

Mathematical Knowledge in a Generation of Pupils

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Abstract:

Educational outcomes are mostly studied in the short term (i.e. during the same phase of schooling). This paper focuses on the longitudinal monitoring of the mathematical knowledge of the same generation of pupils. At the school level, this can be performed by teachers, and at the state level by relevant institutions or experts in the field. Monitoring the progress in pupils' attainments is one way of determining and assuring the school's quality educational work. At the state level, pupils in Slovenia take National Assessments of Knowledge which can be used to monitor progress within the same generation of pupils. The introduction explains the difference between internal and external assessment and presents studies that focus on comparing the attainments in both types of assessment. This study focuses on the same generation of pupils, monitoring their attainments in mathematics in the 6th and the 9th year of schooling. The aim was to determine whether there was a gap between internal and external assessment. The research into external assessment was based on the attainments on the National Assessment of Knowledge in Mathematics, while the research into internal assessment was based on the attainments on the same subject. It has been determined that more than half of the pupils would have the same final grade in the internal and external assessment; less than a fifth of the pupils would have received a lower grade in the external assessment.

Keywords: external knowledge assessment, final grade, hypothetical grade, internal knowledge assessment, mathematics, elementary school

INTRODUCTION

Monitoring mathematical knowledge or progress in the mathematical knowledge of the same generation of pupils not only provides an insight into the pupils' progression in mathematical knowledge but also enables different stakeholders to detect knowledge gaps and to eliminate them. It also provides an insight into the added value, i.e., the progress pupils or schools make between individual measurements of knowledge. The same generation of pupils can be monitored during a specific period at different levels: at the class level, at the school level, and at the state level. At the school level, monitoring the progress of the same generation of pupils, is in the hands of teachers who conduct analyses, interpret results, and -based on their findings- plan future teaching practice.

Determining and assuring the quality of classwork at the school level is also associated with monitoring the students' progress (OECD, 2012). In Slovenian educational context, monitoring students' mathematical knowledge at the state level is facilitated both by the National Assessment of Knowledge in elementary education and by the final state graduation examination (Matura Exam) in general secondary education. In elementary education, schools can review the students' progress from one test to another and can (in addition to achievement, i.e., the scores pupils receive on specific tests or examinations) also delve into comparisons of the progress achieved (Cankar et al., 2019). Researches

often focus on the scores attained by an individual pupil or the whole generation of pupils at a specific point in time (e.g., in the same class or year of schooling), perhaps because such studies are conducted for a shorter period of time than longitudinal studies. Monitoring the attainments of the same generation of pupils throughout their schooling is a lengthy process and the results of such research provide a source of information for designing the educational process, including the course design, implementation and evaluation stages. Claessens and Engel (2013) and Shanley (2016) studied the correlation between the demonstrated mathematical knowledge in the preschool period and in elementary education, confirming a moderate to strong positive correlation in mathematics results. In a longitudinal study, Vanwynsberghe et al. (2017) intensely monitored a group of 6,000 Flemish pupils from kindergarten to the 7th year of schooling. The same group participated in data collection again at the age of 17. The aim of the study was to research the long-term effects of elementary school on students' mathematics attainments at the end of secondary education. The researchers have discovered that students with higher attainments in mathematics at age 17, thus confirming that the attainment at the end of elementary education was the main predictor of attainment in this subject at the age of 17.

From the abovementioned pieces of research, we conclude that assessing students' mathematical knowledge by two successive examinations which contain a certain number of identical test items could provide an insight into how students' progressed, whether their knowledge has increased, which errors and misconceptions are still present among them, and how to help them overcome challenges in mathematics. The main aim of the present study is to examine mathematical knowledge of the same generation of students who attended the 6th year of schooling in the 2015/16 school year and the 9th year of schooling in the 2018/2019 school year.

THEORETICAL FRAMEWORKS

In Slovenia, the educational system makes use of both internal and external knowledge assessment. Internal knowledge assessment is defined as knowledge assessment performed by each teacher autonomously in the class that he/she teaches. This type of assessment can take different formats using the tasks, test items and evaluation criteria prepared by the teachers. They, however, do not possess additional information about the comparability of this type of assessment with those used by teachers in other schools, nor about the comparability of the results of such assessment, i.e., of school grades of their pupils, with the grades awarded by other teachers (Žakelj & Grmek, 2011). In the Slovenian education system, internal knowledge assessment takes place throughout the school year in accordance with the current regulation. Pupils' attainments can be assessed in different ways:

- 1. by written examinations (tests);
- 2. by oral examinations;
- 3. by assessing pupil's performance of activities (e.g., oral presentation, performance of sporting activities, etc.);
- 4. by assessing the pupils' outcomes (e.g., research papers, seminars, reports), and
- 5. with a combination of the above (Marentič Požarnik, 2010).

According to the "rules on knowledge assessment and students' progress to a higher class standing in elementary schools" (UL RS, 2013, No. 52/13, Article 16), the grades obtained throughout the school year are converted into the final grade when the course is completed, typically at the end of the school year. From the 3th to the 9th year of schooling, teachers decide on the final numerical grade in each subject thus reporting how well the pupils have met the target knowledge-the standards defined in the curricula, and taking into account the grades received by pupils in the subject throughout the school year.

