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A Systematic Review of Treatment Acceptability in Mathematics Interventions for Students With Learning Disabilities

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Treatment acceptability is an aspect of social validity that refers to participants' beliefs and perceptions about the intervention, such as the helpfulness of the strategies or the interventions' efficacy to improve performance. The purpose of this study was to conduct a systematic review of treatment acceptability measures administered during mathematics interventions for students with learning disabilities (LD). We sought to identify the characteristics of the measures and the treatment acceptability outcomes. To be included in this review, studies had to (a) focus on testing the effectiveness of a mathematics intervention, (b) include preschool through 12th grade students who were diagnosed with LD, (c) administer student or teacher measures of treatment acceptability, and (d) employ a single case or group design. This systematic review included 23 studies (22 included student measures, 8 included teacher measures). The majority of studies that reported information about treatment acceptability were single case design (91%), used interview-based measures (47%) and reported qualitative results (90%). Fewer studies used measures that allowed for researchers to provide quantitative results (20%) of treatment acceptability. The results of this systematic review indicated students' and teachers' perceptions about mathematics interventions for students with LD were overwhelmingly positive. We discuss implications of our findings in relation to improving the efficacy of mathematics interventions for students with LD.

Keywords: **Social Validity, Treatment Acceptability, Mathematics, Intervention, Learning Disability**

Introduction

Children with learning disabilities (LD) make up the largest proportion of students with disabilities who receive special education services in the United States (33%; Snyder et al., 2019), and a smaller subset of these students (6%) are specifically identified as having a *mathematics* learning disability (Barbaresi et al., 2005; Reigosa-Crespo et al., 2012). Students with LD and disabilities generally, have lagged behind their peers in mathematics (Geary et al., 2012; National Center for Education Statistics, 2019; Nelson & Powell, 2018; Vanbinst et al., 2014). Given research consistently shows early mathematics performance impacts later mathematics achievement (Claessens & Engel, 2013; Geary et al., 2013; Koponen et al., 2019) and adulthood outcomes (Davis-Kean et al., 2021; Rose & Betts, 2004), it is not surprising so many

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mathematics intervention programs exist. In addition to understanding the effectiveness of these intervention programs on achievement outcomes, it is important to understand treatment acceptability as a way to bridge research to practice. Treatment acceptability is a component of social validity. Higher levels of treatment acceptability may impact the degree to which students comply with the intervention (Reimers et al., 1987) and whether teachers integrate the intervention into regular classroom practice (Leko, 2014; Strain et al., 2012).

The purpose of this study was to examine how treatment acceptability is measured in mathematics intervention studies, to identify the characteristics of these measures, and to report the treatment acceptability outcomes of the interventions. Specifically, we focused on treatment acceptability measures in mathematics interventions for students with LD. In the following sections, we define and describe the importance of social validity and treatment acceptability in academic interventions. Then, we describe the social importance of mathematics achievement and discuss previous systematic reviews and meta-analyses that investigated the effectiveness of mathematics interventions. Finally, we provide the research questions guiding the current study.

Social Validity

In 1978, Montrose M. Wolf published a seminal article encouraging researchers in the field of applied behavior analysis to conduct work that was considered socially important. Specifically, he called for research to be validated on three levels:

> 1. The social significance of the *goals*. Are the specific behavior goals really what society wants? 2. The social appropriateness of the *procedures*. Do the ends justify the means? That is, do the participants, caregivers and other consumers consider the treatment procedures effective? 3. The social importance of the *effects*. Are consumers satisfied with the results? *All* the results, including any unpredicted ones? (Wolf, 1978, p. 207).

The judgments of the social importance of research on these three levels have since been referred to as "social validity." Since the publication of Wolf (1978), several other researchers in fields beyond applied behavior analysis have conceptualized models of social validity (see Carter & Wheeler, 2019 for a review), and have further operationalized other aspects of social validity. *Treatment acceptability* is a component of social validity that refers to the participants' perceptions of the intervention as to whether the treatment was "appropriate, fair and reasonable" (Kazdin, 1981, p. 493). Treatment acceptability is central to the judgment of the appropriateness of the intervention procedures; however, it is often difficult to parse judgments of treatment procedures from effects (Kazdin, 1981). Models of treatment acceptability acknowledge the important relations between high treatment acceptability, high compliance with the intervention, high effects, and high maintenance. Similarly, negative impacts of low treatment acceptability are low compliance, low effects, and low maintenance (Reimers et al., 1987).

Social Validity and Intervention Implementation

Aspects of social validity have the potential to improve the implementation and sustainability of an intervention program (Leko, 2014; Strain et al., 2012). For example, researchers have reported on the valuable contribution of collaboration between researchers and practitioners in the development of intervention programs (Denton et al., 2003; Fuchs & Fuchs, 2001) and the importance of using typical intervention agents (i.e., teachers) within intervention studies (Horner et al., 2005). Although this study focuses on social validity and treatment acceptability, it should be noted that other aspects of intervention research also play an important role in improving implementation and sustainability of intervention effects, such as teacher training and professional development (Burns et al., 2013; Denton et al., 2003) and teachers' self-efficacy (Han & Weiss, 2005).

