MODESTUM

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Epistemological Obstacle in Transformation Geometry Based on van Hiele's Level

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Abstract

Each level on van Hiele's model has its characteristics, such as terminology, concept, and reasoning strategies. This study attempts to reveal epistemological obstacles on all van Hiele's levels of geometric thinking in the topic of geometric transformation. Thus, a student could overcome the epistemological obstacles in transformation geometry and develop appropriate reasoning strategies. That way, van Hiele's geometric thinking could promote level by level. This research is a case study that investigates and analyzes the epistemological obstacles on the topic of transformation geometry for each level of thinking of the van Hiele model of geometry. The study was conducted on ten prospective mathematics teachers who had received lectures in transformation geometry at a university in Indonesia. The results found in this study indicated epistemological obstacles at each level of geometric thinking in van Hiele's model.

Keywords: epistemological obstacles, transformation geometry, van Hiele's model

INTRODUCTION

Stages of geometric thinking in van Hiele's model are divided into five hierarchical levels. There are Level 1 (Visualization), Level 2 (Analysis), Level 3 (Abstraction), Level 4 (Deduction), and Level 5 (Rigor) (Abdullah & Zakaria, 2013; Clements & Sarama, 2011; Yilmaz & Koparan, 2016). The stages should progress sequentially. The fixed sequential nature in which participants must go through the level in order with van Hiele's theory is valid (Kutluca, 2013; Yilmaz & Koparan, 2016). Therefore, unfixed personal geometric thinking is probably demonstrating random answers to the research test. Nevertheless, since participant's responses indicate participant's way of geometric thinking, it is possible to trace one's actual level, although one's geometric thinking test results were in a random fit. Since the problem-solving process manifests in participant's answers to a problem, the participant's actual van Hiele's level could be traced from one's answers by the problem-solving process one demonstrates that is participant use of terminology, concept, or reasoning strategies. Each level has its characteristic values (Abdullah & Zakaria, 2013; Clements & Sarama, 2011; Yilmaz & Koparan, 2016).

Students demonstrate their terminology, concepts, and reasoning strategies (Abdullah & Zakaria, 2013; Armah & Kissi, 2019). These values characterize the thinking process that one should possess, such that each level has its problem-solving approach. The values also indicate that each van Hiele's level has different obstacles. Therefore, recognizing a proper obstacle that someone faced would be necessary to drive somebody thinking of van Hiele's level.

Considering different characteristics of the van Hiele's levels, it would be difficult for people at various levels to understand each other. Abu and Abidin (2012) inferred that someone at a higher level would have difficulty correctly expressing their idea in a lower-level terminology. By contrast, someone at a lower level has not developed critical reasoning for a higher thinking process. That gap becomes a didactical problem in the learning process since the instructor, who is at a higher level, should perfectly design the learning for their pupils at a lower level (Abdullah & Zakaria, 2013).

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Contribution to the literature

- A participant who successfully overcame epistemological obstacles will contribute to establishing a high level in the practice of constructivist-based teaching.
- Various epistemological obstacles which participants must overcome in studying transformation geometry.
- In the stage of geometric thinking level, the obstacles were cascaded from drawing, calculating, deriving formulas, and exploring a new system to developing the constructivist-based teaching in the future.

Reviews on pedagogical, didactic, and epistemological theories and practices are required to discuss. It is meant to conquer the causes of difficulties in teaching technological sciences to enlighten the conceptual framework (Hegedus & Moreno-Armella, 2011). An epistemological obstacle in the teaching-learning of technical drawing dampens the affection and causes the learner to fail (Pelligrini et al., 2020). From the standpoint of a broader understanding of drawing, it will be a good way to approach the three conceptual fields (technology, space, and code or semic variables) in terms of technological significance. The graphical aspect enters the geometric analysis process. The problem is spatial and dependent on semic variables, favoring the representation of all geometric requirements of the item in semiotic instruments.

Many geometrical concepts connected to images have names. It does not always pertain to the geometrical concept which can impede understanding of the concepts when utilizing properties. Similarity, for example, is an equivalency relationship between shapes/figures (Zang & Wong, 2020). The similarity issue is not only related to the measurement thread (length, angle, area, and volume) to geometric figures (form and space). It also promoted the development of students' cognitive abilities such as problem-solving reasoning and proof, communications, and linkages (Zang & Wong, 2020).

As mentioned earlier, there are different terminologies and reasoning strategies between instructors and pupils. In addition, Artigue et al. (2014) revealed the knowledge could induce difficulties such as epistemological obstacles. Consequently, inappropriate instruction will not give a significant result of pupils' knowledge constructions. However, according to Turk and Arslan (2012), pupils' construction of new knowledge cannot be separated from the existence of epistemological obstacles. Thus, overcoming epistemological obstacles is one of the key points in closing the gap between knowledge and pupils' terminology.

Purpose of the Study

Accordingly, this study reveals participant students' epistemological obstacles on all van Hiele's levels of thinking in the geometric transformations. Thus, the main background is knowledge of the epistemological obstacles students encounter which becomes a requirement for instructors to design a sufficient flow of learning, sufficient scaffolding, and sufficient instructions. Therefore, students could overcome the epistemological obstacles they encounter and develop appropriate reasoning strategies. Thus, van Hiele's geometric thinking of students can be improved level by level.

METHOD

Research Design

This research is a case study, and the approach used is a qualitative approach of collecting data from the participant's point of view and findings. As Rivas and Gibson-Light (2016) described, case studies allow exploration and understanding of complex issues. This research intends to conduct in-depth research of various epistemological obstacles that can only be analyzed through the participants' thinking processes.

Participants

This research was conducted at a university in Indonesia in a transformation geometry course. Sixty-three participants took the test; then reduced the number to only ten participants based on similarities in learning obstacles they faced. The assessment adopted Karakus and Peker (2015) van Hiele's geometry test, which was administered to pre-service teachers in their research. The test was then adjusted based on several studies (Noto, 2018; Sunariah & Mulyana, 2020) on transformation geometry in Indonesia. These adjustments were made so that the test could provide an overview of the obstacles at each van Hiele's level. Several experts first validated the assessment test to guarantee validity, and a research supervisor was consulted.

Data Collection

Obstacles test phase

Sixty-three participants from a university in Indonesia took the learning obstacles test. The participants were given five questions on a geometric transformation subject. The performed test contained questions that reflected every level of van Hiele's geometric thinking. A similar test was conducted by Abdullah and Zakaria (2013), Clements and Sarama (2011), also Yilmaz and Koparan (2016). A brief description of the levels involved in this research is as follows:

Level 1: The participants need to identify, name, compare, and operate on geometric figures.