External assessments are both international (e.g., the TIMSS and PISA studies) or national (state). The main characteristics of the external knowledge assessments are: all pupils solve the same or comparable tasks; the tests are administered according to uniform criteria, and the tests are metrically validated, at least to some extent (Bucik, 2001). The National Assessments of Knowledge are external knowledge assessments in the nine-year elementary school in Slovenia in the form of a written examination (test). They are administered towards the end of the second and the third educational periods (in May in the 6^{th} and the 9^{th} year of schooling), each time in three subjects. In the 6^{th} year of schooling the pupils' attainments are assessed in three compulsory subjects, that is Slovenian (Italian or Hungarian in border areas), mathematics and first foreign language. While the first two subjects remain the same also in the 9th year of schooling, foreign language, however, becomes one of compulsory elective subjects, which are selected annually by the Ministry of Education. The tests are created by teams of experts (Subject Testing Committees). The National Assessment of Knowledge has been compulsory for students of the final year of schooling since 2001, and for the students of the 6th year of schooling since 2016. The National Assessment of Knowledge test papers contain tasks which cover the knowledge of the same year and of all previous years of schooling, while teachers' internal assessments incorporate solely that years' knowledge. For this reason, comparing the attainments on the National Assessment of Knowledge with the final grades presents certain challenges. Students' attainments on the National Assessment of Knowledge are normally presented in the form of percent; thus, in this paper we are referring to them with the term "attainments".

Measuring and monitoring the attainments of the same generation of pupils in different years of schooling is a long-term process. Most studies were conducted in the same class or in the same year of schooling. Having reviewed the previous studies conducted in the field of internal and external assessment that are listed in the references section, we focused on comparing the outcomes of internal and external and external assessments in elementary schools.

In the lower years of elementary school, i.e., in the 3th and the 5th year of schooling, Martínez et al. (2009) confirmed that the teachers' grades in mathematics strongly correlate with the results of the standardized tests; however, they also discovered that the ratio differs greatly between teachers and is related to certain assessment practices in the classroom. Pupils with higher grades in mathematics attained better results on the test administered for the purpose of the research than the pupils with lower grades in mathematics, according to Cotič (2010). However, her study demonstrated considerable differences between groups of pupils with different grades in mathematics in terms of test results in the following areas: arithmetic, geometry with measurement, and logic with data processing.

A comparison of the attainments on TIMSS 2003 and the National Assessment of Knowledge showed that, of the six countries involved in TIMSS 2003 that also administered national assessments of knowledge, Slovenia demonstrated the biggest gap. A large percentage of our pupils did not even attain the lowest level of knowledge and very few attained a high level of knowledge in the TIMSS study. However, the percentage of pupils who have just managed to attain a low level of knowledge or even below it on the national assessment was 2 to 5 times lower than that measured by the international TIMSS study. Nevertheless, more than half of the population received an above-average final school grade, i.e. higher than "good" (3); moreover, an even greater number of pupils received grade "excellent" than "very good" (Zupanc, 2005).

A comparison of the final school grades in the 8th year of schooling and of the attainments on the TIMSS 2007 study indicates that, with regard to the final grades, almost a half of the generation (47.1%) possesses above-average mathematical knowledge of mathematics, while the results of TIMSS 2007 show that above-average pupils ("very good", "excellent") amount only to one quarter (25%). In this generation, very few negative final grades "insufficient" (0.1% in Mathematics) were awarded., one fifth of the pupils (21.2%) received grades "insufficient" and "sufficient" in mathematics, which can be interpreted as poor knowledge, while international measurements indicate that more than a third (35%)

of the same generation of pupils attain the lowest level of knowledge or even below that (Žakelj et al., 2009; Zupanc, 2010).

A comparison of the final grades and the attainments of the same generation of students in the PISA 2006 study shows that the discrepancy in the highest attainments is smaller. At the end of elementary school, 44.2% of the pupils in generation demonstrate above-average knowledge of mathematics, according to their final school grades, while international measurements indicate that 32.9% are above-average (levels 5 and 6). In the following, the mathematics attainments measured by PISA indicate that 41.2% of the generation are below-average, while school grades indicate only a 25.2% share of below-average pupils ("insufficient", "sufficient"). This gap amounts to almost one-sixth of a generation (16%) (Žakelj et al., 2009; Zupanc, 2010).

Felda (2018) compared final grades in mathematics and the levels attained in TIMSS study in 8th or 9th year of schooling and found that the proportion of pupils with the final grade "insufficient" is more or less constant, while the proportion of pupils with the grade "very good" or "excellent" is on the rise. Focusing only on the final grades in 2011 and 2016, he has deduced that the trend of grade inflation is not stagnating.

