Addressing Agency and Achievement in a Multiplication Intervention

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Understanding increased achievement as a dynamic relationship between student engagement and mathematical understanding, we designed an intervention to address both factors. Our intervention was the instructional routine number strings, led by an undergraduate tutor. In this paper, we provide a case study of the development of a fourth-grade student (Inez) who participated in our number strings intervention. Despite a complex set of initial partial understandings about multiplication and arrays, Inez demonstrated growth both in her conceptual understanding of multiplication and engagement in mathematical discussion. Shifts in engagement were related to the tutor providing physical models of arrays and the tutor checking in frequently with the student. Shifts in conceptual understanding were prompted by her scaffolded engagement in mathematical discourse in which she was asked to reflect on her mathematical understanding of the spatial structuring of arrays.

Keywords: **Equity and Diversity, Mathematics Intervention, Number Concepts and Operations, Instructional Activities and Practices, Engagement, Mathematical Discussion**

Introduction

A perennial problem for schools and teachers is addressing student learning for students who are achieving significantly below grade level in mathematics, a problem exacerbated by the loss of learning opportunities during COVID (Lambert & Schuck, 2021). The approach to intervention in mathematics in the special education literature focuses on remediation through direct or explicit instruction (e.g., Gersten et al., 2009). Boyd & Bargerhuff (2009) critiqued the use of this approach for a lack of integration with standards-based instruction that focuses on student-centered problem solving rather than teacher-directed instruction (Common Core State Standards Mathematics (CCSS-M). In addition, a one-size fits all intervention such as a scripted direct instruction lesson may not be appropriate for students who need individualized assessment and intervention. Such approaches are not designed to deepen student engagement

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in sense-making or develop student strategies necessary to shift students towards a more productive disposition in mathematics (Lambert, 2018). Intervention should address both agency and achievement.

Building from a tradition of constructivist learning experiences, mathematics and special education scholars have proposed interventions designed around student thinking (Hord et al., 2016; Hunt & Ainslie, 2021; Hunt & Tzur, 2017). These approaches to teaching build on long-established learning progressions and are designed to be sensitive to individual differences. We concur that these types of interventions can address partial understandings, support student engagement and agency, and allow for individualized support which can be necessary for supporting students in unfinished learning. However, we situate our intervention in a small group, rather than a one-on-one intervention. We do so for pragmatic reasons, as schools can rarely offer one-on-one instruction. In addition, we believe that small group intervention can offer students the strategic models of their peers and an opportunity to engage in mathematical community. Finally, our model of intervention addresses engagement in mathematical practices along with mathematical ideas (Lambert & Sugita, 2016).

This paper presents a case study of a student with a complex set of prior understandings about multiplication. The student, Inez, was a participant in a mathematics intervention designed around small groups engaging in multiplication number strings. Number strings are designed to engage students in mathematical problem-solving and discussion, building both student number sense and participation in the Standards for Mathematical Practice, as well as computational fluency grounded in mathematical models (Lambert et al., 2017). We became interested in Inez as a case study as her tutor, an undergraduate, became concerned that Inez was not participating in mathematical discussion. As the eight teaching sessions progressed, the tutor successfully engaged Inez in mathematical discussion, however, the tutor was challenged by Inez's complex partial understandings. We aim to reframe the narrative that suggests students with lower achievement in math with learning disabilities are less engaged in mathematical discussion than nondisabled peers (Baxter et al., 2001) by promoting agency in our intervention.

Our research questions are:

1. For a student with limited engagement in mathematics, can a small group number strings intervention support increased participation in mathematical sense-making and talk? How do shifts in participation develop?
2. For the same student with complex partial understandings of multiplication, can a small group number strings intervention support conceptual and strategic development in multiplication? How do these shifts occur?

Literature Review

Strengths-based Lens

We take a strengths-based approach to addressing intervention for all students. We see disability through a disability studies lens, moving beyond medical or deficit conceptions of learners to understand disability as both socially constructed and embodied (Siebers, 2008). We use neurodiversity as a lens to understand cognitive disabilities, such as learning disabilities, as a set of both strengths and challenges (Lambert & Harriss, 2022). We understand multilingual learners not from a deficit lens but a strengths-based lens through which multilingualism is a strength and benefit (Moschkovich, 2002). We acknowledge that the students in this study are also positioned as Latinx students in a highly racialized society, which can position students in need of help, and as less capable than other students (Ochoa, 2013). We reject these deficit notions. We draw from work in education that positions students as at-promise rather than at-risk (Mireles-Rios et al., 2020). This strengths-based positioning is central to the way we not only conceptualize learners, but how we design instruction for them. A recent position paper from NCSM & TODOS entitled *Positioning Multilingual Learners for Success in Mathematics* (2021) described an asset-based view of multilingual learners within which multilingual learners come with their own resources and understandings. The position paper conceptualized the instruction and systems around these students as needing change rather than fixing the students as individuals. We conceptualize common practices in intervention, often inflexible and scripted, as systems that need to be reformed.

Mathematical development is complex, and not uniform. All students make their own way through mathematical topics such as multiplication, learning as they use and revise strategies (Carpenter et al., 2015; Fosnot & Dolk, 2001). CCSS-M describe developmental trajectories in which concepts like multiplication develop first through open-ended problem-solving. Multiplicative thinking is a cognitive shift away from thinking about ones to thinking about repeated groups (Finesilver, 2017). Fundamental to this shift is unitizing (Fosnot & Dolk, 2001), or the ability to see ten ones as one group of ten. Children begin multiplicative tasks through counting by ones or direct modeling (Carpenter et al., 2015). As they begin to unitize, children use skip counting or repeated addition to find a total for a multiplication problem or counting strategies. With practice, children often shift towards the more abstract strategy of derived facts, in which they may break a problem into partial products to solve.

To understand the complexity of student development, scholars have proposed the idea of partial understandings (Johnson et al., 2019), or instances in which a learner demonstrates understanding of a mathematical principle, while the understanding may not be fully developed. Partial understandings honor that there is sense-making in errors and that understanding complex mathematical ideas takes time and multiple experiences.

