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Abstract

Mathematical modeling has been positioned at the forefront of many educational levels globally because modeling reinforces purposeful problem-solving skills, connects mathematics to the real-world, and makes mathematics more meaningful and relevant. This article investigated K–12 practicing teachers' knowledge and attitudes toward mathematical modeling, while considering the relationships between them and some selected demographic variables. The study involved 335 teachers from Midwestern United States of America (U.S.), and data was collected through an Online self-administered survey. The data collected were quantitatively analyzed with the guidance of the research questions, and using statistical analyses such as correlation, multiple regression, and ANOVA. Results showed that teachers had a satisfactory knowledge and attitudes toward mathematical modeling. Teachers' knowledge of modeling was found to be positively related to their overall attitudes toward such modeling, with an R^2 = .25 suggesting a moderate relationship. Overall, gender and grade level band were identified as strong indicators of teachers' knowledge and attitudes toward mathematical modeling. In particular, female teachers on average showed a relatively stronger knowledge and attitudes toward mathematical modeling than their male colleagues. Similarly, elementary teachers demonstrated a relatively stronger knowledge of mathematical modeling and attitudes toward such modeling than middle or high school teachers. Findings from this study have implications for integrating mathematical modeling standards into teacher preparation programs and organizing professional development on mathematical modeling education.

Introduction

Mathematical modeling has been positioned at the forefront of many educational levels globally because modeling reinforces purposeful problem-solving skills, connects mathematics to the real-world, and makes mathematics more meaningful and relevant. Additionally, in the United States of America (U.S.), the development and implementation of the Common Core State Standards for Mathematics (CCSSM; National

Governors Association Center for Best Practices [NGA Center] & Council of Chief State School Officers [CCSSO], 2010), *Principle to Action: Ensuring Mathematical Success for All* (National Council of Teachers of Mathematics [NCTM], 2014); and the *Guidelines for Assessment and Instruction in Mathematical Modeling Education* (GAIMME) report (COMAP & SIAM, 2016) emphasize the importance of mathematical modeling as an essential component of mathematics instruction. Teachers' knowledge and attitudes toward mathematical modeling are crucial for effective teaching and learning of school mathematics (Aydogan-Yenmez et al., 2017; Blum, 2015; Lesh, 2012). However, the GAIMME report indicates that most practicing and future teachers have limited experience with mathematical modeling practices and how to integrate it into their teaching of mathematics (COMAP & SIAM, 2016). Nonetheless, research studies related to mathematical modeling lack a strong focus on exploring teachers' mathematical knowledge and attitudes for mathematical modeling instruction (Asempapa, 2018; Borromeo Ferri, 2018; Gaston & Lawrence, 2015). Therefore, the purpose of this study was to examine practicing teachers' of mathematics knowledge and attitudes toward mathematical modeling.

Mathematical modeling holds a unique place in mathematics curriculum because modeling is a drive to STEM learning and has the potential to enable students use mathematics in flexible, creative, and powerful ways (Blum, 2015; Lesh, 2012; Pollak, 2011; Zawojewski, 2016).These benefits of mathematical modeling are likely to occur when teachers understand modeling and how to use it in their practice. Additionally, teachers knowledge of mathematical modeling is important because modeling enables students to connect mathematics to real life and to learn it in a more meaningful way (Kaiser, 2017; Schukajlow et al., 2018; Tan & Ang, 2012). Moreover, teachers' attitudes and knowledge for mathematical modeling instruction can determine to a large extent how teachers perceive and respond to curriculum innovation efforts to infuse mathematical modeling learning experiences in mathematics teaching (Galbraith, 2017; Stillman et al., 2016).

However, research that explores practicing teachers' knowledge and attitudes toward mathematical modeling is limited (Kaiser et al., 2010; Tan & Ang, 2012). Research studies indicate that most teachers have misconceptions about mathematical modeling and the modeling process (Gould, 2013; Spandaw & Zwaneveld, 2010; Wolfe, 2013), and lack knowledge about mathematical modeling practices (Blum, 2015; Borromeo Ferri, 2018). In order to continue to advance mathematical modeling research, and in particular, an understanding of teachers' knowledge and attitudes, it was necessary to conduct a study that allows for the modeling of relationships between teachers' knowledge and attitudes toward mathematical modeling.

Throughout this article, teacher refers to practicing teachers who teach mathematics in the K–12 setting. Clearly, understanding teachers' use of mathematical modeling practices is complex and depends on many factors including knowledge and attitudes. However, there is paucity of recent research into the relationship between teachers' knowledge and attitudes toward mathematical modeling. Additionally, questions about teachers' knowledge and attitudes toward teaching mathematical modeling effectively in the classroom have prompted many discussions in the literature on mathematical modeling education (Borromeo Ferri, 2018; Blum, 2015; Lesh, 2012). Moreover, teacher education has been criticized for not adequately addressing mathematical modeling in the area of teachers' knowledge and attitudes (Kaiser et al., 2010). Therefore, this study became necessary and sought to examine the relationships among teachers' knowledge and attitudes toward mathematical modeling.

Background and Related Literature

Education in mathematical modeling has increasingly been promoted as crucial to mathematics education (Berromeo Ferri, 2018; Blum, 2015; COMAP & SIAM, 2016; NGA Center & CCSSO, 2010). This point is further reinforced when one examines the influence of teachers' knowledge and attitudes on students' mathematics learning. Research studies have shown that teachers' knowledge of and attitudes toward mathematics (including mathematical modeling) and their teaching are important factors influencing the competence and attitudes of students towards mathematics in general (Di Martino & Zan, 2011; Thiel, 2010; Zan, 2013). The next four subsections focus on the interconnected concepts of teachers' knowledge and attitudes, mathematical modeling, and teachers' knowledge and attitudes toward mathematical modeling in mathematics education.

