Five Freedoms That All Math Explorers Should Enjoy

By Francis Su

I thought I had chosen the right story for the occasion. Assembled before me was a group of eager Latino and African American children from an impoverished Los Angeles neighborhood. On this Saturday morning, I was serving with a program in which volunteers read books to kids. I had selected a delightfully illustrated picture book about going to the beach, and I thought it would be received well. But after reading a few pages in a most spirited voice, I could tell that the kids were not sharing my enthusiasm.

I paused, and asked: "How many of you have ever been to the beach?"

To my surprise—though this part of LA is just 15 miles from the ocean—only one of the eight children raised a hand. Wasn't going to the beach a quintessentially Californian thing to do?

Upon reflection, I realized that in a low-income neighborhood, parents often work multiple jobs to make ends meet, so they may not have the time or the resources to get to the coast. And when an African American friend of mine heard this story, he explained how African American people were systematically excluded from beaches and swimming pools because of Jim Crow segregation, not only in the South but all over the United States, including Los Angeles. I was completely unaware of this.

Alas, I had missed important historical, cultural, and economic contexts that made the beach inaccessible to these kids. It made me reflect on how I motivate my students to pursue mathematics. What contexts am I missing that I should be more aware of? What are the primary experiences that have shaped or are shaping them, and do those present obstacles to or opportunities for learning math? What are the unique strengths they bring to mathematical pursuits? And in what ways do mathematical spaces, like beaches, say "Open to all" but still feel restricted?

For me, the beach became a metaphor for various freedoms that are hallmarks of doing mathematics—freedoms delivered to some and denied to others. Just as they should be manifest at every beach, the freedoms we'll discuss should be present in every mathematical space. They are part of the allure of doing mathematics for those fortunate enough to experience math as it should be experienced. Conversely, the denial of those freedoms contributes to the fear and anxiety that many people feel toward math.

Freedom is a basic human desire. It is a central idea behind historic human rights movements and a sign of human flourishing. We seek freedom in big ways—think of the Four Freedoms that President Franklin D. Roosevelt said all people should have: freedom of speech, freedom of religion, freedom from want, and freedom from fear. We also seek freedom in small ways that can feel just as important, such as freedom with our time or freedom to make our own decisions.

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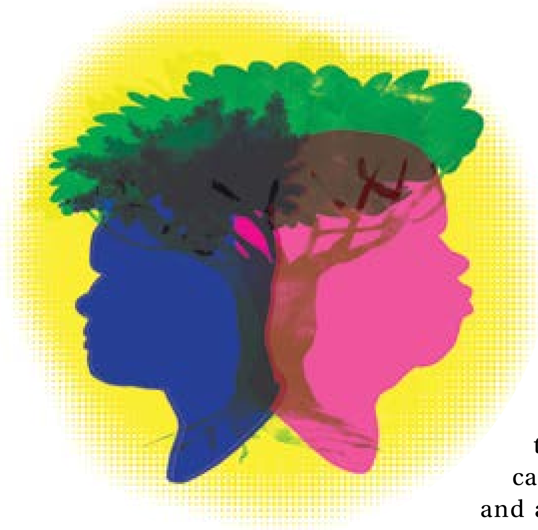
ILLUSTRATIONS BY JOHN JAY CABUAY



I want to highlight five freedoms that are central to doing mathematics: the freedom of knowledge, the freedom to explore, the freedom of understanding, the freedom to imagine, and the freedom of welcome. As a math explorer, you should be aware of these freedoms so that you can claim them for yourself and aspire to fulfill them for everyone you encounter.

The Freedom of Knowledge

The freedom of knowledge is easy to underestimate, because if you have this freedom, you take it for granted, and if you don't have it, you are completely unaware of what you're missing. You have to know about the beach and know its many options for recreation—how to swim, surf, dive, tan, picnic, play volleyball, etc.—if you are to experience its freedom. These seem obvious to anyone who's been there, but if you are like the kids who didn't know about the beach, either because you were never told or because someone prevented you from going, you will not know the joys that await there.

Within mathematics, the freedom of knowledge is also fundamental. If you know just one method for attacking problems, you are limited, because that method may not work well for your particular problem. But if you have several strategies, you have the freedom to choose the option that is the simplest or most enlightening. Mathematics equips you to look for multiple ways to solve problems.