Throughout this process, mathematical models such as arrays, number lines, and ratio tables support (Fosnot & Dolk, 2001). Arrays are particularly important in early multiplication because they can be used to model all levels of strategies, from counting to skip counting to the formula for area. Children do not see arrays initially as a set of rows and columns expressed as *a* x *b*; instead, they often initially count arrays as one-dimensional paths (Battista et al., 1998). They must construct ideas such as "row-as-composite," necessary to understand skip counting using an array. Battista and colleagues see this not as a passive conceptual reorganization, but as a process through which children take actions (their physical counting, gestures to indicate structure, etc.), and through these actions and then their reflections on them, the children develop understandings of the spatial structure of the 2D array.

Mathematics Intervention

Schools in the US are being asked to provide intervention within the Multi-Tiered System of Support (MTSS) in mathematics. However, interventions are often not aligned with classroom instruction based on Common Core State Standards (CCSS-M), creating difficulties for students who must make sense of different approaches to mathematics (Boyd & Bargerhuff, 2009). In addition, one significant difference between recommended intervention practices in math education is explicit or direct instruction versus inquiry-based instruction (Lambert, 2018).

Explicit instruction is not skill and drill, rather a set of practices that include clearly defined goals, well-paced instruction with opportunities for student interaction, feedback, and practice (Gersten et al., 2009). These elements of explicit instruction are quite useful for a wide variety of learners and can be integrated into inquiry-based instruction. However, one aspect of explicit instruction that is incommensurate with current research in mathematics education is that students should be told how to solve mathematics problems *before* they are allowed to think for themselves, particularly as they begin to learn about a new concept or develop a new strategy. Students with disabilities are intellectually capable of thinking mathematically, and their mathematics instruction should always be built on this fact. Otherwise, they are offered a less rigorous curriculum, one with fewer opportunities to develop their own agency and identity as mathematical thinkers (Lambert, 2018).

Student-centered Strategic Instruction

In their book, *Designing Effective Math Interventions*, Hunt and Ainslie (2021) described intervention in mathematics as individualized and strengths-based, driven by the learner's understanding. They argue for students as active constructors of meaning, rather than passive recipients of adults' strategies. They write,

> All learners— including learners whom we describe as struggling with mathematics— use what they already know to understand and make sense of new ideas. It is the use of this existing knowledge that can pave the way towards new understandings (p. 37).

Hunt and Ainslie caution that interventions that are not designed around student thinking are based on the thinking of adults, which may not make sense to the learners. They argue that when instruction is not tailored to the way students currently think about mathematical concepts, misconceptions or partial conceptions arise. When students are encouraged to follow rules that they do not understand, they can develop significant issues with the mathematical content and are encouraged to believe that mathematics does not make sense.

Understanding how strategy change occurs is particularly important for students with difficulties in mathematics, who tend to use inefficient strategies longer than nondisabled peers (Geary, 1990). Siegler's (1998) Overlapping Waves theory provides a way to understand both the tremendous variability across student strategies as well as the process through which change in strategies occurs. Students are unlikely to shift strategies until problems become cumbersome using original strategies. Students may use unitary counting, for example, until given multiplication problems that become time consuming to count (Zhang et al., 2013). Zhang and colleagues (2013) drew on Siegler's Overlapping Waves theory to design individualized intervention for 3 students with mathematical disabilities. They note that instruction should first offer students the opportunity to use their own strategies. If students do not have a strategy, the teacher can model one. Agency is offered, but support is given if a student is stuck. This may be a problem when doing intervention one-on-one, in which students do not have access to other students' strategies. Most of this research on intervention is one-on-one, meaning that if the student is stuck, there is no opportunity for student sharing of strategies; thus, the teacher steps in.

Our approach to intervention integrates constructivism with sociocultural theories of learning. In the latter, learning occurs through increased participation in communities of practice, as learners become increasingly adept at the practices of the community (Lave & Wenger, 1991). The role of the group, including group norms, is invaluable in how participation and learning develops. Partially, that is through the sharing and uptake of mathematical strategies. There is evidence that students' achievement increases in correlation to how much they engage in the strategies of other students and how often others engage in their strategies during discussion (Ing et al., 2015). Some studies have suggested that students with lower achievement in mathematics and learning disabilities may be less engaged in problem-solving and mathematical discussion than nondisabled peers, suggesting that engagement may be critical for learning for these students (Baxter et al., 2001; Bottge et al., 2002).

In addition to the benefits of learning from and with peers, we see practical benefits for the use of small groups. School districts are far more likely to fund interventions that use small groups rather than one-on-one support. This requires research in the setting implemented, to better understand the additional complications of facilitating small group work in mathematics.

Our Intervention: Number Strings

A number string is a short (15–20-minute) daily instructional routine in which a teacher presents a carefully designed sequence of problems one at a time for children to solve mentally to a group of students (can be whole class or small group) (Lambert et al., 2017). Figure 1 is an image of a chart paper after a number string in the small group studied in this paper, in which the problems were: 3 x 10, 3 x 9, 10 x 8, 9 x 8, and 9 x 7. This string was designed by the tutor so that students could develop the strategy of partial products using the distributive property. As students shared their strategies for each problem, the tutor represented student thinking using arrays and equations.

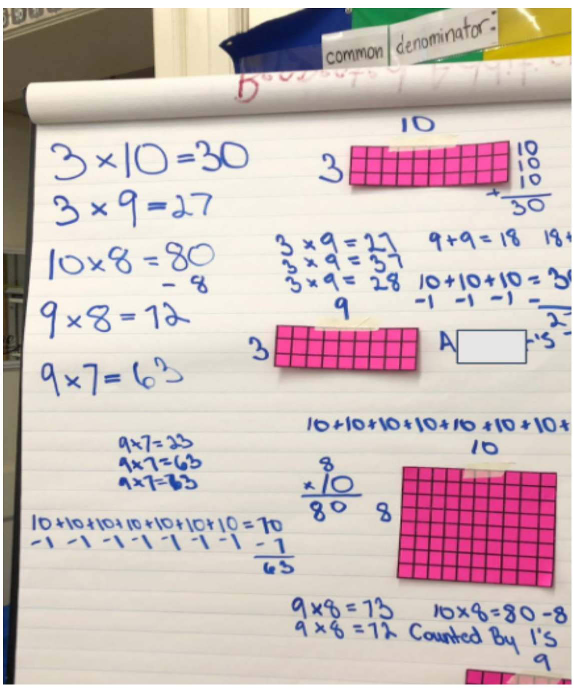


Figure 1. **Chart Paper at the End of a Number String**

Instead of interventions that focus on direct instruction, number strings provide opportunities for students to engage in mathematical discourse, both in describing their strategies and connecting with the mathematical strategies of others. Number strings provide structured opportunities to discuss and develop strategies, as the sequence of problems is specifically designed to elicit particular strategies. Strategic development is closely aligned with conceptual understanding, focusing on models and students explaining why their strategies work. All strategies are shared by the students and represented by the teacher.