Teachers' Knowledge and Attitudes

Research on knowledge and attitudes have been motivated by the belief that teachers' attitudes and knowledge play a significant role in the teaching and learning of mathematics (Ball & Bass, 2000; Zan & Di Martino, 2007). Teachers play a vital role in students' achievement—so an insufficient knowledge and negative attitudes can be detrimental to the practices in mathematics classrooms (Youngs & Youngs, 2001). Additionally, teachers' knowledge and attitudes both overtly and covertly influence the classroom atmosphere, which affect their behaviors and responses in relation to classroom practices (Zan & Di Martino, 2007). Moreover, research shows that there are positive relationships among teachers' knowledge, attitudes, and students' achievement (Hill et al., 2004; Zan & Di Martino, 2007). Therefore, teachers' mathematical modeling knowledge and attitudes are critical in assisting students in developing the mathematical skills and competencies required for the 21st century.

Teachers' knowledge was defined as the understanding practicing teachers have of mathematical modeling standards and practices taught in K–12 schools as measured by the Mathematical Modeling Knowledge Scale [MMKS] instrument (Asempapa, 2016, 2018, 2020). Similarly, teachers' attitude was defined in this study as the feeling of practicing teachers toward modeling and the teaching of modeling standards and practices as measured by the Mathematical Modeling Attitude Scale [MMAS] (Asempapa, 2016, 2019; Asempapa & Brooks, 2020). Teachers' knowledge and attitudes influence what and how they teach in the classroom (Ball & Bass, 2000; Guskey, 1988; Zan & Di Martino, 2007). Teachers' knowledge contributes to both the quality of instruction and student performance (Shulman, 1986) and attitudes affect their instructional practice and allocation of instructional time (Guskey, 1988). Meijer et al. (1999) explained that it is essential for teachers to have a rich knowledge of the subject they teach. Additionally, "effective attitudes and actions employed by teachers ultimately can make a difference on the lives of their students" (Gourneau, 2005, p. 4). Moreover, Thomas (2008) explained that teachers' knowledge and attitudes toward educational reform have significant influence on inquiry instructional practices, which includes mathematical modeling.

Wilkins (2008) proposed a theoretical model relating teacher knowledge and attitude to instructional practices. This theoretical model is based on Ernest's (1989) model regarding the knowledge, belief, and attitudes of teachers. It shows that teachers' knowledge and attitudes are influenced by their background characteristics including experience, education, and training (Wilkins, 2008). As suggested by Wilkin's model, teachers' knowledge and attitudes have a direct relationship with their behavior in their instructional practices, which involve mathematical modeling. Therefore, knowledge and attitudes changes are important and unavoidable part of any instructional innovation, and mathematical modeling practices are no exception, which is worth examining.

Mathematical Modeling and Its Process

Mathematical modeling is not a new phenomenon in the area of mathematics and has been a central theme in mathematics education during the past 30 years. Although there is no one agreed-upon definition of mathematical modeling, it is a process where one identifies a situation in the real world, makes certain assumptions and choices, and then uses a mathematical model to obtain a solution that can be translated back into the real world. Mathematical modeling is defined in a variety of ways, but in this study, it is described as a mathematical process in which problem solvers build explanations of mathematical information in the form of non-physical models in order to make sense of scenarios (Chamberlin, 2019; Chamberlin & Parks, 2020). In the GAIMME report, mathematical modeling is defined as "a process that uses mathematics to represent, analyze, make predictions or otherwise provide insight into real-world phenomena" (COMMAP & SAIM, 2016, p. 8). According to Blum (2015), mathematical modeling is important for students because mathematical models and modeling are everywhere around us. Using mathematical modeling in mathematics instruction gives students the opportunity to understand mathematics more meaningfully, learn mathematics by relating with real life, and eliminate inadequacy of available problems.

Mathematical modeling is more than simply presenting students with a word problem. Whereas problem solving usually begins with the idealized real-world situation in mathematical terms, mathematical modeling begins in the "unedited" world, and after engaging in problem formulation and problem solving, the modeler moves back into the real world where the results are considered against the original context of the problem (Pollak, 2012). Additionally, Zbiek and Conner (2006) explained that mathematical modeling is a non-linear process that involves elements of both a treated-as-real world and a mathematics world. Fundamentally, the mathematical modeling process is defined by the iterative negotiation of learning between the real and mathematical worlds. Thus, modeling is an iterative process that involves revisions before one arrives at an acceptable conclusion, and the process involves movement among elements such as a real-world situation, a mathematical entity, and a mathematical solution. Mathematical modeling as a process converges to a translation between the real world and mathematics (Blum, 2015; Blum & Borromeo Ferri, 2009). In such a process, one identifies a situation in the real world, makes certain assumptions and choices, and uses a mathematical model to obtain results that can be translated back into reality and validated by their practicality (Lesh, 2012; Pollak, 2011).

