathematics. It is essential to examine the roles and influence of family, parents, community, teachers, administration, and teacher educators as they collaborate to support learners. Their role should be considered in preparing and supporting teachers to develop curriculum, plan instruction, and implement strategies that support students' development of language in the mathematics classroom. An examination of the literature regarding the effects of language on children's understanding of mathematics was conducted around six areas: 1) impact of language on understanding and meaning making; 2) symbols, expressions and language connections; 3) effects of teachers' listening orientation; 4) language development, play and family influences; 5) implications for multilingual learners; and 6) technology and digital media. Implications for teacher education and future research are presented. We offer readers a potential framework to consider for guiding teacher educators' practices and future research efforts. In so doing, we display various connections and interplays between language and children's mathematical meaning making and understanding.

Keywords: mathematics, language, teacher education, family, play

Introduction

As a profession, teacher educators have a moral and civic obligation to examine in a formal, directive manner ways in which language and mathematics are connected and how one supports the understanding of the other. It is essential that we examine the roles and influence of varying constituents such as family, parents, community, preservice teachers (PSTs) and inservice teachers (ISTs), school administration, teacher education and the public in general. We should commit to examining how to create opportunities for collaboration among and between these constituents in order to have a focused community of practice that supports learners' meaning making in mathematics. Further, teacher educators should consider their roles in preparing and supporting both

PSTs and ISTs to develop curriculum, plan instruction, and implement strategies that sustain the development of language competencies in the mathematics classroom.

The Association of Teachers Educators (ATE) developed standards for teacher educators (2018), “to help all teacher candidates and other school personnel impact student learning” (p. 1). These standards serve as a guide for teacher educators. *Standard 1* on teaching indicates that we, as teacher educators, should model research-based practices in teacher preparation and professional development. This entails having a strong knowledge of both content and pedagogical practices that include the appropriate use and application of language in mathematics. In *Standard 2,* cultural competence, ATE notes teacher educators have a role and responsibility to support PSTs and ISTs in connecting to communities, families, and cultures where both language and mathematics can be effective vehicles. Finally, ATE’s *Standard 6* addresses collaboration, underscoring the need to engage all stakeholders in teacher education. These partnerships can facilitate teaching, learning, and research in the fields of mathematics and language education.

The Association of Mathematics Teacher Educators (AMTE) also ascribes to the important role of language in teaching and learning mathematics, as noted in the 2017 *Standards for Preparing Teachers of Mathematics*. AMTE advises teachers to “use mathematical language with care and precision” (p. 9), promote equitable practices by considering the “everyday, informal language of students that support or hinder the specialized use of language in mathematics” (p. 14), and attend to language and familiar contexts and experiences that include opportunities and support for multilingual students to use their own language (p. 22). AMTE further advocates that teacher educators should assist teachers in seeing “their role as helping children mathematize their world, nurturing understanding of mathematical concepts and relationships, and developing language to talk about those observations” (p. 59) while helping learners connect their informal language to formal mathematical language and notation, offering multiple opportunities for learners to communicate. In the context of both ATE and AMTE teaching standards, we approached the use of language and children’s meaning making in mathematics from multiple dimensions.

Overview of Process

As teacher educators, many questions come to mind: What is language? What is mathematical language? Are there different contexts such as home, classroom, or playground to be examined? What is the role of language development--reading, writing, speaking and listening--and of academic language in the conceptual learning of mathematics? How do we consider the relational, computational, symbolic, and syntactical areas of language, including register, to foster meaning making and develop shared meaning? An eight-member Commission on the Effects of Language on Children's Conceptual Understanding of Mathematics appointed in 2015 by the Association of Teacher Educators considered these and other questions. The commission was tasked with examining the current theories, practices, and research on the effects of language on children’s conceptual understanding of mathematics and determining implications for teacher education. To undertake this task, the following questions were identified:

1. What are the effects of language on children’s meaning making and conceptual understanding of mathematics?
2. What implications do these effects have for mathematics teacher education?

The commission, made up of experienced mathematics teacher educators with PK-12 teaching experience, reviewed both the AMTE 2017 *Standards for Preparing Teachers of Mathematics* and the 2018 ATE Standards for Teacher Educators mentioned previously, along with mathematics curriculum standards noted in the National Council of Teachers of Mathematics’ *Principles and Standards for School*

Mathematics (2000) and the *Common Core State Standards for Mathematics* (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010). From this, six areas were identified to explore further (listed below). Then each committee member took an area and read and reviewed the literature, both theoretical and research studies, related to that area. These resources, along with the authors' individual areas of expertise and related research, helped frame the facets explored around mathematics and language. The commission met regularly to share summaries of findings, engage in further in-depth discussion to synthesize literature, as well as determine emerging patterns. This provided opportunities for verifying findings and coming to common agreements. Additionally, the group met with other teacher educators at the ATE annual conference for two years to gather feedback on the direction of the Commission's work and identify areas for further exploration regarding the effects of language on children's meaning making and conceptual understanding of mathematics. Through this process, the commission determined the following areas to discuss in this paper.

1. What mathematical language is and how it connects to meaning making and understanding
2. Symbols, expressions and language connections
3. Effects of teachers' listening orientation on students' mathematical learning
4. Mathematics language development: Play and family influences
5. Mathematics and multilingual learners
6. Technology and digital media

The Commission is not making the claim that these are the only areas of mathematics and language connections. For the purposes of this paper, however, it is critical to examine ways of forming and supporting a community of practice that aims to synthesize and summarize research on effective practices for developing students' conceptual mathematical understanding, meaning making, and effective communication of their understanding through language.