All the above-mentioned studies focus on a generation of pupils in specific years of elementary school and indicate an inconsistent distribution of final grades in mathematics compared to the attainments on external knowledge assessments. Have the same conclusions at the secondary level of education in the Slovenian school setting? Cankar (2016) and Japelj Pavešić and Cankar (2019) examined the comparison or correlation between grades obtained on the Matura examination, the final grades, and the TIMSS Advanced attainments, discovering a discrepancy between the Matura grades and the final grades, and a discrepancy between the obtained grades and the TIMSS Advanced attainments. They also discovered a gender-based difference in the attainments. Similar findings were already presented by Japelj Pavešić (2009) and Zupanc (2010) in the TIMSS study for secondary school graduates of 2008. 35.2% of grammar school students demonstrated above-average knowledge in the form of school grades ("very good", "excellent"), which is more than 20% higher than indicated by the TIMSS assessment for secondary school graduates of 2008. At the end of the 4th year of upper secondary schooling, 36.7% of grammar school students were awarded the grade "sufficient", while the TIMSS study indicates that 59% of grammar school students were incapable of solving routine tasks to attain the basic level of knowledge. The gap between the two measurements amounts to more than one-fifth of the generation (22.3 percentage points).

Research Problem

In the Slovenian school setting, there seems to be no longitudinal study focusing on monitoring the mathematical knowledge of the same generation of pupils. No systematic research has investigated the comparison of the same generation of pupils in terms of their attainments at different levels, e.g. school grades and attainments in the TIMSS and PISA studies. Our longitudinal study attempts to provide a more realistic insight into the trends in the mathematical knowledge of the same generations of pupils; it facilitates comparisons between different generations and between important social events taking place in specific periods that might influence the attainments. Each school has access to all the data required to monitor the progress within the same generation of their pupils or the added value. At the state level, it is possible to monitor the generation of pupils in years 6 to 9 of schooling, based on the data on their final grades in mathematics and their attainments on the National Assessment of Knowledge, but only from the 2015/2016 school year onward. For previous school years we can only monitor their attainments on the National Assessment of Knowledge. This longitudinal study into the mathematical knowledge of the same generation of pupils provides starting points for further comparisons, an analysis of comparability of attainments with the final grades obtained at school, and indicates how to plan future educational measures at the state level.

Research Objective

The research objective is to analyse the attainments of the same generation of pupils in mathematics and to compare them with one another. The attainments include the achievements on the National Assessment of Knowledge in Mathematics, the final grades in mathematics, and the hypothetical grades the pupils would be awarded on the National Assessment of Knowledge. The hypothetical grade is the grade awarded to a pupil according to a defined methodology or procedure known in advance, which is described in chapter "data processing". We analysed the attainments of the generation of pupils who attended the 6th year of schooling in the 2015/2016 school year, and then three years later, i.e. in the 2018/19 school year, when they were finishing the 9th year of elementary school. The study focuses on differences between the internal and external knowledge assessment. For this purpose:

- 1. We compared the achievements on the National Assessment of Knowledge in Mathematics and the final grades in mathematics of the same generation of pupils to determine whether the percentage of pupils with better final grades has been increasing, and to determine the correlation between the attainments on the National Assessment of Knowledge and the final grades in mathematics within the same generation of pupils.
- 2. We compared the final grades of the same generation of pupils with the hypothetical grades (the attainments on the National Assessment of Knowledge in Mathematics were converted into hypothetical grades) to determine whether the results indicate a potential subjectivity in the final grades in mathematics.

METHODOLOGY

Methods

The descriptive and causal non-experimental methods of empirical pedagogical research were used in the study. The quantitative approach we used was based on the statistical processing of numerical data obtained from the National Examinations Centre (in Slovenian, Republiški Izpitni Center or RIC; the central institution established in the early 1990's for measurement in education. Today they administer various external assessments of pupils, apprentices, students and adults in Slovenia). This longitudinal study includes data taken in years 2016 and 2019.

The Sample

The study makes use of the data for the generation of pupils in the 6th year of schooling in the 2015/2016 school year who took the National Assessment of Knowledge in Mathematics, and for the same generation of pupils who attended the 9th year of schooling three years later, i.e. in the 2018/2019 school year, and who took the National Assessment of Knowledge in Mathematics in the their final (9th) year of schooling.

For the 6th year of schooling generation, the National Examinations Centre obtained the final grade for 86.3% of the pupils who took the National Assessment of Knowledge in Mathematics; in the 9th year of schooling generation, the percentage of pupils for whom it also obtained the final grade is higher, i.e., 99.5%. The number of pupils of the same generation who took the National Assessment of Knowledge in Mathematics in their 6th year of schooling and then again in the 9th year of schooling has been reduced by 614 pupils (RIC, 2016, 2019). Thus, the sample included only those students with both their known final grades in mathematics and their attainments on the National Assessment of Knowledge. There is also a noticeable difference between the number of pupils who took the National Assessment of Knowledge in Mathematics and the number of pupils with known final grades in mathematics (**Table** 1). For reasons unknown to us, some pupils of the 6th year of schooling in 2015/16 did not take the National Assessment in their 9th year of schooling in 2018/19. Representativeness was achieved by processing the data for the entire population.