The mathematician Art Benjamin is a human calculator—he can multiply five-digit numbers in his head. While this sounds impressive, the mathematical fun for him is not in the calculation. The fun is in thinking of multiple strategies for doing a calculation easily and choosing the one that works best.[1] I'm not as practiced as he is, but I also rely on such skills to do calculations. For instance, if I want to multiply 33×27 in my head, I can think of four different ways to do it.

I can do it the "standard" way, which means taking thirty 27s and three 27s and adding them. That's $(30 \times 27) + (3 \times 27) = 810 + 81 = 891$. I don't find it so easy here to hold all the intermediate calculations in my head.

Or I can do it by factoring 27 as 3×9 and first multiplying 33 by 3, then multiplying that product by 9. That's $(33 \times 3) \times 9$, which is $99 \times 9 = (100 \times 9) - (1 \times 9) = 900 - 9 = 891$. This seems easier than the standard way.

Or I can factor 33 as 3×11 and first multiply 27 by 3 (which is 81) and then multiply that product by 11. That's 81×11, which is easy if I know the shortcut for multiplying by 11: take the digits 8 and 1 and insert their sum, 9, in between to get 891.*

Or I can see that this algebra identity might be helpful: $(x - y)(x + y) = x^2 - y^2$. So if I recognize $27 = 30 - 3$ and $33 = 30 + 3$, then the desired product of 27×33 is just $30^2 - 3^2 = 900 - 9 = 891$.

If someone asks me to do this calculation quickly, I will look at the quiver of arrows I have and choose the best arrow to attack this problem. For me, that would be the last way. The freedom of knowledge gives us a large quiver.

The Freedom to Explore

A second basic freedom that should be present in mathematical learning is *the freedom to explore*. Just like the wide expansiveness of the beach—with its shells, its sounds, and the treasure we fancy buried underneath—the learning of mathematics should be a place for exploration, so as to stimulate creativity, imagination, and enchantment. But some teaching styles don't offer this kind of freedom. I think of the difference between how my mom and my dad taught me math, a study in contrasts between obligation and exploration.

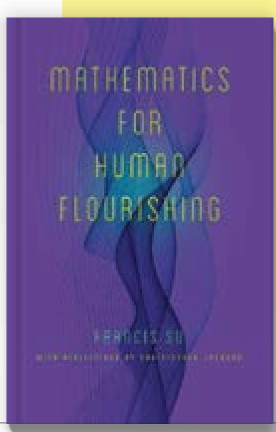
My parents wanted me to learn math at a young age, so even before I went to school, my father would teach me numbers and arithmetic. Since he was busy with his own work, he would make up long worksheets of addition problems to keep me occupied. I did them as a dutiful kid, but didn't find them very much fun. "Do this one over," he would say. "You can't go out to play until you get every one right."

My father's approach was a one-way transmission of information. He showed me what to do, but left me to myself to do worksheets. I followed the rules he taught me for arithmetic, often without understanding. I learned how to add numbers bigger than 10 by "carrying," but I didn't have any idea what I was doing. I was following recipes. And my father's praise and rewards were always connected to my performance. Now, my dad, in all fairness, was a good dad, but in an immigrant Asian American family, I could

*This shortcut for multiplying by 11 will require a "carry" if the sum of the digits is 10 or more. For instance, to compute 75×11, you should add 7 and 5 to get 12, put the 2 between the 7 and the 5, and then carry the 1 by adding it to the 7, to get 8. Thus, the answer is 825. If you know some algebra, you can use it to show why the shortcut works: the number $10a + b$ is the number with digits a and b. Then $(10a + b) \times 11 = 110a + 11b = 100a + 10(a + b) + b$. This last expression does indeed suggest adding the two digits and putting their sum between them.

Mathematics for Human Flourishing

In *Mathematics for Human Flourishing*, from which this article is drawn, Francis Su shares the beauty, creativity, and power of mathematics with everyone—even those of us who have found the subject intimidating. As the first president of the Mathematical Association of America who is not white (all prior presidents were white males), Su is determined to make mathematics welcoming. Knowing that students from marginalized communities often do not have access to advanced courses and enter college feeling unprepared, Su writes, "I remind my students that grades are a measure of progress, not a measure of promise." To learn more about the book, visit francissu.com/flourishing.



be shamed for turning in anything less than a perfect paper. That's not freedom.