For many students, mathematics is experienced or seen as a foreign, unfamiliar language (Kenney et al., 2005). Mathematical language is used informally in some contexts, such as home, and more formally in school settings. In school contexts, some students struggle to make meaning of the formal mathematical language and the related mathematical ideas, especially those whose first language is not the spoken language of the classroom. Even further, when blending their formal and informal experiences, students encounter words and expressions having multiple meanings, such as *odd, mean,* and *fraction* (Wilkerson et al., 2015). It is important for teacher education programs to consider ways to prepare PSTs and support ISTs in identifying effective ways to encourage students in academic language development and applications. Additionally, it is important to examine the diverse environments where mathematical meaning making occurs, and their affordances, to identify effective ways of supporting students' language development and to further envision the roles of teacher education.

Hence, the commission examined this complex issue in the context of mathematical meaning making and understanding with the goals of (1) identifying current strategies used in practice that effectively support language and mathematical learning and (2) proposing further directions in terms of innovative approaches and research methods. This paper is a culmination of the Commission's explorations and syntheses, where ideas related to language and mathematics are examined through the lens of teacher education. We offer readers a literature review that showcases multiple facets of mathematics and language, illuminates the problems of practice, reports on related research, and points to areas for further research. Then, we offer a potential framework for consideration.

What is Mathematical Language and How Does it Connect to Understanding and Meaning Making?

We begin by offering our shared understanding of mathematical language. In the context of the mathematics education of young children, language is understood to be an evolving, developmental system. This system is comprised of spoken, written, visual, or bodily signs or actions and structures that are used in a cultural community for informal mathematical talk, exploration, or meaning making as well as formal mathematical communication and representation. For the purposes of this paper, the authors define mathematical language as the mathematics content, its literacy ramifications, and modes of discourse that occur within and among school, home, and community contexts. Specific facets include vocabulary, symbols, heuristics, questions, technology, critical literacies, and social interactions that influence children's reading, writing, listening, and speaking competencies and proficiencies with respect to communicating mathematical meaning making and understandings. Factors influencing mathematical language include linguistics and linguistic diversity, comprehension, and complexities in mathematics language acquisition (e.g., socioeconomic status (SES), students with differing abilities).

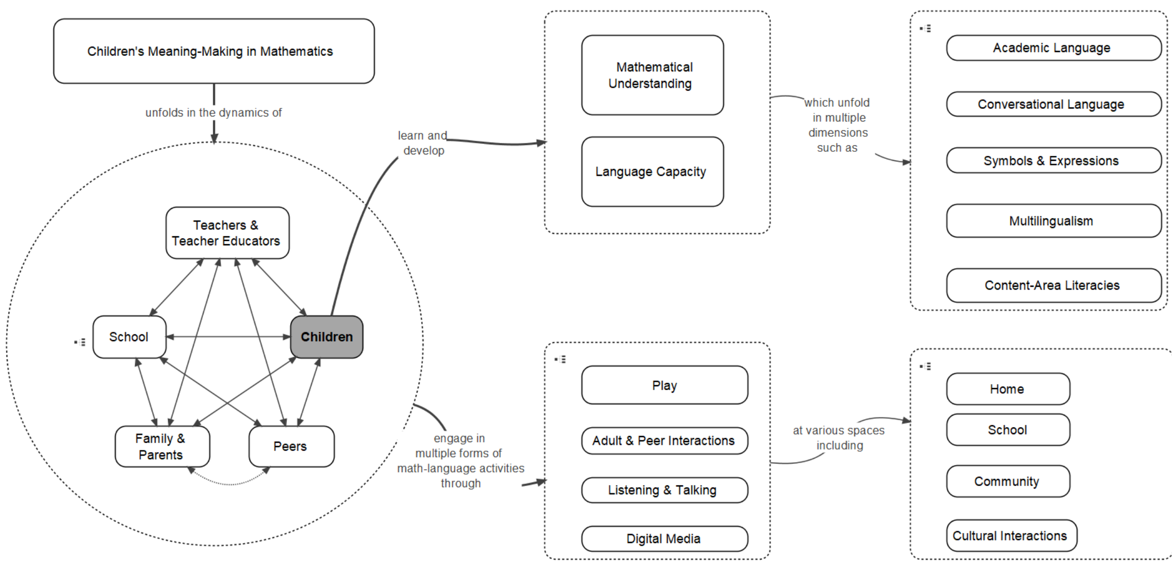
Additionally, we posit that understanding is a result of "learning as meaning making … a process by which people [actively] interpret situations, events, objects, [and/]or discourses, in light of their previous knowledge, experience, [and available cultural resources]" (Zittoun & Brinkmann, 2012, p. 106). Ivkovic (2019) further mentioned that as we engage in this process, we use multiple modalities to make meaning, interact, and communicate. Lee and Stephens (2020) summarized the key findings from a 2018 National Academies of Sciences, Engineering, and Medicine consensus report, entitled *English Learners in STEM subjects: Transforming classrooms, schools, and lives*. They noted that there have been parallel shifts in the fields of STEM subjects and second language acquisition away from a "focus of learners' mastery of discrete elements of content" towards an "emphasis on students' mak[ing] sense of phenomena and problems" (p. 429). A key construct from the consensus report that undergirds the crucial connections and interplays between language and children's mathematical understanding (see Figure 1) is that "[a]s English Learners engage in STEM disciplinary practices (e.g., developing models, arguing from evidence, constructing explanations), they use language for the purpose of making meaning of STEM subjects through social interactions with peers and the teacher in the classroom community" (p. 426).

In 2010, Moschkovich led an effort to consider various perspectives for research in mathematics education and language. She noted the complexity of mathematical language (Pimm, 1987), which includes specialized vocabulary and complex discourse (Crowhurst, 1994; Halliday, 1978; Moschkovich, 2007) associated with the mathematical language. She states, "I use the phrase 'the language of mathematics' not to mean a list of vocabulary words or grammar rules but the communicative competence necessary and sufficient for competent participation in mathematics discourse practices" (p. 3).