Level 2: The participants need to describe figures in terms of their components and relationships among components and discover properties/rules of a class of shapes empirically.

Level 3: The participants need to interrelate previously discovered properties/rules by giving or following an informal argument.

Level 4: The participants need to prove theorems deductively and establish interrelationships among networks of theorems.

Level 5: The participants need to explore and establish theorems beyond their available knowledge.

Screening phase

The obstacle test phase result of students was identified and analyzed. The analysis involved students' way of geometric thinking in obtaining answers. First, it could be deduced participants' achievement levels of geometric thinking. Then, the analysis ways of geometric thinking were grouped based on their similarity. Finally, in each method of geometric thinking, a representative participant is selected to proceed to the interview phase.

Interviewing phase

After the screening phase, 10 participants were selected for the interview phase. The 10 participants were considered to represent the geometric thinking that had been analyzed in the screening phase. The selected participants were then confronted with the corresponding geometric thought based on their answers to the test. Finally, they were asked to describe their answers. It is necessary to decide whether it is a misconception or an improper way of geometric thinking.

Data Analysis Technique

The data obtained from the assessment test and interviews were then analyzed qualitatively. The researcher determined the association and identified the problem-solving procedure employed by participants in the first stage. Next, the researcher deleted, combined, or subdivided the coding categories found in the first stage in the second stage for the problem-solving process used by participants. In qualitative research, the coding process is a data processing technique (Saldana, 2016). A grounded analysis was used in this study. As defined by Phillips-Pula et al. (2011), a grounded analysis is a generic methodology for examining patterns, themes, and common categories in this research until a specific code can be assigned to an epistemological barriers criterion. The test assessment had been adjusted to the van Hiele's levels at the test development stage; therefore, the epistemological obstacles that appeared had been coded at the van Hiele's level. The interview data were contrasted with the epistemological obstacles coded to meet the credibility and validity criteria (Darawsheh, 2014). Credibility or reliability determines the extent to which study results are consistent over time and authentic to represent in accordance with a total population under study. Then, validity criteria determine how truthful the result of the study (Darawsheh, 2014).

RESULTS

In this study, the epistemological obstacles in transformation geometry were analyzed in five levels of thinking (Abdullah & Zakaria, 2013; Clements & Sarama, 2011; Yilmaz & Koparan, 2016). The analysis was carried out based on the learning obstacles test results. In addition, it was adjusted to the van Hiele's levels, and interviews were conducted to confirm the participants' answers and thinking processes. An overview of the learning obstacle test results in transformation geometry is presented in Table 1.

Epistemological Obstacles at the Visualization Level

In the test, Level 1 (Visualization) is divided into two indicators: recognizing the types of transformations and visualizing (sketching/drawing) transformations in the Cartesian plane. This definition is based on Alex and Mammen (2016), and it includes the following:

1. "Figures are judged according to their appearance."
2. "When one has shown … he can produce those figures without error on a geoboard of Gattegno, even in difficult situations."

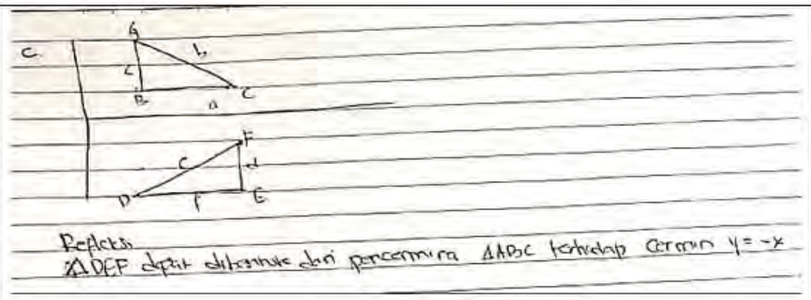
Likewise, Clements and Sarama (2011) argued that drawing geometric objects is essential before students reach non-visual aspects and aspects of the logical arrangement, conclusion, and proof. Moreover, in geometric transformation, students are required to dynamically imagine the transformation of two-dimensional images before proving logically (Moss et al., 2015). Therefore, the visualization indicators in this study were divided into two indicators of action (Figure 1).

Level 1 for indicators recognizing the types of transformations represented by question number 5a. In this indicator, all participants did not have difficulty recognizing the types of transformations that have been learned. However, participants had difficulty answering non-routine questions, such as recognizing the composition of transformations. In the interview session,

Table 1. Result of the learning obstacle test in transformation geometry

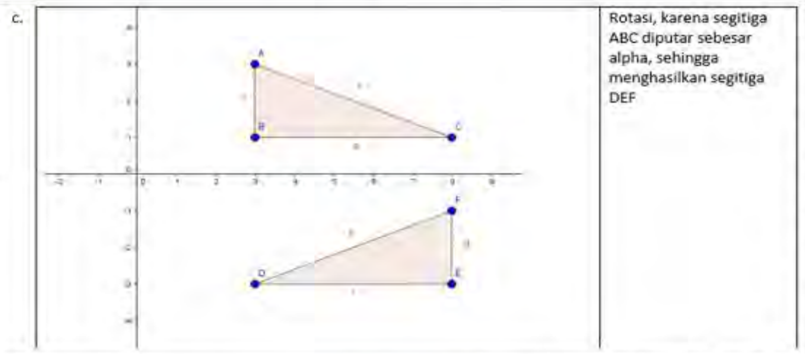
Subject	Level					% TO IDEAL	% TO MAX
	Lv. 0	Lv. 1	Lv. 2	Lv.3	Lv. 4		
Subject-1	50%	68%	66.50%	60.00%	28.00%	56%	75.50%
Subject-2	90%	80%	93.50%	72.00%	20.00%	73%	100.00%
Subject-3	65%	56%	20.00%	48.00%	48.00%	43%	57.40%
Subject-4	35%	32%	20.00%	20.00%	12.00%	22%	32.00%
Subject-5	45%	40%	7.50%	48.00%	0.00%	24%	32.30%
Subject-6	45%	76%	50.00%	40.00%	48.00%	46%	61.70%
Subject-7	35%	32%	20.00%	48.00%	12.00%	28%	39.60%
Subject-8	45%	68%	7.50%	20.00%	12.00%	24%	32.50%
Subject-9	35%	32%	13.50%	20.00%	0.00%	18%	26.70%
Subject-10	45%	44%	93.50%	40.00%	48.00%	60%	81.30%
Average	53%	52.80%	39.15%	41.60%	22.80%	39.40%	53.90%
Maximum	90.00%	80.00%	93.50%	72.00%	48.00%	73.00%	100.00%
Minimum	35%	32%	8%	20%	0%	18%	27%
Variance	0.0288	0.0366	0.1171	0.0310	0.0370	0.0361	0.0640

1)



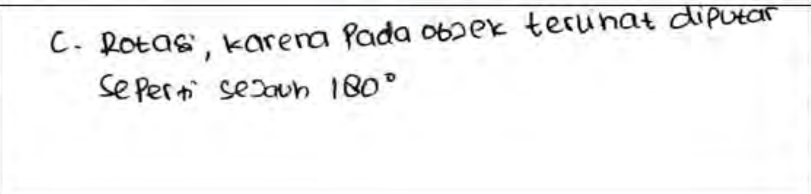
Answer: Reflection ΔDEF could be constructed from the reflection of ΔABC into line $y = -x$.