By contrast, my mother's approach was relational. We played games that encouraged numerical thinking and pattern recognition. She sat with me, and together we read books about counting. And the books we read were also relational—full of wonder and delight. They invited more questions. Like: Why does that Dr. Seuss character have eleven fingers? It's not even five fingers on one hand and six fingers on the other, as you might expect, but rather four and seven! This fanciful strangeness invited further imagination. With my mother, I had the freedom to explore, and the freedom to ask questions, the freedom to think ridiculous thoughts. Questions and fanciful thoughts were praised.

This freedom is also at the heart of mathematics at higher levels of learning. As a high school senior, I attended a lecture for prospective students at the University of Texas at Austin. The topic was infinity, and the speaker was the math professor Michael Starbird. His lecture style was different than anything I had experienced in high school. It was highly interactive, and he was constantly asking questions of the audience—as if to invite us to be explorers together. I'd never before been in a room with 300 people where everyone was engaged and paying attention. This kind of interaction exemplifies a teaching style known as *active learning*. I left that lecture thinking: wow, if every class here is like that, college is going to be a lot of fun.

So I enrolled at Texas. Having placed out of calculus, and thinking I was "good" at math, I jumped into the subsequent course. It was in a traditional lecture style—this meant that the professor lectured without much interaction, and we took notes. On the first day, he began talking about matrices, a topic that I had never seen before and that wasn't on the list of prerequisites for the course. (A matrix is an array of numbers, and usually discussed in the class *after* this one.) And then he started *exponentiating* matrices, which means he took the number *e* and wrote an array of numbers as an exponent. For me, that was category confusion: like asking me to use an avocado to brush my teeth or putting my cat in my wallet.

I looked around and assumed that everyone else knew what was going on. I was intimidated, fearful of asking any questions because no one else was and the professor wasn't inviting them. Symbols were flying by as if a broken keyboard were stuck on the Symbol font. I dutifully took notes, but I had no idea what I was writing down. And that was just the first day of class. All semester I struggled to keep up, my understanding always two weeks behind. That was not fast enough to help me on my homework and exams, where I was often guessing at solutions I didn't understand. I was a hamster on a wheel powered by someone else, fearful that any mistake would mean I'd tumble off and do poorly in my first math class in college. This was not freedom.

The Freedom of Understanding

This story highlights a third kind of freedom that mathematics offers: *the freedom of understanding*. I was learning that if you go through life pretending you understand, you will always be limited by the things you don't understand. You will continue to feel like an impostor, believing that everyone else knows what is going on and you're the one who doesn't belong. By contrast, true understanding means you have to devote fewer brain cells to remembering formulas and procedures, because everything fits together meaningfully. Math education should promote, rather than inhibit, this freedom, but as learners we must strive for deep understanding even when our education isn't promoting it. This is where the hard work is.

After that first course, I almost didn't become a math major. But I decided to give it one more try. I took a course with a professor who was much more interactive and approachable, and I began to feel more confident again. Then, the next year, I took a class with Starbird. The class topic was topology—the mathematics of stretching things. Or, a little more accurately, it's the study of properties of geometric objects that don't change when you continuously deform the objects. For that reason, it is sometimes called "rubber-sheet geometry." This meant that drawing pictures was very important in this course, while numbers were almost nonexistent!

To my delight, Starbird was teaching in an "inquiry-based learning" format. There were no lectures. Instead, we were given a list of theorems and provided the challenge of discovering their proofs for ourselves. Through guided interaction with him and with one another, we learned how to present our ideas and subject them to constructive scrutiny by peers. But the underlying strength of the course was how the professor used this format to encourage a different classroom culture. He created an environment where questions were praised and unusual ideas were welcome. He was giving us the freedom to explore.

The learning of mathematics should be a place for exploration, so as to stimulate creativity, imagination, and enchantment.