Since the 1980s, the National Council of Teachers of Mathematics (NCTM) has emphasized the critical role of communication with particular attention to the role of discourse and its contribution to the development of language by learners, and the connection to their ability and opportunity to express mathematical ideas (NCTM, 1989; 1991; 2000; 2014; 2018; 2020a, 2020b). NCTM (2014) identified eight effective mathematics teaching practices, one of which specifically targets discourse, stating, "Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments" (p. 29). NCTM notes that discourse is "central to meaningful learning of mathematics" (p. 35). Further in the recent release of *Catalyzing Change* (NCTM 2018; 2020a; 2020b) one of the four recommendations made points specifically to broaden the purposes of learning mathematics so that each and every learner develops a deep understanding of mathematics, builds a positive mathematical identity, and is able to use mathematics to understand and critique their world. This focuses on meaning making in mathematics for which mathematical language is central. There is consequently a need for teachers to attend to the development of mathematical language to support students' meaning making and mathematical understanding.

Figure 1

Design Framework for Exploring Language Effects on Children's Mathematical Understanding



In seeking mathematical understanding through discourse, student-teacher talks play a central role in the development of mathematical language. Khisty and Chval (2002) examined the role of pedagogic discourse related to teacher talk, in particular to second language learners. Interactions of two teachers with their students were analyzed. In the environment where the teacher used engaging mathematical talk, including rich mathematical words, students were able to demonstrate their understanding of the words and their meanings. The authors go on to offer the premise that the teacher plays a critical role in the communication process that forms the context for learning since the teacher is obviously present in the classroom, as the 'more capable other' (Vygotsky, 1978), and is the person who engineers the learning environment (p. 167). These authors contend that it is essential to better understand student and teacher interactions that occur in the classroom, that is, the influence of pedagogic discourse.

Research by Hebert and Powell (2016) found that fourth grade students had varied levels of success with the use of mathematical vocabulary. While mathematical vocabulary is only one aspect of mathematical language proficiency, it is important to examine how students acquire and use vocabulary in different contexts, as well as to analyze the implications for students' meaning making in mathematics. These authors found that many students' use of vocabulary was limited to procedural rather than conceptual aspects of the concept, although some students did include references to mathematical properties. Further, Lemke (2003) posits a semiotic relationship between mathematics, natural language and visual representations to support meaning making. He argues that "semiotics helps us understand how mathematics functions as a tool for problem-solving in the real world, and how this function may have played a key role in the historical evolution of mathematics" (p. 215), advocating for bridging verbal language, visual representations, symbolics of mathematics and the real world.

These studies and perspectives offer insights into the rich connections among language, meaning making, and mathematical understanding. As will be described in this paper, mathematics educators should attend to multiple aspects of teaching and learning mathematical language, including: mathematical understanding; vocabulary and classroom discourse; role of play and influence of families, language and culture; connections to reading and writing; effects of teachers' language and listening orientations on students' learning; second language learning; differing learning abilities; instructional practices; role of the teacher; role of technology and multimodal representation; structure and use of representations in mathematics; curriculum development; and equity. There is much to learn in terms of student support and engagement, considering the foundational nature of language in students' meaning making and conceptual understanding of mathematics.

Symbols, Expression, and Language Connections

Teachers use language, including both verbal and non-verbal expressions, to communicate mathematically with their students. Teachers focus on mathematical conceptualization, essential skills, and reasoning aimed at engaging students' developing mathematical understanding. Inside the classroom learning community, language conventions should be carefully chosen as an agreed-upon mutuality – where the teacher chooses words and phraseology; students simultaneously grapple with and then make sense of those concepts (Zhang et al., 2015). Teacher educators (mentors, supervising teachers, professors, teacher-leaders, evaluators, etc.) then provide a common ground of shared understanding in order to enter the dialogue. Thus, the language of symbolic expression factors into the teacher educator's practice. This ties directly to ATE *Standard 1* (ATE, 2018) that highlights the importance of modeling inside of instructional practice. Furthermore, all five content standards and corresponding process standards from NCTM (2000) link to the importance that symbolism, expression, representation, and notation play in mathematics instruction and the development of mathematical language.

The Importance of Symbols in Mathematics

Symbolic comprehension and symbolic number processing serve as critical corner stones of mathematics achievement (Sasanguie et al., 2013). When it comes to symbols in mathematics, students should be able to identify the meaning and the function of any given symbol. Teachers support by drawing connections to those meanings and functions in order that students can grasp the abstraction of those symbols (Zhang et al., 2015). Other studies show the predictive nature of symbolic processing on mathematical achievement, particularly relative symbolic number ability and later skill (Lyons et al., 2014).

The Influence of Language on Mathematical Comprehension of Symbols

Firmender et al. (2014) link the connection between student mathematical achievement and the instructional practice of using verbal communication alongside targeted mathematical language practice. This significant correlation reinforces the importance that language plays in mathematics aptitude. For teacher educators, this direct link should be understood and emphasized. Teacher educators can guide practice toward the effective use of academic vocabulary, mathematical meaning making, and discourse inside the classroom.

Teachers use language to communicate, and therefore teach, complex ideas which are often abstractly represented by symbols (Burns, 2006; Caglar, 2003). When introducing abstract concepts, teachers are able to support students in making meaning of key vocabulary, while also providing a visual example of how such words and symbols interact in the processing of the concept. Think about our commonly shared understanding of the equal symbol ("=") and the complexities of interpretation that arise around equivalence. Studies have shown that not only is symbolic representation a critical area, but when taught for meaning, support deeper understanding and provide opportunities for students to effectively communicate their understandings (Bishop et al., 2022; Stephens et al., 2021;). Teacher educators should discuss the need for instructional precision and appropriate use of mathematical symbols with connections with their students.

Implications for Teacher Education and Research

It is incumbent upon teacher educators to provide PSTs and ISTs with opportunities to develop the language skills needed in supporting students in making sense of symbols. This should exist in a robust learning environment in which practitioners have multiple and varied opportunities to practice the language of mathematics instruction (verbal, representational, assessment) through problem solving, resource utilization, and professional/pedagogical development (Socus & Hernandez, 2013). Thus, maintaining a classroom environment suitable for academic language where the discussion of and meaning making of symbolism can thrive.