Research on number strings has found that students participating in number string routines adopt new strategies and make connections between conceptual understanding and procedures (O'Loughlin, 2007). In addition, number strings support students in making connections between conceptual understanding and procedures (Callandro, 2000). Murata et al. (2017), in a qualitative study comparing student strategy development in two first-grade classrooms using number strings, found that the classroom in which students developed new strategies (almost all students were improvers rather than non-improvers), included a wider variety of strategies, including strategies that were still emergent. This classroom also provided process-oriented representations (visuals), rather than a complete, finished visual. Finally, teacher facilitation in this classroom offered greater opportunities for students to connect their thinking with the strategies of their peers, often mediated by the visual or a model.

Methods

Participants

The study was situated in grades 3 – 5 at an elementary school in California. Demographics are as follows: 76.3% are Socioeconomically Disadvantaged, 14.4% are Students with Disabilities, 58.9% are English Learners, and 9.9% of students are Homeless. The majority of students at the school are Latinx (88.3%) with the second largest demographic category being White students (7.5%). The full study included 12 student participants in 3rd grade, 6 students in 4th grade, and 18 students in 5th grade. 12 students had current IEPs, with 4 students in the referral process. After analyzing the first Curriculum Based Measurement Multiplication Division Assessment (MD-CBM), the first researcher met with the classroom teachers to decide students' placement in the intervention. Tutors were undergraduates enrolled in an education practicum course.

Intervention

The 36 student participants were placed into 7 small groups for the intervention. There were 3 groups in third grade, 1 in fourth grade, and 3 in fifth grade. The reason for the single group in fourth grade was because a tutor left the study. Each group met for 8 sessions, twice a week for 4 weeks. The intervention took place during the students' math class, often while the classroom teacher was also doing small group work.

The intervention consisted of 8 sessions of number strings (Lambert et al., 2017) designed and facilitated by undergraduate tutors after 6 hours of professional development led by the first author. Each tutor was observed 2-4 times by a member of the research team and offered feedback. In addition, all tutors participated in a session in which they analyzed the participation of the students in their small group.

Data Collection

Curriculum Based Measurement Multiplication Division Assessment (MD-CBM)

The first author created this assessment with four iterations/versions, all designed to be equivalent in item difficulty. There were three one-page sections, each of which addressed the CCSS expectations for multiplication and division in grades 3, 4 and 5 respectively. Students in the intervention took the assessment four times (pre, during and post-intervention, with a follow-up assessment).

Video recordings and transcripts

We video-recorded all intervention sessions with one camera with a wide-angle lens. Members of the research team transcribed the videos, adding descriptions of gestures and other visual content.

Field notes

Tutors wrote field notes after each session. They were asked to reflect on the design of their number string, student participation and strategies, as well as any issues they had in facilitation. In addition, tutors took images of their chart paper and other artifacts and included those in their field notes. Members of the research team also wrote field notes for sessions they observed.

Teacher interviews

The first author interviewed the teachers whose students were in the intervention. Topics included students' engagement in math class, current mathematics achievement, and services provided by the school. These interviews were transcribed.

Data Analysis

We assessed student use of strategies and participation in mathematical problem-solving and discussion through analysis of transcripts of number string sessions, using a modified version of the coding scheme by Ing et al. (2015). Two authors each coded the small group we present in this paper, resolving any discrepancies. Interrater reliability was determined at .83. According to Landis and Koch (1977), Cohen's rule-of-thumb suggests that a 0.61-0.80 score reflects a good agreement and a 0.81-1.0 score reflects a very good agreement. We coded student engagement for each problem presented in the number string, which ranged from 4 to 7 problems per session.

Coding engagement

While we coded for several categories of engagement, in this paper, we focus on the level of shares of a student. A *Complete Share* was an answer that was accurate and explained in enough detail that we could confidently code the strategy. A *Partial Share* was either inaccurate or did not include enough detail that researchers could determine the exact strategy of the student. We added the last two categories to the coding scheme of Ing and colleagues to track students who had nonverbal engagement in the problem. *Nonverbal Engagement* captured moments in which we could see evidence of nonverbal engagement, yet students did not verbally share in discussion (such as students counting on fingers). *No Engagement* was coded if the student did not demonstrate verbal or nonverbal engagement that we could discern.

Coding student strategies

Analysis of student strategies was done through multiple evidence sources. We coded strategies used in the number string session within Dedoose, an on-line coding software program. We also coded strategies based on the written assessments (MD-CBM), analyzing student work. The first author did this analysis.

Integrating analyses

Figure 2 provides a visual of how we integrated multiple forms of data into a case study. After each separate analysis, we wrote a memo on the findings in that area. These memos also integrated field notes, which contained the analysis of both tutors and researchers. The first author then synthesized findings across the different areas of analysis, writing an Integrated Case Study Memo on Inez, which included charts of her participation, images of her written work and artifacts from the intervention session, as well as integrating these data points into a narrative of Inez's development over time. The memo was shared with the full research team for analysis and critique.