Relationships with each other were central to our explorations. We learned in this environment how to proclaim, "My proof is wrong," without shame or judgment. Indeed, a wrong proof was always a point of delight, because it meant we were seeing something subtle, and it was a springboard to further investigation.

I've seen professors foster this kind of culture in more traditional lecture formats too, using active learning methods. In such classes, every day can be like a Dr. Seuss poem, filled with surprise and wonder, where the fanciful is celebrated.

The Freedom to Imagine

A fourth freedom present in mathematics is *the freedom to imagine*. If exploration is searching for what's already there, imagination is constructing ideas that are new, or at least new to you. Every child who has ever built a sand castle on the beach knows the limitless potential of a bucket of sand. Similarly, Georg Cantor, whose groundbreaking work in the late 19th century gave us the first clear picture of the nature of infinity, said, "The *essence* of *mathematics* lies precisely in its *freedom*."[2] He was saying that, unlike in the sciences, the subjects of study in mathematics are not necessarily tied to particular physical objects, and therefore mathematicians are not constrained like other scientists in what they can study. A math explorer can use her imagination to build any mathematical castle she wants.

How much more fun could mathematical learning be if, at every stage, we had opportunities to use our imagination?

My topology class taught the practice of imagination. Topology, as I mentioned earlier, is the study of properties of geometric objects that do not change when you stretch the objects in a continuous way. If I take an object and deform it without introducing or taking away "holes," I haven't changed it topologically. So a football and a basketball are the same in topology, because one shape can be deformed into the other. On the other hand, a doughnut is not the same as a football in topology, because you can't turn a football into a doughnut without poking a hole in it. Topology is an entertaining subject because we get to construct all sorts of groovy shapes by cutting things apart, gluing things together, or stretching things in weird ways.

How much more fun could mathematical learning be if, at every stage, we had opportunities to use our imagination? You don't need to be doing advanced mathematics to do this. In arithmetic, we can try to construct numbers with fanciful properties. What's the smallest number divisible by all the digits in your date of birth? Can you find 10 numbers in a row that are not prime? In geometry, we can design our own patterns and explore their geometric natures. What kinds of symmetry exist in the patterns you like? In statistics, we can take a data set and come up with creative ways to visualize it. Which ones have the best features? If you're learning mathematics from a dull textbook, see if you can modify the questions so that they increase your imaginative capabilities. In doing so, you are exercising your freedom to imagine.

The Freedom of Welcome

Unfortunately, the prior freedoms—the freedoms of knowledge and of understanding, the freedoms to explore and to imagine—are difficult to secure without the last freedom, which is *the freedom of welcome*. This is a freedom missing from many mathematical communities.

Beaches, as I learned, have a historical association that is exclusionary, which keeps people, even today, from enjoying those spaces. Imagine this scenario at the beach. There's no longer a sign saying that you aren't allowed, but you don't come very much, because your parents never came at all. There's no one chasing you out, but you get sideways stares. People question whether you meant to go to *another* beach. Some think you're the service staff at the beach showers and ask for more paper towels in the restroom. Others avert their gaze and clutch their children tightly when you walk past. People make up seemingly arbitrary rules for you, telling you that you can't cook *that* food for your picnic, or play *that* game on this beach. You go instead to the volleyball courts for a pickup game, but no one invites you to play. They don't expect that you know or will want to learn the game. The beach may be open to you, but you aren't really welcome.

Sadly, mathematical communities can be like that. We say we value diversity, but there are exclusionary undercurrents. Consider these examples.

Your name is Alejandra, and you've noticed that in every math textbook since grade school, the names in generic examples are all white male names. In middle school, you come up with novel ways to solve problems, but your teacher never seems interested in solutions other than the ones she knows. Your high school math teacher is lecturing, and he makes eye contact only with boys.

You place into an advanced math class in college and find the work challenging, but the professor encourages you to drop to a less advanced class rather than encouraging you to continue. You're a college athlete with a demanding practice schedule every afternoon, but the professor makes himself available only for afternoon appointments. A professor calls a proof "trivial" and "obvious"; you think there's something wrong with you because it isn't trivial and obvious to you. He notes that "students like you don't often do well in this major." A math competition is going to take place; the organizer invites every math student but you to the practices. You come from a culture that prizes community and storytelling, yet your math professors talk about math as if it's completely devoid of any history or culture, and all the assigned work is to be done alone.

