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The effect of using PDEODE teaching strategy supported by the e-learning environment in teaching mathematics for developing the conceptual understanding and problem-solving skills among primary stage students

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Abstract

The aim of the current research is to know the effect of teaching mathematics using the PDEODE teaching strategy supported by the e-learning environment in developing the conceptual understanding and problem-solving skills among the fourth grade of the primary stage. To achieve the research objectives, the experimental approach was in its quasi-experimental design. The research sample consisted of 76 students, divided into two groups; experimental, 38 students who studied using PDEODE supported by the e-learning environment, and control, 38 students who studied in the traditional method. The results showed firstly, there are statistically significant differences at the significance level 0.05 between the mean scores of the students in the two groups: experimental and control in the conceptual understanding test in favor of the experimental group students. Secondly, there are statistically significant differences at the significance level 0.05 between the mean scores of the students in the two groups: experimental and control in the test of problem-solving skills in favor of the experimental group students. Thirdly, there is a positive correlation between the scores of experimental group students in the conceptual understanding test, and their scores in the problem-solving skills test in mathematics.

Keywords: PDEODE strategy, e-learning environment, teaching mathematics, conceptual understanding, problem-solving skills, primary stage students

INTRODUCTION

Mathematics curricula are among the school curricula that educational reform movements have focused on in various countries of the world due to their role in achieving the requirements of comprehensive community development. Educators have emphasized the great role of mathematics curricula in enabling students to develop their abilities which enable them to acquire mathematical concepts and mathematical problem-solving.

Al-Saeed (2018) indicated that the interest in conceptual understanding is positively reflected in the students' ability in logical inference, and in transferring their mathematical knowledge to other situations through which they solve the problems they face and build new knowledge. Conceptual understanding is an important mathematical learning outcome and a component of mathematical proficiency which were recommended by the National Research Council (NRC), meaning the learner's ability to understand the various aspects of the mathematical content of the concepts, generalizations and mathematical connections, so that the learner can know the content in which mathematical ideas are used, which represents a deep understanding of mathematics (Al-Menoufy & Al- Moatham, 2014).

Conceptual understanding is shown through the learner's ability to understand mathematical ideas from concepts, generalizations, relationships, and procedures, knowing information and procedural steps in a coherent manner, knowing the importance of the mathematical idea, knowing the content in which the mathematical idea is used, knowing the interrelationships between mathematical ideas, being able to represent mathematical situations in different ways, rebuilding ideas to solve mathematical problems and situations, and producing new knowledge (Al-Menoufi & Al-Moatham, 2019).

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Contribution to the literature

- This research is consistent with all literature reviews in terms of handling the PDEODE teaching strategy variable. However, it differs in that it explores the effect of teaching mathematics using the PDEODE teaching strategy supported by the e-learning environment in developing the conceptual understanding and problem-solving skills among the fourth grade of the primary stage.
- The Education Evaluation Commission (EEC, 2016) report indicated a low level of students in the primary stage, and the study of Al-Anazi and Al-Nazir (2016), Al-Menoufi and Al-Moatham (2018), Al-Salouli et al. (2010), indicated a low level of conceptual understanding among the students. In addition, the researcher applied a test of 60 students of the fourth grade of the primary stage, the results indicated a low level of students in the test of problem-solving in adding and subtracting decimals based on their learning through the traditional method and without technology. Therefore, this research came to help those students in developing the conceptual understanding and problem-solving skills through the PDEODE strategy with modern technological methods represented in electronic environment.
- According to the results of the current research, we can say that the use of the PDEODE strategy, supported by an e-learning environment in the teaching of mathematics, is proven to be effective in developing the conceptual understanding and problem-solving.

Many studies have emphasized the importance of conceptual understanding in mathematics as one of the important outcomes of learning mathematics, and recommended the need to use a variety of teaching strategies to develop it among students, and among these studies (Abdul Hamid, 2019; Abdullah, 2015; Ahmed, 2015; Al-Qatatsheh & Al-Miqdadi, 2018; Kaware, 2017; Shafie, 2013).

One of these strategies is the PDEODE strategy which is based on the ideas and principles of constructivist learning. This strategy is defined as a set of teaching procedures that encourage the creation of an interactive atmosphere in the classroom through discussion, and the expression of diverse points of view. In addition, it helps students to understand everyday situations. This strategy consists of six steps, namely predict, discuss, explain, observe, discuss, and explain (PDEODE) (Costu, 2008).

Employing the PDEODE strategy in mathematics education is not limited to enhancing students' conceptual understanding by enabling them to access, understand, construct, and apply mathematical knowledge in different contexts, but also contributes to enabling them to be involved in solving the problem (task), far from learning practices that are based on memorization (NCTM, 2014). Therefore, the effective use of the PDEODE strategy plays an important role in developing students' abilities to solve mathematical problems, particularly since this aspect is one of the basic operations standards that they must learn (NCTM, 2000).

Khabour (2019), through her review of educational literature, mentioned that the mathematical task is a mathematical problem whether it is symbolic or verbal, that is from the reality of students as it requires them to possess sufficient skills and experience, and to be able to deal with different concepts and laws.

With the development of e-learning and the technological innovations, it has become necessary to take advantage of these technological innovations in teaching and learning mathematics, and e-learning environments are among the most important technological innovations because of their possibilities of suspense and attraction to the learning process, and provide a kind of communication and interaction between learners, and allow them to work as a team and build their distance learning under the supervision of their teacher (Azmy et al., 2014). Therefore, the idea of the current research came to build a strategy that combines the ideas of the constructivist theory represented in the PDEODE strategy with modern technological methods represented in electronic environments, and to measure its effectiveness in developing the conceptual understanding and problem-solving skills among the fourth grade of the primary stage.

Research Problem

The Kingdom of Saudi Arabia was keen to participate in the Trends in International Mathematics and Science Study (TIMSS) in order to improve the level of achievement of its students, but the results of the Kingdom's students in this study were disappointing. The reports of this study revealed that the results of the performance of the students in Saudi were lower than the international average during the period 2003 to 2019 (IEA, 2019). The main objective of the participation of Saudi in the TIMSS is for students to practice the scientific inquiry to solve problems, with putting of hypotheses, drawing and evaluating results, making decisions about the problems they face through discussion, and applying mathematical knowledge in life situations that depend on the method of critical thinking, discussion, explanation, and analysis away from methods of memorization in the teaching and learning of mathematics, which is consistent with the philosophy of constructivist theory.

The Education Evaluation Commission (EEC, 2016) report indicated a low level of students in the primary stage, and the study of Al-Anazi and Al-Nazir (2016) showed that the level of conceptual understanding among sixth-grade students was average, and the study of Al-Menoufi and Al-Moatham (2014, 2018) found that the students were not able to understand conceptually, and the study of Al-Malouhi (2018) indicated a low level of conceptual understanding among the students. The study of Al-Salouli et al. (2010) showed a low level of students at the primary stage of understanding the concepts related to arithmetical operations.