(2)



Answer: Rotation Because ΔABC rotated in the amount of alpha (degrees, $\alpha°$) resulting ΔDEF

(3)



Answer: Rotation Because the object appears to be rotated by 180°

Figure 1. Some of participants' answers in the visualization level

the participants admitted that they did not expect the problem to include the composition of transformations.

The second indicator for Level 1 was given in question 1a, which visualizes one transformation type through a picture (drawing). Participants were asked to draw the result of a reflection from a straight line to the line. In the process, participants had difficulty describing the reflection process. Some of the participants were not even able to draw a suitable straight line. Meanwhile, other participants were unable to draw a correct normal line. Participant difficulties are shown in Figure 2.

Even though it is only a sketch, the visualization process from reflection or other types of transformation involve several rules or procedures that must be carried out appropriately. Some of the procedures are creating a normal line and determining the proper distance to appear according to the reflection process. Thus, several stages are needed to obtain an image that visualizes the transformation process correctly. So, these stages have pictured the participant's difficulties, and it can be said to be epistemological.

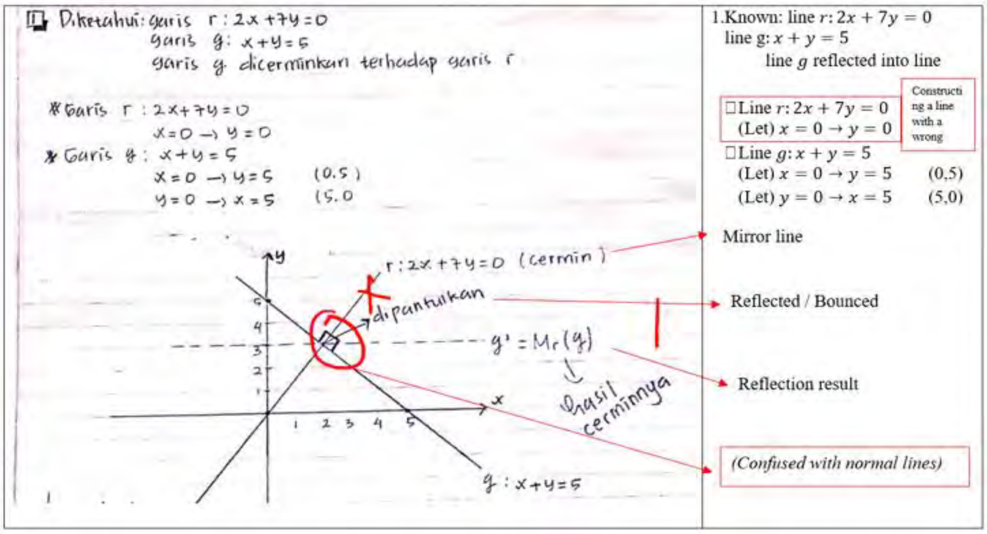


Figure 2. The epistemological obstacle at the visualization level

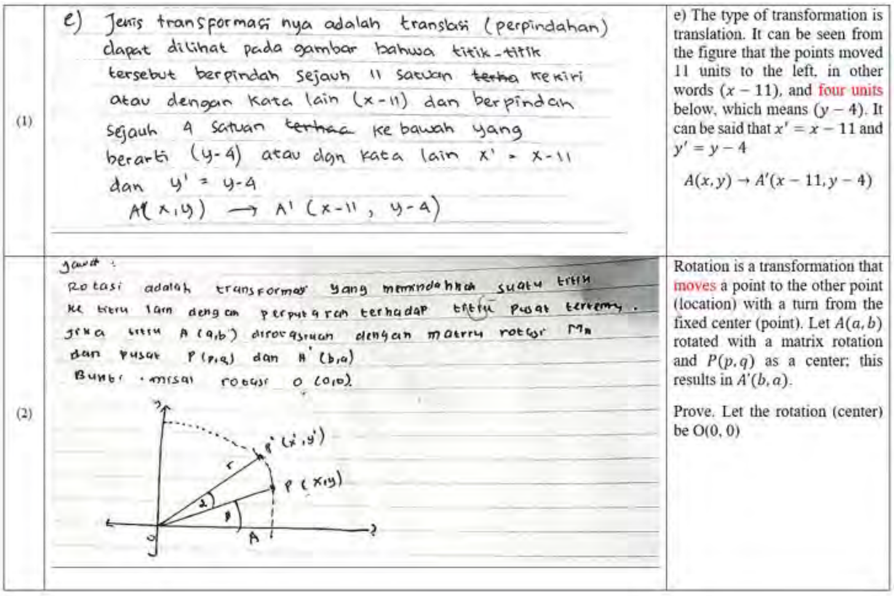


Figure 3. Participants' answers in the analysis level

Epistemological Obstacles at the Analysis Level

At the analysis level (Level 2), the way of geometric thinking by van Hiele is the appearance of participants' recognition and understanding the properties of transformations. Hence, geometric thinking is recognized and associated according to the characteristics of transformation (Abdullah & Zakaria, 2013; Clements & Sarama, 2011; Yilmaz & Koparan, 2016). However, in this research, only a few participants fulfilled the appropriate way of geometric thinking. Many of them try to solve problems for this level of analysis by doing deductions and calculations. Figure 3 (1) presents an answer from a participant who gave a detailed calculation about "What kind of transformation is the figure, and then provide features or properties to back up your response?" Instead of mentioning translation properties, the participant rather guessed and gave a detailed calculation.

In Figure 3 (2), a participant's answer gives a few unclear properties. The participant also has not shown his understanding of the order of transformations. Again, the participant tried to provide a detailed calculation, which was not required because the question is "What is a rotation? How could it be constructed from a combination of transformations?"

Such a way of geometric thinking is not entirely wrong, especially if there are no miscalculations. However, with such a way of geometric thinking, participants will face various complexities that involve computation and may be related to higher levels, thus giving rise to certain epistemological obstacles.