You decide to go to graduate school in math, but there are few women in the program and no Latina students like yourself, and certainly no Latina women on the faculty. No one knows how to pronounce your name; they call you "Alex" without your permission. The graduate student lounge in your department has no art or plants or color; it feels sterile, and you certainly don't want to hang out there. The other students seem very competitive, and quick to point out others' mathematical errors in unsupportive ways. Your advisor seems uninterested in your life outside work, even when you signal that you're struggling with child care. Yes,

you've made a decision to start a family in grad school, but the administration seems inflexible in handling that.

You become a mathematician, and are thrilled to get a job at a college where teaching is valued, but your friends at research universities ask with pity, "Are you happy there?" When you go to conferences, your small stature and dark complexion mean that you are often mistaken for "the help" at conference hotels. When you publish papers collaboratively, people always think that the other person did more of the work. So you feel pressure to publish papers alone. You love all the things that mathematics offers, but it doesn't feel like it's worth *this*.

Taken together, Alejandra's experiences can feel life draining and oppressive, even though the people involved may have had the best intentions and been completely unaware of what she was going through. Collectively, they are a coercive use of power. Alejandra does not have the freedom of welcome. You might wonder why she hangs in there at all.

To be welcoming means more than just allowing people to coexist. It means extending an invitation of welcome—to say, "You belong," and follow it up with supportive actions. It means maintaining high expectations and providing high support.

Expectations can influence how a student does in class. There is substantial research on "expectancy effects," which shows that teacher expectations can affect how students learn. The most famous is the 1966 Rosenthal-Jacobson study, in which students were given a fake aptitude test and their teachers were told which students were expected to "bloom" (when in reality the so-called good students were randomly selected). Over the next year, those students did better than their classmates.[3]

This is a silent captivity of expectations. It holds both student and teacher captive. Teachers are bound by a limited imagination of a student's potential. Students are bound to someone else's idea of who they can be, and they don't have the freedom to be free. A freedom of welcome would say, "I believe you can succeed, and I will help you get there."

In the book *Teaching to Transgress*, bell hooks discusses her experience as a student in an all-Black school in segregated America. She praises the teachers who were on a mission to help the students reach their highest potential.

> To fulfill that mission, my teachers made sure they "knew" us. They knew our parents, our economic status, where we worshipped, what our homes were like, and how we were treated in the family....
>
> Attending school then was sheer joy. I loved being a student. I loved learning.... To be changed by ideas was pure pleasure.... I could ..., through ideas, reinvent myself.[4]

You can hear how those teachers practiced the freedom of welcome. They got to know everything about their kids, not just their academic performance. These students' education was rooted in community. Because of the freedom of welcome, hooks had other freedoms: the freedom to explore ideas and the freedom to imagine a new identity for herself.

By contrast, after schools were integrated and she changed schools,

> knowledge was suddenly about information only. It had no relation to how one lived, behaved.... We soon learned that obedience, and not a zealous will to learn, was what was expected of us....
>
> For [B]lack children, education was no longer about the practice of freedom. Realizing this, I lost my love of school.[5]

The beach was now open, but there was no welcome, no community or hospitality. hooks was captive to expectations: always feeling like she had to prove herself. She was afraid that if she spoke up, she would be perceived as stepping out of bounds. Education felt like domination. Without the freedom of welcome, she lost all the other freedoms.

Having the freedom to explore builds *fearlessness in asking questions* and *independent thinking*, and we experience the joy of discovery.

To be clear: I'm not advocating segregation. I'm saying that real welcome must involve real freedom, especially when that freedom has been denied in the past.