To determine the levels of problem-solving in mathematics learning environments, the researcher applied a test of 60 students of the fourth grade of the primary stage. For the purpose of correcting the test, the solo taxonomy was adopted as stated by Biggs and Collins (1982). The results indicated a low level of students in the test of problem-solving in adding and subtracting decimals based on their learning through the traditional method and without technology where 70% of the students got less than half of the mark.

Moreover, the researcher examined the three mathematical abilities (procedural knowledge, conceptual understanding, and problem-solving skills) in 30 students of secondary school and 60 students of primary school. Students were given five separate tests in each domain of mathematics (number and operations, algebra, geometry, measurement, data analysis, and probability) and each test covered the three mathematical abilities. It was found that, in general, students performed well in procedural knowledge and poorly in conceptual understanding which led those students to make many errors in solving problems. This is confirmed by Hasnida and Zakaria (1991) where the authors conducted a survey study on the results of high-school graduates. They found that the students' level of procedural understanding was high, whereas the level of conceptual understanding was low.

According to the results of the five separate tests the researcher found that the main reasons behind this are the fact that the procedural knowledge problems are those types of problems that are familiar to the students from their K-12 education in schools in Saudi. In addition, the problem in the teaching approach in a traditional classroom setting, students are made to sit passively while the teacher delivers a lecture. This led to students' focus being set in the wrong direction; in taking notes rather than understanding and absorbing new concepts. The researcher noticed that mathematics teachers focused on presentation with little time left for practice. Since a teacher has to deliver a fixed number of concepts within a limited time, most classroom activities are sufficed to the presentation stage only. Practice is left for the student to do as homework. Also, a teacher's lecture is generally one-size-fits-all. Not every student has the same pace of learning.

Developing conceptual understanding and solving all mathematical problems requires teaching strategies that encourage students to understanding and absorbing new concepts, and staying away from procedural memorization of the steps. Research suggests that once students have memorized and practiced procedures that they do not understand, they have less motivation to understand their meaning or the reasoning behind them (Hiebert, 1999). The use of the PDEODE teaching strategy supported by the e-learning environment may be reflected in the improvement of students towards learning mathematics, particularly since gaining positive attitudes is one of the goals of teaching mathematics (Al-Maliki, 2002), so this research came to know the effect of this strategy on students' learning of mathematics.

The Importance of the Research

This research is important as it adopts a modern teaching strategy based on learners' practice and interaction with the e-learning environment which makes the learning environment more vital and active. In addition, this strategy will help students to stay away from procedural memorization of the steps. This will be reflected in the improvement of students towards learning mathematics. Furthermore, the primary stage occupies a prominent position in forming the various aspects of the learner's personality because it is the medium in which the acquisition of various learnings begins. Primary education in all countries is the base for all the following educational levels, and whenever the primary education stage is strong, the output will be greater for the following stages. Primary education is the basis through which young adults are prepared for the following stages of education (Saadat, 2014). Moreover, the results that this study will bring up are expected to assist curriculum supervisors, educators, teachers, and researchers by providing them with the appropriate strategy of presenting mathematical content. Finally, the research keeps pace with global and local trends which advocate the need to benefit from the use of modern strategies supported by the e-learning environment in mathematics education.

Research Questions

This study seeks to answer the following main questions:

1. What is the effectiveness of using the PDEODE strategy supported by the e-learning environment in teaching mathematics on conceptual understanding?
2. What is the effectiveness of using the PDEODE strategy supported by the e-learning environment in teaching mathematics on improving

mathematical problem-solving skills among primary stage students?

3. What is the connection between conceptual understanding and problem-solving skills in mathematics learning among primary stage students?

Study Hypotheses

The current study sought to verify the validity of the following hypotheses:

1. There are statistically significant differences at the significance level 0.05 between the mean scores of the students in the two groups: experimental and control in the conceptual understanding test in favor of the experimental group students.
2. There are statistically significant differences at the significance level 0.05 between the mean scores of the students in the two groups: experimental and control in the test of problem-solving skills in favor of the experimental group students.
3. There is a positive correlation between the scores of the experimental group students in the conceptual understanding test, and their scores in the problem-solving skills test in mathematics.

Research Limits

1. Objective limits: The chapter *Adding and Subtracting Decimals* from the fourth-year primary school students' mathematics book in the first semester.
2. Time limits: The research was conducted in the first semester of the academic year 2019-2020.
3. Place limits: The research was conducted in Dammam, Kingdom of Saudi Arabia.
4. Human limits: The current research was limited to a targeted group of fourth-year primary school students in a government school in Dammam.
5. Tools limits: The study tools were limited to the application of the conceptual understanding test (developing conceptual understanding skills at the levels of explanation, interpretation, application, perspective, empathy, and self-knowledge), and the problem-solving test.

THEORETICAL FRAMEWORK AND LITERATURE REVIEW

The First Axis: PDEODE Strategy in Mathematics Teaching

Ibrahim (2016a) defines the PDEODE strategy as an interactive teaching strategy based on theory constructivism, and includes six steps: prediction, discussion, interpretation, observation, discussion, and interpretation. The strategy provides an environment supported by group discussions, prediction about the problem, its interpretation, and puts solutions to it. It aims to make the learner aware of his thinking, and an observer of his own thoughts.

Muhammad (2018) mentioned that the PDEODE strategy is based on when the teacher raises a question or set of questions or a mathematical problem that provokes thought, which requires the learner to carry out a set of operations from prediction, discussion, exchange of opinions, analysis, comparison, observation and data collection, and its interpretation, to arrive at a definition of the concept, an explanation of the phenomenon, or a solution to the mathematical problem.

Several studies have focused on PDEODE's strategy and its implementation in the educational process to achieve many learning outcomes, including: Al-Kubaisi and Fakhri (2016) aimed to find out the effect of the PDEODE strategy on the achievement and mental motivation in mathematics of the fourth literary students. The research sample consisted of two groups, the first is an experimental group, 36 students; and the second is a control group, 34 students. The two groups were equalized with some variables. The researcher used two tools; the first is an achievement test and the other is a mental motivation scale, and he used the appropriate statistical means. The results showed that there were statistically significant differences at the level 0.05 between the two groups in the mean achievement test and the mean scores of the motivation scale in favor of the experimental group.

Al-Shahrani (2018) conducted a study aimed to identify the effectiveness of using the PDEODE strategy in science teaching on developing achievement and creative thinking among primary school students. The units (electricity and magnetism) have been analyzed, and a teacher's guide prepared in line with teaching according to the strategy. The empirical approach was used. The quasi-experimental design, with a pre- and post-measurement, was used. The sample consisted of 68 students from the sixth grade of primary school, divided into two groups: experimental, 33 students, who studied the unit (electricity and magnetism) using the PDEODE strategy, and a control group of 35 students, who studied in the traditional method. The results found that there were statistically significant differences at the significance level 0.05 between the mean scores of the students of the two groups, experimental and control in the achievement test in favor of the students of the experimental group, and that there were significant statistical differences at the level of significance 0.05 between the mean scores of the experimental and control groups in the test of creative thinking as a whole, and its sub-skills, except for originality, for favor of the experimental group students.