Given the role treatment acceptability may play in the long-term effectiveness of interventions (Leko, 2014; Strain et al., 2012), it is not surprising that the field of special education has seen an increase in the acknowledgment of social validity in research and research funding. In special education research, experts have recognized the importance of selecting dependent variables with social significance and including measures and reporting results related to social validity (Horner et al., 2005; Council for Exceptional Children, 2014). Specifically, Horner et al. (2005) called attention to selecting socially important dependent variables, demonstrating that the independent variables can be applied by typical intervention agents in typical settings, and selecting interventions that are feasible and cost effective. Related to treatment acceptability, Horner et al. (2005) emphasized reporting participant perceptions of acceptable procedures, effective procedures, feasibility, and continued use of the procedures. With the rise in social validity research and heightened awareness related to overall research study quality, educational funding agencies have also started encouraging researchers to consider social validity in grant proposals. Both federal (U.S. Department of Education, Office of Special Education Programs, 2020) and private (Nuffield Foundation, 2021) grant funding agencies call for researchers to administer and collect social validity data.

Previous Research on Social Validity and Interventions

Despite increased awareness of the importance of considering social validity in education-related research, several reviews of research have highlighted too few studies address social validity. Authors have conducted reviews of social validity in several fields such as reading interventions (Lindo & Elleman, 2010), early language interventions (Larson et al., 2020), behavior interventions (Park & Blair, 2019), and single-case design generally (Snodgrass et al., 2018). Collectively, the results of these previous studies reported the prevalence of and extent to which social validity was measured in intervention studies was low. For example, Snodgrass et al. (2018) reviewed publications in six top special education journals from 2005 to 2016 and determined only 26.8% of single-case design studies included a measure of social validity. This is surprising given the emphasis of social validity in single-case design quality indicators for special education research (Horner et al., 2005).

To date, we were not able to locate a systematic review focused on examining measures of social validity in mathematics interventions. Conducting a systematic

review of the treatment acceptability of mathematics interventions for students with disabilities can identify the degree to which measures of student perceptions of the intervention are administered and what the characteristics of those measures are. Moreover, the results of this review can shed light on the relation between treatment acceptability and intervention outcomes, thereby having the potential to inform the development of future interventions for students with LD.

Mathematics Achievement for Students with Disabilities

Mathematics achievement for all students is important to society for a variety of reasons. Perhaps most salient is the fact that professional and daily living skills increasingly rely on the use of science, technology, mathematics, and problem-solving skills. Adults need to demonstrate mathematics proficiency in order to enter careers with the highest expected rates of growth, such as technicians for alternative forms of energy, healthcare professionals, security analysts, and software developers (U.S. Bureau of Labor Statistics, 2019). Basic mathematics skills are also required for adults to assess short- and long-term benefits of health care decisions (Peters et al., 2007), as well as independent skills such as paying bills, acquiring loans, and creating a budget.

Despite mathematics' central role in professional and everyday life, students with LD lag behind their peers in mathematics achievement, with gaps in achievement widening over time (Nelson & Powell, 2018). Previous researchers have reported initial and continued gaps in achievement across different domains in mathematics and grade levels (Chong & Siegel, 2008; Geary, 2011; Schwenck et al., 2015; Vanbinst et al., 2014). Given the generally stagnant mathematics performance of students with disabilities, researchers continue to develop and test the efficacy of interventions for students with or at risk of disabilities (e.g., Moser Opitz et al., 2017; Toll & Van Luit, 2012; Zhang et al., 2021), with encouraging and positive results.

With the rise of empirical investigations of mathematics interventions, the number of research syntheses focused on mathematics interventions is expected. Meta-analyses and systematic reviews have the potential to identify effective instructional components (e.g., explicit and systematic instruction, using concrete and pictorial representations) that yield larger effects for students with disabilities. Often, the results of research syntheses are considered in identifying evidence-based practices for students with disabilities (Therrien et al., 2020). In a review of the literature from 2000 to 2020, Nelson, Crawford et al. (2022) identified 36 syntheses (systematic review, evidence-based review, or meta-analysis) focused on mathematics interventions for students with LD or mathematics difficulty. Nelson, Crawford et al. (2022) reported the syntheses focused on several content areas and instructional strategies including, for example, word problem solving (e.g., Cook et al., 2020; Lein et al., 2020), fractions (e.g., Hwang et al., 2019; Shin & Bryant, 2015), computer-assisted learning and technology (e.g., Kiru et al., 2018; Ran et al. (2020), and schema-based instruction (e.g., Jitendra et al., 2015). Across syntheses, Nelson, Crawford et al. (2022) predominantly reported moderate and large summary effect sizes on mathematics achievement. Despite positive effects on mathematics achievement, the authors also reported only three (8%) of the included syntheses reported intervention outcomes related to social validity (Jitendra et al., 2015; Misquitta, 2011; Nelson, Hunt et al., 2022).

This brings into question whether the authors of the 36 syntheses had enough information from included studies to report on social validity (i.e., are intervention studies reporting social validity outcomes for researchers to syntheses?). Nelson, Hunt et al. (2022) reported that one study in their synthesis reported social validity outcomes; therefore, they were not able to report a *synthesis* of treatment acceptability results across the nine proportional reasoning interventions in their synthesis. Misquitta (2011) simply reported that two of 10 studies included in the review on fraction interventions interviewed students at the conclusion of the intervention regarding their perceptions; the authors did not provide any further details. Jitendra et al. (2015) examined the quality of strategy instruction priming across 28 studies. The authors reported a common problem across the studies was not meeting the social validity quality indicators. Studies consistently failed to provide "evidence of acceptability, feasibility, effectiveness, or continued use of the instructional practice…" (Jitendra et al., 2015, p. 68).