Epistemological Obstacles in the Abstraction Level

The epistemological obstacles findings at Level 1 are also related to Level 3 (Abstraction), where question number 1b mentions the procedure for drawing the reflection process. As a result, the participants had difficulty mentioning the procedure. Figure 4 presents an answer from a participant who finished the calculation. As we can see in Figure 4, the reflection procedure is complex and involves various prerequisite knowledge. Despite spending his calculations, the participant in Figure 4 reached a wrong conclusion. Due to their complexity, most participants stopped doing calculations because they felt something is odd or feel it is unnecessary to continue with the calculation.

The participant in Figure 4 completed the task with correct procedures. But, unfortunately, he made several miscalculations that resulted in the wrong answer. Table 2 is an excerpt from the participant in Figure 4.

Table 2. Participant's view on the procedure that is prone to errors

Researcher	Participant
I see that you left your answer on reflection and its procedures problem.	Yes, Sir. I was trying my best.
Okay, lets take a look at your final sketch. Could you tell me, where is the line and its images?	[Pointing with his finger] This is the line, it is reflected to this mirror (line) resulting … this line … [a bit unsure] but I'm not really sure about my answer.
How could it be?	Personally, I feel odd and not really sure with my calculations.
Which steps are you unsure of?	I am not sure; I think the procedure is too long, making it prone to errors. There must be some mistakes, somewhere.
What is odd in your answer?	The images, it does not look like it was reflected. I had considered to skip the problem, but it looks like the other problems will not be easy either, and there are the images I got.