The mathematical modeling process is an iterative problem-solving procedure in which mathematics is used to investigate and develop a deep understanding of real-world situations. Through mathematical modeling, students are able to transition between the real-world (as they know it) and mathematics. Mathematical modeling as a process occurs after a task is situated in a meaningful context and continues until a model is complete, and results are known. This process contains the following steps: (a) understanding the phenomenon, (b) constructing a representation or model, (c) mathematizing the phenomenon and performing computations, (d) interpreting results, (e) validating them in reality, and (f) disseminating them through discussions and in writing (Blum & Leiss, 2007; Integrating Mathematical Modeling Experiential Learning and Research through a Sustainable Infrastructure and an Online Network [IMMERSION], 2016; NGA Center & CCSSO, 2010). Just as there is no one agreed-upon definition for mathematical modeling, there is no one agreed-upon modeling process. Therefore, mathematical modeling involves a multistep and an iterative process as illustrated in Figure 1, and can be defined as a process that uses mathematics and real-world context that situates mathematics between a way of making sense of our world and a set of formal mathematical structures and representations.

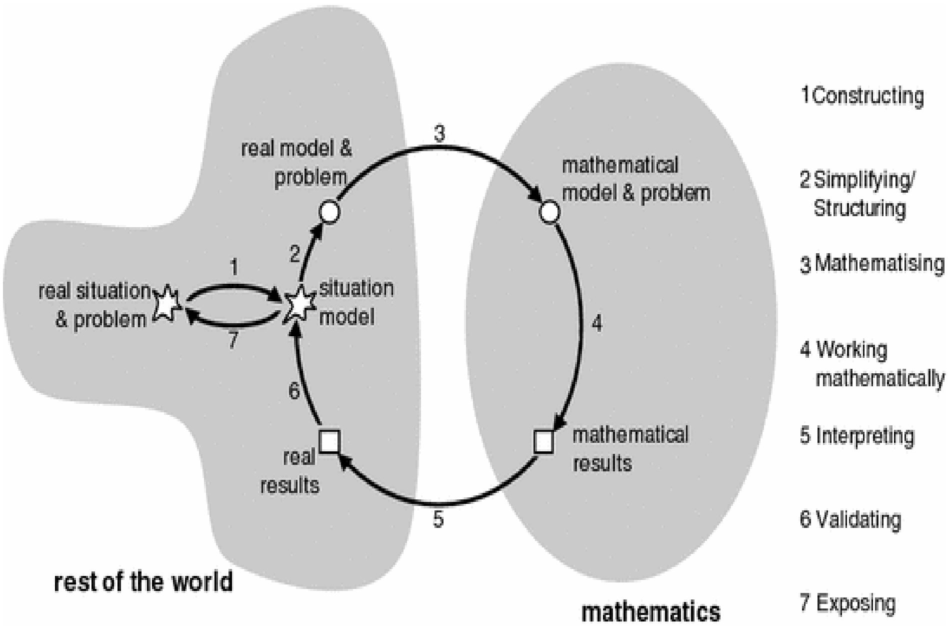

Figure 1. Mathematical Modeling Process (adapted from Blum, 2011, p. 18)

Benefits of Mathematical Modeling

The broad attention and growing usefulness of mathematical modeling warrants the need to examine teachers' knowledge and attitudes toward it. A more compelling case about the benefits of mathematical modeling is the emphasis on modeling in recent research, standards, and reports (Blum, 2015; COMAP & SIAM, 2016; NGA Center & CCSSO, 2010; Next Generation Science Standards [NGSS] Lead States, 2013; Zawojewski, 2016). Mathematical modeling brings many immediate benefits for both teachers and students. Research shows that when students engage in mathematical modeling activities not only does such engagement help in their achievement in mathematics (Boaler, 2001), but also it assists them with their social interaction and makes the learning of mathematics relevant and meaningful (English & Watters, 2004; Pollak, 2011). During mathematical modeling activities, students get the opportunity to engage in mathematical tasks that match their current conceptual understanding while simultaneously presenting opportunities for challenge and growth (Flevares & Schiff, 2013). Mathematical modeling brings an important perspective to K–12 mathematics education, in that the real world is not just a context to highlight the value of mathematics; rather, both the real world and mathematics are taken seriously (Pollak, 2011). Research shows that mathematical modeling promotes students' understanding of a wide range of key mathematical concepts and "should be fostered at every age and grade… as a powerful way to accomplish learning with understanding in mathematics" (Romberg et al., 2005, p. 10).

Through mathematical modeling, students' conceptual understanding and proficiencies in mathematics are better developed because modeling provides great opportunities for both teachers and students to create tasks that are of high cognitive demand. The high cognitive demand of modeling tasks offers students richer learning experiences and multiple points of entry in solving real-life problems. Mathematical modeling plays an important role in mathematics, because through modeling, students begin to appreciate the relevance and utility of mathematics to individuals and the society (Pollak, 2011). Therefore, exploring teachers' knowledge and attitudes toward mathematical modeling will provide much more "powerful and effective ways to help students become (a) better problem solvers, and (b) better able to use mathematics in real life situations beyond school" (Lesh, 2012, p. 197).

Teachers' Attitudes and Knowledge of Mathematical Modeling

Attitudes are an important aspect of a person's personality. Teachers' attitudes are resilient to change, because they were developed and shaped over several years of their experiences (Maasepp & Bobis 2014). Teachers' attitudes toward mathematical modeling instruction can determine to a large extent how teachers perceive and respond to curriculum innovation efforts to infuse mathematical modeling learning experiences in the classroom. To think about attitudes toward mathematical modeling instruction requires going beyond knowing the practices of mathematical modeling, but may require understanding the complex interplay among aspects of other forms of teacher attitudes in the mathematical modeling instruction (Hannula et al., 2016). Thus, teachers' attitudes toward mathematical modeling have a significant impact on their mathematics instruction. According to Veloo and Chairhany (2013), attitude is the main factor in a person's orientation to learning a topic. Therefore, it is important for researchers and mathematics educators to identify teachers' attitudes towards mathematical modeling, especially those with which they are less familiar, and to use this knowledge in taking appropriate measures to improve these attitudes (Hannula et al., 2016; Veloo & Chairhany 2013).