Lim (2016) underscores the importance of transitioning mathematics students from working with numbers to working with symbols. In addition, emphasizing the relevance and meaning of working with symbols, especially in algebra is critical. This requires an emphasis in language, particularly in explicit instruction. Cain and Faulkner (2011) draw the conclusion that teaching symbols along with other mathematics concepts requires teachers to think in terms of building background knowledge and comprehension, not unlike the work that literacy teachers do in teaching reading. Teacher educators then execute the particulars of that methodology in order that the skill of symbolic language transfers.

Further research would look closely at the interplay between student achievement (both individual and collective data) and SES relative to student mastery of the language of symbols. Rutherford et al. (2010) suggest research that looks at both longitudinal collective data as well as individual student data in order to provide greater understanding of the instructional intervention that their software/pedagogically-driven view suggests. Furthermore, Lyons and colleagues (2014) echo the same desire in research for longitudinal studies on students' understanding of mathematics skills that are heavily reliant on adeptness with symbolism. This reinforces that a gap in research exists when studying symbolic system mastery and mathematics achievement (Sasanguie et al., 2013). Perhaps a deep look at the way in which those who are multilingual learners grapple with mathematical symbolic mastery would yield a greater understanding of language, symbols, and mathematical meaning making.

Classroom interchanges of talking and listening are mediated by sign systems, in general, and language, in particular. In considering the role of symbols, expressions, and language, it is important to examine the role of listening and its connection to students' mathematical learning. This is explored in the next section.

Effects of Teachers' Listening Orientation on Students' Mathematical Learning

Listening Orientations and Equity

Talking about one's mathematical ideas and listening to others' is an essential feature of interactions and communications that support meaning making. When communicating their mathematical thinking, the manner in which students feel listened to (or not) by their teacher and peers affects their academic and emotional engagement with the content. This has implications for equity pedagogy. Many researchers (Confrey, 1991; Davis, 1997; Steffe & D'Ambrosio, 1995) have described ways of listening to students as a vital part of a constructivist philosophy of teaching mathematics. Davis (1997) noted that many teachers engage in 'evaluative' listening, where they listen for an expected correct answer and respond with feedback aimed at fixing any ideas that stray from that expected, correct answer. Nathan and Petrosino (2003) called this an expert blind spot. With this manner of listening, students' thinking is often disregarded and the sources of their ideas are not often uncovered. Davis (1997) contrasts evaluative listening with hermeneutic listening, where he and others (Confrey, 1991; Steffe & D'Ambrosio, 1995) offer the concepts of de-centering, or the ability to take on another's perspective, and give reason to the child's thinking. Such listening involves immense intellectual effort and practice and serves to validate the student as a thinker and doer of mathematics. Franke and Kazemi (1991) found that listening to students in this manner was fundamental in advancing their mathematical thinking and understanding, an implication for equity pedagogy. Moschkovich (2004) offered the notion of 'appropriation', which is based on mutual teacher-student or student-student listening. "Appropriation involves joint productive activity, a shared focus of attention, and shared meanings…[as well as] taking what someone else produces during joint activity for one's own use subsequent productive activity" (p. 51). With regard to equity pedagogy, she also noted when working with multilingual learners the importance of listening with a mathematical reasoning focus rather than one of mathematical language correctness (Moschkovich, 2012). Davies and Walker (2007) studied "how four mathematics teachers listened to and made sense of students' ideas and the influence of content knowledge on their capacity to listen. [They found] that the depth of teachers' content knowledge—both subject matter knowledge and pedagogical content knowledge—mediated their enactment of effective listening practices" (p. 217).

Implications for Teacher Education and Research

Looking into the future, it would be beneficial for a study of teachers' listening orientations to focus on mathematics teacher preparation programs, as well as ISTs' continuing professional development. AMTE *Standards P3*, *P4* and *C2* focus on providing beginning teachers with opportunities to hone their knowledge, skills and dispositions toward teaching mathematics to support students' sense-making, understanding and reasoning. ATE *Standard 4* encourages teachers to engage in career-long professional development where they systematically reflect on their own teaching practices with the goal of improving. To achieve these ends, in addition to engaging PSTs and ISTs in experiences to deepen their mathematics content and pedagogical content knowledge, Arcavi and Isoda (2007) point teacher educators and professional development specialists to *the importance of an explicit focus on listening*. These authors define listening to students as "giving careful attention to hearing what students say (and to see what they do) and trying to understand it and its possible sources and entailments" (p. 112). This orientation toward listening "is not a passive undertaking." Instead, these authors offer the following components to mathematics teaching that is supportive of such listening:

1. Detecting, taking up and creating opportunities in which students are likely to engage in freely expressing their mathematical ideas;
2. Questioning students in order to uncover the essence and sources of their ideas;
3. Analyzing what one hears (sometimes in consultation with peers) and making the enormous intellectual effort to take the 'other's perspective' in order to understand it on its own merits; and
4. Deciding in which ways the teaching can productively integrate the students' ideas. (p. 112).

We urge mathematics teacher educators and professional development specialists to integrate the above components offered by Arcavi and Isoda (2007) so that listening becomes an important foci of effective mathematics teaching and learning. This is further underscored in research about teacher noticing, as teachers actively listening to students is one key component. It connects to making instructional decisions, understanding student thinking and lesson planning among other areas (Jacobs et al., 2011; Munson, 2020). We also recognize that for young mathematicians, play is a natural medium through which mathematical ideas are nurtured and communicated. In the next section, we discuss the importance of play as a conduit for Funds of Knowledge (Gonzalez et al., 2005) to facilitate mathematical language development.

Mathematics Language Development: Play and Family Influences

Importance of Play for Language Development

The NCTM Communication Process Standard states that a learner should be given the opportunity to grasp mathematical concepts before being forced to use formal mathematical terms. It is through exploring the ideas and working through their informal meanings that a learner becomes engaged and takes ownership of the ideas (NCTM, 2000). Play, the most natural way of learning for young children, is the means through which this process is conducted (National Association for the Education of Young Children [NAEYC], 2009).