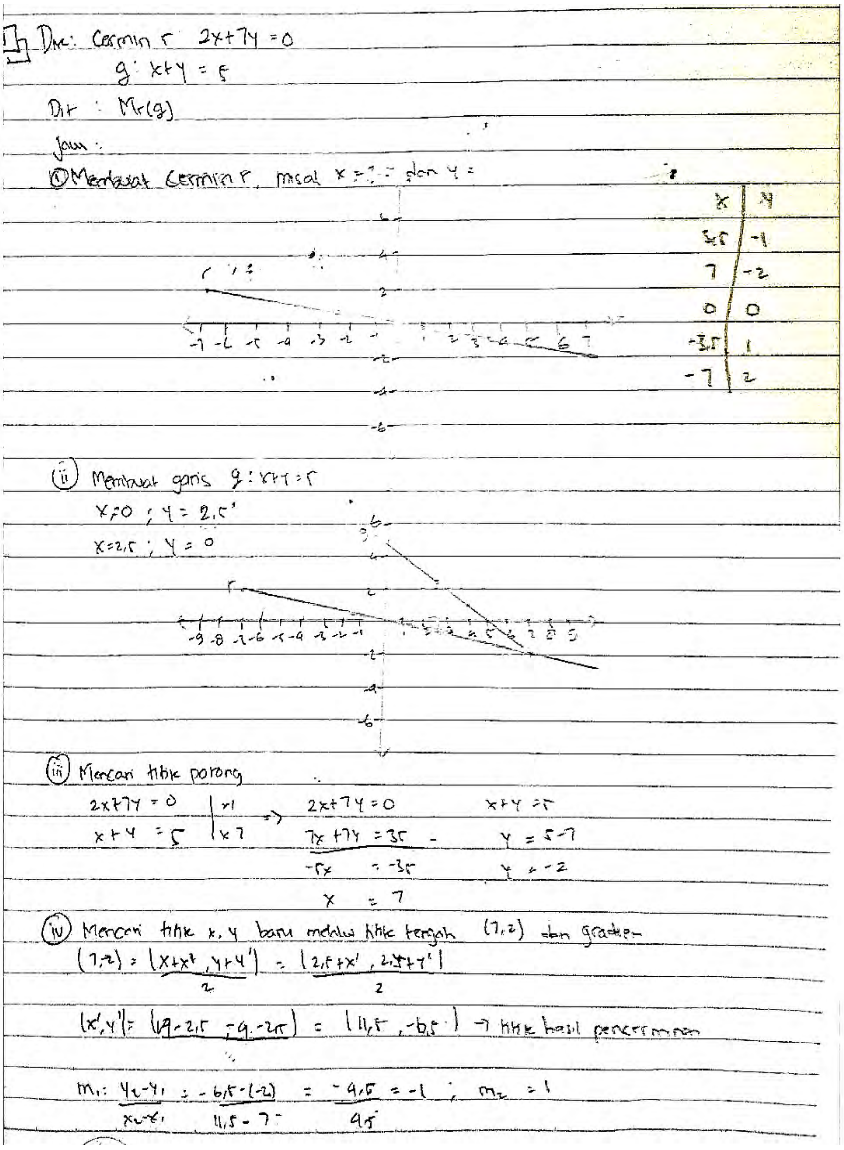


Figure 4. Participants' answers reflect a line and its procedures

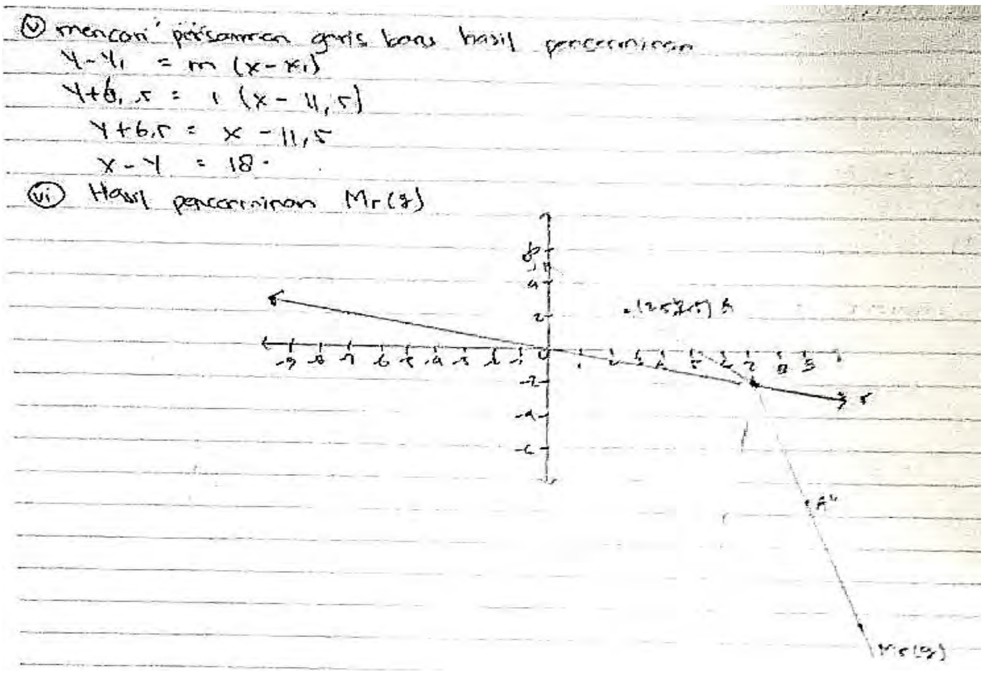
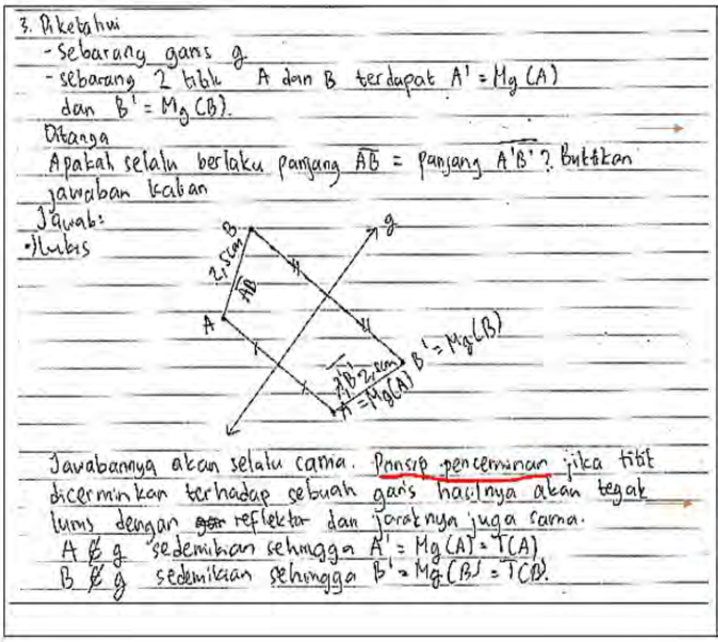


Figure 4 (continued). Participants' answers reflect a line and its procedures



English Translation

3. Known:
- any line *g*
- for any two points A and B, exist A' = M_g(A) and B' = M_g(B)

Question:
Is length of $\overline{AB}$ = length of $\overline{A'B'}$? Prove.

The answer, they will always be equal. (Because of) reflection principle (definition), a reflection of a point into a straight line yields an image perpendicular to the straight line (reflector) and **the length would also be equal.**
A ∈ *g* such that A' = M_g(A) = T(A)
B ∈ *g* such that B' = M_g(B) = T(B)

Figure 5. Answer from one of the participants who argued that the isometric theorem was already clear (proven) confirmed by the existence of the principle (definition) of reflection

Problem number 2 for the level of abstraction asks participants to argue about the rotation, which is the composition of reflection. Participants also have difficulty providing appropriate arguments at this level. The problem in question number 2 is suitable for participants who have difficulty understanding the reflection procedure. Other types of transformations, such as translation, rotation, and dilation, also require similar procedures. Procedures that are too complex make it difficult for participants to understand relationships between transformations. The transformation procedure that is quite complex and involves various prerequisite knowledge can be categorized as an epistemological obstacle at Level 3 (Abstraction).

Epistemological Obstacles in the Deduction Level

Difficulty also appeared at Level 4 (Deduction). In contrast to other levels, at this level of deduction, the participants made fewer deductions. Many participants used informal arguments, examples, generalizations, and conclude pictures. As shown in Figure 5, one participant argued that the length of a line segment $\overline{A'B'}$.

Table 3. Interview excerpt that shows the participant's belief about definition of reflection

Researcher	Participant
Do you still remember your sketches in proving problem number 3?	Yes. I was trying to sketch possibilities of a line segment $\overline{AB}$. There maybe other possibilities, but I thought those were enough to represent and prove the theorem.
Well, regardless the possibilities, they were clean and good sketches. Could you tell me, what is your idea to prove the statement with the sketches?	It can be seen that the distance between point B to line g is equal to the distance between line g and point B'. Also the distance between point A and line g is equal to the distance between line g and point A'. This case is guaranteed by the definition of reflection. Thus, the length of $\overline{AB}$ would always be equal to the length of $\overline{A'B'}$.
[Showing the definition of reflection] Here, the definition is only guarantee that the distance between any point (Point B) to a line (line g) is equal to the distance between the line (g) to the resulting image (point B'). So, the definition does not say anything about the length of $\overline{AB}$ and $\overline{A'B'}$.	Umm… [Thinking] I remember that reflection is preserve length and distance.
Okay, could you explain the proof for me?	[Smiles] No. I give up. I cannot give the proof, Sir.

The result of the reflection of a line segment $\overline{AB}$, would be equal to line segment $\overline{A'B'}$ because the distance of point A to line g is equal to the distance of point A' to line g. The arguments presented by participants in Figure 5 are incomplete, and the participant probably misinterpreted or sidetracked from the question to their understanding of the definition of reflection. In Figure 5, the participant wanted to prove the expression by using the rule of congruence. Segment $\overline{AA'}$ will be the perpendicular bisector the rule of congruence, resulting in the length of the segment $\overline{A'B'}$ = length of the segment $\overline{AB}$. The participant's way of geometric thinking can be said to be correct. However, the participant's answer began to sidetrack probably because of the perpendicular bisector's sign and speculation about the consequence of the reflection definition. Instead of using the congruence rules to finish their good set up of the way of geometric thinking and drawing, the participants argued that it was evident that the length of $\overline{A'B'}$ = length of $\overline{AB}$ from the reflection definition. That argument was presumptuous and without some rationale to explain "why was the length of $\overline{A'B'}$ = length of $\overline{AB}$ when $d(A, g) = d(A', g)$ and $d(B, g) = d(B', g)$?."

These findings were then confirmed through an interview session. Based on this question, the participant had difficulty expressing rules, theorems, or logical thinking as the basis for his answers. Table 3 presents an excerpt of the interview with the participant.

In accordance with the previous study, the participant insists that the length of a segment $\overline{A'B'}$ as the length of the segment $\overline{AB}$ as a result of the definition of reflection, which states that the distance of point A to the mirror equals the distance of point A' to the mirror. This answer indicates a cognitive bias in the participant's way of geometric thinking. Cognitive bias is a type of False Consensus Effect, where the information or knowledge obtained is considered clear and generally understood (Dwyer, 2018; Rønning, 2021).