Al-Kassi and Al-Qahtani (2018) aimed to know the effectiveness of science teaching using the PDEODE strategy in the achievement and development of metacognition skills for first-grade intermediate students in the Asir region, and to achieve this, the quasi-experimental approach was used, designing the experimental and control groups with a pre- and post-measurement. The research tools represented in the achievement tests and metacognition skills—after verifying their validity and reliability—on a sample of first-grade intermediate 40 students, were divided into two groups: the experimental group consisting of 19 students, and the control group consisting of 21 students. The results showed that there are statistically significant differences at the 0.05 level between the mean scores of the experimental group students and the control group's scores in the post-measurement of the achievement test, and the metacognitive skills test in favor of the experimental group, and the existence of a large impact size of the PDEODE strategy in the achievement and development of metacognition skills among first-grade intermediate students.

Muhammad (2019) conducted a study aimed to identify the effectiveness of the PDEODE strategy in teaching mathematics on developing mathematical confidence and divergent thinking among first year middle-school students, and the sample consisted of two groups, experimental and controlling, and the research tools were applied before (confidence scale, mathematics, and divergent thinking test). Then teaching was according to the PDEODE strategy for the students of the experimental group, while the students of the control group studied using the traditional methods, then applied the research tools afterwards.

The results of the research concluded that there are statistically significant differences between the mean scores of the students of the experimental and control groups in both the mathematical confidence scale, and that the divergent thinking test is in favor of the group of experimental, as well as there being statistically significant differences between the two applications, the pre- and post-applications, in both the mathematical confidence scale and the divergent thinking test in favor of the post application for the students of the experimental group. There was no correlation between the scores of the experimental group students on both the mathematical confidence scale and the divergent thinking test in the post application.

PDEODE strategy steps

Costu (2008) stated that the PDEODE strategy includes six steps, as follows:

1. Prediction: The teacher presents the phenomenon or concept to be learned to the students, and then gives them the opportunity to predict the outcomes or results of the presented phenomenon. This should be done individually, and to provide logical justifications for their predictions.
2. Discussion: In this step, the teacher creates a good atmosphere for the students, which allows the exchange of opinions through discussion groups to present their ideas and discuss them.
3. Explain: The teacher asks the students of each group to come up with explanations for the phenomenon presented to them and exchange the results with other groups through group discussions.
4. Observe: The students notice the changes in the phenomenon, and the teacher should guide them to make observations related to the new concept presented to them.
5. Discuss: The teacher asks the students to modify their predictions through the actual observations they recorded in the previous step, and this requires the students to do the process of analysis, comparison, and criticism of each other's ideas.
6. Explain: Students face the contradictions between observations and predictions, through contradictions, they get the information correctly.

The role of the teacher in the PDEODE strategy

Muhammad (2019) indicates that the teachers play many roles in the PDEODE strategy, as follows:

1. A guide to the learning process, an organizer of the learning environment, participating in the management, and evaluation of learning;
2. Providing a social atmosphere by making the classroom a safe environment for learning;
3. Making available appropriate educational opportunities for learning in small collaborative groups, group discussion of ideas and predictions to collectively solve, and rule out wrong predictions;
4. Motivating students during discussions, learning in the group or between groups and each other;
5. Guiding the group' students to share their opinions with other groups in a group discussion for the class as a whole, deciding whether or not the predictions, solutions, and ideas of each group are true or false;
6. Not criticizing the students' opinions and allowing them to freely express all their opinions and suggestions;
7. Interest in exploring the tribal knowledge of students, and taking it as a base to gain new mathematical experiences and build on them; and
8. Helping students solve inconsistencies between their actual predictions and observations, and formulating the aspects of their learning which they have reached.

The role of the learner in the PDEODE strategy

Some studies such as Kamal (2017) and Mohamed (2019) indicate that the students play many roles under this strategy, including:

1. Discoverer, seeker of knowledge, access to meaning, and building knowledge by himself;
2. Making predictions about the proposed solutions for the problems that are posed or asked by the teacher;
3. Providing explanations for the predictions that were presented by the problems;
4. Cooperating with his colleagues in the group while solving the problem or answering a question asked by the teacher;
5. Discussing with his colleagues in their answers about the questions asked by the teacher; and
6. Analyzing, comparing and critiquing ideas among the predictions he made for the solutions to the problem posed by the teacher, and between the observations he reached through his performance of the activities assigned to him by the teacher.

Advantages and disadvantages of PDEODE strategy in teaching mathematics

Much research conducted aimed at identifying the effectiveness of the PDEODE strategy in teaching mathematics at various stages with the aim of developing multiple educational outcomes, and among these studies (Costu et al., 2010; Demircioglu, 2017; Saleh, 2019).

The PDEODE strategy has many advantages in teaching mathematics, the most important of which are, as follows:

1. It provides a learning environment that supports discussion and a diversity of views among students in the classroom and incorporating individual work and teamwork of students;
2. It encourages collaborative work;
3. It connects the learner's previous knowledge with the new knowledge, providing a safe learning environment without fear of punishment while learning mathematics lessons;
4. It motivates students to participate and be positive in mathematics teaching situations and acquire mathematical knowledge in an active way; and
5. It provides appropriate opportunities to make meaningful learning, and to help the student to engage in learning for as long as possible (Costu, 2008).

On the other hand, there is some criticism of the PDEODE strategy included:

1. Teaching using the PDEODE strategy is difficult and requires effort from the teacher, these difficulties represented in the teacher's roles in the PDEODE strategy as mentioned in the above section and
2. Repeated complaints from students that they did not immediately get the correct answers (Kolari et al., 2004).

The researcher think that a teacher's doubts is can be expected when teaching tools and strategies change. However, the bottom line is that all changes must be made with the best interest in mind. With proper training, relevant data, teacher incentives, and awareness, you can give your teachers the gentle nudge they need to explore these tools and strategies. Regarding the second criticism, we can see from the above section ‘‘the role of the learner in the PDEODE strategy’’, the existence of the students' active role in constructing knowledge, students making connections between ideas, and then sharing the results of their thoughts becomes a learning experience, that is, student-centered learning. In this learning, the focus is on the learning process, that is, “how to learn” instead of “how much is learned.” This learning environment encourages students to develop their thinking skills. Student-centered learning can be used to develop thinking skills.

The Second Axis: E-Learning Environments

E-learning environments have received increasing attention since the emergence of technology-based learning in the educational process. Therefore, it is interesting in the next two sections to answer the following questions:

What is the e-learning environment?