Teachers' attitudes can influence mathematical modeling practices in the classroom and research findings in the international community on mathematics education indicate that teachers' attitudes toward mathematics affect mathematics instruction (Thiel, 2010), and students' attitudes and achievement (Tella, 2009). The motivation behind research on attitudes emanates from the belief that attitude and knowledge play a significant role in the teaching and learning of mathematics (Di Martino & Zan, 2015). When new standards or topics are introduced, they may demand on the part of teachers to revise, refine, or change their attitudes, which is a challenge for most teachers. To improve teaching and the curriculum "it is not enough for people to act differently, which is a surface phenomenon, they may also be required to change the way they think about certain issues, which is a deeper and more complex change" (Kennedy, 1988, p. 329). "Teachers' educational attitudes, although in many cases unconsciously held, have an effect on their classroom behavior, influence what students actually learn, and are a potent determinant of teachers' teaching style" (Karavas-Doukas, 1996, p. 188).

Equally important to teachers' attitudes is teachers' knowledge of mathematical modeling. Research suggests that teacher's knowledge is essential for teaching mathematics (Ma, 1999). Ball et al. (2008) suggested that mathematics teachers need certain knowledge domains–subject matter knowledge and content knowledge. Teachers' ability to teach depends on their knowledge, so if teachers do not possess strong knowledge of mathematical modeling, we cannot expect them to promote mathematical modeling practices effectively in their classrooms as outlined and emphasized in the CCSSM and the GAIMME report. In recent years, the knowledge of teachers regarding mathematical modeling practices has received much discussion in the literature (Borromeo Ferri, 2018; Paolucci & Wessels, 2017). However, within mathematics education, defining the knowledge of mathematical modeling could seem as a complex construct because of the discrepancy in the components associated with the mathematical modeling process usually used as a criterion in teaching mathematical modeling.

Teachers' knowledge of mathematical modeling was conceptualized as their understanding, interpretations, familiarizations, and minimal competencies associated with the Common Core standard of mathematical practice—model with mathematics—and teaching and learning of mathematical modeling (Borromeo Ferri, 2018; Blum, 2015; NGA Center & CCSSO, 2010). Teachers' knowledge is crucially important to the improvement of teaching and learning mathematical modeling. Theoretical and empirical research into the work of teaching (Ma, 1999; Shulman, 1986, 1987) has prompted greater attention to the role knowledge of the topic plays in teacher education. Shulman emphasized the importance of teacher's subject matter knowledge as a central aspect of teachers' professional competence. As Ponte and Chapman (2008) explained, "having strong content knowledge does not produce an effective mathematics teacher, [but] teachers with subpar knowledge struggle to promote relational and conceptual understanding" (p. 226). Almost 22 years later, Ball et al., (2008) explained that teachers need strong knowledge to teach mathematics effectively, because their knowledge does influence student learning. Therefore, examining teachers' knowledge of mathematical modeling cannot be overlooked. As Ma (1999) articulated, "the quality of teachers subject matter knowledge directly affects student learning" (p. 144). One can only teach what he or she knows, therefore, "teachers must have an in-depth knowledge not only of the specific mathematics they teach, but also of the mathematics that their students are to learn for the future" (Fennema & Franke, 1992, p. 147). In the past few years, teachers' knowledge and attitudes toward mathematical modeling has become an object of concern (Jacobs & Durandt, 2017; Tan & Ang, 2012). The lack of teachers' in-depth knowledge, productive attitudes, and skills for teaching mathematical modeling led to the development of the modeling standards in the Common Core and the GAIMME report. At the time of writing this article, there is no empirical research in the literature that had explored teachers' knowledge and attitudes toward mathematical modeling.

The Present Study

Despite the importance of mathematical modeling education, there is no known empirical research that investigates the association between practicing teachers' knowledge and attitudes towards mathematical modeling. For this reason, this study sought to examine what teachers know about mathematical modeling practices, their attitudes toward such modeling, and modeled the relationship among these variables. In doing so, the focus of this research reported on examining the relationship between teachers 'knowledge and attitudes toward modeling, and the connections made between teachers' knowledge, attitudes, and some selected demographic variables on mathematical modeling as a construct. Therefore, to accomplish this purpose, the following research questions guided the study:

Research Question 1(RQ1): What does the scale reveal about teachers' knowledge and attitudes toward mathematical modeling and is there a relationship between these two constructs?

Research Question 2(RQ2): Do age, teaching experience, gender, and grade level band predict teachers' overall attitudes toward mathematical modeling?

Research Question 3 (RQ3): Are there differences between teachers' grade level band and gender on their knowledge of modeling and overall attitude towards such modeling?

Method

Participants and Study Setting

The participants in this current study were practicing teachers of mathematics from Midwestern U.S. The study participants teach mathematics to students in the following grade level bands: elementary school: K–Grades 5, middle school: Grades 6–8, and high school: Grades 9–12. Additionally, participants were from nine public school districts, and the school districts were classified as rural, small-town, suburban, and urban. The number of participants who consented to participate in the study were n = 335, with a response rate of about 25% via an Online, self-administered survey.

Of the 335 participants and given this sample size, 268 (80.0 %) self-identified as females, and 67 (20.0 %) as males. The characteristics of the sample proportionately reflected the gender of the larger teacher population. The average age of the participants was 40.42 years, with a SD = 10.84 and the average years of teaching experience was 13.78 with a SD = 8.75. Majority of the participants self-identified as elementary school teachers (66.9%, n = 224). Almost 16.7% (n = 56) of the participants were middle school teachers, and the remaining 16.4% (n = 55) were high school teachers. Participants had different backgrounds with respect to race, highest degree earned, teaching credentials, and school district typology.