Even with the large number of syntheses focused on mathematics interventions for students with LD and mathematics difficulty, a systematic review and synthesis focused on treatment acceptability has not been conducted. Although practitioners have access to a plethora of effective interventions, trends in mathematics achievement for students with LD indicate gaps in mathematics knowledge and skills remain. One potentially significant contribution of bridging research to practice is to consider the role social validity plays in mathematics interventions. A critical first step is to conduct a systematic review of treatment acceptability measures used in these intervention studies.

Purpose and Research Questions

The purpose of this study was to conduct a systematic review of treatment acceptability measures (i.e., student and teacher perceptions of the intervention) used in mathematics intervention studies with school-aged children with LD. The research questions were:

1. For mathematics intervention studies that administered measures of treatment acceptability, what is the representation of study design (i.e., group design, single case design), mathematics content area (e.g., word problem solving) and grade level?
2. What are the characteristics (e.g., what aspects of student perceptions are measured, types of items, response format) of the treatment acceptability measures used in mathematics intervention studies with students with LD?
3. What are the reported treatment acceptability outcomes as related to the implementation of mathematics interventions for students with LD?

Method

Literature Search

We conducted an electronic search of the literature on academic interventions for students with LD in which researchers also administered a student measure

of treatment acceptability. We did not restrict the date of publication for our electronic search; however, we concluded our electronic search in November 2020 and the oldest study that surfaced during our search was published in 1983. First, we conducted an electronic search of the dissertations and peer-reviewed journal articles using Academic Search Premier, Education Research Complete, Educational Resources Information Center (ERIC), PsycARTICLES, and PsycINFO. Next, we identified studies for this systematic review using the following Boolean search string: ("social validity" OR "treatment acceptability" OR "treatment feasibility" OR "social importance" OR "social acceptability") AND intervention AND ("learning disabilit*" OR "learning difficult*" OR disabilit*). This search initially resulted in 1,069 abstracts, with 621 abstracts after duplicates were removed across databases.

Second, we identified five peer-reviewed journals focused specifically on publishing research on students with learning disabilities and scanned their electronic table of contents (*Learning Disabilities: A Contemporary Journal*; *Learning Disabilities Research & Practice*, *Learning Disability Quarterly*, *Journal of Learning Disabilities, The Journal of Special Education*). From the table of contents review we identified an additional 241 abstracts for review. Finally, we reviewed the curriculum vitae of two authors who conduct research in mathematics intervention and had three or more publications identified for inclusion.

Inclusion Criteria

We included studies published in English that met the following inclusion criteria.

1. The focus of the study was on testing the effectiveness of a mathematics content or instructional strategy intervention. Other types of interventions, such as those focused on reading, writing, spelling, behavior, social skills, communication, etc. were excluded.
2. The participants in the study were in preschool through 12th grade and had diagnosed LD. Studies that included participants who were typically achieving, at-risk for learning disabilities, or had other diagnosed disabilities (e.g., Autism spectrum disorders, ADHD, emotional behavior disabilities) were excluded unless (a) participants with diagnosed LD made up 50% or more of the total participants, or (b) the study provided disaggregated treatment acceptability results for students with LD.
3. Students or teachers were administered a measure of treatment acceptability related to the implementation of the academic intervention. The study also included either qualitative or quantitative results related to treatment acceptability.
4. Studies employed a quantitative design, either a group design or single-case design. Qualitative studies were excluded.