These freedoms in mathematics are associated with several virtues. The freedom of knowledge leads to the virtue of *resourcefulness*. We can take the tools we know about and bend them to solve our problem. Having the freedom to explore builds *fearlessness in asking questions* and *independent thinking*, when we are not shamed for brainstorming aloud and we experience the joy of discovery. It also builds in us the skill of *seeing setbacks as springboards*, as we learn to not simply discard wrong ideas but explore how they can lead us to good answers or push us off into new areas of investigation. The freedom of understanding builds our *confidence in knowledge*, because understanding builds a firm foundation of facts

I See You

The Importance of Having a Mentor

We assume that learning mathematics doesn't involve culture. This is a common assumption, especially if you are not part of a marginalized group. It leads to inaccurate assessments of student knowledge. A mathematician friend shared this example with me:

> On an examination, I asked the classical Fermi problem "Estimate how many piano tuners live in this city." A student timidly raised his hand. He whispered to me, "Is a piano tuner a device or a person?" Other students thought good piano players would tune their own pianos, like guitar players do. Some students thought piano tuners would work in a music store. Few students had a sense of how often a piano might need tuning, or how long it would take to tune a piano. This example opened my eyes to how important background experience can be in dealing with questions that may appear to be mathematical, but instead bring up all sorts of cultural or experiential issues.

I could have been one of those perplexed students, since a piano wasn't a household item for me. Now imagine a student without the requisite cultural experiences who constantly encounters obstacles like these. Would they feel like they belong? Cultural barriers are impossible to avoid, but if we are aware of them, we can mitigate their effects.

The math education professor William Tate points out that such experiences are common to African American children in mathematical spaces, who often encounter instruction based on white middle-class norms, and he contends that connecting pedagogy to the lived realities of African American students is essential for equitable instruction.[1] He advocates that teachers take a "centric" perspective: allowing and expecting students to center their problem solving in terms of their own cultural and community experiences, and encouraging students to think about how the same problem might be viewed from the perspectives of different members of the class, school, and society. For example, a teacher could reframe the problem about piano tuners as an estimation problem whose subject the students choose as relevant to their daily lives or struggles.

In 2015, I had the great pleasure of running MSRI-UP (Mathematical Sciences Research Institute—Undergraduate Program), a summer research program for students from underrepresented backgrounds: Hispanic, African American, and first-generation college students. Later, I asked them to tell me about obstacles they'd faced in doing mathematics. One of them, who did wonderful work that summer, told me about her experience in an analysis course after she returned to her university:

> Even though the class was really hard, it was more difficult to receive the humiliations of the professor. He made us feel that we were not good enough to study math, and he even told us to change to another, "easier" profession.

As a result of this and other experiences, she switched her major to engineering.

Let me be clear: there is no good reason to tell someone that she shouldn't be doing mathematics. That's her decision—not

cemented together by meaning and insight. And the freedom to imagine encourages the virtues of *inventiveness* and of *joyfulness*, because that freedom gives you room to explore and to take delight in all the wild things your mind might conjure up.

When I encountered difficulties in graduate school, feeling underprepared and out of place, with professors questioning my ability to succeed, these virtues rescued me. I knew I had the beginnings of research skills, because I'd experienced independent thinking. I leaned on my fearlessness in asking questions to speak up when I didn't understand. I knew the joyfulness in creating mathematics for myself. And the confidence in the knowledge I already had helped me to trust that I would eventually catch up, through earnest effort and hard work.

Freedom is a crucial ingredient of learning and doing mathematics, so we ought to consider what freedom entails. I know that some people define freedom as "the absence of constraints," as if it means "do whatever you want." I don't believe that's what true freedom is.

True freedom never comes without cost, relationship, or responsibility. Think of that teacher who poured their time and energy into you, gave you the space to ask questions, and showed you how to explore the beach and imagine the castles you could yours. You may not know what she's capable of. One of my friends who is now a math professor described this incident that happened to him when he was a student:

> This faculty member had one of those private in-the-office conversations with me that begins with "I think it may be only a kindness to tell you that…," followed by a stated concern that I was not really cut out for a career in mathematics. I've not done all that badly since then, and in fairness I have to add that the faculty member sought me out in later years to apologize for the comment. I consider the person a friend, but when I'm working with our graduate student training program, I do stress that any conversation that begins with "I think it only a kindness to tell you that" will almost never be a kindness.

Look at my friend now—a successful mathematician. It's too easy for such pronouncements to reflect personal biases and limited information.