The emergence of this cognitive bias comes from within a person. However, considering the order of reflection and isometry definitions, there is a close relationship between them. Isometry is a type of transformation with several criteria, and reflection is one type of transformation that meets the isometric criteria. This association makes the isometric criterion as if it is a particular property of reflection, in other words, as if isometry were the consequence of reflection. This finding emerges in Figure 5, where participants though that their answers are evident because of the implications of the definition of reflection. Therefore, epistemological obstacles were found as the emergence of biases in transformation geometry.

Epistemological Obstacles in the Rigor Level

Level 5 (Rigor) is the most challenging stage to achieve. In Table 1, it appears that no one has been able to solve this problem well. Besides, level 5 has the lowest average score of all the learning obstacle questions. In this study, question number 4 asked participants to find a translational vector with a reflection composition. Even though the participants had received a lecture on transformation geometry, the translation content formed from the design of two reflections and their translation vectors fulfills the question criteria for level 5. The content of question number 4 involves various aspects, such as visual (sketch/drawing), analytic (function and sketch), deduction (formula), and abstraction (procedure).

Based on the participants' answers, all participants could not visualize the problem in question number 4. As shown in Figure 6, the participants did not provide a visualization for the problem. Although the visualization in question number 4 does not guarantee

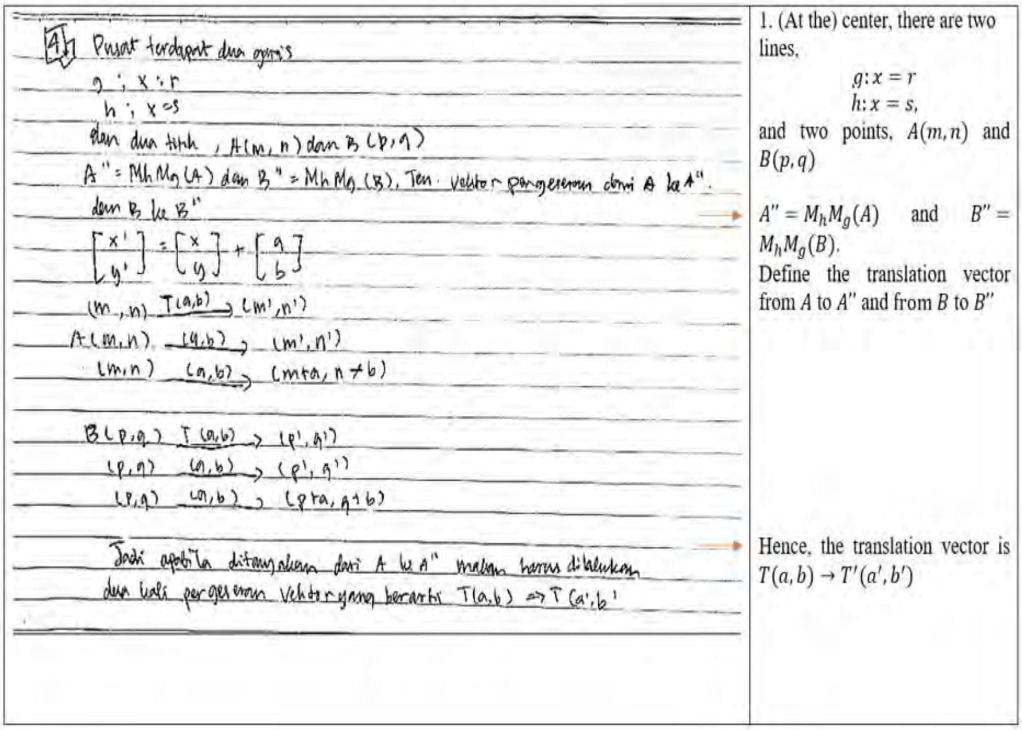


Figure 6. The answers of participants who try to find translation vectors through the translation formula

the correctness of the answer's results, it can provide a short description and understanding of the problem. Consider Figure 7, another participant's answer who "discovered" the translation vector through visualization. Unfortunately, the participants were unable to relate their visualization to their analytical skills.

In Figure 6 and Figure 7, both participants answered question number 4 using a translation formula, not the composition of reflection. In contrast, in the matter, it was clear that they were asked to do two reflections. Participants argued that the translation vector could be obtained by reversing the translation formula without the need to do a reflection composition. This way of geometric thinking has a point, though question number 4 does not meet the sufficient and necessary conditions to find a translation vector through the translation formula.

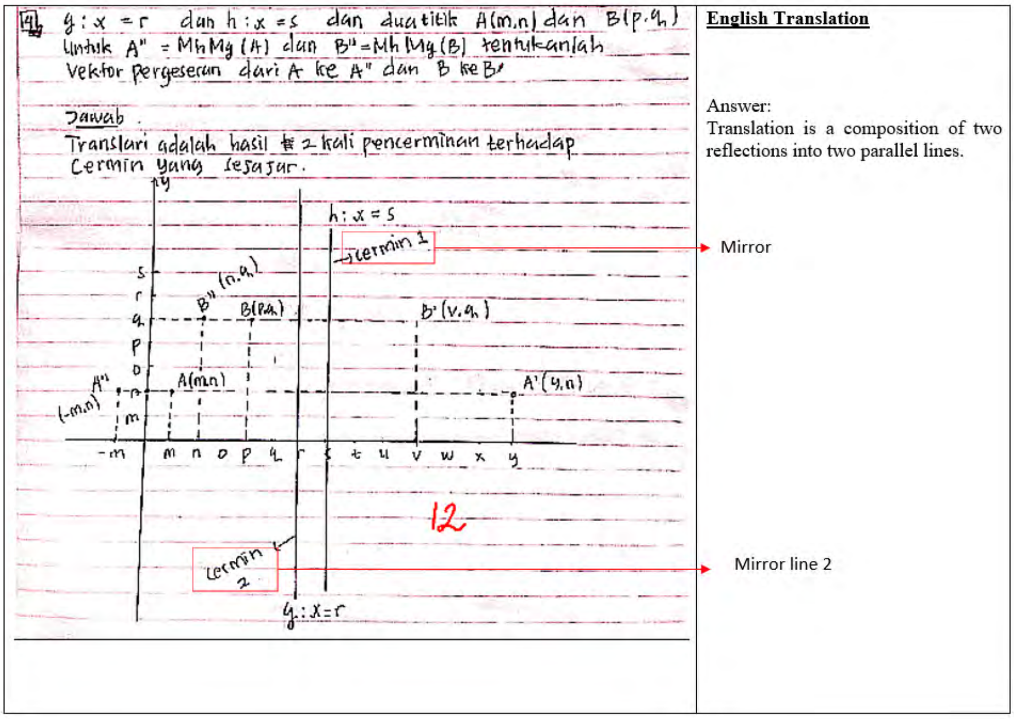


Figure 7. The answer of a participant who tried to find a translation vector through a translation formula