Ibrahim (2016b) mentions that e-learning environments refer to pages published on the Internet for specific educational contents, allowing interaction and communication between students, and between them and the teacher using modern technologies in a large way such as: social networks, blogs, files, and slides shared, relying on learning management systems and educational content management systems. Therefore, the current research attempts to design an e-learning environment using some of these tools (the second-generation web tools).

What are the advantages of employing e-learning environments?

Employing electronic e-learning environments in teaching leads to the achievement of many advantages, including: helping students to maintain the information for a long time (Al-Omari, 2017); flexibility of learning in time and place, and according to the student's own speed. Learning is student centered. The student searches for information and creates his own knowledge.

It provides the opportunity for the learner to work on his own and be responsible for his learning, or exchange information with his colleagues, analyze and interpret it, and come up with solutions to problems collaboratively. It is also a way of synchronous and asynchronous communication between the teacher and his students (Al-Imam & Muhammad, 2010).

The Third Axis: Conceptual Understanding

Mathematical concepts are the basis of mathematical construction, upon which other elements of mathematical knowledge depend such as generalizations and skills in their formation, absorption, and acquisition (Abbas & Al-Absi, 2017). Abu Zina (2010) defines the concept as a mental abstraction of a group of phenomena, experiences, or things. Abu Zina and Ababneh (2011) define it as a perception, mental construction, or mental abstraction which represents the mental image formed by the individual as a result of generalizing common characteristics deduced from similar things which are examples of that concept.

NRC (2001) defined conceptual understanding as the comprehension of mathematical concepts, processes, and mathematical relationships. In addition, Syukriani et al. (2017) defined conceptual understanding as the student's ability to use terminology, equations, and mathematical symbols in the classroom and daily life. And Freund (2011), defines it as the ability of the student to deal with arithmetical operations by understanding the relationships and by realizing the connections between mathematical equations.

Al-Saeed (2018) indicated that students with conceptual understanding beyond their knowledge of facts and the basic methods realize the importance of the mathematical idea and the appropriate contexts for the idea. They organize their knowledge in a coherent way that leads them to know new ideas, in addition to that, conceptual understanding helps them to keep learning for a long time (Badawi, 2019). On the other hand, the low level of conceptual understanding leads students to be mathematically incompetent (NCTM, 2000).

Al-Saeed (2018) indicated that the interest in conceptual understanding is positively reflected on the students' ability in logical inference and transfers their mathematical knowledge to other situations through which they solve the problems they face and build new knowledge.

Dimensions of conceptual understanding in mathematics

Wiggins and McTighe (2005) stated that there are six dimensions of conceptual understanding, and the following is an explanation of those dimensions:

1. Explanation: This dimension refers to the learner's ability to use facts, generalizations, and the illustrative examples to describe the aspects of mathematical learning in a good description, supporting that with justifications and appropriate facts, and expressing them briefly and clearly. This means that this aspect of understanding is not just repeating a definition of a term mentioned in the textbook or mentioned by the teacher during the lesson (Al-Bahith Foundation for Research Consultations, 2018).
2. Interpretation: This dimension refers to the student's ability to employ concepts and generalizations to make arguments about a particular mathematical idea or position, and explanations in mathematics take various forms, including: clarification, causation, and statistics, and the ability to explain is shown by reading between the lines of the problem or situation, describing the idea in a more appropriate way to the topic.
3. Application: This dimension refers to the student's ability to employ what he has learned about concepts, principles, and laws to solve problems presented to him in new situations and different contexts, and he is able to determine where. How can the knowledge or skill you have learnt be used? How can previous ideas be developed and improved to take advantage of this new knowledge or skill? (Mahmoud, 2019).
4. Perspective: This dimension refers to the student's ability to form critical points of view, seeing things from a critical perspective, non-emotional or biased, by examining different points of view and seeing the whole picture.
5. Empathy: This dimension refers to the student's ability to understand the views of others, and realize their emotions, perceiving things from the point of view of others.
6. Self-knowledge: This dimension refers to the student's ability to become aware of his mental habits and the personality that shapes his own understanding, his awareness of the limits of his knowledge, and the ways that help him to develop them.

It is evident through the presentation of the literature that dealt with the dimensions of conceptual understanding: the conceptual understanding includes cognitive and mental aspects which is represented in explanation and interpretation, and practical aspects represented in application and perspective, and emotional aspects represented in empathy and self-knowledge, and these six dimensions have been adopted in the current research; The researcher will adopt these dimensions when constructing a conceptual understanding test in mathematics, due to its suitability for the fourth grade of the primary stage, and its suitability to the nature of Mathematics.

The teacher's role in developing conceptual understanding in mathematics

Sabry (2020) states that a teacher can develop a student's conceptual understanding through: clarifying the main aspects of learning included in the lesson and indicating their importance from the beginning of the lesson; presenting mathematical concepts and relationships in a coherent manner; guiding the attention of students to the importance of linking different mathematical ideas; providing information and procedural steps in a sequential and coherent manner; providing continuous feedback to ensure understanding of mathematical concepts, processes and relationships; giving students an opportunity to demonstrate their mathematical understanding in different ways and in new mathematical contexts and situations, and diversity in teaching strategies to deliver mathematical knowledge to students. Moreover, Jazuli et al. (2017) concluded that most students find it difficult to understand and to apply the concepts of mathematics to real-life problems. Therefore, mathematics teachers should help students to understand and apply the concepts of mathematics to life problems and situations.

The Fourth Axis: Problem-Solving in Mathematics Learning (The Mathematics Tasks)

There are many definitions of the mathematical problem (task), one of them was defined by Abu Zina and Ababneh (2011) as a new educational situation that the learner is exposed to, and does not have a ready solution in his mind, and that it is not necessary to be the educational situation mathematical task for all students. And if the educational situation is a mathematical task, it must be completely clear and unambiguous, and there must be an obstacle that requires the student to make an effort to solve the problem, and the problem is solvable.

The researcher defines it procedurally. It is that skill that is used to analyze and develop strategies to solve a difficult question, a complex situation, or a problem that hinders progress in one aspect of life, using the four steps to solve the problem (understand the problem, develop a solution plan, implement the solution plan, validate the solution).

Al-Kurd (2017) and Sayail (2017) indicated that mathematical tasks are divided into several categories, which are: arithmetical tasks, algebraic tasks, and geometrical tasks. The tasks in mathematics were also categorized into two, as follows:

1. Verbal tasks which are the problems that are presented to the student in the form of vocabulary, and linguistic symbols which need the student to translate from a symbolic problem into arithmetic tasks to solve them;
2. Non-verbal tasks which are problems that are presented to the student in a quantitative manner and need a mathematical operation to solve them.

Some educators (Debono, 2009; Khashan & Helmy, 2009) have addressed the importance of solving problems in general, and mathematics in particular. This importance can be summarized in the following points:

1. By solving problems, the student applies what has been learnt of the concepts, generalizations and skills in new mathematics or life situations;
2. Arousing motivation to learn, as they generate the desire to think in order to reach the right solution;
3. Developing the ability to think logically and other thinking skills;
4. Developing learners' confidence in themselves and their ability to face obstacles and difficulties which brings happiness to themselves and enhances their morale; and
5. Keeping the information in the mind of the student and reducing the rate of forgetting information.