Abstract Screening Process

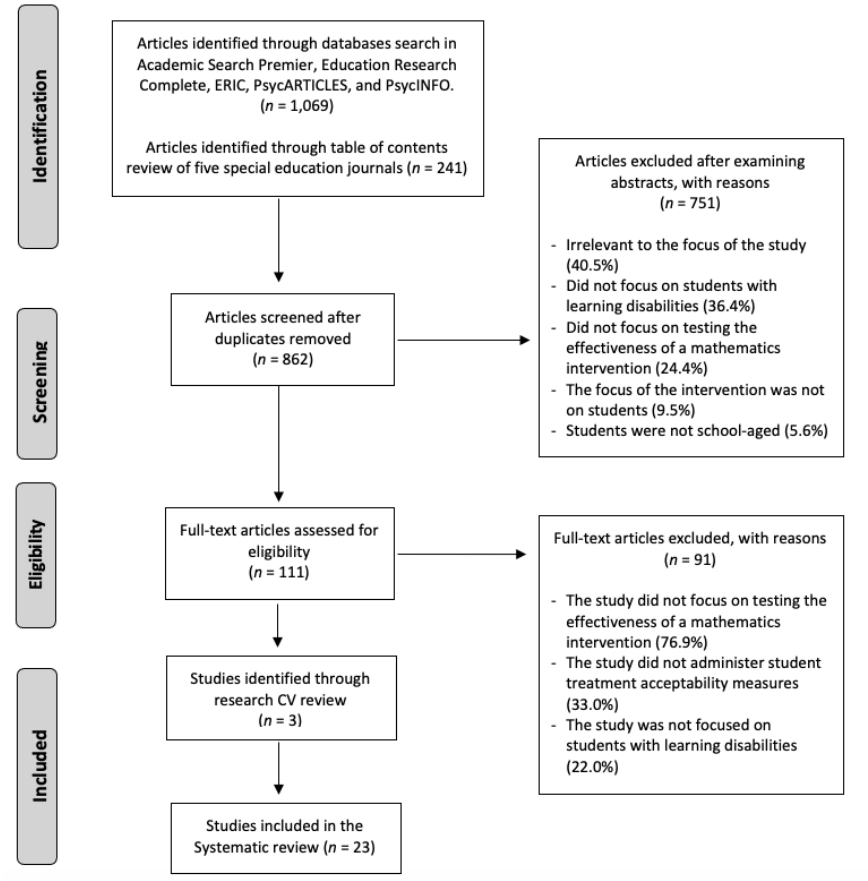

Figure 1. **Prisma Diagram Documenting the Literature Search Procedures**

The electronic search of databases resulted in 1,069 total abstracts, with 621 abstracts to review after duplicates were removed across databases; the electronic search of table of contents from five journals resulted in an additional 241 abstracts to review. Therefore, we reviewed 862 titles and abstracts. (See Figure 1 for a PRISMA diagram documenting the literature search process). At this stage, we excluded 751 studies for the following reasons: irrelevant (40.5%; e.g., not focused on interventions for school aged children); did not focus on students with LD (36.4%); did not focus on testing the effectiveness of a mathematics intervention (24.4%); the focus of the intervention was not on students (9.5%); students in the study were not school-aged (5.6%). The first author trained the second and third authors, a graduate student and an undergraduate student, to conduct the abstract screening using a code book with operational definitions and explanations of the inclusion criteria. For this stage of the

screening, we also double-screened 21% of abstracts ($k = 183$); agreement was 96.7%. The first author reviewed the 6 abstracts with disagreements and none were identified for inclusion in this study.

Then, the first author reviewed the full-texts of the 111 studies identified for further review. We excluded 91 studies after completing a full-text review for the following reasons (most were excluded for more than one reason): the study was not focused on testing the effectiveness of a mathematics intervention (76.9%; most intervention studies were related to literacy); the study did not administer a treatment acceptability outcome measure (33.0%); and, the study was not focused on students with LD (22.0%; i.e., studies were focused on other disability categories or students who were at-risk). Finally, the first author located three additional studies from the review of two authors' curriculum vitae. In total, 23 studies were included in this systematic review.

Coding Procedures

The first author developed the coding manual; she has experience developing coding manuals and corresponding databases for systematic reviews. The first author reviewed the coding sections of systematic reviews with a similar focus on social validity (e.g., Larson et al., 2020; Park & Blair, 2019) to identify relevant variables to code. The first author selected codes directly aligned to answering the research questions posed in this study. After the first author developed the coding manual, all authors coded the same article independently. Then, the authors discussed any issues with the codes and explanations in the coding manual and discussed any missing relevant codes that needed to be added. From there, the first author revised the coding protocol by adding codes related to intervention setting, implementation fidelity, the type of social validity measure (e.g., scale, interview), and the number of items on the measure. Then, the three authors coded two more articles and refined the coding manual after discussing any remaining concerns with the coding manual (i.e., added details to the code explanations). After the coding manual was finalized, all articles were coded by two of the authors and agreement of each assigned code was compared to determine interrater agreement. Across all articles, the interrater agreement was an average of 87.0%. The authors held meetings to discuss each of the discrepancies; the authors reviewed the code together, reviewed the original study to identify any information one of the authors missed, and agreed on a final code. The final codes were used in data analyses.

General Codes

In addition to treatment acceptability measure information, we coded each study for variables in the following categories: (a) general study information (authors, year of publication, journal title, and publication type); (b) methodological information (total sample size, sample size for students with LD, design, if the study included a research question or purpose statement related to social validity;, (c) student participant characteristics (age or grade, gender, race/ethnicity, and socioeconomic status, dual language learner status), and (d) intervention features (instructional arrangement, setting, intervention agent, treatment or implementation fidelity, content area focus).

Treatment Acceptability Measures

We coded each study for information related to the treatment acceptability measures. We recorded if the measure was researcher-developed or if the authors provided a name of a specific social validity measure, as well as information about the reliability and validity of the measure. We recorded the response format of the measure (i.e., scale, open-response questionnaires, interviews, focus group), and the number of questions and if the questions were provided (if questions were provided, we recorded all questions). After reviewing the studies, we also identified common categories that represented the type of treatment acceptability aspects researchers measured. Then, we recorded if studies addressed those common treatment acceptability categories. For student measures, these categories included: beliefs about the effectiveness or helpfulness of the intervention, opinions about the design or format of the intervention, belief they (students) would use the strategies again, enjoyment of the intervention activities, if the intervention content was easy to understand, perceptions about the engagement of the activities, and recommendations for improvement of the intervention. For teachers measures, these categories included: cost of the intervention, beliefs that procedures were easy to follow, beliefs that students enjoyed the intervention, beliefs that student performance improved as a result of the intervention, statements that they (teachers) would use the program again or recommend it, and recommendations for improvement of the intervention program. We also recorded when the researchers collected social validity data (e.g., before, during, or after the intervention).

Treatment Acceptability Results

Finally, we recorded any qualitative or quantitative results studies reported related to treatment acceptability. Qualitative results were recorded as summaries of what the authors reported and quantitative results were recorded as averages or ranges of scores the authors reported for treatment acceptability results.

Data Analysis

To answer research question 1, we calculated the overall frequency with which treatment acceptability was measured according to different study features, including design, mathematics content area, and grade level. To answer research question 2, we calculated frequencies according to the characteristics of treatment acceptability measures. To answer research question 3, we recorded the number of studies that reported results in the treatment acceptability categories to identify patterns.