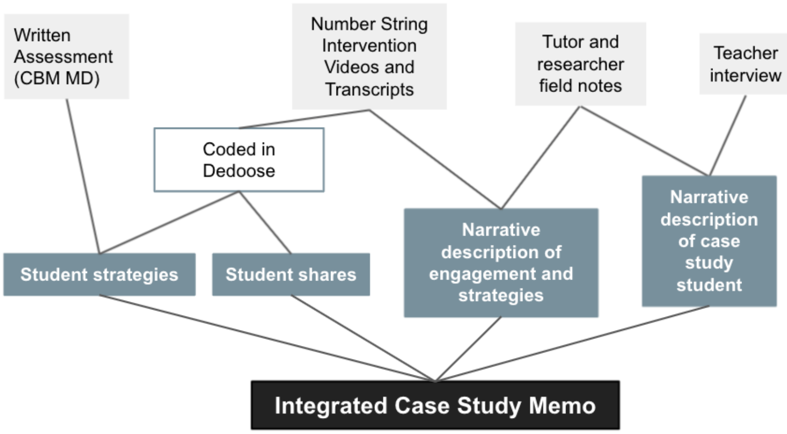


Figure 2. **Study Analysis Design**

Findings

This paper is a case study that focuses on one fourth-grade student (Inez) (all names are pseudonyms). Inez is Latina and classified as a Multilingual Learner. For a few sessions, Inez showed some hesitancy working and sharing her strategies with her peers but was able to converse with the tutor on multiple occasions privately. We did not see Inez using Spanish during class or in social settings. We regret that we did not design instruction to capitalize on her multilingual strengths, a major limitation of our study that we will address in future iterations.

Inez scored below our cutoff in the initial screener, and her classroom teacher also recommended that she participate in the intervention. In the teacher interview, Inez's classroom teacher described her as a student of significant concern in mathematics. The teacher shared that Inez had been through the special education referral process once in previous grades and had not qualified for services. The teacher planned to recommend a second evaluation. The teacher noted that Inez rarely shared in mathematics class and seemed to have significant issues with number sense. The teacher described Inez as having strategies that the teacher struggled to understand.

Inez was placed in a small group of 6 students taught by Yola, an undergraduate tutor. Comprised of students with and without disabilities, the students in this group had the lowest scores on multiplication in their class. Inez appeared eager to participate in this small group, even when she did not initially share. She seemed to particularly enjoy talking to Yola before and after the number string.

We chose Inez as a case study because she was the primary focus of her tutor, Yola, who initially was concerned about both Inez's lack of participation and her understanding of multiplication as evidenced by field notes. Both her tutor and our research team saw evidence of growth during the intervention from Inez, based on evidence from our field notes. Yet we also noticed that Inez presented pedagogical challenges for Yola that seemed fruitful to pursue, as such difficulties are common when teaching mathematics to students who may have a history of difficulty in the subject.

Complex Initial Set of Engagement Practices

Inez was very engaged in talking with Yola about non-math topics. In field notes, Yola noticed that this talkativeness seemed to disappear when the number string began. In the video recording of the first two number string intervention sessions, Inez did not volunteer to answer questions. She shared twice when called on and did not elaborate on her answers. Unlike her peers, she did not use her fingers to keep track as she skip counted and lost track of her counting. The first author, observing this session, became interested in how Inez seemed to subvocalize skip counting and resisted using her fingers even when prompted.

In discussions after the first number string, the first author and Yola wondered if Inez needed support to help her keep track of her count. Yola and the first author decided to support Inez through physical card stock versions of the arrays that might allow her to count the boxes. Starting in the second session, Yola passed out a card stock array for the students. Inez soon began using these arrays to keep track of her counting.

Supporting Shifts in Participation

Yola continued to write about Inez extensively in her field notes, noting the need for additional support to help Inez engage in the number string. In field notes after the third session, Yola wrote,

> The participation, while good, has been the same three kids. [Students] are always very engaged and willing to share answers while Inez is not. I think for the next number string I'm going to have Inez sit up front next to me so I can speak to her more. When I present the problems I can see the other kids working on it in their heads or on their hands but she seems to not be participating even in trying for an answer. Next time I'm going to situate her next to myself and girls who are more willing to share their strategies. I plan on making two sets [of card stock arrays] from now on, one set specifically just for Inez. I will continue with making goals for the group but also making special goals for her, starting with getting her to understand the purpose of an array.

In addition to changing her seat to be next to Yola and girls who were sharing their strategies, Yola also began to speak to Inez during every turn and talk, which seemed to support Inez sharing in the small group. This shift seemed to mark a pronounced difference in engagement from a lack of engagement in the first two sessions to a more sustained engagement in mathematical discussion in the subsequent sessions, as documented in Figure 4. It also meant that Yola had a much more complete understanding of Inez's strategies through these one-on-one conversations.

Analysis of Inez's whole group shares documented a clear shift away from a lack of participation towards a deeper involvement in mathematical discussion (Fig. 4). We coded engagements by each problem within the number string. In the initial sessions, Inez's engagements were coded primarily as no engagement or only non-verbal engagement, across multiple problems of each number string. As the sessions progressed, Inez increased the numbers of complete and partial shares of her mathematical thinking, and for Session 5 Inez provided a partial share for every problem in the number string. Again, complete shares were both mathematically correct and fully detailed explanations of thinking. This was then a positive and clear shift towards deeper engagement in mathematics during the intervention.

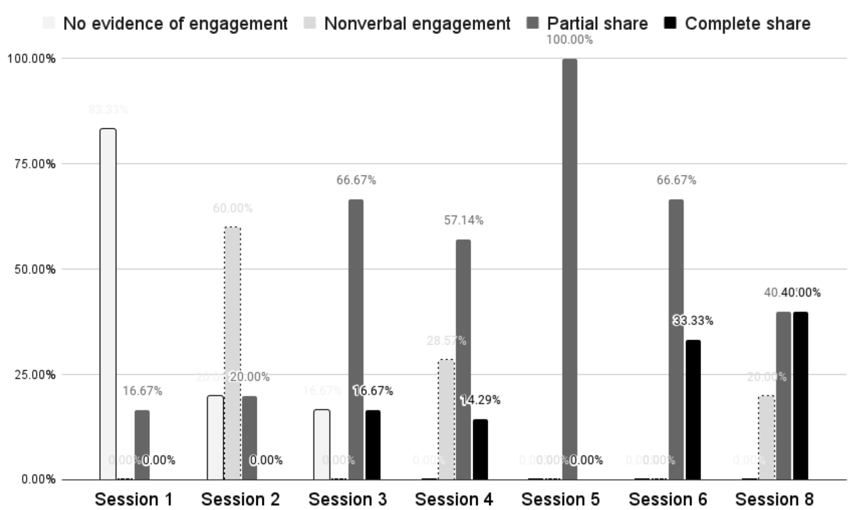


Figure 3. **Shifts in Inez's Participation in Mathematical Discussion by Percentage of Problems**