Solving a mathematical problem is not just finding a correct answer, but rather it is a way to develop mathematical thinking, and gives a context for mathematics and opportunities to transfer the learning of new concepts and ideas in problem-solving situations.

METHODOLOGY

Study Approach

To achieve the research objectives, the experimental approach, in its quasi-experimental design, was employed which is based on applying pre- and post-test to two equal groups; one experimental which studied using the PDEODE strategy supported by e-learning environment, and the other control which studied in the traditional method.

The Research Sample

Al-Bassam (2015) defined the research sample as "a subset of vocabulary of population of the study in question, to be selected properly to represent the study population" (p. 12). The sample for this research comprised 76 students enrolled in the fourth grade of the primary stage in one public school in Dhahran district (**Table 1**). The experimental group comprised 38 students and the control group comprised 38 students. The sample was purposive, because this school was equipped with the devices and tools needed to conduct this research.

Research Variables

The current research included the following variables:

1. Independent variable: It is represented in the teaching of mathematics according to the strategy

Table 1. Number of students in the experimental and control group

Group	Number of students registered at the beginning of the experiment	Number of students excluded from the experiment	Number of students within the statistical treatment
Experimental	40	2	38
Control	42	4	38
Total	82	6	76

of PDEODE supported by the e-learning environment.

1. Dependent variables: The dependent variables in this research are as follows: first; conceptual understanding skills in mathematics which are explanation, interpretation, application, perspective, empathy, self-knowledge; second, problem-solving skills in mathematics.

Preparation of Experimental Processing Tools and the Measurement for the Research

the preparation of the experimental processing tools went through the following steps:

1. Choosing the educational content: The following unit was selecting *Adding and Subtracting Decimals*, because the unit contains information, knowledge, and activities which are easy to formulate into a PDEODE strategy which may have an impact on students' concepts and their problem-solving skills. In addition, the unit contains abstract mathematical concepts which allow students to discover information by conducting prediction, discussion, observation, and interpretation related to these concepts in the light of work in small groups. Furthermore, this content is interesting, effective, and related to many of the students' daily-life applications.
2. Analysis of the content of the selected unit: The content of the selected unit was analyzed in the mathematics course for the fourth grade of primary in the second semester of the academic year (2018-2019) with the aim of determining the concepts, generalizations and skills contained in this unit. After conducting the analysis process, the reliability and validity of the analysis were calculated, as follows:
    a. The validity of the analysis: The results of the analysis were presented to a group of professors of curricula and teaching methods of mathematics, and some mathematics teachers with the aim of judging the comprehensiveness of the results of the analysis. The validity of the analysis in the light of the unit of knowledge building was referred to, and the opinions of the arbitrators confirmed the comprehensiveness of the analysis of the learning aspects included in the selected unit.
    b. Analysis reliability: The reliability of the analysis was calculated by the researcher and three of my colleagues. The reliability coefficient of the analysis was calculated using the Holsti equation (Holsti, 1968), and was equal to 0.98 which indicates the reliability of the analysis.
3. Prepare the teacher's guide: A teacher's guide has been prepared for teaching the unit *Adding and Subtracting Decimals*, and it consists of the following: The main title of the research, unit title introduction includes: a brief idea for the teacher about the six-dimensional strategy and how to apply it in teaching mathematical topics; general objectives for teaching mathematics; special objectives for teaching the unit; a brief theoretical overview of the study variables; the list of mathematical concepts included in the unit; growth characteristics for fourth-grade students elementary; unit timetable. In the unit lessons, each lesson includes title, main idea, teaching aids, time, lesson components, homework, lesson evaluation. The evaluation questions that it presents after the learners answer the lesson activities, and the evaluation questions take into account that they are comprehensive aspects of learning in the lesson, and include problems in which the student reaches conclusions, and the final evaluation is represented in the tests of the conceptual understanding and problem-solving skills in mathematics.
4. Preparation of the learner's handbook: The lessons of the selected unit have been reformulated according to the foundations of the PDEODE strategy; the book included: an introduction; the general framework for the content of the booklet (procedural objectives, presentation of the lesson content using the PDEODE strategy supported by an e-learning environment).
5. Designing the e-learning environment: By using a set of second generation web tools which are: educational blog, an educational blog entitled *Enjoyment of Mathematics* in which the content of the selected unit was displayed, and the *Blogger* site was used to create blog, and the researcher used wonder share quiz creator 4.5 to create interactive electronic tests for students, and link tests to researcher's e-mail, so that once the student finishes answering the test, the test result

is sent to the researcher's email. In addition, educational group on Facebook; a closed group has been created on Facebook with the title *Enjoyment of Mathematics* in order to discuss the tasks and activities of the lessons of this unit.

6. Sending to the arbitrators: The researcher presented the teacher's guide and student's handbook to a group of experts and specialists in the field of curricula and methods of teaching mathematics and a group of mathematics teachers and supervisors to make sure it is suitable for the application. Some modifications were made in the light of the arbitrators' opinions to come out in its final form.

A Proposed Perspective for Implementing the PDEODE Strategy Through an E-Learning Environment

Through what was presented in the first and second axes in the literature review, the researcher extracts a proposed perspective for the implementation of the PDEODE strategy through an e-learning environment, as follows:

A. The face-to-face learning stage: It consists of the following steps:
   1. The teacher poses a question or problem that provokes students' thinking, and gives them the opportunity to predict the solution individually, and justify those predictions (Prediction stage);
   2. Dividing the students into groups, and the students of each group consult and exchange experiences together to discuss their predictions and their thoughts among themselves; To get predictions that all the students in the group agree on (Discussion stage); and
   3. Students of each group provide justifications and reasons for the solutions they agreed upon, and the groups exchange solutions among themselves with other groups (interpretation stage).

B. Completing the study of the lesson by taking advantage of the e-learning environment, which is represented in the following steps:
   1. The teacher makes the lessons available through the e-learning environment.
   2. The teacher directs the students to enter the e-learning environment and implements the following:
      a. Look at the content of the lesson;
      b. Watching videos related to the lesson assignments;
      c. Implementation of the activities associated with each task of the lesson;
      d. Interact with what the teacher asks through comments;
      e. Search online, collect more resources and share them with others; and
      f. Coming to conclusions, testing the correctness of their opinions and solutions about the problem (observation stage).
   3. Group discussion and exchange of dialogue with students and each other, and with the teacher electronically through the comments or chat room available in the e-learning environment, and implement the following:
      a. Compare the solutions they have come up with by conducting activities and online research with their predictions and
      b. Modifying the predictions in the light of the result of the research and the conducting of activities, and explain it (discussion stage).
   4. Provide explanations through the e-learning environment, and implement the following:
      a. Discuss the reasons for agreement and disagreement between observations and predictions and
      b. Solve inconsistencies that they may have (interpretation stage).
   5. Preparing the final evaluation, sending it to the students, and based on the students' results in the final evaluation; it determines its path. If he passes the test, he moves to the next lesson, and if he does not reach the required level, he re-studies the lesson again, and at the end of the unit lessons, it moves to the post-test for the whole unit.