	School year					
Number of numile	2015/2016	2018/2019				
Number of pupils	6 th year of schooling	9 th year of schooling				
who took the National Assessment of Knowledge in Mathematics	17 358	16 744				
for whom the National Examinations Centre obtained the final grade	14 982	16 668				

Table 1. Number of pupils who took the National Assessment of Knowledge in Mathematics and whose final grades in mathematics are known for the select school year

Data Collection

The National Examinations Centre provided the relevant data. They consist of the aggregate attainments of pupils on the National Assessment of Knowledge in Mathematics and the final grades in mathematics for the same generation of pupils, namely for the 6th year of schooling of elementary school at the end of the 2015/16 school year and for the 9th year of schooling pupils at the end of the 2018/19 school year. Initially, we had access to data presenting the number of pupils with specific attainments (expressed in percent) on the National Assessment of Knowledge in Mathematics who were awarded the following final grades in mathematics: insufficient (1), sufficient (2), good (3), very good (4), or excellent (5). The statistical data on the National Assessments of Knowledge in Mathematics for the 6th and the 9th year of schooling (number of pupils who took the National Assessment of Knowledge, the average attainment, the standard deviation, the reliability index and the difficulty index) were obtained from the Annual Reports published on the National Examinations Centre website.

The National Examinations Centre obtains the data on final school grades in individual subjects from the Ministry of Education, Science, and Sport each year at the end of July. Thus, the situation after the remedial exams at the end of the school year has not been taken into account. For the 9th year of schooling, the National Examinations Centre supplies data on the final school grades from year 2009 onwards, and for the 6th year of schooling from year 2016 onwards (Cankar et al., 2020, p. 50); however, the quality of data depends on how accurately the schools entered the data. This is the reason why we find differences in the number of pupils who took the National Assessment of Knowledge in Mathematics and the number of pupils with final grades. As 2016 was the first year that data were collected for the 6th year of schooling, the difference is relatively significant.

Data Processing

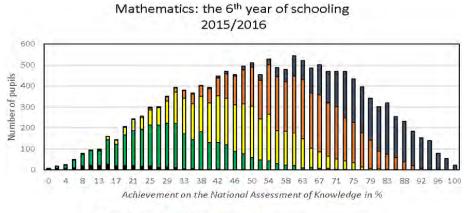
Pupils' attainments on the National Assessment of Knowledge are presented in percent form. Thus in order to compare them with their final mathematics grades which are present in the students' report cards on scale from 1 to 5 where 1 is sufficient and 5 is excellent. It is necessary to establish appropriate methodology, which would permit us to convert percentages from the National Assessment of Knowledge into 1-5 school grades. These grades are referred to as "hypothetical grades". The same methodology was used to calculate the hypothetical grade that had been used by the Subject Testing Committee for Mathematics in the school years from 2001/2002 to 2004/2005, and in the study from 2018 (Felda, 2018). In the school years from 2001/2002 to 2004/2005, the Subject Testing Committee for Mathematics retained the same distribution of grades as were awarded to national examination pupils by their mathematics teachers at the end of the 9th year of schooling (RIC, 2005). Let us provide an example for the 9th year of schooling, the 2018/19 school year: out of 16,668 9th year of schooling pupils, 187 pupils (1.1%) received the final grade "insufficient". The same percentage of pupils (i.e., 1.1%) would be awarded the hypothetical grade "insufficient" on the National Assessment of Knowledge. 183 pupils attained 10% of all points or less, while 291 pupils attained 12% of all points or less. In order to retain the same percentage, a correction of benchmarks was made. For the number of pupils with the final grade "insufficient" in mathematics, i.e., 187, it is true that 183<187<291. A moderated version of benchmarks was chosen and the hypothetical grade "insufficient" was assigned to pupils who had scored 10% of all points or less. The lower benchmarks were set for other hypothetical grades in a similar way.

The hypothetical grades were then compared with the final grades obtained in mathematics class. Each student had then two grades: his/her final grade (i.e. the teacher's grade) and the hypothetical grade that was obtained from his/her attainment on the National Assessment of Knowledge. The two grades are not necessarily the same, thus we were interested is studying possible differences between those two grades. Pearson's correlation coefficient was used to calculate the correlation between the pupil's final grade in mathematics and the attainment on the National Assessment of Knowledge in Mathematics, and between each pupil's final grade in mathematics and his hypothetical grade. A *t*-test was used to determine whether the difference between the final and hypothetical grades was statistically significant. Individual variables were analysed using the basic descriptive statistics: arithmetic mean, standard deviation (SD), standard error, and median. All the data was analysed with the aid of the SPSS version 27.0 statistical program.

RESULTS AND INTERPRETATION

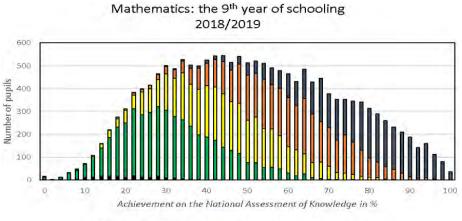
Final Grade in Mathematics

A detailed relationship between final grades in mathematics and attainments on the National Examination of Knowledge for the 6th and the 9th year of schooling are presented in **Figure 1** and **Figure 2**, and for both years they are similar. For example, among 510 students in the 6th year of schooling who scored 50% at the National Assessment of Knowledge, 20 students were awarded grade "excellent" in their report cards, 186 students were awarded grade "very good", 244 received grade "good", 60 received grade "sufficient", and no students received grade "insufficient".