Complex Initial Set of Partial Understandings

Through analysis of the first two MD-CBM assessments (pre-intervention and during intervention) and Inez's strategies during the initial number string sessions, we determined that Inez was able to skip count by 2s and 5s and was able to multiply single digit numbers by ten. As the intervention progressed, we saw Inez skip counting by 2s and 5s for problems such as 6 x 4, without adjusting for the group size she was counting for. She also counted the boxes on arrays as units of 2, rather than units of 1.

Inez's first MD-CBM assessment had 8 problems correct out of a possible 22 multiplication and division problems. There was no work visible. She seemed to have some understanding of patterns when multiplying and dividing by tens. Inez offered an answer for every problem on the assessment, with many of the answers (with the exception of the tens problems) possibly being guesses, without a discernible pattern. For example, for the problem 6 x 15 she wrote 12.

On the second written assessment Inez used a skip counting by 2s and 5s strategy, with much more written work throughout the assessment. She made ratio tables to help her keep track of her skip counting, similar to what Yola had done in representing the students' thinking on chart paper. Her strategy worked for problems with factors of 2 or 5. It did not work for 3 x 8, 6 x 9, and 7 x 8. Table 2 shows an example of these written strategies.

Shifting Understandings of Multiplication

Through close analysis of Inez's strategies across the 7 sessions (the final session was not video recorded because of technical difficulties), we saw evidence that Inez developed additional spatial structure for arrays. While in the beginning she did not count squares by ones successfully (by counting boxes as 2s or 5s), she was able to do so by Session 5. This developed in tandem with an increased understanding of the connection between skip counting and multiplication by groups. The evidence for this claim came from transcripts of exchanges between Inez and Yola, when Yola called on Inez to describe her strategy.

There were three instances in which Inez used this partial understanding of skip counting on the array by a different factor during the number strings. In Session 3, Inez got an answer of 50 for 9 x 4, skip counting by 5s. Yola represented her skip counting next to the array, making connections between the two models. Inez stared at the array and the skip counting represented next to it, and said, "What the heck?"

In Session 4, Inez counted an array by 2s, getting an answer of 62 for 6 x 5. Here is their exchange for the problem 6 x 5, after Yola asks Inez what she got for the problem:

> **Inez**: 62.
>
> **Yola**: Do you want to tell me how you got 62?
>
> **Inez:** Yeah I counted the array by 2's.
>
> **Yola:** You counted the whole array by 2's. Did you give each square 2? Did you go 2,4,6,8,10? [points to one square at a time while counting by 2's]
>
> **Inez:** No I did it like this [runs her finger along each row but does not count out loud or point to individual squares]. I did it like this.
>
> **Yola:** Okay you counted by 2's. That's a lot of 2's to write so I'm just going to write counted by 2s. So let's look at it on the array again so we can double check our answers. So how many does each column have in it?
>
> **Inez:** 5.
>
> **Yola:** 5 right. And they're all equal right? So they all have five on them. So if I go like this and I make two column [covers the array so that only 2 columns are showing] how many do we have?
>
> **Inez:** 10.
>
> **Yola:** 10 right. And we can keep doing that so we can use it to skip count right? Just like we did before. 5, 10, 15, 20, 25, 30. [other students join her in counting out loud]. Inez does that make any sense? How can we use it to count like that? I know you're really good at counting by 5's.
>
> **Inez:** [Inez takes the array and silently counts on it] Yeah, I went really really far [smiling].
>
> **Yola:** [laughing with Inez]: yeah you went really really far, that's okay.

Yola listened to Inez's strategy, ensuring she understood it without critiquing it. Yola noted the counting by 2s strategy (Fig. 3) but appeared to decide in the moment to build on Inez's strengths in multiplying by 5s, drawing attention to the columns of 5. Yola then gave Inez a card stock array and waited while she recounted using 5s. Inez seemed to be provoked into disequilibrium by this exchange, saying, "I went really really far." This comment appeared to link her strategy to the much higher number it resulted in. Connecting visual and numerical representations of her own strategy appeared to make Inez's own thinking visible to her, thus allowing her to understand her own thinking as reflected by the tutor's representations and Inez's actions of recounting in the moment.

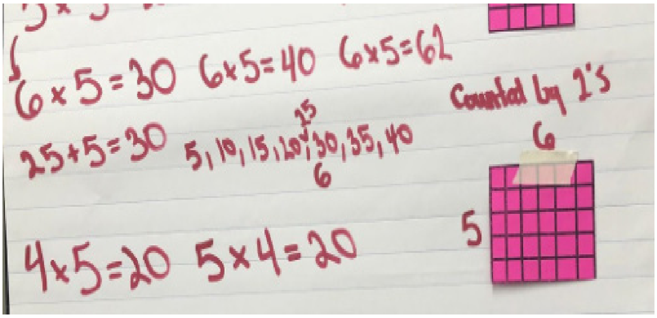


Figure 4. **Representation of Inez's Strategy for 6 x 5 in Session 4.**

In the next session (Session 5), Inez used this strategy once but twice counted an array by ones, successfully for 6 x 6 and one number off for 9 x 6 (answer of 55). Also, in this session Inez developed her understanding of the commutative property. During this session, one student got frustrated with Yola for writing 6 x 8 on the back of an array that the student saw as 8 x 6. This started an ongoing conversation about problems that were the same answer, just "switched around." Inez twice explained this property in her own words, that the array was "backwards" but it was the "same problem."

Shifts in Assessments

Table 1 presents scores and some sample written work for all four assessments.