Oscar, another student from MSRI-UP, told me about his experience as a math major who, unlike his peers and because of his background, did not enter college with any advanced placement credit:

> I noticed how different my trajectory was, however, while I was in a Complex Analysis course. A student was presenting a solution on the board which required a bit of a complicated derivation halfway through. They skipped over a number of steps, saying, "I don't think I need to go through the algebra … we all tested out of Calculus here anyway!," with my professor nodding in agreement and some students laughing. I quietly commented that Calculus was my first course here. My professor was genuinely surprised and said, "Wow, I did not know that! That's interesting." I was not sure whether to feel proud or embarrassed by the fact that I was not the "typical math student" that was successful from the beginning of their mathematical career. I felt a sense of pride in knowing that I was pursuing a math degree despite my starting point, but I could not help but feel as though I did not belong in that classroom to begin with.

The reason Oscar was in that class to begin with was the active support of another professor. Oscar said:

> She presented me with my first research opportunity and always encouraged me to study higher math. I was also able to confide in her about a lot of the internal struggles I had with being a minority in mathematics since, as a female, she had a similar experience herself! My complex analysis professor became one of my mentors as well. I think it was just an interesting moment because she didn't realize how her reaction to the situation could have hurt me (and I don't think she's necessarily at fault!). It was more that her reaction piled onto the insecurities I held in regards to being a minority with a weak background in math.

Actually, Oscar didn't have a "weak" background—he had a standard background. I'm pleased to say that Oscar and his team from that summer have published a paper on their research, and he is now in graduate school.

You hear in Oscar's story the importance of having an advocate, someone who says, "I see you, and I think you can flourish in mathematics." Everyone can use this encouragement, but this can be especially important for marginalized groups who already have so many voices telling them they don't belong. Can you be that advocate?

We must be mindful to not set up structures for learning that disadvantage people with weaker backgrounds or make them feel out of place. When I was teaching at Harvard, there was a regular calculus class, an honors calculus class called Math 25, and on top of that—for those with very strong backgrounds—a super honors class called Math 55. Ironically, I regularly encountered students *in the honors track* who felt that they didn't belong in the math major, because they hadn't placed into the super honors track. I had to keep reassuring them that "background is not the same as ability." I wish that college and grad school admissions would remember this too. The mathematician Bill Vélez says this about barriers at the graduate level: "In mathematics we value creativity, yet we evaluate students on knowledge. Departments erect barriers to keep down applications, and it works. Top-rated departments have few minority students."

We must not set up structures for learning that disadvantage people with weaker backgrounds or make them feel out of place.

–F. S.

Endnote

1. W. Tate, "Race, Retrenchment, and the Reform of School Mathematics," *Phi Delta Kappan* 75, no. 6 (February 1994): 477–84.

build. Think of that parent who provided whatever resources they had so that you could get to the beach, and instilled in you a resolve to overcome any obstacles. Think of the real welcome that some provided, helping you to enter, the welcome that took real energy and care. And think of the cost of your own commitment to invest yourself in a subject that is deep and wide and beautiful, so that you now have the freedom to flourish on that beach in ways you never could have otherwise.

Those of us who have experienced the freedoms of mathematics have a significant responsibility to welcome others to those freedoms as well. ☐

Endnotes

1. To learn some of his shortcuts, see A. Benjamin and M. Shermer, *Secrets of Mental Math* (New York: Three Rivers, 2006).

2. G. Cantor, "Foundations of a General Theory of Manifolds: A Mathematico-Philosophical Investigation into the Theory of the Infinite," trans. W. Ewald, in *From Kant to Hilbert: A Source Book in the Foundations of Mathematics*, vol. 2, ed. W. Ewald (New York: Oxford University Press, 1996), 896. Italics in the original.

3. R. Rosenthal and L. Jacobson, "Teachers' Expectancies: Determinants of Pupils' IQ Gains," *Psychological Reports* 19 (1966): 115–18. It is worth noting that this study has attracted controversy. An interesting account of this study, including critiques and follow-up studies, can be found in K. Ellison, "Being Honest About the Pygmalion Effect," *Discover*, October 29, 2015, go.aft.org/wr7.

4. b. hooks, *Teaching to Transgress: Education as the Practice of Freedom* (New York: Routledge, 1994), 3.

5. hooks, *Teaching to Transgress*.