Instrumentation

The survey used to collect information on teachers' knowledge and attitudes toward mathematical modeling consisted of two validated scales. The two validated scales used to collect data were the MMKS (Asempapa, 2016, 2018, 2020) and MMAS (Asempapa, 2016, 2019; Asempapa & Brooks, 2020). The MMKS is a 12-item unidimensional Likert survey that measures practicing teachers' knowledge of mathematical modeling. The items on the MMKS were derived from current and relevant literature including the CCSSM modeling standards, the GAIMME report, and the NCTM's annual perspectives in 2016 on mathematical modeling (COMAP & SIAM, 2016; Hirsch & Roth McDuffie, 2016; NGA Center & CCSSO, 2010). The MMKS is single-factor scale and was supported in its development and validation by a scree plot, factor structure, exploratory factor analysis (EFA) (Asempapa 2018, 2020) and confirmatory factor analysis (CFA) (Asempapa & Brooks, 2020). The reliability measure of the scale's internal consistency for the 12 items (without the demographic information) is reflected in the Cronbach's alpha of .84 (Asempapa, 2016, 2018, 2020). The goodness of fit indices for the single-factor model based on the CFA including TLI = .96; GFI = .95; RMSEA = .05; CFI = .97; and SRMR = .04 indicated a good model.

Similarly, teachers' attitudes toward mathematical modeling were measured with the MMAS. The MMAS used in this present study is a 28-item multi-dimensional Likert scale with four subscales. The MMAS items were created from recent and relevant literature including the CCSSM modeling standards, the GAIMME report, and the NCTM's annual perspectives in 2016 on mathematical modeling (COMAP & SIAM, 2016; Hirsch & Roth McDuffie, 2016; NGA Center & CCSSO, 2010). The development and validation of the MMAS was supported by a scree plot, factor structure, and exploratory factor analysis (EFA) (Asempapa 2019, Asempapa & Brooks, 2020) and confirmatory factor analysis (CFA) (Asempapa & Brooks, 2020). The goodness of fit indices for the four-factor model based on the CFA including TLI = .90; GFI = .90; RMSEA = .05; CFI = .92; and SRMR = .05 indicated a good model. The overall reliability of the 28 items on the scale was .91 (Asempapa, 2016, 2019; Asempapa & Brooks, 2020).

Procedure and Data Analysis

According to Creswell (2009), the research design of a study summarizes the basic methods that researchers employ to answer their research questions. Likewise, it is essential for researchers to select the appropriate research design to meet the objectives of the study. The quantitative approach to this study is as a result of the belief that attributes, phenomena, or variables in human behavior can be studied empirically. Thus, a survey research design—specifically a cross-sectional survey research design was used in this study (Fowler, 2014). Additionally, this study employed purposeful sampling (Creswell, 2009) to identify participants. An Online self-administered survey was chosen because it is believed the participants in the study have the skills and access to use the internet (Dillman et al., 2009). Moreover, with self-administered surveys, participants do not have to share answers with an interviewer, which makes collection of sensitive data likely valid (Fowler, 2014). Because of the nature of the data collection non-responses were unexamined and incomplete surveys were discarded.

Because Likert scales were used in the data collection process, it gave the researcher the ability to combine the scale items into a single composite score during the data analyses (Boone & Boone, 2012; Joshi et al., 2015; Subedi, 2016), which helped provide quantitative measures of teachers' knowledge and attitudes toward mathematical modeling. Data analyses for examining teachers' knowledge and attitudes toward mathematical modeling employed different statistical methods. The statistical analyses included descriptive statistics, multiple regression, and ANOVA. Specifically, linear correlation, bivariate, and multiple regression analyses were used to explore the relationship between the means score of teachers' knowledge and overall attitudes. The difference between mean scores of teacher's knowledge or overall attitudes and other demographic variables were examined by using ANOVA. Tukey's test was performed for multiple comparison testing (post-hoc analysis) where applicable (Warner, 2013). All data were analyzed using SPSS version 25. In all statistical analyses, a $p < .05$ was considered to be statistically significant.

Results and Discussion

RQ1: Teachers Knowledge and Attitudes toward Mathematical Modeling and their Relationships

RQ1 explored what the survey revealed about teachers' knowledge and attitudes toward mathematical modeling and the relationship between the two constructs. The mean score of participants on the MMKS was 9.51 (SD = 2.81), with a 1.12 standard error of measurement (*SEM*), showed satisfactory knowledge of mathematical modeling. Thus, with this particular sample, the MMKS sores should fluctuate by about a point, on average, due to random measurement error in measuring teachers' knowledge of mathematical modeling.

Additionally, descriptive statistics on the MMKS revealed that among K–12 female teachers' (M = 9.56) had a higher mean score than the male teachers (M = 8.84). Similarly, with grade level bands, elementary teachers (M = 9.46) had a higher score compared to both middle school (M = 8.36) and high school (M = 8.43) teachers. The Cronbach's alpha measure of internal consistency for MMAS was .91. Similarly, teachers' mean MMAS score of 4.82 (SD = .48), showed a slightly positive overall attitudes toward mathematical modeling. Moreover, the scores on the MMAS revealed that female teachers' (M = 4.83) had a higher score than male teachers (M = 4.66). Again, middle school teachers (M = 4.84) had a higher score than elementary (M = 4.77) and high school (M = 4.61).