Table 1. ***Inez Assessment Scores and Written Work***

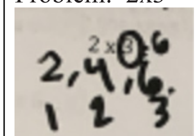
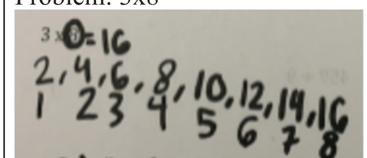
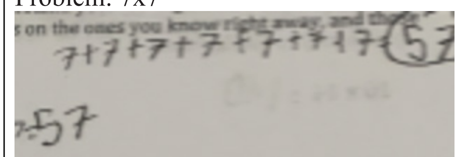
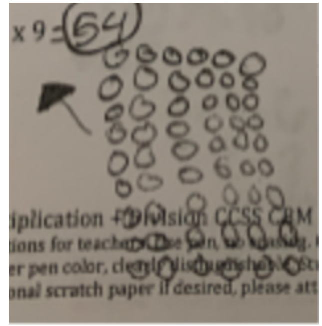
Assessment Tool	Raw Score (number of problems correct disaggregated by grade level alignment)	Researcher Analysis	Image of Student Work
MD-CBM 1 Pre-intervention	8/22 Total (7/12 3rd grade problems 1/10 4th grade problems)	No work is visible on any problem.	
MD-CBM 2 Mid-intervention	5/22 Total (4/12 3rd grade problems 1/10 4th grade problems	Work is visible on many problems. Dominant strategy is skip counting by 2s and 5s without adjusting	Problem: 2x3  Problem: 3x8 
MD-CBM 3 Immediately Post-intervention	10/22 Total (8/12 3rd grade problems 2/10 4th grade problems)	Only one problem shows work. That problem shows use of repeated addition. Factor of 7 used correctly.	Problem: 7x7 

Table 1. ***Inez Assessment Scores and Written Work (continued)***

MD-CBM 4 2 months Post-intervention	10/22 Total (8/12 3rd grade problems 2/10 4th grade problems)	Work shows use of repeated addition and accurately drawn arrays.	Problem: 6x9 

In the third assessment, there was no longer evidence of inaccurate skip counting by 2s or 5s. There was very little work on this assessment, with the exception of the use of repeated addition for 7 x 8. This showed a shift towards using the factors of a problem, rather than only 2s and 5s. In the final assessment, Inez began drawing dot arrays to solve problems. Considering the research of Battista and colleagues (1998) on the difficulty that students have drawing arrays, and how this skill develops after a student has developed spatial structuring for the array, this is significant development. On this final assessment, we did not see any more inaccurate skip counting by 2s and 5s. Inez seemed to understand that one unit in an array is one unit, and that skip counting is by a group of those units. This understanding of skip counting (and repeated addition) appeared to co-develop with her understanding of the array, culminating in her ability to draw multiple accurate arrays on her final assessment. We summarize her development in Figure 5.

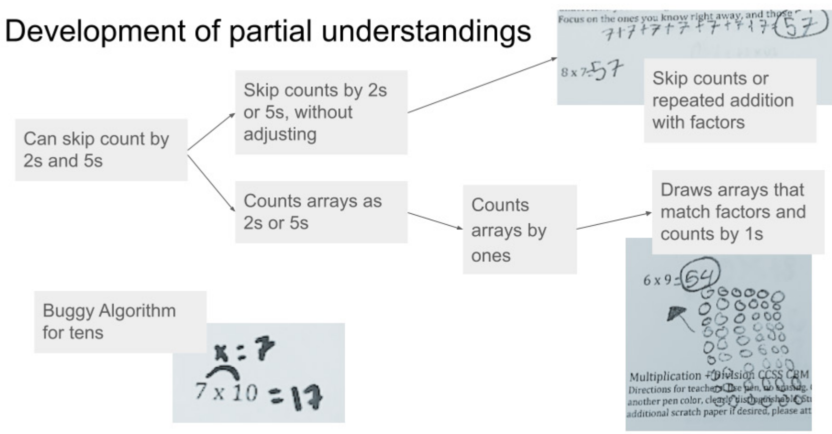


Figure 5. **Development of Inez Partial Understandings**

Challenges in Pedagogy

While Yola seemed to make supportive moves to increase Inez's participation, as well as to model her thinking to make it visible, Yola described in her field notes having significant difficulty understanding and representing Inez's strategies. While there were instances in which Yola pressed for explanation, there were more instances in which Inez shared an incorrect answer and Yola did not ask her to elaborate. We suspect that pressing a student for further explanation when that student has a pattern of strategies that do not make sense to the teacher might be a particularly challenging teaching move to enact. From our observation of teachers, we think that Yola took risks to call on Inez, risks that teachers do not always choose to take. Yola reflected on this challenge in her field notes:

> The most challenging thing for today was trying to balance both working with the group and working with Inez. I was really happy she was volunteering answers but it was difficult for me to follow her thought process and represent it on the board. I didn't want to leave her behind and felt like I needed to ask her questions in order to point her in the right direction but I don't know if I was taking too much time away from the rest of the group. I want her to continue to feel comfortable sharing her thoughts but also make sure she is seeing why her methods are incorrect.

We also note that Yola paid attention to her relationship with Inez, chatting with her and building a relationship. We wonder about the role this relationship may have played in Inez's participation in the small group.

Discussion

Our intervention aimed to increase the mathematics achievement of students with disabilities and students whose performance was significantly below grade level, but not with instructional practices that focused on memorization or procedural learning. Instead, we investigated the use of a number string to develop multiplication and division computation simultaneously with number sense. Our intervention also prioritized the role of engagement in a small learning community within the learning process since there is evidence that student achievement is correlated with student engagement with others' strategies (Ing et al., 2015). This paper demonstrates how one student with a complex partial understanding of mathematics developed her engagement and conceptual understanding of multiplication.