As mentioned in the introduction, ATE *Standard 1* guides teacher educators to inform, or remind teachers that there are best ways to facilitate student development. AMTE *Standard EC.8* calls for "Well-prepared beginning teachers of mathematics at the early childhood level create mathematical learning environments characterized by exploration…[and] draw upon children's mathematical, cultural, and linguistic strengths thereby developing conceptual …." Thus, the importance of learning about and providing mathematical learning environments where math-talk is encouraged and used is supported by national standards for teacher educators. In addition, Gonzalez et al.'s (2005) work with Funds of Knowledge support the importance of play and knowledge of family interactions to help develop children's mathematical language.

Types of Play

Play is often described as either free-play, or guided-play where an adult helps, supports, guides and tutors to scaffold the child's play. Johnson et al. (1987) discuss the characteristics of free-play where internal reality takes over external reality (termed nonliterality). Play is freely chosen and produces pleasure and enjoyment, and in free-play the process is experienced as more important than the product. Saracho (2012) reports that children who engage in symbolic play, such as pretending a pot is a hat, are beginning to use symbolic representation to communicate their thought process. When children engage in this symbolic thinking, they are better able to engage in abstract thinking, which contributes to mathematical understanding.

Fisher and colleagues (2013) found that guided-play helped scaffold students' understanding. Additionally, students engaged in curriculum involving and revolving around play activities linked to learning outcomes obtained higher scores than those who were not (Holmes et al., 2015). Likewise, some research supports that free-play alone is not as efficient as guided-play in promoting mathematical understanding (Ginsberg et al., 2008). It is important that students have ample amounts of time for play and exploration to connect their play to mathematical concepts (Beaver et al., 2017).

Importance of Play in Child Development

Ramani and Eason (2015) found that by engaging in play, children may enhance their counting skills, spatial skills, and geometry understanding. These authors recommend teachers seek out curricula that encourages free-play in centers, encourages math-talk amongst students, and includes parents in mathematical play opportunities. Clements and Sarama (2005) claim, "High quality math learning emerges from children's play, their curiosity, and their natural ability to think" (p. 25), while Emfinger (2009) reinforces the value of "pretend play as a curricular tool to facilitate the development and consolidation of numerical skills" (p. 333). Others, such as Bulotsky-Shearer et al. (2014), report that through cooperative play (symbolic or constructive play activities) early skills in expressive and receptive language, problem solving, creativity, mathematics and spatial skills, are practiced, modeled, reinforced and extended by peers.

Implications for Teacher Education and Research

Purpura and Logan's (2015) study found a child's mathematical language was a predictor of mathematics performance. Linking this with the findings from Duncan et al. (2007) show a preschool child's mathematical abilities, not literacy skills, are a better predictor of success later in school, with critical implications for teacher education. These studies provide the impetus to continue research in early childhood mathematics play and how to prepare teachers and parents to provide these learning experiences. Parks and Blom (2014; 2015) call for teachers to become more familiar with children's development of important concepts by engaging in math-talk during guided- or free-play. Future research should address the connection between mathematics-specific literacy and play. There are natural opportunities for different types of challenging talk and these could be investigated in relation to Funds of Knowledge (Gonzalez et al., 2005) to see how they may facilitate or constrain child-directed talk (Gest et al., 2006). Additionally, Razfar (2012) calls for research on the use of play and games to facilitate mathematical learning for multilingual learners.

Influence of Family and Home Regarding Mathematics and Language

Young children's experiences with mathematics at home vary. Understanding these differences aligns with two aspects of the ATE Standards (2018). Knowing practices based on research and how families support children's mathematical engagement is aligned to *Standard 2*, cultural competence and *Standard 6*, collaboration. These two elements are critical for teacher educators to consider when working with future early childhood teachers. As identified below, the family's role is complex and early childhood teachers need to collaborate with families, while being culturally responsive.

Common mathematical activities families support include counting objects, oral counting, printing numbers, and activities involving shape (Missall et al., 2015). Families engage children in different ways depending on the activity. Vandermaas-Peeler et al. (2009) indicate more numeracy exchanges during children's play than when reading books. Siblings also can influence young children's mathematical development by how they interact during play. Howe et al. (2016) reviewed play interactions of sibling dyads and reported that siblings, depending on age, taught concepts such as number, measurement, and geometry.

Some research focused on ways to support families' mathematical conversations. Fenton et al. (2016) explored how to engage preschoolers in mathematical thinking at home. Using a Strengths Approach, the teacher and families co-constructed possible activities that related to the child's interest. For instance, one child's interest in dinosaurs, engaged the child in sorting them by size. Vandermass-Peeler et al. (2016) studied conversations of preschool children and their parents at a science museum. The families, provided with guidance in supporting children's reasoning, used more specific terms related to the mathematics presented in the exhibit. Eason and Ramani (2020) investigated how preschoolers and parents used mathematical language depending on the amount of support provided when using the same materials. These authors write, "formal learning yielded the greatest amount of math talk, guided play still showed an advantage over unguided play in eliciting parent and child math talk" (p. 559). These findings highlight the importance of helping families to support children's emerging mathematical knowledge and language development. Early childhood teachers' role is a critical element outlined in the AMTE standards. *Standard EC.6* describes the importance of early childhood teachers collaborating "with families in a mutually respectful, reciprocal manner to enhance and connect children's in-school and out-of-school mathematical development." Knowing different ways to support families in this process is important to support children's mathematical meaning making.