Study Tools

The study used the following tools:

1. Conceptual understanding test: To verify the validity of the test, it was presented in its initial form to a group of specialists and experts in the field of curricula and methods of teaching mathematics, and a group of mathematics supervisors and teachers to take advantage of their opinions to ensure the accuracy of the mathematics, and the integrity of the linguistic formulation. Based on the opinions of the arbitrators, the questions that got 97% or more of the arbitrators' agreement were adopted. The conceptual comprehension test items reached seven items, each item included two parts, one of which was a multiple choice and the other an essay. The total test scores reached 14 degrees where a scale (estimated scale) was designed to

Table 2. The results of the Mann-Whitney-U test for two independent samples

Variables	Group	Number	Arithmetic mean	Standard deviation	Value of T	Value of sig.	Significance
Conceptual understanding	Experimental	38	7.61	6.81	0.64	0.54	Not significant
	Control	38	8.59	7.12			
Problem solving	Experimental	38	9.98	4.49	0.56	0.58	Not significant
	Control	38	10.53	4.15			

calculate the score of each item according to the possible answers. Then, the reliability of the test was calculated using Cronbach's alpha coefficient; the value of the reliability coefficient of the test as a whole reached 0.89.

The difficulty and ease coefficients for the test items ranged from 20% to 80%; these are suitable coefficients. Bühner (2012) clarified that the questions that students answer correctly in the range from 20% to 80% are considered acceptable for the coefficients of ease and difficulty, while the discrimination coefficients for the items ranged between 0.66 and 0.24 which are suitable coefficients (Wendler & Walker, 2006). To determine the appropriate time for the test, the average total time for answering the questions was calculated by calculating the average response time of the exploratory sample members, with 28 students from outside the study sample and from the same school. It has been shown that the appropriate time for the conceptual understanding test is 25 minutes.

2. The problem-solving test: The problem-solving test was prepared with the aim of revealing the effectiveness of teaching with the PDEODE strategy in improving learning to solve the mathematical problem. To verify the validity of the test, it was presented in its initial form to 14 specialists and experts in the field of curricula and methods of teaching mathematics, and a group of mathematics supervisors and teachers. Based on the opinions of the arbitrators, the questions that got 97% or more of the arbitrators' agreement were adopted.

   The problem-solving test was 10 questions which was an essay. A score was assigned to each skill of the problem-solving skills (understand the problem, develop a solution plan, implement the solution plan, and validate the solution). This means that the question's score was 4, and the final score of the test was 40.

   The difficulty and ease coefficients for the test items ranged from 20% to 80%, while the discrimination coefficients for the items ranged between 0.50 and 0.40 which are suitable coefficients. Then, the reliability of the test was calculated using Cronbach's alpha coefficient; the value of the reliability coefficient of the test as a whole reached 0.80-0.71. This value of the reliability coefficient is considered acceptable (Shahat et al., 2013). To determine the appropriate time for the test, the average total time for answering the questions was calculated by calculating the average response time of the exploratory sample members with 28 students from outside the study sample and from the same school. It has been shown that the appropriate time for the problem-solving test is 30 minutes.

Equivalence of the Two Groups

To verify the equivalence of the two groups, a pre-application of the research tools was carried out, and then the differences between the two groups were calculated using the Mann-Whitney-U test to identify the level of significance of the differences between two independent groups. **Table 2** shows the results of the Mann-Whitney-U test for two independent samples to indicate the differences between the scores of the students of the experimental and control groups in the pre-application of the two tests of conceptual understanding and the problem solving.

It is evident from **Table 2** that there are no statistically significant differences between the experimental and control groups in the pre-application of the two tests of conceptual understanding and problem-solving skills in mathematics learning which indicates the parity of students of the two groups in the research variables before experimentation.

RESEARCH RESULTS AND DISCUSSION

The First Question:

To test the validity of the first hypothesis of the research, which states: "There are statistically significant differences at the significance level 0.05 between the mean scores of the students in the two groups: experimental and control in the conceptual understanding test in favor of the experimental group students." The value of T was calculated to compare the mean scores of the experimental and control groups in the post-application to test the conceptual understanding as a whole and its sub-components, and the value of the effect size was calculated using the coefficient. The results were as shown in **Table 3**.

Table 3 shows that there is a statistically significant difference (α=0.05) between the experimental and control groups in the post-test. This indicates that there is a statistically significant difference between the mean

Table 3. Results of the post-application of the mathematics conceptual understanding test and its sub-dimensions

Skill	Group	N	Arithmetic mean	Standard deviation	T value	Squared Eta	Effect size
Explanation	Experimental	38	11.33	2.339	7.959	0.382	Big
	Control	38	6.70	3.365			
Interpretation	Experimental	38	6.28	0.748	9.377	0.56	Big
	Control	38	3.98	1.112			
Application	Experimental	38	10.55	2.773	5.920	0.255	Big
	Control	38	6.89	3.39			
Perspective	Experimental	38	7.86	4.95	0.20	0.21	Big
	Control	38	3.55	1.15			
Empathy	Experimental	38	4.81	1.18	13.50	0.25	Big
	Control	38	3.60	0.94			
Self-knowledge	Experimental	38	6.2	3.1	4.3	0.18	Big
	Control	38	3.00	3.7			

scores of the students of the experimental and control groups in the dimensional application in the dimensions included in the conceptual comprehension test in mathematics, as well as the total score of the test as a whole favor the experimental group, and this indicates acceptance of the first hypothesis of the research.

It is also evident from **Table 3** that the coefficient values are bigger than 0.14 for each of the dimensions of the conceptual understanding test in mathematics and the test as a whole. This indicates a significant and effective effect of experimental treatment (teaching with the PDEODE strategy supported by an electronic environment) in developing the conceptual understanding in mathematics as a whole and its sub-dimensions among students of the experimental group. This result is consistent with the results of several studies that found the effectiveness of the PDEODE strategy in developing conceptual understanding among students at different educational stages among the studies (Costu et al., 2012; Demircioglu, 2017; Saleh, 2019).

The researcher attributes this result to the following reasons. The PDEODE strategy provided the students with the opportunity to learn the unit in a meaningful way by linking the new knowledge with the previous knowledge of the students which helped in developing their conceptual understanding. The learner's ability to control the number of times the educational content is displayed, as well as the ability to control the momentary pause, forward, or rewind to see the most important details in the educational lesson. The PDEODE strategy provided the opportunity for the students to practice the skills of analysis and comparison, criticizing their colleagues in other groups, and modifying their predictions by doing many activities which led to the development of different conceptual understanding skills. The e-learning environment provided various visual elements which helped the students to link and represent their ideas and to find relationships between the different aspects of mathematics learning. This helped in the growth of conceptual comprehension; this is consistent with Costu (2008).