[■]insufficient ■sufficient □good ■very good ■excellent

Figure 1. Distribution between final grades in mathematics and attainments on the National Assessment of Knowledge in the 6th year of schooling (2015/16 school year)



■insufficient ■sufficient ■good ■very good ■excellent

Figure 2. Distribution between final grades in mathematics and attainments on the National Assessment of Knowledge in the 9th year of schooling (2018/19 school year)

A more detailed analysis leads us to the following conclusions. The smallest deviation in the percentage of pupils with regard to the final grade is identified in the final grade "excellent"; 22.1% of pupils received this grade in the 2015/2016 school year, and 22.3% in the 2018/2019 school year, which makes up a difference of 0.2 percentage points. The biggest increase is identified in the final grade "sufficient", i.e., 6.7 percentage points. 20.6% of the 6th year of schooling pupils received the final grade "sufficient" in the 2015/2016 school year, and as many as 27.3% of the 9th year of schooling pupils in the 2018/2019 school year. The biggest drop in the percentage of final grades can be seen in the final grade "very good", where 29.6% of the 6th year of schooling pupils received this grade, and 24.8% in the 9th year of schooling, which makes up a difference of 4.8 percentage points. It has been established that the percentage of pupils with regard to a specific final grade in the same generation of pupils in the examined school years (2015/2016 and 2018/2019) is rather constant in the final grades "insufficient", "good" and "excellent", while the biggest difference is evident in the final grades "sufficient" and "very good" (**Figure 3**).

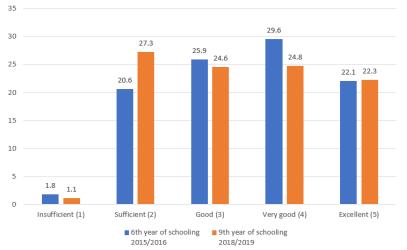


Figure 3. Percentage of pupils of the same generation with the final grade in mathematics in the 6th and the 9th year of schooling

Distribution of attainments on the National Assessment of Knowledge in Mathematics with regard to the final grades in mathematics

The relationship between the final grades and the attainments on the National Assessment of Knowledge can be presented by the distribution of attainments in each group of students with the same final grade in mathematics (**Figure 4**). For example, students in the 6th year of schooling with the final grade "good" scored from 4% to 94% on the National Assessment of Knowledge. Similarly, students with the final grade "good" in the 9th year of schooling scored from 0% to a 94% on the National Assessment of Knowledge.



Figure 4. Distribution of attainments on the National Assessment of Knowledge in Mathematics in relation to final school grades

Both in the 6th and the 9th year of schooling, the pupils of the same generation with a specific final grade in mathematics achieved dispersed percentage on the National Assessment of Knowledge in Mathematics. Thus, in the 6th and in the 9th year of schooling, the pupils with the final grade "insufficient" attained 0% on the National Assessment of Knowledge. However, in the 9th year of schooling pupils attained higher results (64%) compared to the 6th year of schooling pupils (52%), or 12 percentage points higher. The pupils with the final grade "sufficient" also scored 0%, but the difference in the highest attainments is only 4 percentage points (82% in the 6th year of schooling, 78% in the 9th year of schooling and in the 9th year of schooling; the lowest percentage (0%) was attained by the 9th year of schooling pupils, while the 6th year of schooling pupils attained 4%. The pupils with the final grade "very good" attained the same lowest percentage as the pupils with the final grade "good". The highest attainment in the 6th year of schooling was 98%, while in the 9th year of schooling it was two percentage points lower. The pupils with the final grade "excellent" scored from 0% to 100% in the 6th year of schooling, while the 9th year of schooling pupils had the lowest attainment 22%, and the highest of 100%.

It can be concluded from the data that the 6th year of schooling pupils who scored 0% on the National Assessment of Knowledge received final grades "insufficient", "sufficient", and "excellent" in class. The 9th year of schooling pupils who scored 0% on the National Assessment of Knowledge received the whole range of final grades, apart from the final grade "excellent". The 6th year of schooling and the 9th year of schooling pupils with the final grade "excellent" attained 100% on the National Assessment of Knowledge, leading us to conclude that pupils with the final grade "excellent" achieve the highest levels on the National Assessment of Knowledge in Mathematics. It has also been established that the attainment ranges are relatively large.