To determine whether there was a statistically significant relationship between teachers' knowledge and overall attitudes toward mathematical modeling, first, the data set was examined for bivariate correlation. A Pearson product-moment correlation coefficient was conducted to assess the relationship between teachers' knowledge of mathematical modeling and overall attitude toward such modeling. The scatter plot of knowledge of modeling and attitude towards such modeling suggested a moderate positive relationship between teachers' knowledge of mathematical modeling and overall attitude towards such modeling as shown in Figure 2. The correlation between the two variables was statistically significant, $r(335) = .41$, $p < 0.05$ (two-tailed), with an effect size of .25.

Additionally, a bivariate regression was performed to evaluate how well overall attitude toward modeling could be predicted from knowledge of mathematical modeling. The correlation between knowledge of mathematical modeling and attitudes toward such modeling for this sample data was statistically significant $r(335) = .41$, $p < 0.05$. The regression model ($Attitude = 4.47 + .25 \times Knowledge$) for predicting teachers' attitudes toward modeling from their knowledge indicates that a relatively moderate significant amount of variation (25%) in teachers attitudes can be accounted for by knowledge, $F(1, 333) = 15.67$, $p < .05$. The bivariate analysis revealed that knowledge of modeling was statistically significantly correlated with attitudes toward modeling, and the R^2 for this model was about .25. Therefore, about 25% of the variance in teachers' overall attitudes toward mathematical modeling was predictable from knowledge of mathematical modeling from this particular sample.

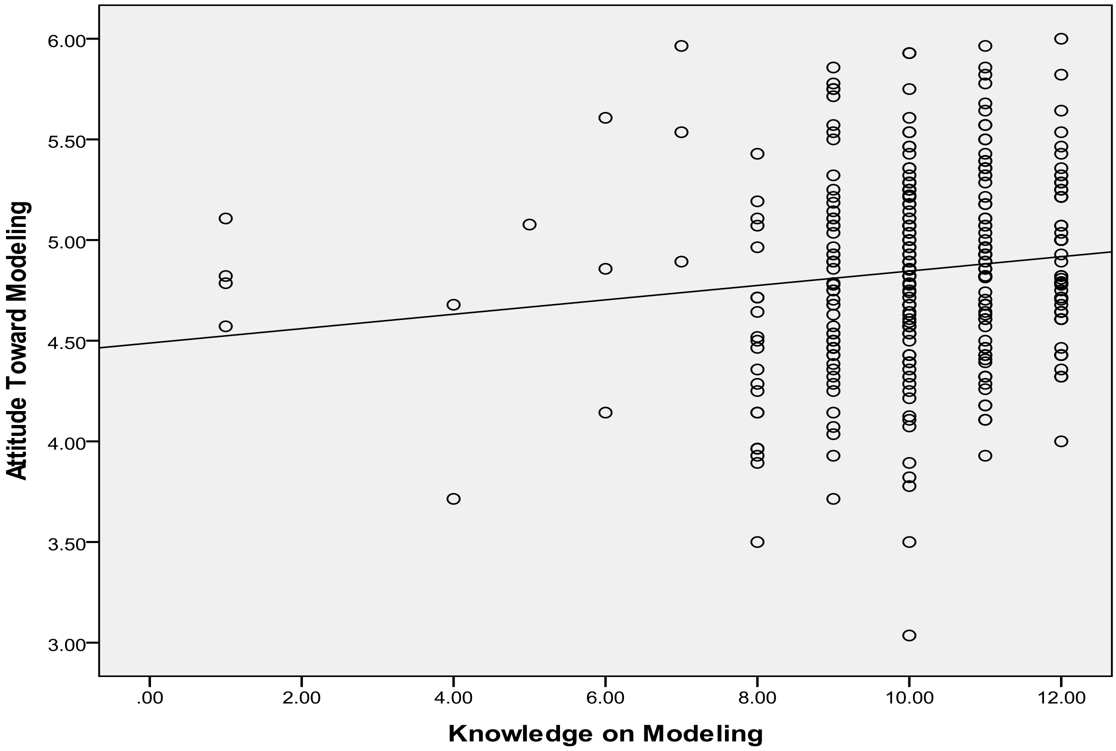

Figure 2. A scatter Plot of Teachers' Knowledge and Attitude toward Modeling

RQ2: Predicting Attitudes from Selected Variables

To answer RQ2, a multiple linear regression was performed to determine whether teachers' overall attitude towards mathematical modeling (Average MMAS) could be predicted from selected demographics variables including age, years of teaching experience, gender, and grade level band. Preliminary data screening had been done to assess whether the assumptions for multiple linear regression were seriously violated. Examination of the histogram of scores on the dependent variable suggested that Average MMAS scores had a normal distribution. Visual examination of the plot of standardized residuals showed that the assumption of homoscedasticity was not violated. Likewise, multicollinearity was not an issue, therefore, all the assumptions were tenable for the multiple regression.

The SPSS enter regression method in the multiple regression analysis was used for this analysis. A multiple linear regression analysis was run to predict Average MMAS from age, years of teaching experience, gender, and grade level band. Because gender and grade level band were categorical variables, they were dummy coded for the multiple regression analysis. Basic descriptive statistics and regression results are summarized in Table 1 with gender and grade level band recoded into dummy variables. The predictor variables that had a statistically significant zero-order correlations and partial effects ($p < .05$) in the full model are marked with the symbol (*) as shown in Table 1. The five predictors in the model were able to account for about 10% of the variance in Average MMAS scores, $F(5, 329) = 3.57$, $p < .05$, $R^2 = .10$.