For instance, the teacher asked those students who studied through traditional methods to solve 29.3+8.367 and then explain why their answer made sense, and seven of those students knew the "rules" for adding and subtracting decimals. These are a few of their responses:

> "0.12007 makes sense because I tried my best and, if I remember correctly, in addition problems you don't need to line the decimals together."
>
> "0.7948 makes sense because when I added I knew that it doesn't matter how it's lined up."
>
> "7.841 makes sense because with adding you only have to add the decimals on the top. Then you add and finally add the decimal back in."
>
> "79.41 makes sense because you do it just like an addition problem (that's how I remember it anyway)."
>
> "37.667-this makes sense to me because this is how I learnt it. You do simple addition, but line up the decimal points."
>
> "37.667 makes sense because I used what you taught me, line up decimals, add zeros so everything is lined up and then solved."
>
> "37.667-I do not know how it makes sense, but it's how I learned to do it."

On the other hand, the other students who were taught through the PDEODE strategy were supported by the e-learning environment. The teacher used a set of second-generation web tools such as Blogger which is a site that students and teachers can use to communicate with the whole class. A teacher can create a class network where the students can work together on problems, projects, and share ideas and work with each other. This allowed students to take more of a hands-on approach when learning; this is consistent with Ibrahim (2016a). The researcher noticed that students also have many different learning styles. Therefore, this teacher used

Table 4. Results of the post application of the problem-solving skills test

Variable	Group	N	Arithmetic mean	Standard deviation	T value	Squared Eta	Effect size
Problem-solving skills test	Experimental	38	4.02	72.02	25.50	0.90	Big
	Control	38	5.30	41.94			

videos, manipulatives, worksheets, apps, and pictures to help students of various learning styles learn about adding and subtracting decimals. All these tools helped teachers and students to easily answer the following questions: ‘‘What's the point?'' ‘‘What's the purpose of lining up the decimal points?'' Actually, we need this strategy to help students to increase their conceptual understanding because if students don't understand the point of lining up the decimal points when adding and subtracting decimals, then somehow they've missed the idea of place value. And what do we do if the numbers don't even have a decimal point? (Some students get pretty lost when that happens). Indeed, the researcher found this environment coupled with the PDEODE strategy to be motivating to students and supportive of their growing understanding of decimal ideas.

Before moving to the second question, it is interesting to mention that the researcher noticed that teaching using the PDEODE strategy requires effort from the teacher, but the researcher thinks that this strategy can be seen as a challenging but rewarding experience. A very important enhancement is the increase of student activity during lectures. In addition, the researcher noticed that two of the students complained they did not immediately get the correct answers. The researcher thinks that those students complained because they do not get the correct answers immediately because they have to think on their own, and these two notes are consistent with what was also noticed by Kolari et al. (2004).

The Second Question

To test the validity of the second hypothesis of the research which states, “There are statistically significant differences at the significance level 0.05 between the mean scores of the students in the two groups, experimental and control, in the test of the problem-solving skills in favor of the experimental group of students”. The value of T was calculated to compare the mean scores of the experimental and control groups in the post-application to test of the problem-solving skills, and the value of the effect size was calculated using the coefficient. The results were as shown in **Table 4**.

Table 4 shows that there is a statistically significant difference (α=0.05) between the experimental and control groups in the post-test in which the coefficient values are bigger than 0.8. This means that the effect of teaching based on the PDEODE strategy supported by the e-learning environment was great on the problem-solving skills of mathematics. This means that the students of the experimental group increased their mathematical problem-solving skills than the students of the control group, and this confirms the validity of the second hypothesis. Actually, the researcher did find that in the Saudi context, studies so far use this variable, and the studies in the literature such as Al-Kubaisi and Fakhri (2016); Al-Kassi and Al-Qahtani (2018); Al-Shahrani (2018), and Muhammad (2019). These studies used the PDEODE strategy in developing different mathematical learning outcomes.

The significance of these differences and the size of the large impact on the mathematical problem-solving skills can be explained as follows. The use of the PDEODE strategy to teach the unit *Adding and Subtracting Decimals* helped students through (the prediction stage) to link their ideas and information to the teacher's questions and the data available in the situation. Then, (during the discussion stage), they cooperated and worked in small groups to discuss the ideas presented, excluded errors from them, and worked on their interpretation. This was followed by the interpretation stage during which the students of the groups presented and interpreted the results of their ideas that they had reached within their groups to the other groups. It was then followed by the discussion stage where the students discussed their ideas within their group to verify them using the skills of analysis, comparison, and criticism within the groups. In the end, the students made the contradiction between their ideas and what had been observed until they reached the correct solution to the problem or situation, all of which helped them to develop their skills to solve the mathematical problem.

This means that the problem-solving skill requires the ability to (predict, discuss, explain, observe, discuss, and explain) of the elements of the problem or situation the individual faces, and we can find this in the PDEODE strategy supported by the e-learning environment. In addition, the e-learning environment also helped to develop students' skills to solve the mathematical problem as follows: retaining knowledge for as long as possible (Al-Omari, 2017). This allows students to practice their own individual learning style, and according to the students' own speed, it put the students at the center of focus, rather than the teachers (because online learning is fundamentally student-centered, due to the easy implementation of student discussion boards and peer grading systems). This is consistent with Al-Imam and Muhammad (2010). Both collaborative e-learning and synchronous online learning promote student-to-teacher interaction. The value of student-centeredness must not be understated.

Table 5. Pearson correlation coefficient between the results of the conceptual understanding test and the test of problem-solving for the experimental group

Variable	Number	Correlation coefficient	Significance level
Conceptual understanding test & Problem-solving skills test	38	0.70	0.05

The researcher noticed that mathematics teachers when teaching students with traditional methods in this classroom typically set the pace of their instruction based on the average student in the class. This means that the brighter students will be bored, and being taught what they already know. Meanwhile, low-performing students will be unable to keep up and will continue to lag behind. By allowing students to access lesson videos (or to record online sessions), students have more flexibility to skip the material they already understand, or pause to focus on what they find to be particularly challenging, and this helped the teacher to increase mathematical problem-solving skills for all students. This increase is very important for the students, and this is consistent with Debono (2009) and Khashan and Helmy (2009).

The researcher noticed also that when this teacher taught with traditional learning, books were the only sources of knowledge. In relation to these students' knowledge and experience, this was limited to the information of textbook curriculum. When he taught using the PDEODE strategy supported by the e-learning environment, the students could easily access any type of knowledge, and focus on increasing mathematical problem-solving skills. Although reliability and credibility are issues that need to be considered when we go to use information from the internet as there are thousands of unreliable pages, the researcher noticed that the school was very keen to follow the teacher in not using any site except the trusted sites. Therefore, all of the websites and materials used in this presentation have been checked, and are reliable sources for using to teach fractions.