Saracho (2012) adds that before children enter formal schooling, they have acquired large amounts of mathematical knowledge. This knowledge is based on their interaction with others and their environment. These experiences may support children's mathematical development in varying ways. Pupura and Reid (2016) showed a child's mathematical language is a significant predictor of numeracy skills for preschoolers and kindergarteners. Anderson and Gold (2006) indicate that home practices influence how children approach mathematical tasks at school, such as strategies for playing board games or completing puzzles. While Segers and colleagues (2015) found that the home literacy environment did not affect numeracy skills; but instead, parent's numeracy expectations and activities did. Factors such as parental education seem to influence the development of this knowledge (Pupura & Reid, 2016). Conflicting evidence related to families' SES influence emerged from a review of the literature. Vandermaas-Peeler et al. (2009) indicated more numeracy exchanges between parents and children in higher SES groups, while Missall et al. (2015) found no significant difference in such exchanges among families from different SES groups.

Implications for Teacher Education and Research

As children transition into elementary school, it is important to consider the possible influences that might support children's mathematical meaning making. Early childhood teachers should understand the role of the family in order to create high quality learning environments as outlined in the AMTE (2017) standards. Teacher education programs need to consider how to support preservice early childhood educators. Programs should provide opportunities for candidates to interact with families and children. These interactions can lay the foundation for understanding how to support families in developing mathematical understanding and language. Having experiences in a supportive environment about how to prepare family friendly materials are critical.

Future research in this area could focus on understanding more about how families support young children's mathematical development. The studies reviewed included small sample sizes, so replication or expansion would provide more evidence of factors that might influence children's mathematical meaning making prior to elementary school. Observational data of family interactions during play and other everyday experiences could help to understand the many layers of these ideas. Furthermore, family questionnaires about the type of support that would be helpful could guide early childhood professionals in building supportive and culturally responsive ways to facilitate families' approaches to engaging children in mathematical conversations.

In the next section, we explore how families interact with children as they progress through elementary and secondary school in supporting ways to communicate mathematically.

Older Children, Family, and Play

As previously emphasized, family members are those social factors that can positively influence mathematical language. Hence, it is important that the home interactions described above continue into the middle and secondary grade levels. Equally essential is awareness of and responsiveness to challenges family members may face when trying to support children with mathematics. For example, when family members come from different learning environments, unfamiliarity with current practices can cause anxiety and frustration (Vukovic et al., 2013). In response, researchers recommend providing exploratory support for families with the use of open-ended tasks that are free from clear, predetermined procedures.

Mangram and Metz (2018) specifically promote playful mathematics experiences as vehicles for engaging families in mathematical problem solving, regardless of parents' mathematics proficiency and comfort level. Mistretta (2013) reports on such an experience entitled Which One Doesn't Belong? (Fuys & Welchman-Tischler, 1979). She found this task to help reduce family members' stress, and develop confidence in their ability to communicate about mathematical ideas in ways that can transfer well into the middle and secondary grade levels.

The following example is not a unique classroom idea; however, it is one suggested for teacher educators to consider using or adapting when preparing teachers with resources for engaging older families in mathematics meaning making and related language use. The task involves comparing and contrasting four examples to determine one that is different from the others. Specifically, when shown four computation examples such as -5 + -2, 8 x -2, -5 + (-7), and -9 ÷ 3, responses may include a) -5 + -2 because it is the only example that has both an even and an odd integer b) 8 x -2 because it is the only example that begins with a positive integer, d) -5 + (-7) because it is the only example that has parentheses, or d) -9 ÷ -3 because it is the only example with a positive solution.

The existence of different correct solutions affords families multiple entry points into the conversation that do not necessarily hinge on parents' level of mathematics proficiency. Such verbal discourse provides family members opportunities to individually make their thinking audible as well as recognize, and celebrate, the reasoning of others. In turn, a strength-based stance on family engagement can emerge that acknowledges family members as partners in mathematics meaning making.

Implications for Teacher Education and Research

Edwards et al. (2019) report new teachers' unfortunate lack of experience working with families. Given such a circumstance, along with the vast amount of playful tasks that exist for supporting mathematical language, teacher educators' use of and inquiry around how such tasks can help shape PST learning experiences with respect to working with families warrants attention. In addition, Froiland and Davison (2016) report family member expectations and intrinsic motivation as contributing significantly to the development of mathematics achievement in 9th through 11th grades. In turn, these researchers suggest studying interventions designed to enhance family member expectations and motivation. Playful tasks may serve as such an intervention to study.

Another research venue stems from O'Sullivan et al. (2014) who report family members' self-efficacy as associated with family engagement among low-income urban junior high school families. The authors suggest lack of confidence as possibly contributing to these families not providing direct assistance with mathematics. Hence, inquiry around how playful tasks can influence self-efficacy among middle and secondary grade level families has potential for contributing to the knowledge base.

It will be important to also consider how students are engaged in mathematical language in their home language and/or in their new language. The following section provides insight into English language learning populations and related opportunities for cultivating mathematical language.

Mathematics and Multilingual Learners

Many researchers have studied the effects of language learning on mathematical learning. Although language learners can refer to a variety of students, this section specifically considers students who are learning English in addition to their primary language and attending schools taught in English. Throughout the paper we use the term multilingual learner (ML) to reflect students who have a primary language, perhaps learned at home, but are learning new languages and in particular being taught in a language that is not their primary language. Many people may think that MLs do not face additional struggles in mathematics classes since mathematics uses symbols, which are understood across many languages. However, this is a myth (Janzen, 2008). Many face substantial barriers in mathematics when it is not taught in their primary language.

Mathematics Assessment of MLs

Multilingual learners often score significantly lower on mathematics assessments. However, do lower test scores mean less mathematical understanding or are we assessing the students' language understanding? MLs usually score lower than their native English-speaking peers on large-scale assessments, and the achievement gap is larger on language-heavy questions. Studies by Martiniello (2008) and Wolf and Leon (2009) further support that MLs struggle more on linguistically complex test items. However, when language accommodations are made, such as presenting simplified language, ML students' performance improves (Abedi & Lord, 2001; Alt et al., 2014; Martiniello, 2008; Newkirk-Turner & Johnson, 2018; Wolf & Leon, 2009). Kurz et al. (2017) claim that elementary PSTs need experience working with MLs to better understand the complexities of teaching these students. The authors provide a framework to assist teachers in making accommodations to mathematics word problems. This framework describes language adaptations, mathematical adaptations, tool/visual adaptations, and structural adaptations. With modifications to mathematics assessments, mathematics material is more accessible to MLs and they can be more accurately assessed on their mathematical understanding.