Table 1. *Summary of Studies Included in the Systematic Review*

Authors (year)	*N*	LD *N*	Grades	Math Content Focus	Interventionist	Fidelity
Brawand et al. (2020)	9	6	7, 8	Proportional reasoning WPS	Researcher	100%
Bryant et al. (2015)	6	6	4	Multiplication facts	Researcher	94%
Calhoon & Fuchs (2003)	92	68	9, 10, 11, 12	Computation, concepts applications	Peer	90.30%
Case et al. (1992)	4	4	5, 6	Addition, subtraction WPS	Undergrad student	NR
Cass et al. (2003)	3	3	7, 9, 10	Area, Perimeter	Teacher	100%
Cuenca-Carlino et al. (2016)	6	3	8	Multi-step equations	Researcher	100%
Dennis et al. (2016)	6	6	2	Basic facts	Researcher	92-97%
Flores et al. (2014)	4	4	4, 5	Multiplication	Researcher	100%
Freeman-Green et al. (2015)	6	6	8	WPS	Researcher	97.50%
Kellems et al. (2020)	7	7	8	Multistep problems	NR	89%
Milton et al. (2019)	5	4	4, 5, 6	Multiplication, division	Teacher	100%
Ok & Bryant (2016)	4	4	5	Multiplication facts	Researcher	98%
Owen & Fuchs (2002)	24	20	3	WPS	Researcher	91.9-97.0%
Park et al. (2021)	3	2	6, 7	Multiplication	Researcher	100%
Satsangi, Billman et al. (2020)	3	3	10	Algebra, linear equations	Researcher	100%
Satsangi & Bouck (2015)	3	3	9, 11	Area, perimeter	Researcher	100%
Satsangi et al. (2016)	3	3	11, 12	Algebra	Researcher	100%

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Table 1. *Summary of Studies Included in the Systematic Review (continued)*

Authors (year)	*N*	LD *N*	Grades	Math Content Focus	Interventionist	Fidelity
Satsangi, Hammer et al. (2020)	3	3	9, 10	Geometry WPS	Researcher	100%
Satsangi, Hammer, & Evmenova (2018)	3	3	9	Multistep equations	Researcher	100%
Satsangi, Hammer, & Hogan (2018)	3	3	9	Geometry WPS	Researcher	100%
Satsangi, Hammer, & Hogan (2019)	3	3	9	Multistep linear equations	Researcher	100%
Shin & Bryant (2017)	3	3	6, 7, 8	Fraction WPS	CAI	100%
Strickland & Maccini (2013)	3	3	8, 9	Multiplying linear algebraic expressions	Researcher	100%

Note. CAI = computer administered intervention; Fidelity = fidelity of implementation; LD *N* = number of students in the sample with a documented learning disability; NR = not reported; SCD = single case design; WPS = word problem solving.

Before answering our three research questions, we provide a descriptive overview of the studies included in this systematic review. Table 1 includes a summary of the 23 studies. Across the studies, There were 206 total participants, of which 170 (83%) had a diagnosed LD. The majority of studies ($k = 17$) included only students with LD; other disabilities represented in the studies included: intellectual disability ($n = 13$), other – not specified ($n = 8$), emotional behavioral disability ($n = 4$), speech or language impairment ($n = 2$), ADHD ($n = 2$), autism spectrum disorder ($n = 2$), other health impairment ($n = 1$), Deaf or hard of hearing ($n = 1$), and auditory processing disorder ($n = 1$). Students ($n = 195$) were predominantly White (46.7%) and Black (37.4%), with fewer students identifying as Hispanic (12.3%) or more than one race (2.6%). The majority of students ($n = 193$) were male (64.3%).

The Degree to Which Treatment Acceptability is Measured

With our first research questions, we explored the extent to which treatment acceptability is measured in mathematics intervention studies across design type, grade levels, and mathematics domains. The 23 studies that met inclusion criteria overwhelmingly used single case design methods ($k = 21$). Studies focused on participants from second through 12th grades (oftentimes with a focus of participants across different grade levels), with the following distribution of studies per grade: second ($k = 1$), third ($k = 1$), fourth ($k = 3$), fifth ($k = 4$), sixth ($k = 4$), seventh ($k = 4$), eighth ($k = 6$), ninth ($k = 8$), 10th ($k = 4$), 11th ($k = 3$), 12th ($k = 2$). Studies addressed a variety of content including: word problem solving ($k = 6$), computation or basic facts ($k = 7$), Algebra ($k = 4$), geometry ($k = 4$), multi-step equations ($k = 3$), proportional reasoning ($k = 1$), and fractions ($k = 1$). The majority of studies used researchers or graduate students as intervention agents ($k = 17$), followed by teacher ($k = 2$), mix of researchers and teachers ($k = 1$), peer tutoring ($k = 1$), computer administration ($k = 1$), and not reported ($k = 1$). Finally, although all studies included in this systematic review included a measure of treatment acceptability, seven studies (30%) did not include a research question or purpose statement related to social validity.

Characteristics of Treatment Acceptability Measures

With our second research question, we investigated the characteristics of the treatment acceptability measures. First, we report characteristics related to student measures, then we report characteristics teacher measures.