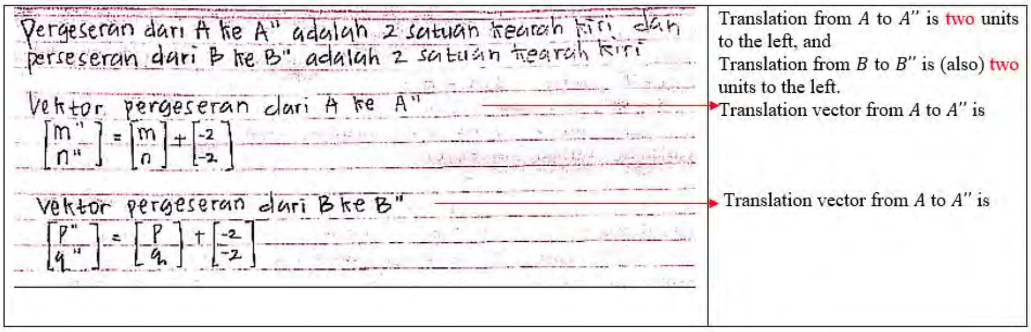


Figure 7 (continued). The answer of a participant who tried to find a translation vector through a translation formula

Table 4. Epistemological obstacles findings based on van Hiele's level of geometric thinking

Van Hiele's Level of Geometric Thinking	Epistemological Obstacles
Level 1 (Visualization)	It takes several steps so that the resulting image can visualize the transformation process properly.
Level 2 (Analysis)	The properties and types of transformations tend to be recognized by the inclusion of calculations.
Level 3 (Abstraction)	The transformation procedure is quite complex and involves a variety of prerequisite knowledge.
Level 4 (Deduction)	There is a bias between definitions and theorems because some types of transformations are derived from other transformations.
Level 5 (Rigor)	The combination of systems in transformation geometry with axiomatic thought processes on content construction is still not solid.

DISCUSSION

Based on the findings described, the process of acquiring knowledge for the topic of transformation geometry in van Hiele's which level of geometry thinking has its characteristics adjusted to five levels of thinking (Abdullah & Zakaria, 2013; Clements & Sarama, 2011; Yilmaz & Koparan, 2016). Each of the five levels has epistemological obstacles that should be overcome to acquire knowledge and reach the next level. The epistemological obstacles that have been found and described previously are presented in Table 4.

A learning obstacle whose source is epistemological is not something that can or should not be avoided (Trouche, 2016). An epistemological obstacle is a way of knowing, which helps acquire knowledge under certain conditions, although it can lead to errors (Artigue, et al., 2014; Trouche, 2016). The way of knowing, namely acquiring knowledge, is a side effect of epistemological obstacles (Sleep, 2012; Turk & Arslan, 2012). However, this does not mean that we can deliberately focus on the impact of these obstacles. The characteristic variations of one's conceptual understanding, from the point of view of a way of geometric thinking and understanding, can create an incompatible way of knowing (Abdullah & Zakaria, 2013; Armah & Kissi, 2019). Regularities and patterns argued that abstraction impedes local understanding situations is an unproductive claim. It excludes theory and existing knowledge (crucial apparatuses) from the heuristic toolbox (Albert et al., 2018). Both in the phases of recognizing, building-with, and constructing, epistemological difficulties might occur. False intuition in determining prior knowledge might lead to epistemological problems. To respond to the question or make a less straightforward generalization (Subroto & Suryadi, 2018; Mol & Primiero, 2014). This study also related to Hazzan's descriptive framework according to abstraction level reduction. Reducing the amount of abstraction is a coping mental activity with abstraction. Students may approach a task on a lower level of abstraction than the one intended by the teacher or the assignment itself in an attempt to deal with a circumstance. Hazzan's descriptive framework employs many abstraction notions and elaborates on them in three ways: 1) The quality of the relationships between the object of thought and the thinking person is defined as the abstraction level. 2) The amount of abstraction as a reflection of the process–object duality. 3) The abstraction level is defined as the mathematical concept's degree of complexity (Wijeratne & Zazkis, 2015).

The limitation of this research is only related to the qualitative case study. It means that the results of the epistemological obstacles that arise are limited to our participants. However, this study provided an idea for epistemological obstacles with more participants. The interrelationships between van Hiele's levels are not the research question in this article. Therefore, in this study, we do not perform any statistical tests that can show how much ability at one level can affect the next level.

This study supports the knowledge which has been previously investigated. This study demonstrates the epistemological impediment based on the van Hiele thinking model cannot be overcome without a good design or learning path. van Hiele's level of thinking causes it should proceed sequentially.

CONCLUSIONS

The level visualization is the principal capital of participants in understanding the concept of transformation geometry. Making visualization sketches with the help of algebra-analytic is not at all easy. It complicates the visualization of transformation geometry concepts. Sometimes a transformation geometry concept requires visually equalizing perceptions between teachers/lecturers and students/students. As long as visualization is only mental action, this perception equation is challenging to occur. Therefore, it is necessary to convey the concept of geometry visually appropriately. The analysis level cannot be separated from the visual level because various essential geometric characteristics or properties can only be identified through visuals. The level of abstraction (informal reduction) is critical in achieving the highest level of understanding, namely accuracy. In this study, the complexity of the procedure at the abstraction level prevented participants from fully understanding the concept. A structured learning design and the help of certain media are needed so the students can overcome obstacles at the level of abstraction. In this study, the participants' weak deduction and rigor levels resulted from inadequate previous levels. It is hoped that overcoming the various epistemological barriers of the earlier levels can result in reduced levels and better rigidity.