Our first research question looked at student engagement. We asked, for a student with limited engagement in mathematics, can a small group number strings intervention support increased participation in mathematical sense-making and talk? How do shifts in participation develop? We saw that Inez did not initially participate in the number string, but shifted towards a much more engaged stance, sharing multiple answers per session. Those shifts seemed to be the result of some teaching moves by her tutor. For example, Yola not only printed out cardstock arrays for students to count, she eventually printed a separate copy for Inez because it was important for her engagement. Because Inez used skip counting but would not use her fingers, she lost track of her counts. Focusing on the arrays not only helped Inez count, but also allowed her partial understanding of the arrays to emerge so that Inez could reflect on it. In addition, Yola moved Inez so she was right next to her. During the sessions, Yola checked in with Inez to hear Inez's strategy, which may have supported whole group shares. Baxter and colleagues documented a similar increase in engagement when students were able to do rehearsals with a paraprofessional (Baxter et al., 2002).

Our second research question asked: Can a small group number strings intervention support conceptual and strategic development in multiplication for a student with complex partial understandings of multiplication? We established that Inez had a complex set of partial understandings, some of which were in evidence in the initial written MD-CBM assessment, and some that emerged through her participation in the small group and in subsequent written assessments. Building from her ability to skip count by 2s and 5s, Inez first applied this strategy to all multiplication problems. Through discussion of her strategies in the small group, including the tutor making her strategies visible through modeling them on arrays, Inez began to develop a more coherent spatial structure for the array. By the follow-up assessment, Inez was able to draw arrays and count them. She also developed an understanding of the commutative property of multiplication. Using Siegler's overlapping wave theories, Inez strategy usage shifted away from ineffective and time-consuming beginner strategies to more complicated strategies (Siegler, 1998).

How did these shifts occur? We believe that the critical movements were when Yola called on Inez to explain and justify her answers in the small group. Engagement was critical here, as Inez was willing to explain what she was thinking. That took bravery, and also a sense of comfort in the small group. Yola also made the move to call on Inez, even when she felt confused by how Inez was solving problems. This also took a kind of pedagogical bravery, to take a risk and allow a possibly confusing or opaque strategy to emerge. These moments allowed Inez to reflect and act on her own strategies, and, we believe, to shift towards understanding the spatial structure of the array. Students make their own paths in understanding mathematical concepts as they are learning and revising their strategies (Carpenter et al., 2015; Fosnot & Dolk, 2001).

Limitations

While this case study does benefit from multiple data sources (4 written MD-CBM assessments, 7 sessions of teacher interviews, and field notes of tutor and researchers), we note that we lack the perspective of Inez in this case study. We believe our findings would be more robust if we had interviewed Inez, perhaps multiple times across the intervention. In addition, we note that we did not design instruction to capitalize on multilingual strengths, which could have benefited Inez's engagement. For future iterations, we can strive to use students' multilingual status as a strength. There could be additional training for the tutors such that there is more support for multilingual learners by incorporating more of a translanguaging format where we encourage students to use all of their language repertoire during number strings (Creese & Blackledge, 2010; García & Wei, 2014). We also note that we did not know what impact the mathematics classroom instruction that occurred during and after our intervention had on her development.

Implications

This MTSS tier 2 intervention was focused around number strings, which appeared to support Inez in her development of more complex strategies and boost her engagement with others. The number strings allowed for students to have more agency and self-expression through mathematical discourse. In addition, number strings allowed for development of more complex strategies without the presence of direct instruction from the teacher. Overall, the intervention of small group instructions built around number strings greatly benefitted Inez and should benefit other students similar to Inez by improving achievement, engagement, and agency.

This case study documents the interrelation between engagement and cognitive shifts in a small group intervention. It is quite possible that if Inez was in a one-to-one setting, she could have developed conceptually further because instruction would be individualized, based solely on what Inez knew about multiplication. However, in the small group Inez tried strategies suggested by other students (skip counting and counting by ones on the arrays) and engaged in conversation about the commutative property. Our intention is not to promote one or the other approach, but to bring dialogue between constructivist teaching experiments with the affordances of small group settings.

In further work, we will turn our attention to the teaching moves of novice teachers to determine the effectiveness of the professional development we provided for tutors and what aspects of teaching number strings were most challenging for tutors to enact. We also plan to analyze how to better leverage multilingualism in her learning. Most importantly, we seek to better understand how to provide mathematics intervention for students who need more support engaging in meaningful mathematics, interventions that develop agency along with achievement.

References

Battista, M. T., Clements, D. H., Arnoff, J., Battista, K., & Borrow, C. V. A. (1998). Students' spatial structuring of 2D arrays of squares. *Journal for Research in Mathematics Education*, *29*(5), 503–532. https://doi.org/10.5951/jresematheduc.29.5.0503

Baxter, J. A., Woodward, J., & Olson, D. (2001). Effects of reform-based mathematics instruction on low achievers in five third-grade classrooms. *The Elementary School Journal*, *101*(5), 529–547. https://doi.org/10.1086/499686

Baxter, J., Woodward, J., Voorhies, J., & Wong, J. (2002). We talk about it, but do they get it? *Learning Disabilities Research & Practice*, *17*(3), 173–185. https://doi.org/10.1111/1540-5826.00043

Bottge, B. A., Heinrichs, M., Mehta, Z. D., & Hung, Y. H. (2002). Weighing the benefits of anchored math instruction for students with disabilities in general education classes. The *Journal of Special Education*, *35*(4), 186–200. https://doi.org/10.1177/002246690203500401

Boyd, B., & Bargerhuff, M. E. (2009). Mathematics education and special education: Searching for common ground and the implications for teacher education. *Mathematics Teacher Education and Development*, *11*, 54–67.

Callandro, C. (2000). Children's inventions for multidigit multiplication and division. *Teaching Children Mathematics*, *6*(6), 420–426. https://doi.org/10.5951/TCM.6.6.0420

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (2015). *Children's mathematics: Cognitively guided instruction* (2nd ed.). Heinemann.

Creese, A., & Blackledge, A. (2010). Translanguaging in the bilingual classroom: A pedagogy for learning and teaching? *The Modern Language Journal*, *94*(1), 103–115. https://doi.org/10.1111/j.1540-4781.2009.00986.x

Finesilver, C. (2017). Between counting and multiplication: Low-attaining students' spatial structuring, enumeration and errors in concretely-presented 3D array tasks. *Mathematical Thinking and Learning: An International Journal*, *19*(2), 95–114. https://doi.org/10.1080/10986065.2017.1295418

Fosnot, C. T., & Dolk, M. (2001). *Young mathematicians at work: Constructing multiplication and division.* Heinemann Press.