Actually, the researcher heard many times during the class time that the students said, ''That's not fair, 0.5 is greater than 0.43!'' Some students will usually choose longer decimals as larger, but will make correct choices when the initial decimal digits are zero. For example, these children will say 0.43 is greater than 0.5, but will know that 0.043 is smaller than 0.5. This common protest is an excellent teachable moment to introduce decimals to your students. But in the traditional method, the teacher simply told them that 0.5 is greater than 0.43. I do not think that this is going to be enough, so what should teacher do? Indeed, a good strategy to introduce decimals is to connect to prior knowledge. In this strategy, you explain that we use decimals throughout our day, sometimes without realizing it! For example, the teacher who taught these students by using the PDEODE strategy supported by the e-learning environment, asked students to think about what is the point of "studying" the decimal? And then he used the same set of second-generation web tools mentioned in the answering of the first question. The researcher found this environment coupled with the PDEODE strategy to be motivating to students and supportive of mathematical problem-solving. Employing the PDEODE strategy in mathematical education is not limited to enhancing students' conceptual understanding, but also contributes to enabling them to be involved in solving the problem (task), far from learning practices that are based on memorization (NCTM, 2014). Therefore, the effective use of the PDEODE strategy plays an important role in developing students' abilities to solve mathematical problems, particularly since this aspect is one of the basic operations standards that they must learn (NCTM, 2000).

The Third Question

To test the validity of the third hypothesis of the research, which states, "There is a positive correlation between the scores of the experimental group students in the conceptual understanding test and their scores in the problem-solving skills test in mathematics". The researcher used the Pearson correlation coefficient as shown in **Table 5**.

Table 5 shows that the correlation coefficient in the case of the experimental group reached 0.70, at a significance level of 0.05. There is therefore a positive correlation relationship between the conceptual understanding and problem-solving skills among fourth primary grade students who were taught through the PDEODE strategy, supported by the e-learning environment. It is apparent that if the students do not possess an appropriate level of conceptual understanding in *Adding and Subtracting Decimals*, this adversely impacts on their problem- solving capabilities. This confirms the urgent need to focus on this issue and to find a solution to improve the conceptual understanding of students, as will ultimately improve their problem-solving skills.

The researcher attributes the existence of a close relationship between the conceptual understanding and the development of problem-solving skills to the following. The low level of conceptual understanding leads students to be mathematically incompetent (NCTM, 2000). This eventually leads to many misconceptions and errors in problem-solving. This is also aligned with the results found in Jazuli et al. (2017) where the authors concluded that most students find it difficult to understand and to apply the concepts of mathematics to real-life problems. They argue that the difficulty is due to conventional learning strategies which are unable to improve the students' abilities. Therefore, the current research found that conceptual understanding and practical mathematical problem-solving skills certainly should not be mutually exclusive; nor should one be sacrificed for the other because conceptual understanding helps students to avoid many errors in solving problems.

The current study is consistent with what the researcher mentioned in the ''research problem section''. He conducted study to examine the three mathematical abilities (procedural knowledge, conceptual understanding, and problem-solving skills) with 30 students of secondary school and 60 students of primary school. He found, in general, that students performed well in procedural knowledge and poorly in conceptual understanding. This is also confirmed by Hasnida and Zakaria (1991), and led those students to make many errors in solving problems.

The researcher found 14 cases of students who were taught by traditional methods in this research, where students could efficiently use their procedural knowledge and reason to eliminate misconception, but failed to do so. One such example is a question asking why 0.5 is greater than 0.43 where students could easily compare the two decimals to reason why 0.5 is greater than 0.43. It has been found that many of the participants did not use the proper logic to state a valid reason. Therefore, based on the results of this question, the conceptual understanding and the problem-solving skills are positively and significantly correlated with each other according to the Pearson correlation. This confirms the urgent need to focus on this issue and to find a solution to improve the conceptual understanding of students as such will ultimately improve their problem-solving skills. This is consistent with Abbas and Al-Absi (2017) who mentioned that mathematical concepts are the basis of mathematical construction, upon which other elements of mathematical knowledge depend, such as generalizations and skills in their formation, absorption, and acquisition. In addition, Al-Saeed (2018) indicated that the interest in conceptual understanding is positively reflected in the students' ability in logical inference and transfers their mathematical knowledge to other situations through which they solve the problems they face and build new knowledge. Khabour (2019), through her review of educational literature, mentioned that the mathematical task is a mathematical problem. It requires students to be able to deal with different concepts and laws.

CONCLUSIONS & RECOMMENDATIONS

The results showed firstly, there are statistically significant differences at the significance level 0.05 between the mean scores of the students in the two groups: experimental and control in the conceptual understanding test in favor of the experimental group students. Secondly, there are statistically significant differences at the significance level 0.05 between the mean scores of the students in the two groups: experimental and control in the test of problem-solving skills in favor of the experimental group students. Thirdly, there is a positive correlation between the scores of the experimental group students in the conceptual understanding test, and their scores in the problem-solving skills test in mathematics.

According to the above results the researcher can say that the use of the PDEODE strategy, supported by an e-learning environment in the teaching of mathematics, is proven to be effective in developing the conceptual understanding and problem-solving. Therefore, in the light of the findings, the researcher recommend the following: The interest of the faculties of education in training student-teachers to employ the PDEODE strategy, and various technological innovations in mathematics teaching; the PDEODE strategy supported by an e-learning environment must be included in mathematics curricula; organizing workshops for mathematics teachers on how to use the PDEODE strategy supported by an e-learning environment in the teaching of mathematics; encouraging the mathematics teachers to take advantage of what is presented in this study from the teacher's guide and student's handbook, and trying apply it to their students; holding workshops for supervisors of mathematics teachers at the primary stage, through introducing them to the results of this study, of the importance of developing conceptual understanding and increasing problem-solving skills among students, and the necessity of these supervisors to encourage teachers to employ this in schools; and benefiting from the applications and experiences of developed countries who have used e-learning in teaching mathematics to keep pace with contemporary developments. Education has always been a powerful agency in any society, and since good teachers are considered as the backbone of good education, efforts to develop and improve teaching strategies are highly needed and appreciated in the field of education.

The researcher also suggest conducting further research in the following areas: Identifying the effectiveness of the PDEODE strategy in other stages, and on other dependent variables such as critical thinking, scientific thinking, numerical sense skills, and to modify misperceptions of mathematical concepts; conducting studies dealing with the impact of integrating the PDEODE strategy and some other innovations technologies in developing different mathematics learning outcomes; studying the relationship between the training of mathematics teachers on methods of employing the PDEODE strategy, and its effect on cognitive achievement and various thinking skills; evaluating the content of school mathematics curricula in the light of conceptual understanding and problem-solving skills in mathematics.

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