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REFERENCES

Abdullah, A. H., & Zakaria, E. (2013). The effect of van Hiele's phases of learning geometry on students' degree of acquisition of van Hiele levels. *Procedia-Social and Behavioral Sciences, 102*, 251-266. https://doi.org/10.1016/j.sbspro.2013.10.740

Abu, M. S., & Abidin, Z. Z. (2012). Improving the levels of geometric thinking of secondary school students using geometry learning video based on van Hiele theory. *International Journal of Evaluation and Research in Education, 2*(1), 16-22. https://doi.org/10.11591/ijere.v2i2.1935

Albert, M., Mylopoulos, M., & Laberge, S. (2018). Examining grounded theory through the lens of rationalist epistemology. *Refllections, 24*, 827-837. https://doi.org/10.1007/s10459-018-9849-7

Alex, J. K., & Mammen, K. J. (2016). Lessons learnt from employing van Hiele theory based instruction in senior secondary school geometry classrooms. *Eurasia Journal of Mathematics, Science & Technology Education, 12*(8), 2223-2236. https://doi.org/10.12973/eurasia.2016.1228a

Armah, R. B., & Kissi, P. S. (2019). Use of the van Hiele theory in investigating teaching strategies used by college of education geometry tutors. *EURASIA Journal of Mathematics, Science and Technology Education, 15*(4), em1694. https://doi.org/10.29333/ejmste/103562

Artigue, M., Haspekian, M., & Corblin-Lenfant, A. (2014). Introduction to the theory of didactical situations (TDS). In: A. Bikner-Ahsbahs, & S. Prediger (Eds.), *Networking of theories as a research practice in mathematics education. Advances in mathematics education*. Springer, Cham. https://doi.org/10.1007/978-3-319-05389-9_4

Clements, D. H., & Sarama, J. (2011). Early childhood teacher education: The case of geometry. *Journal of Mathematics Teacher Education, 14*, 133-148. https://doi.org/10.1007/s10857-011-9173-0

Darawsheh, W. (2014). Reflexivity in research: Promoting rigour, reliability and validity in qualitative research. *International Journal of Therapy and Rehabilitation, 21*(12), 560-568. https://doi.org/10.12968/ijtr.2014.21.12.560

Dwyer, C. (2018). 12 common biases that affect how we make everyday decisions. *Psychology Today*. [Online]. https://ssh.expoplanner.com/files/12/SpeakerFiles/27241_Handout_12_Common_Biases_That_Affect_How_We_Make_Everyday_Decisions.pdf

Hegedus, S. J., & Moreno-Armella, L. (2011). The emergence of mathematical structures. *Educational Studies in Mathematics, 77*, 369-388. https://doi.org/10.1007/s10649-010-9297-7

Karakuş, F., & Peker, M. (2015). The effects of dynamic geometry software and physical manipulatives on pre- service primary teachers' van Hiele levels and spatial abilities. *Turkish Journal of Computer and Mathematics Education, 6*(3), 338-365. https://doi.org/10.16949/turcomat.31338

Kutluca, T. (2013). The effect of geometry instruction with dynamic geometry software; GeoGebra on van Hiele geometry understanding levels of students. *Educational Research and Reviews, 8*(17), 1509-1518. https://eric.ed.gov/?id=EJ1017439

Mol, L. D., & Primiero, G. (2014). Facing computing as technique: Towards a history and philosophy of computing. *Philosophy & Technology, 27*, 321-326. https://doi.org/10.1007/s13347-014-0169-4

Moss, J., Hawes, Z., Naqvi, S., & Caswell, B. (2015). Adapting Japanese lesson study to enhance the teaching and learning of geometry and spatial reasoning in early years classrooms: A case study. *ZDM Mathematics Education, 47*, 377-390. https://doi.org/10.1007/s11858-015-0679-2

Noto, M. S., Priatna, N., & Dahlan, J. A. (2019). Mathematical proof: The learning obstacles of pre-service mathematics teachers on transformation geometry. *Journal on Mathematics Education*, *10*(1), 117-126. https://doi.org/10.22342/jme.10.1.5379.117-126

Pelligrini, E. F., Nesso, A. C., Zaccaro, A. M. S., Berenguel, D. C. C., Campos, E. J., & Rodrigues, L. S. A. (2020). Between the frontier of learning and school failure in attention deficit hyperactivity disorder. *International Journal of Humanities Social Sciences and Education, 7*(12), 1-14. https://doi.org/10.20431/2349-0381.0712001

Phillips-Pula, L., Strunk, J., & Pickler, R. H. (2011). Understanding phenomenological approaches to data analysis. *Journal of Pediatric Health Care, 25*(1), 76-71.https://doi.org/10.1016/j.pedhc.2010.09.004

Rivas, R., & Gibson-Light, M. (2016). Exploring culture through in-depth interviews: Is it useful to ask people about what they think, mean, and do? *Cinta de Moebio, 57*, 316-329. https://doi.org/10.4067/S0717-554X2016000300007

Rønning, F. (2021). Opportunities for language enhancement in a learning environment designed on the basis of the theory of didactical situations. *ZDM Mathematics Education 53*, 305-316. https://doi.org/10.1007/s11858-020-01199-x

Saldana, J. (2016). *The coding manual for qualitative researchers*. SAGE. https://emotrab.ufba.br/wp-content/uploads/2020/09/Saldana-2013-TheCodingManualforQualitativeResearchers.pdf

Sleep, L. (2012). The work of steering instruction toward the mathematical point: A decomposition of teaching practice. *American Educational Research Journal, 49*(5), 935-970. https://doi.org/10.3102/0002831212448095

Subroto, T., & Suryadi, D. (2018). Epistemological obstacles in mathematical abstraction on abstract algebra. *IOP Journal of Physics: Conference Series, 1132*, 1-6. https://doi.org/10.1088/1742-6596/1132/1/012032

Sunariah, L. & Mulyana, E. (2019). The didactical and epistemological obstacles on the topic of geometry transformation. *IOP Journal of Physics: Conference Series*, *1521*, 032089. https://doi.org/10.1088/1742-6596/1521/3/032089

Trouche, L. (2016). Didactics of mathematics: Concepts, roots, interactions and dynamics from France. In *Tools and mathematics: Instruments for learning*. Springer, Cham. https://doi.org/10.1007/978-3-319-02396-0_10

Turk, Y., & Arslan, S. (2012). Examining creative drama based mathematics course with respect to theory of didactical situations in mathematics. *Procedia-Social and Behavioral Sciences, 46*, 4782-4785. https://doi.org/10.1016/j.sbspro.2012.06.334

Wijeratne, C., & Zazkis, R. (2015). On painter's paradox: Contextual and mathematical approaches to infinity. *International Journal of Research in Undergraduate Mathematics Education, 1*, 163-186. https://doi.org/10.1007/s40753-015-0004-z

Yilmaz, G. K., & Koparan, T. (2016). The effect of designed geometry teaching lesson to the candidate teachers' van Hiele geometric thinking level. *Journal of Education and Training Studies, 4*(1), 129-141. https://doi.org/10.11114/jets.v4i1.1067

Zhang, Q.-P., & Hong, N. Y. (2021). The learning trajectories of similarity in mathematics curriculum: An epistemological analysis of Hong Kong secondary mathematics textbooks in the past half century. *Mathematics, 9*, 2310. https://doi.org/10.3390/math9182310

Zorin, B. (2011). *Geometric transformations in middle school mathematics textbooks* [Graduate theses and dissertations, The University of South Florida].

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