García, O., & Wei, L. (2014). Translanguaging: Language, bilingualism, and education. *Bilingual Research Journal*, *37*(3), 366–369. https://doi.org/10.1080/15235882.2014.965361

Geary, D. C. (1990). A componential analysis of an early learning deficit in mathematics. *Journal of Experimental Child Psychology*, *49*(3), 363–383. https://doi.org/10.1016/0022-0965(90)90065-G

Gersten, R., Beckmann, S., Clarke, B., Foegen, A., Marsh, L., Star, J. R., & Witzel, B. (2009). *Assisting students struggling with mathematics: Response to Intervention (RtI) for elementary and middle schools* (NCEE 2009-4060). What Works Clearinghouse.

Hord, C., Tzur, R., Xin, Y. P., Si, L., Kenney, R. H., & Woodward, J. (2016). Overcoming a 4th grader's challenges with working-memory via constructivist-based pedagogy and strategic scaffolds: Tia's solutions to challenging multiplicative tasks. *The Journal of Mathematical Behavior*, *44*, 13–33. https://doi.org/10.1016/j.jmathb.2016.09.002

Hunt, J. H., & Ainslie, J. (2021). *Designing Effective Math Interventions: An Educator's Guide to Learner-Driven Instruction* (1st ed.). Routledge. https://doi.org/10.4324/9781003014744

Hunt, J., & Tzur, R. (2017). Where is difference? Processes of mathematical remediation through a constructivist lens. *The Journal of Mathematical Behavior*, *48*, 62–76. https://doi.org/10.1016/j.jmathb.2017.06.007

Ing, M., Webb, N. M., Franke, M. L., Turrou, A. C., Wong, J., Shin, N., & Fernandez, C. H. (2015). Student participation in elementary mathematics classrooms: The missing link between teacher practices and student achievement? *Educational Studies in Mathematics*, *90*, 341–356. https://doi.org/10.1007/s10649-015-9625-z

Johnson, N. C., Turrou, A. C., McMillan, B. G., Raygoza, M. C., & Franke, M. L. (2019). "Can you help me count these pennies?": Surfacing preschoolers' understandings of counting. *Mathematical Thinking and Learning*, *21*(4), 237–264. https://doi.org/10.1080/10986065.2019.1588206

Lambert, R. (2018). "Indefensible, illogical, and unsupported"; Countering deficit mythologies about the potential of students with learning disabilities in mathematics. *Education Sciences*, *8*(2), 72. https://doi.org/10.3390/educsci8020072

Lambert, R., & Harriss, E. (2022). Insider accounts of dyslexia from research mathematicians. *Educational Studies in Mathematics*. https://doi.org/10.1007/s10649-021-10140-2

Lambert, R., Imm, K., & Williams, D. A. (2017). Number strings: Daily computational fluency. *Teaching Children Mathematics*, *24*(1), 48–55. https://doi.org/10.5951/teacchilmath.24.1.0048

Lambert, R., & Schuck, R. K. (2021). "The wall now between us": Teaching math to students with disabilities during the COVID spring of 2020. *The Asia-Pacific Education Researcher*, *30*, 289–298. https://doi.org/10.1007/s40299-021-00568-8

Lambert, R., & Sugita, T. (2016). Increasing engagement of students with learning disabilities in mathematical problem-solving and discussion. *Support for Learning*, *31*(4), 347–366. https://doi.org/10.1111/1467-9604.12142

Landis, J. R., & Koch, G. G. (1977). The measurement of observer agreement for categorical data. *Biometrics*, *33*(1), 159–174. https://doi.org/10.2307/2529310

Lave, J., & Wenger, E. (1991). *Situated Learning: Legitimate Peripheral Participation*. Cambridge University Press. https://doi.org/10.1017/CBO9780511815355

Ochoa, G. (2013). *Academic Profiling: Latinos, Asian Americans, and the Achievement Gap*. University of Minnesota Press. https://doi.org/10.5749/minnesota/9780816687398.001.0001

O'Loughlin, T. A. (2007). Using research to develop computational fluency in young mathematicians. *Teaching Children Mathematics*, *14*(3), 132–138. https://doi.org/10.5951/TCM.14.3.0132

Mireles-Rios, R., Rios, V. M., Auldridge-Reveles, T., Monroy, M., & Castro, I. (2020). "I was pushed out of school": Social and emotional approaches to a youth promotion program. *Journal of Leadership, Equity, and Research*, *6*(1), 1–21. https://eric.ed.gov/?id=EJ1293284

Moschkovich, J. (2002). A situated and sociocultural perspective on bilingual mathematics learners. *Mathematical Thinking and Learning*, *4*(2–3), 189–212. https://doi.org/10.1207/S15327833MTL04023_5

Murata, A., Siker, J., Kang, B., Baldinger, E. M., Kim, H. J., Scott, M., & Lanouette, K. (2017). Math talk and student strategy trajectories: The case of two first grade classrooms. *Cognition and Instruction*, *35*(4), 290–316. https://doi.org/10.1080/07370008.2017.1362408

Siebers, T. (2008). *Disability Theory*. University of Michigan Press. https://doi.org/10.3998/mpub.309723

Siegler, R. S. (1998). *Emerging minds: The process of change in children's thinking*. Oxford University Press.

NCSM & TODOS. (2021). Positioning Multilingual Learners for Success in Mathematics. Joint position statement from NCSM: Leadership in Mathematics in Education and TODOS: Mathematics for ALL. https://www.todos-math.org/assets/images/NCSM-TODOS%20Multilingual%20Learners%20Position%20Paper%202021.pdf

Zhang, D., Xin, Y. P., & Si, L. (2013). Transition from intuitive to advanced strategies in multiplicative reasoning for students with math difficulties. *The Journal of Special Education*, *47*(1), 50–64. https://doi.org/10.1177/0022466911399098

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