Characteristics of Student Measures

Table 2. ***Summary of Student Treatment Acceptability Measures from Mathematics Interventions***

Authors (year)	Measure Type	Total Questions or Items	Questions Provided	Quantitative Results
Brawand et al. (2020)	Scale	NR	No	No
Bryant et al. (2015)[a]	Open-ended questionnaire or Interview	8	Yes	No
Calhoon & Fuchs (2003)	Scale	11	Yes	Yes
Case et al. (1992)	Interview	NR	No	No
Cass et al. (2003)	Interview	NR	No	No
Cuenca-Carlino et al. (2016)	Interview	NR	No	No
Flores et al. (2014)	Open-ended questionnaire	NR	No	No
Freeman-Green et al. (2015)	Scale, Open-ended questionnaire	7-10	No	No
Kellum et al. (2020)	Open-ended questionnaire	9	No	No
Milton et al. (2019)	Scale	NR	No	No
Ok & Bryant (2016)	Scale, Interview in-person	20	Yes[b]	Yes
Owen & Fuchs (2002)	Scale	5-8	Yes	Yes
Park et al. (2021)	Interview	NR	Yes	No
Satsangi, Billman et al. (2020)	Interview	NR	No	No
Satsangi & Bouck (2015)	Interview	4	Yes	No
Satsangi et al. (2016)	Interview	9	No	No
Satsangi, Hammer et al. (2020)	Interview	NR	No	No
Satsangi, Hammer, & Evmenova (2018)[a]	Open-ended questionnaire or Interview	NR	No	No

Table 2. ***Summary of Student Treatment Acceptability Measures from Mathematics Interventions (continued)***

Authors (year)	Measure Type	Total Questions or Items	Questions Provided	Quantitative Results
Satsangi, Hammer, & Hogan (2018)	Interview	NR	No	No
Satsangi, Hammer, & Hogan (2019)	Interview	NR	No	No
Shin & Bryant (2017)	Scale, Open-ended questionnaire	5, 9	No	Yes
Strickland & Maccini (2013)	Scale, Interview in-person	NR	Yes	Yes

Note. NR = not reported
a = the study did not provide a clear description of the social validity measure or used multiple terms to describe the measure.
b = questions were provided for a subset of items

All but one of the included studies (Dennis et al., 2016) included a student measure of treatment acceptability. Table 2 provides a summary of each of the student measures we coded as part of this systematic review. Three studies specifically reported using researcher-developed measures, whereas, the remaining 19 studies did not report a measure name or report that the measure was researcher-developed. One study reported reliability for the scoring of the student responses to the treatment acceptability measure, but no studies reported any other form of reliability. Only one study reported validity information for the development of the measure. The majority of studies administered only one type of treatment acceptability measure, but four studies employed a mix of different types of treatment acceptability measures. Studies utilized scales (e.g., yes, no, maybe agreement, Likert scales; $k = 8$); interviews ($k = 12$), open-ended written questionnaires ($k = 4$) and two studies did not provide a clear description of the measure to assign the measure to one of our predetermined codes. Less than half of all studies ($k = 9$; 40.9%) provided the number of total items asked as part of the treatment acceptability assessment, and authors reported asking between 4 and 20 questions. Seven studies (31.8%) provided the actual treatment acceptability questions that researchers asked students. Finally, all authors reported they collected treatment acceptability at the conclusion of the intervention; eight studies (36%) also reported collecting data prior to the start of the intervention. No studies reported collecting treatment acceptability data during the intervention period.

Regarding student perceptions about the interventions, all studies collected or reported data regarding at least three different aspects of treatment acceptability (according to the aspects of treatment acceptability we coded as part of this study), with an average of 5.0 aspects ($SD = 1.1$). Studies collected or reported results of aspects of treatment acceptability from students' perspectives with the following frequencies: beliefs the intervention was effective or helpful (90.9%), opinions about the design or format of the intervention (95.5%), belief they (students) would use the strategies again (72.7%), description of the enjoyment of the intervention activities, (90.9%), perceptions about whether the intervention content was easy to understand (63.6%), perceptions about the engagement of the activities (50.0%) and recommendations for improvement of the intervention (36.4%).

Characteristics of Teacher Measures

Table 3. ***Summary of Teacher Social Validity Measures from Mathematics Interventions***

Authors (year)	Measure Type	Total Questions or Items	Questions Provided	Quantitative Results
Cass et al. (2003)	Interview	1	No	No
Dennis et al. (2016)	Questionnaire	NR	No	No
Flores et al. (2014)	Questionnaire	NR	No	No
Freeman-Green et al. (2015)	Scale	NR	No	No
Milton et al. (2019)	Scale	NR	No	No
Owen & Fuchs (2002)	Scale	9	Yes	Yes
Park et al. (2021)	Interview	4	Yes	No
Satsangi, Hammer & Hogan (2018)	Interview	1	No	No

Note. NR = not reported.

Eight studies collected teacher treatment acceptability data (see Table 3). None of the studies reported the measure name, whether it was researcher-developed, or information about reliability or validity. Researchers administered scales ($k = 3$), conducted interviews ($k = 3$), and collected questionnaires ($k = 2$) to capture perceptions of treatment acceptability. Only two of the seven studies provided the questions researchers asked teachers and only one study provided quantitative results. On average studies collected information about 2.4 aspects ($SD = .74$) of social validity related to teachers' perceptions. The aspects of social validity that authors reported for teacher measures included: recommendations for using the intervention, improvements in student performance, easy to implement, engaging activities, cost effective, and recommendations for improvement. Of the eight studies that also administered teacher measures, all reported that they collected treatment acceptability information after the conclusion of the intervention; two studies also collected data prior to the start of the intervention.

Treatment Acceptability Outcomes

With our third research question, we investigated trends in the reported treatment acceptability outcomes as related to the implementation of the mathematics interventions. First, we report results based on student measures, then we report results based on teacher measures.