MLs in the Classroom

A student's performance on an exam is far more complex than their language competency. Ercikan et al. (2015) explain that many factors of language, culture, and context can affect student performance even when the questions are linguistically simple. The comma and decimal point are used differently for place value. Ordinal numbers are notated differently in different languages. Some languages, such as Japanese, use numerical classifiers, which have no literal translation to a language that does not use numerical classifiers such as English. There are many additional challenges for MLs (Miura & Okamoto, 2003). Driver and Powell (2017) encourage the combined use of culturally and linguistically responsive instruction where teachers "consider the unique learning characteristics of their students including native language, English language proficiency, race and ethnicity, home and community culture, and past educational experiences" (p. 43). Chval and colleagues (2021) stress the importance of academic language for MLs and their need to develop specialized mathematical language and the need to engage them in experiences where they can distinguish between multiple meanings of words (e.g., mean or change). These authors advocate for implementing instructional strategies that help students distinguish between everyday language that may be used at home or in their community and specialized mathematical language, and help them to connect or transition their use of everyday language to specialized mathematical language.

Implications for Teacher Education and Research

Teacher educators should strive to develop culturally efficacious mathematics instructors. This can be done by encouraging teachers to better know themselves, as well as to know their students and their communities (Flores et al., 2015). Teachers can support MLs by encouraging student discussion and small group communication. Group work will allow students to hear and speak the language of mathematics in English, which will increase their mathematical understanding as well as develop their

English proficiency. Teachers should not discourage students from speaking in their native language when working in small groups (Gann et al., 2016; Janzen, 2008; Murrey, 2008; Zahner, 2012). Murrey (2008) recommends that teachers explain the mathematics conceptually in an appropriate context before introducing the academic vocabulary. The use of manipulatives may be helpful; however, it is important for teachers to know their students' English language proficiency level. Using manipulatives does not help develop a contextual mathematics understanding if the language barrier is quite large (Janzen, 2008). Similarly, technology can be a great tool in the classroom, but language ability is a critical factor in student understanding with its use (Ganesh & Middleton, 2006). All students, especially MLs, will benefit when teachers get to know the students and their cultures (Janzen, 2008). Multiple instruments are available to educational leaders and teachers to assess teacher beliefs, attitudes and knowledge about MLs (Fernandez & McLeman, 2012; Flores et al., 2015; Gann et al., 2016; Reeves, 2006). Survey results can inform district leaders in planning effective professional development on cultural and linguistic issues.

This section has examined mathematical teaching and learning of students who are learning the English language. Although MLs tend to score lower on assessments, much of the achievement gap is due to language deficiencies. In the classroom, there are several techniques that teachers can implement to better support MLs. In the next section, we discuss a language that many young students are fluent in before kindergarten--technology.

Technology and Digital Media

Digital technologies and interactive media have transformed almost all aspects of our social and cultural practices, including children's meaning making with both language and mathematics. Blending visual, auditory, and haptic representations in a multimodal and interactive environment, digital technologies are particularly appealing to young children. When used strategically, digital technologies can change the nature of children's linguistic and mathematical experiences, promoting their competencies in meaning making using language and mathematical resources (Beschorner & Hutchison 2013; Chmiliar, 2017; Clements et al., 2008; Couse & Chen 2010; Falloon 2013; Falloon & Khoo 2014; Kucirkova et al., 2014; NAEYC, 2012; NCTM, 2015; Patchan & Puranik, 2016). In teacher education, both the ATE and the AMTE recognize the essential roles of digital technologies in instruction and assessment. ATE (2018) characterizes technology use from a social and cultural perspective, encouraging teacher educators to model best practices and technology integration in the global context of teaching, learning, and assessment. AMTE (2017) emphasizes technology as a type of instructional tool that should be strategically used by educators to promote students' sense-making and understanding of mathematics. On the other hand, inappropriate technology use or abuse may cause harm to children's cognitive development and social emotional well-being (Twenge et al., 2018). In fact, children today are spending an alarming amount of time on digital media, raising serious concerns among caregivers, educators, and policy-makers (Vulchanova et al., 2017).

The Emergence of Digital Literacy

Mathematics is a kind of academic language, calling for reading, writing, and sense-making strategies in quantitative reasoning and inquiry. A lack of literacy competence in mathematics can seriously hinder the academic and social performance of school children as well as citizens at all levels (Madison & Steen, 2003; Steen, 2001). Accordingly, there have been persistent efforts to re-conceptualize and re-design literacy instruction in content areas, including school mathematics (Draper & Siebert, 2010; Manzo et al., 2009; Siebert & Hendrickson, 2010). This shift toward a focus on literacy and language in mathematics education highlights, on the one hand, the mediating role of language in all aspects of mathematical cognition and, on the other hand, the foundational role of quantitative reasoning in daily language use and other social and cultural settings. Indeed, mathematics has become inseparable from the evolving scope of literacies in what is often called digital literacy, a synergistic integration of traditional literacy, media literacy, quantitative literacy (de Lange, 2003), and interactive technologies. The emergence of digital literacies has not only broadened conventional literacy education and research but also opened new frontiers for educational development in assessment, instruction, and interventions (Goodman et al., 2010).

Digital technologies, versatile by design, support a wide spectrum of traditional and contemporary media and pedagogical paradigms, including the integration of audio, visual, and other multimodal representations. Both the design and the content of digital media such as mobile apps are significant factors in determining an app's educational value and the quality of children's experience (Couse & Chen, 2010; Falloon, 2013; Falloon & Khoo, 2014; Kucirkova et al., 2014). More importantly, a teacher's pedagogical choices and guidance play critical roles in ensuring the effective use of technology and digital media in enhancing children's learning and development, including meaning making in mathematics (Clements et al., 2008; Couse & Chen, 2010; Falloon & Khoo, 2014).