In the detailed study of the data we observed major discrepancies between the attainments of a small number of students on the National Assessment of Knowledge and their final grades awarded by classroom assessment. The attainments of these, mostly individual students on the National Assessment of Knowledge were significantly lower than the attainments of the majority of students with the same final grade. Let us illustrate this by students in the 6th year of schooling with the final grade »excellent«. Only one of this group of students scored 0% on the National Assessment of Knowledge, while the next two students scored 25%, and then the attainments continue from 29% onwards (**Table 2**). In **Table 2**, we illustrate the number of student with the excellent final grade that gained same initial scores on the national Assessment of Knowledge, up to 35% since from 29% onwards these is no attainment with no student attaining it. We refer to these individual students with a single lower achievement as "loners". Eliminating these "loners" from the study would have resulted in the need for adjusting the lowest benchmark in determining the hypothetical grade.

	0			A	chieve	ement o	on the	Natior	nal Ass	sessme	ent of I	Knowl	edge ii	n perce	ent			
Final grade	0	2	4	6	8	10	13	15	17	19	21	23	25	27	29	31	33	35
Excellent (5)	1	0	0	0	0	0	0	0	0	0	0	0	2	0	1	1	1	3

Table 2. Showing the "loners"	" in the 2015/16 school	year, the 6 th year of schooling
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The average value of attainments on the National Assessment of Knowledge in Mathematics with regard to the final grades in mathematics

For further comparisons, we provide some statistical data regarding the National Assessment of Knowledge in Mathematics (**Table 3**), which show that the standard deviation and the difficulty index in both years are comparable.

Table 3. Basic statistical data on the National Assessment of Knowledge in Mathematics

School year	Year	Average achievement on the National Assessment of Knowledge	Standard deviation	Difficulty index	Reliability index
2015/2016	6 th year of schooling	53.79	21.19	0.54	0.93
2018/2019	9 th year of schooling	51.05	21.51	0.51	0.93

We were then interested in the average value of attainment on the National Assessment of Knowledge with regard to the final grade, which was calculated using the weighted arithmetic mean. According to the calculated average value of attainment on the National Assessment of Knowledge in Mathematics with regard to the final grade in mathematics, it has been established that the average values for the same generation are similar, even though the pupils were in different years of schooling (**Table 4**). The biggest deviation can be seen in grades "insufficient" and "very good" (both by 2.6 percentage points). The smallest deviation can be seen in the final grade "sufficient", i.e., by 0.4 percentage points. The average attainment of pupils with the final grade "very good" and "excellent" is above the average percentage on the National Assessment of Knowledge at the state level, whereas the average attainment of pupils with the final grade "good", "sufficient", and "insufficient" is below the state average.

Table 4. Average attainments on the National Assessment of Knowledge in Mathematics with regard to the final grade in mathematics

		Average attainment on the National Assessment of Knowledge in Mathematics (%)						
		Insufficient (1)	Sufficient (2)	Good (3)	Very good (4)	Excellent (5)		
2015/2016	6th year of schooling	18.9	30.8	46.4	61.4	77.3		
2018/2019	9th year of schooling	21.5	31.2	44.6	58.8	75.4		

Lower Benchmarks in Determining the Hypothetical Grade on the National Assessment of Knowledge

Between the 2001/02 and 2004/05 school years, the attainments on the National Assessments of Knowledge in Mathematics in the 9th year of schooling were converted into a grade (excellent, very good, good, sufficient, insufficient), which then influenced the final grade in the subject in the 9th year of schooling. In the above-mentioned years, the Subject Testing Committee for Mathematics emphasised that the distribution of grades awarded to pupils by mathematics teachers at the end of the 9th year of schooling was taken into account when determining the grades (RIC, 2005). Since 2005/06, the attainments on the National Assessments of Knowledge have no longer been converted into grades and serve mostly for formative purposes. Our study examines how the grades would have been distributed if the methodology used by the Subject Testing Committee for Mathematics in the school years from 2001/02 to 2004/05 had been used in the school years 2015/2016 and 2018/2019; this methodology has been described in the section "methodology". To sum up, the same percentage of pupils that received grade "insufficient" as their final grade in school, also received the hypothetical grade "insufficient" on the National Assessment of Knowledge; the same percentage of pupils that received grade "sufficient" as their final grade in school, also received the hypothetical grade "sufficient" on the National Assessment of Knowledge, and so on. We would like to emphasise at this point once again that the percentage of students attaining a specific final grade corresponds to the percentage of students attaining the same specific hypothetical grade. Nevertheless, students' final and hypothetical grades might be different. This procedure was carried out for each school year separately. This way, the lower benchmarks for each hypothetical grade, expressed as a percentage, was determined. As we might see in Table 5, the same generation of pupils got a sufficient hypothetical grade in 2015/2016 school year by obtaining at least 13% in the National Assessment of Knowledge, while the same generation of pupils got a sufficient hypothetical grade at a lower benchmark, i.e., 12%. Similarly, very good an excellent benchmark where lowered, while the good benchmark increased.