Student Treatment Acceptability Outcomes

Of the 22 studies that administered student measures, 20 reported qualitative results and 5 reported quantitative results for student social validity measures. The qualitative results varied in their detail; for example, whereas some studies reported specific student excerpts from the open-ended questions and synthesized the

overall student perceptions of the interventions, other studies simply reported a sentence summary of the student social validity responses. Despite the variability in the level of detail provided by the authors of the studies, the qualitative results of the student social validity measures were overwhelmingly positive. All of the 20 studies generally reported positive results such as student statements about (a) positive perceptions of the intervention, (b) belief the intervention was helpful in learning mathematics, (c) how the intervention encouraged them to check their work, (d) how they planned on continuing to use the strategies taught during the intervention, (e) enjoying receiving teacher feedback, (f) enjoying graphing their data, (g) enjoying working with a partner,

Only five studies reported students made negative comments about the intervention, which included statements regarding (a) the intervention was confusing, (b) the intervention was too slow at times, (c) students did not want to be with the same partner the entire intervention, and (d) the games were too childish, (e) dislike of worksheets, and (f) students did not like leaving their regular class for tutoring.

Of the five studies that reported quantitative results, four studies reported results as an average score or range of averages out of a rating of 5 (with higher scores indicating more positive perceptions of the intervention by students). Averages for these studies included 4.7, 4.6, 3.67, and a range between 2.5 and 4.3. The fifth study reported results as a percentage of the proportion of students who indicated agreement or disagreement with individual social validity prompts; results for this study indicated high levels of agreement with positive statements.

Teacher Treatment Acceptability Outcomes

For teacher measures, the results indicated the following frequencies with which the teachers made statements about the intervention: teachers would use the program again or recommend it to another teacher (71.4%), beliefs that student performance improved as a result of the intervention (57.1%), beliefs that procedures were easy to follow (42.9%), beliefs that students enjoyed the intervention (28.6%), cost effectiveness of the intervention (14.3%), and recommendations for improvement of the intervention program (14.3%). Overall, the results for teachers' perceptions of the intervention programs in this systematic review were deemed by study authors as positive. Two studies reported specific areas of improvement noted by teachers including (a) better aligning the intervention with regular math instruction so that the lessons do not feel like "extra" work, and (b) although the intervention was successful with a small group of students, the intervention may be more challenging to implement with a whole class.

Discussion

Quality indicators in special education acknowledge the role social validity plays in intervention reporting (Horner et al., 2005), and previous research reports on the important relation between treatment acceptability and intervention implementation (Denton et al., 2003). The purpose of this systematic review was to examine 23 mathematics intervention studies for students with LD. This study evaluated characteristics of treatment acceptability measures and patterns in student and teacher perceptions of the interventions. Although researchers have conducted sever-

al other syntheses focused on mathematics interventions for students with LD (Nelson, Crawford et al., 2022), to date, no synthesis has focused exclusively on investigating treatment acceptability for the interventions. Below, we discuss the implications of the results of our systematic review and offer suggestions for researchers to expand the knowledge base related to treatment acceptability of mathematics interventions.

Treatment Acceptability Measures in Mathematics Interventions

We were unable to identify *specific* patterns according to participant grade level or the mathematics domain given that these characteristics varied greatly across the 23 included studies. However, the results indicated researchers are considering treatment acceptability of their interventions across many grade levels and mathematics domains. The variability in intervention studies in which treatment acceptability was considered is encouraging and sets the stage for future researchers to continue to explore treatment acceptability and other aspects of social validity across diverse grade levels and content areas. Our results also indicated the majority of studies that investigated treatment acceptability were single-case design (91%); this is not surprising given the emphasis experts have placed on including social validity in single-case design studies (Horner et al., 2005). Standards for quality in experimental and quasi-experimental studies tend to emphasize authors report intervention effects for outcomes that are aligned to the intervention content, administer proximal and distal outcomes, and measure and report fidelity of implementation (Gersten et al., 2005). These are all critical aspects of group design intervention studies and should be emphasized in quality indicators, however, social validity, including treatment acceptability, is not a design-specific aspect of intervention research. Collecting social validity data will allow future researchers to understand how to enhance or improve interventions for student engagement (e.g., add motivational aspects if students indicate they did not enjoy the lessons), address confusing aspects of the intervention, and ensure students understand how to transfer skills or continue using learned skills beyond the intervention. In general, mathematics intervention research with students with LD may be enhanced if more authors captured and reported treatment acceptability information, including for group design studies.

Similarly, studies in which authors collect data on treatment acceptability may be enhanced by including specific research questions related to social validity. It is encouraging that 70% of studies included in this systematic review included a research question to emphasize the importance of collecting such data. The results of this systematic review related to this aspect of studies is more favorable than what researchers of previous systematic reviews have reported (e.g., 39%, Snodgrass et al., 2018; 46%, Larson et al., 2020). This is an important aspect of a study to consider because it brings into question the authors' purpose or intent of measuring and reporting on social validity if there is not a research question guiding the data collection or analysis of the data. Further, an area for additional research is for future syntheses to examine in-depth how social validity was addressed in other areas of the study. For example, if social validity was mentioned in the introduction, or if the results of the social validity data collection were mentioned or effectively embedded into the discussion of the overall effectiveness of the mathematics intervention (Snodgrass et al., 2018).