Implications for Teacher Education and Research

The evolving nature of digital literacy may lead to practical and theoretical disruptions in the educational enterprise, including the language and mathematical development of young children, as these digital tools find their way naturally and informally into homes, classrooms, communities, and workplaces. Because of the lack of research on the long-term impact of digital media on children, it seems imperative that educators follow the NAEYC (2012) recommendations for developmentally appropriate practices (DAP). This is in light of children's vulnerability and sensitivity to the novelty and multimodal stimuli of digital tools and researchers should continue to study this impact in partnership with practitioners.

Traditionally, human communication is accomplished through face-to-face interactions, supplemented by printed or recorded materials. Today, the prevalence of digital media has provided new possibilities and theoretical perspectives for human communication, blending hearing, vision, touch, and other sensory modalities (Vulchanova et al., 2017). Existing research points to the powerful affordances of digital media in transforming children's educational experiences, which, however, is subject to appropriate design, content, and teacher guidance. Therefore, in teacher education programs, both PSTs and ISTs should be mindful of DAP and the social and emotional ramifications of digital media use while striving to use new tools for linguistic and mathematical understanding and equity (AMTE, 2017; ATE, 2018; NAEYC, 2012).

Summary of Findings

Table 1, while not an exhaustive list, identifies several implications for teacher educators and offers ideas for further research to consider based on this synthesis. The implications and ideas for further research are delineated by the six areas addressed in this paper examining the effects of language on children's understanding of mathematics, particularly related to meaning making of mathematics.

Discussion and Conclusion

The teaching of mathematics and utilization of mathematical language are both very complex phenomena. Many researchers, such as Pengelly (1990), have recognized linkages between mathematics and language with multiple approaches that include nuances such as mathematics as a language, mathematical language, and language and learning mathematics, to name a few. As teachers and teacher educators, we should be aware of all the varying aspects of mathematical language. How best can teachers—both PSTs and ISTs--be prepared for this critical work? The synthesis of research in the area of mathematics and language provided in this paper underscores the importance of the continued study of students' mathematical language, both inside and outside classroom walls, as means for supporting students' mathematics meaning making.

Table 1

Implications for Teacher Educators and Ideas for Further Research Based on the Literature

Areas addressed	Implications for teacher educators	Ideas for further research
1. What mathematical language is and how it connects to meaning making and understanding	• Engage teachers in experiences to broaden their understanding of what mathematical language is and its role in mathematical meaning making.	• Examine types of supports and engagement that are beneficial in supporting language development for students to deepen conceptual understanding and making meaning of mathematics.
2. Symbols, expressions and language connections	• Provide multiple and varied opportunities to support making sense of symbols. • Create a robust learning environment that supports academic language and allows meaning making of symbolism to thrive.	• Consider ways that multilingual leaners grapple with mathematical symbols to yield greater understanding and make significant connections between language, symbols, and mathematical meaning.
3. Effects of teachers' listening orientation on students' mathematical learning	• Provide explicit focus on listening and its connection to teacher noticing to facilitate meaning making of mathematics.	• Examine teachers' listening orientations and how they do or do not focus on mathematical meaning making. • Analyze instructional decision making related to listening orientations.
4. Mathematics language development: Play and family influences	• Ensure that teachers have a deep understanding of their role in play to support student learning. • Provide opportunities for teachers to strategically observe play, engage in play, and have experiences teaching from a play perspective to build mathematical meaning. • Provide opportunities for teachers to interact with families and children to understand how to better support families in developing mathematical understanding and meaning making.	• Examine the connection between mathematics-specific literacy and play. • Study the impact of various math instructional materials and interventions that engage families with their children and can help shape mathematical understanding and collaboration. • Examine how families support young student's mathematical development and how to support positive self-efficacy of families in supporting their children in mathematics.
5. Mathematics and multilingual learners	• Develop teachers understanding of culture and its impact on mathematical instruction and meaning making. • Provide opportunities for teachers to experience the impact of discussions, use of manipulatives, and technology and the role of academic language and discourse in teaching and learning mathematics.	• Study the impact of varied manipulatives and technology as used with multilingual learners. • Examine types of professional development that could impact teachers' beliefs, efficacy, attitudes, and understanding of multilingual learners and how to use this data to develop professional development specifically to target mathematical meaning making.
6. Technology and digital media	• Include a wide variety of technologies in teacher's experiences while attending to the developmentally appropriate use of that technology.	• Design studies to examine the long-term impact of digital media on children's understanding of mathematics and the social and emotional ramifications with a particular focus on current technologies. • Examine how technology tools support or hinder linguistic and mathematical understanding.

We offer readers Figure 1 as a potential framework to consider for guiding teacher educators' practices and future research efforts. In so doing, we display various connections and interplays between language and children's mathematical meaning making and understanding. The framework begins with an overarching theme of children's meaning-making in mathematics and unfolds with the dynamic interplay of teachers and teacher educators, schools, families and parents, and children and their peers. These dynamic connections impact the development of mathematical understanding and help build language capacity through engaging in a myriad of mathematics and language activities such as play, listening, interactions, talking and use of technology. These occur in varied spaces both formal and informal (home, school, community and cultural interactions) and manifest in development of academic language, conversational language, use of symbols, consideration of multilingualism, and mathematical content literacies.

Our synthesis and related representation illuminate symbols as processes; a lens less utilized in some classrooms. We highlight the importance of responding to mathematical language needs during instructional planning, instructional delivery, assessment, and evaluation as language is an important facet in all of these areas as students strive to make sense of mathematics. Finally, we underscore the goal of reaching each and every student, that is, students struggling to understand, gifted students, and those learning English as a new language so all are successful in mathematics learning and develop a positive mathematical identity. Our Commission team urges all stakeholders including mathematics teacher educators, researchers, community/family partners, and professional development specialists to promote and provide support and appropriate training for practitioner inquiry that further develops students' mathematical language in schools and at home.

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