Table 1. Multiple Linear Regression Predicting Average MMAS

	Zero-order *r*								
Variable	AG	TE	GD	GLE	GLH	MMAS	β	*sr*	*b*
GLH						–.19*	–0.26*	–.16	–0.19
GLE					–.58*	.11	–0.04	–.03	–0.04
GD				.29*	–.15*	.14*	0.14*	.12	0.12
TE			.07	.14*	–.18*	.003	0.001	.01	0.01
AG		.76*	.01	.13*	–.11*	–.04	–0.003	–.04	–0.06
							Intercept = 4.88		
M	40.22	13.76	0.78	0.65	0.16	4.83			
SD	10.78	8.76	0.41	0.48	0.37	0.48	$R^2 = .10$*		

Note. AG = Age; TE = Years of Teaching Experience; GD = Gender; GLE = Grade Level Elementary; GLH = Grade Level High School; and MMAS = Average MMAS.
*$p < .05$. $n = 335$.

RQ3: Differences between Selected Variables on Teachers' Knowledge and Attitudes toward Modeling

RQ3 was analyzed in two parts. The first part examined whether there were differences in teachers' grade level band and gender on their knowledge of mathematical modeling. To achieve this, a factorial ANOVA was performed to determine whether there were statistically significant differences between teachers' knowledge of mathematical modeling (Total MMKS) and their grade level (elementary, middle, high), gender (female, male), and interaction between the variables. An examination of the histogram of Total MMKS scores showed a slightly skewed distribution. The Levene's Test for equality of variance was $F(5,329) = 22.83$, $p < .00$, which was a violation of the homogeneity assumption and could be due to the unequal cell sizes. However, no data transformation was applied because Tabachnick and Fidell (2007) stated that ANOVA is robust to modest violations of these assumptions. Table 2 provides descriptive statistics for the ANOVA results.

The two-way ANOVA showed there was a statistically significant main effect of teachers' grade level band on knowledge of the nature of mathematical modeling, $F(2, 329) = 3.61$, $p < .05$, partial $\eta^2 = .02$. Additionally, there was a statistically significant main effect of teachers' gender on knowledge of the nature of mathematical modeling, $F(1, 329) = 15.20$, $p < .01$, partial $\eta^2 = .05$. Moreover, there was a statistically significant interaction between teacher's grade level band and their gender on knowledge of the nature of mathematical modeling, $F(2, 329) = 4.94$, $p < .01$, partial $\eta^2 = .03$. Based on the effect sizes of the main factors (gender and grade level band), the significant interaction effect indicates that the gender effect was greater in teachers' knowledge of modeling than in grade level band.

Table 2. Descriptive Statistics for the 3 × 2 ANOVA on the MMKS

		Total MMKS		
Grade Level	*n*	Gender	*M*	*SD*
K–5	28	Male	9.07	3.88
	196	Female	9.84	1.99
	224	Total	9.74	2.32
6–8	21	Male	6.62	5.31
	35	Female	10.11	1.39
	56	Total	8.80	3.79
9–12	18	Male	8.16	4.21
	37	Female	8.71	3.38
	55	Total	8.52	3.66
Total	67	Male	8.06	4.51
	268	Female	9.73	2.19
	335	Total	9.38	2.90

Examination of the estimated marginal means showed that female teachers in elementary, middle, and high school had higher scores on the knowledge of mathematical modeling on average than their male counterparts. There was a statistical significance in the mean difference (M_{diff}) for gender; $M_{\text{diff}} = 1.61$, 95% CI [.80, 2.42], $p < .001$. Additionally, the Tukey's post hoc test showed that within the grade level band, there was a statistically significant difference in means between elementary and high school teachers, $M_{\text{diff}} = 1.22$, 95% CI [.23, 2.22], $p < .05$. However, there was no statistically significant difference in means between elementary and middle school teachers, $M_{\text{diff}} = .94$, 95% CI [–.06, 1.94], $p = .07$ and middle and high school teachers, $M_{\text{diff}} = .29$, 95% CI [–.99, 1.56], $p = .10$.

The second part of RQ3 investigated whether there were differences in teachers' grade level band and gender on their overall attitudes toward mathematical modeling. To accomplish this, a factorial ANOVA was performed to determine whether there were statistically significant differences between teachers' overall attitude towards mathematical modeling (Average MMAS) and their grade level (elementary, middle, high), gender (female, male), and interaction between the variables. Preliminary data screening and examination of the histogram of scores on the dependent variable suggested that Average MMAS scores had a normal distribution. The probability associated with Levene's test for equality of variance was $F(5,329) = .25$, $p = .938$. Therefore, all the assumptions were tenable for the variable Average MMAS. Table 3 provides descriptive statistics for the ANOVA results.

The two-way ANOVA showed there was a statistically significant main effect of teachers' grade level on attitude toward mathematical modeling, $F(2, 329) = 2.94$, $p < .05$, partial $\eta^2 = .02$. Additionally, there was a statistically significant main effect of teachers' gender on teachers' attitude toward mathematical modeling, $F(1, 329) = 5.58$, $p < .05$, partial $\eta^2 = .02$. However, there was no statistically significant interaction between teacher's grade level and their gender on attitude toward mathematical modeling, $F(2, 329) = 1.25$, $p = .29$, partial $\eta^2 = .01$. This suggests that there was no interaction between the two factors on overall attitude towards mathematical modeling.