Table 5. Lower benchmarks in determining the hypothetical grade on the National Assessment of Knowledge

There all all and de	Lower benchmark in % for	a specific hypothetical grade
Hypothetical grade	6 th year of schooling 2015/2016	9 th year of schooling 2018/2019
Sufficient (2)	13	12
Good (3)	35	36
Very good (4)	52	50
Excellent (5)	71	66

Correlation Between the Final Grades in Mathematics and the Attainments on the National Assessment of Knowledge in Mathematics

The study examined the same generation of pupils to determine the correlation between a pupil's final grade in mathematics and the attainment on the National Assessment of Knowledge in Mathematics, for each year of schooling separately. For this purpose, Pearson's correlation coefficient was calculated. For the 6th year of schooling it yielded r=0.796 (p<0.001), and for 9th year of schooling r=0.770 (p<0.001), which indicates a positive, rather strong, and statistically significant correlation (**Table 6**). It has been established that pupils with higher final grades in mathematics also have higher attainments on the National Assessment of Knowledge in Mathematics, in the 6th and 9th year of schooling alike.

Table 6. Average final grade in mathematics, average attainments on the National Assessment of Knowledge in Mathematics and Pearson's correlation coefficient

	Year -School year					
	6th year of schooling-2015/2016	9th year of schooling-2019/2019				
Average final grade in mathematics	M=3.49	M=3.40				
	SD=1.10	SD=1.14				
Pearson's correlation coefficient between average final grade	r=0.796	r=0.770				
and attainment on the National Assessment of Knowledge	(p<0.001)	(p<0.001)				

In the 6th and the 9th year of schooling, the hypothetical grades were defined and compared to the final grades in mathematics. The hypothetical grade average in the 6th year of schooling is M=3.80 (SD=1.22); in the 9th year of schooling it is M=3.52 (SD=1.18). The average difference between the actual grades and the hypothetical ones in the 6th year of schooling is–0.304 (SE=0.007), and in the 9th year of schooling–0.119 (SE=0.006). The t-test for dependent samples has shown that in the 6th year of schooling this difference is statistically significant (t(14981)=-45.4; p<0.001), which also applies to the 9th year of schooling (t(16667)=-18.7; p<0.001); therefore, it can be concluded that the hypothetical grades are statistically higher than those entered on the pupils' school-leaving certificates.

Comparison of Hypothetical and Final Grades

The study also examines what percentage of pupils would have been given a hypothetical grade higher than the final grade, how many would have been given the same grade, and how many a lower grade than the actual final one (**Figure 5**). The comparison between the 6th and the 9th years of schooling of the same generation of pupils shows that the percentage of pupils who would have been given a hypothetical grade higher than, lower than or the same as the final grade is approximately the same. Just over half of the pupils (54.9% and 53.3%, respectively) would have been given a hypothetical grade matching the final grade. What might cause our concern is the finding that slightly less than a third of the pupils (26.6% and 28.2%, respectively) received a final grade that is lower than the hypothetical grade, while slightly less than a fifth (18.5%) of the pupils received a final grade that is higher than the hypothetical grade.

After converting the attainment (expressed as %) on the National Assessment of Knowledge in Mathematics into hypothetical grades that the pupils would have been given if their attainments on the National Assessments of Knowledge in Mathematics were taken into account, we examined whether there was a correlation between these hypothetical grades and the pupils' final grades. Pearson's correlation coefficient indicates a rather strong, positive, and statistically significant correlation between the variables in the 6^{th} year of schooling (r=0.756; p<0.001) and the 9^{th} year of schooling (r=0.753; p<0.001). In this case too, it can be said that the pupils with higher school grades would have been given higher hypothetical grades.

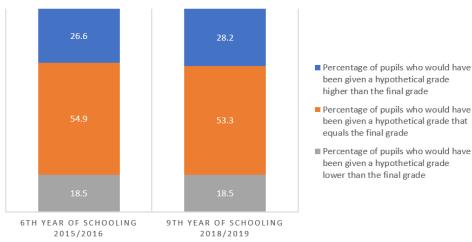


Figure 5. Percentage of pupils with specific hypothetical grades with regard to the final grades

DISCUSSION

One of the quality standards for pupils' attainments is that the pupils progress in the knowledge and skills defined in the national curriculum documents and attain the set goals and objectives. One indicator that demonstrates the achievement of this standard is that the pupils demonstrate quality knowledge and knowledge progression (Cankar et al., 2019). Monitoring the mathematical knowledge of the same generation at the end of each educational period during the course of their schooling may provide more insights into how knowledge attainment changes, how the trends in final grades change, and so on. By a longitudinal survey, both teachers at the school level and policymakers in education at the state level can monitor how well the pupils attain higher levels of knowledge (at different taxonomy levels), and the knowledge trends in individual learning areas of mathematical knowledge (geometry, arithmetic, etc.). Such an analysis provides an insight into the perceived strengths and weaknesses of a generation of pupils and into the mathematical knowledge of pupils advancing to the next year of schooling (based on the attainments on the National Assessments of Knowledge in Mathematics and the final grades). In addition, the knowledge trends of several generations at individual schools can be monitored, as well as the progress in pupils' knowledge compared to other pupils in Slovenia (Cankar et al., 2019).