Table 3. Descriptive Statistics for the 3 × 2 ANOVA on the MMAS

		Average MMAS		
Grade Level	*n*	Gender	*M*	*SD*
K–5	28	Male	4.66	0.46
	196	Female	4.89	0.49
	224	Total	4.86	0.49
6–8	21	Male	4.70	0.40
	35	Female	4.97	0.46
	56	Total	4.87	0.46
9–12	18	Male	4.62	0.49
	37	Female	4.62	0.43
	55	Total	4.62	0.44
Total	67	Male	4.66	0.45
	268	Female	4.87	0.48
	335	Total	4.82	0.48

A follow up analysis showed there was a statistical significance in gender with a mean difference of .17, 95% CI [.03, .30], $p < .05$. The results showed that female elementary and middle school teachers had a relatively positive attitude toward mathematical modeling on average than their male counterparts. Within the high school teachers, both female and male teachers had on average the same positive attitude towards mathematical modeling. Additionally, the Tukey's post hoc test showed that within the grade level, there was a statistically significant difference in means between elementary and high school teachers, $M_{\text{diff}} = .24$, 95% CI [.07, .41], $p < .05$ and middle and high school teachers, $M_{\text{diff}} = .25$, 95% CI [.04, .46], $p < .05$. However, there was no statistically significant difference in means between elementary and middle school teachers, $M_{\text{diff}} = .01$, 95% CI [–.18, .16], $p = .12$.

Conclusion and Implications

The purpose of this quantitative study was to investigate teachers' knowledge and attitudes toward mathematical modeling. The role that mathematical modeling has in school mathematics curriculum is of paramount interest for the achievement of goals put forth by the NCTM, CCSSM and GAIMME report, which are in agreement of promoting mathematical modeling education. Practices associated with mathematical modeling are relatively new and understanding the level of teachers' knowledge and attitudes they bring to the classroom is important when identifying their professional needs related to teaching modeling standards. The findings in the study indicate that K–12 teachers have satisfactory knowledge and slightly positive overall attitudes toward mathematical modeling. Teachers' knowledge and attitudes are important factors in teaching (Hill et al., 2005). The results showed that teacher's knowledge of mathematical modeling was relatively higher among elementary teachers than high school or middle school teachers. This revelation was surprising because mathematical modeling is both a standard of mathematical practice and a conceptual category for high school teachers. Consequently, high school teachers were expected to be well informed and knowledgeable about mathematical modeling. An explanation to this disparity could be attributed to the context of mathematical modeling or difference in sample size. Additionally, the study results indicated a statistical significance on teachers' knowledge and overall attitudes toward mathematical modeling. This finding was not surprising because other similar studies indicate that teachers have positive attitudes toward mathematics in general (Thiel, 2010; Wilkins, 2002). In terms of the relationship between the two main variables, the study results suggest that it is possible to predict teachers' attitudes from their knowledge on the construct mathematical modeling (R^2 = 25%), which is in concurrence with other studies (Hannigan et al., 2013; Wilkins, 2008).

The findings of this study show that the participants have moderate knowledge and overall attitudes toward mathematical modeling and these results are consistent with other previous related studies (Wilkins, 2002, 2008). This indicates that teachers need mastery in the area of mathematical modeling and teaching to improve their teacher efficacy and effectiveness in the classroom. This could be accomplished through the development of preservice coursework and in-service workshops, or professional development specifically designed and devoted to mathematical modeling education. Previous research measuring teachers' knowledge and attitudes regarding mathematical modeling are lacking in the research, making these current findings an important addition to the knowledge base in mathematics education. Related studies show that teacher knowledge and attitudes are strong indicators of teachers' ability to be successful in the classroom (Cakiroglu et al., 2012; Fessehatsion & Peng, 2021; Hill et al., 2005; Reichenberg, 2021).

Although this study was limited to public school teachers of mathematics in a midwestern state in the U.S., and only represents a fraction of the national K–12 teachers of mathematics population, a major strength of this research is the evidence provided regarding teachers' knowledge and attitudes toward modeling, which is scarce in the literature. This provides researchers, policy makers, and teacher educators with an understanding of teachers' current levels of knowledge and attitudes toward mathematical modeling. Even with these limitations, the findings from this study offer interesting insights into the relationship between these variables and provide mathematics educators and researchers with additional evidence of the importance of enhancing teachers' knowledge and attitudes toward mathematical modeling as they relate to instructional practices in the teaching and learning of mathematics.

Future Research Directions

The purpose of this research was to examine and explore the relationship between teachers' knowledge and attitudes toward mathematical modeling. This study provided preliminary insights into K–12 practicing teachers' knowledge and attitudes toward mathematical modeling, while considering the relationships between them and some selected demographic variables. The methods applied in this study could be used in future research in coordination with qualitative methods such as interviews and observation to provide further evidence of validation that would strengthen how the results are interpreted and applied by other researchers. Additionally, an area that needs to be addressed in future research is the focus on the subscales to identify the unique contribution of different factors in explaining the impact of teachers' attitudes on mathematical modeling. The use of subscales in subsequent data collection and to hypothesize, will deepen understanding of the influence of these unique factors (subscales).

Teachers' moderate knowledge and attitudes toward mathematical modeling suggest the need in the area of mathematical modeling professional development for teachers of mathematics (Phillips, 2016). Developers of professional development programs need to create professional growth opportunities for teachers that fit the mathematical modeling standards and guidelines in the CCSSM and the GAIMME report respectively. More research is needed to identify the components to include in in-service training programs related to mathematical modeling education and the impact that these trainings have on classroom instruction. Although this current study points to the need of strengthening teachers' knowledge, attitudes, practices, and experiences with mathematical modeling, future research is needed to determine the specific types of expertise, experiences, and professional support that teachers need to successfully implement the mathematical modeling standards and practices outlined in both the CCSSM and the GAIMME documents.

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