The purpose of this study was to compare the attainments on the National Assessment of Knowledge in Mathematics and the final grades in mathematics in the same generation of pupils at an interval of three years of schooling. This paper does not focus primarily on added value, even though measuring added value in the knowledge of the same generation is important for teaching practice and for educational policies. As a result of a growing emphasis on improving pupils' achievements in mathematics, we expect a growing demand for progress monitoring tools, by which schools will be able to assess the pupils' current learning, predict their future performance, and support the teacher's efforts to prepare effective instruction (Foegen et al. 2007, p. 138).

Comparisons between the attainments in external and internal summative knowledge assessment is challenging because the internal grades are obtained in various ways and over a longer period of time, and each assessment process covers a specific subject matter. On the other hand, external assessment is a one-off event that tests the knowledge developed in several years of schooling. Internal assessment differs from school to school, as has been demonstrated by Martínez et al. (2009), who determined that teachers do not assess the pupils' performance in absolute terms but rather compare it with the performance of other pupils' at the school, and that the teachers might adapt their assessment to certain pupils, perhaps on account of the detected differences regarding their educational needs and/or abilities.

External knowledge assessments, both international and national, have a potential to improve internal assessment by raising teachers' awareness of their classroom assessment procedures, selection of contents and levels of complexity (e.g., taxonomic levels), evaluating outcomes and processes against the established criteria. Moreover, external assessments have a noticeable impact on pupils' attainments on international knowledge assessments. Wößmann (2005) has confirmed that regular standardised tests during schooling have additional positive effects on pupils' performance on external knowledge assessments. Jürges et al. (2005) have determined that the impact of national external examinations on pupils' attainments equals approximately one-third of the grade equivalent, while another international study (Piopiunik et al., 2012) has found that the impact of external examinations between countries may even exceed the entire grade equivalent.

Our study has indicated that approximately half of the pupils would have been given the same grade in the external and internal knowledge assessment, while the other half would have been given a higher or a lower grade. The pupils who receive a lower or a higher grade on the external knowledge assessment compared to the one on the internal knowledge assessment are perhaps deprived of a realistic feedback about their knowledge. Receiving a realistic feedback about the pupils' knowledge would help pupils to improve their learning and, last but not least, the teachers' their teaching practice.

The findings of the present research could contribute to balancing the (too) high and (too) low final (school) grades (Semen, 2010). External assessments at the state level could contribute to a fairer internal grading system and consequently to fairer final grades by incorporating more demanding tasks in the internal assessment. The prerequisite for such assessment is that teachers strive towards attaining higher taxonomy levels in their teaching practice. We believe that this study also has practical effects, as it forms a basis for further research and comparisons. Moreover, the teachers at individual schools are able to compare the whole school and individual pupils' attainments in mathematics with the results demonstrated in the study, and plan their future teaching practice.

Possibilities for Further Research and Research Limitations

This study could potentially initiate further relevant surveys. To our knowledge, in Slovenia no comparison has been made regarding the mathematical knowledge of the same generation; for this reason, it would be advised to direct future research towards a longitudinal comparison of attainments in the mathematical knowledge of the same generations. This study does not analyse the types of knowledge examined by the National Assessment of Knowledge which overlap with the mathematics curriculum (arithmetic and algebra, geometry and measurement, and other areas), nor does it monitor the progress in attainments at different taxonomy levels. Further research possibilities regarding the comparison of final grades and hypothetical grades on the National Assessments of Knowledge could be to study any potentially demonstrated gender differences, seeing that the figures comparing the gender-based distribution of attainments of male and female pupils presented in the "Annual Reports on the Implementation of the National Assessment of Knowledge" do not show any significant deviations. Further research could also focus on determining the differences in attainments between various regions in Slovenia taking into account the comparison of final and hypothetical grades, and considering the fact that certain regions are constantly at the top of the scale while others remain at the bottom of the scale, as presented in the "Annual Reports on the Implementation of the National Assessment of Knowledge". Additionally, a comparison of differences between final and hypothetical grades based on the pupils' socioeconomic status, which have been confirmed by previous research (Nolimal & Cankar, 2015; Žakelj & Ivanuš-Grmek, 2011), is also possible.

The SARS-CoV-2 epidemic with its consequences presents a significant limitation to our study. In the 2019/2020 school year, the National Assessment of Knowledge was not administered, and this limited this study to discussing a single generation of pupils, due to the fact that the data on final grades in mathematics in the 6th year of schooling have been available only for years from 2016 onwards. As a result, when evaluating the research and drawing conclusions it should be observed that only a single

generation is discussed. In light of the SARS-CoV-2 epidemic, a comparison between the generation of 2016-2019 and the generation of 2018-2021 would make an interesting topic for future research. As the trial National Assessment of Knowledge has been reintroduced for the 3rd year of schooling pupils, it would be interesting to monitor the same generation of pupils from the 3rd to the 9th year of schooling.

Another research limitation that should be mentioned is that a correlation between the final grades in mathematics in the 6th and the 9th year of schooling and between the hypothetical grades in the 6th and the 9th year of schooling could not be determined because the samples differed. Future research should consider only the data on those individuals (pupils) who take the National Assessment of Knowledge in both years and whose final grades in mathematics are specified for both years. This also explains the reason why we were unable to compare the attainments of the same students in this study.

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