Characteristics of Treatment Acceptability Measures

Studies rarely reported information related to the development of the measure (i.e., if the measure was researcher-developed), reliability, or validity. Given reliability and validity of a measure impacts the interpretation of the data collected, it is essential that future researchers who administer social validity measures report these characteristics. Studies also reported wide variability in the number of items asked related to treatment acceptability, ranging from 4 to 20 items on student measures and 1 to 9 items on teacher measures. Yet, very few studies provided readers with the specific questions students ($k = 7$) and teachers ($k = 2$) were asked regarding acceptability. Future research can enhance replicability of mathematics interventions by providing readers with the specific questions researchers asked students; moreover, providing this information can give researchers insight into aspects of their own intervention they may explore for areas of improvement. With trends in mathematics achievement that indicate consistent gaps for students with LD, researchers may consider the importance of providing information about how social validity data were collected as a means to improve implementation and sustainability of the interventions.

Studies most often utilized treatment acceptability measures that required students and teachers to respond on a scale (e.g., yes, no, maybe agreement; Likert scales) or to questions during an interview. Open-ended written questionnaires were used less frequently with students, but presumably the interviews also used open-ended questions. Both structured responses (e.g., via scales) and unstructured responses with open-ended questions play an important role in the measurement of treatment acceptability. For example, evaluating the scores on a scale may be more efficient for group design studies where the number of students to collect data from is greater. Scores on a scale are also more easily transferred into quantitative results for interpretation or comparison with the results of other studies. This may be an appropriate option for researchers who want to monitor treatment acceptability of their intervention program over time as they make changes based on student and teacher feedback. In contrast, open-ended written or verbal questions may allow respondents more flexibility in the type of feedback they provide researchers because they are not constrained to numerical or yes/no responses. With open-ended questions, researchers may collect important information about specific aspects of lessons, activities, or the implementation of the interventions they may enhance. For example, it is prudent for a researcher to know if students who generally struggle with learning mathematics are understanding the newly introduced instructional strategies (e.g., schema-based instruction) or tools (e.g., virtual manipulatives). Given the benefits of collecting both qualitative and quantitative information, future intervention studies may consider administering both types of treatment acceptability measures to enhance their interventions.

A final pattern of the treatment acceptability measures we identified was that few studies administered treatment acceptability measures prior to the implementation of the intervention and no studies administered treatment acceptability measures during the intervention. These results are similar to what other researchers have reported (Larson et al., 2020; Snodgrass et al., 2018). An area of future research is for authors of intervention studies to consider collecting treatment acceptability data

before and during the intervention period. Researchers may collect information before an intervention that is related to students' perceptions of mathematics generally given that prior to the start of an intervention they may not have been introduced to different strategies or content. Treatment acceptability data is related to participants' enjoyment of the intervention activities, belief the content is important, opinions about the clarity of the intervention content and procedures, and thoughts about whether they will continue to use the intervention strategies beyond the intervention period. If treatment acceptability information is to be truly important in intervention research, then authors should intend to use the information to make adjustments to the delivery of the content before or during the intervention period (Schwartz & Baer, 1991; Snodgrass et al., 2018). Moreover, although eight studies reported they collected student treatment acceptability data before the intervention, none of those studies reported using the pre-intervention data to inform or adapt the intervention procedures. We found the same result with the two studies that administered teacher measures before and after the start of the intervention. Yet, previous research reports on the value of including teacher collaboration to develop mathematics interventions (Fuchs & Fuchs, 2001). Future intervention studies that include measures of treatment acceptability can be strengthened with a discussion of how pre-intervention treatment acceptability data were used to adapt or refine intervention content, procedures, or goals.

Treatment Acceptability Outcomes

Our final research question was aimed at identifying patterns in participants' perceptions about the interventions in which they participated. It is encouraging that the quantitative and qualitative results of this review indicated students and teachers had overwhelmingly positive perceptions about the interventions on various aspects of treatment acceptability including helpfulness and planned continued use of the strategies learned during the intervention, engagement during the intervention, positive impact on student achievement, and enjoyment of the activities. Despite positive results, an area for future research is for authors to consider the level of detail they provide readers regarding participants' responses on measures of treatment acceptability. There was wide variability in how authors reported results, with some authors only providing a single sentence of social validity results. Providing readers with more detail can help inform the development of other intervention programs.

Very few studies reported participants had any negative perceptions of the interventions. Yet, the specific negative perceptions are important to consider alongside the time of administration of the measures. For example, negative perceptions included a dislike of the worksheets and being pulled out of regular class time to participate in the intervention, as well as reports of the intervention being confusing. If researchers were to collect student and teacher treatment acceptability data during the intervention, they may be able to adjust intervention activities and procedures to increase students' enjoyment and engagement during the intervention. This is especially important given the fact that components of treatment acceptability models underscore the relationship between high treatment acceptability and greater outcomes (Reimers et al., 1987). The same is true of teacher measures. If researchers aim to develop socially important interventions that can feasibly implemented by typical

intervention agents after the duration of the study, researchers should solicit feedback from classroom teachers before, during, and after the development and implementation of the intervention.

Limitations and Future Research Directions

There are limitations of this systematic review. First, we investigated social validity narrowly as student and teacher perspectives of treatment acceptability. Our results do not speak to other stakeholders' perspectives of treatment acceptability, such as parents or administrators. In future reviews we may also consider other aspects of social validity beyond treatment acceptability measures such as determining the cost effectiveness of the programs or if the goals of the intervention were described as socially important in the introduction of the manuscript. Second, our inclusion criteria were narrow as we only considered mathematics interventions for students with LD. Our results may not generalize to other content areas. Our results may also not generalize to mathematics interventions for students without LD. Broadening the search criteria in future syntheses may provide a different picture of how social validity is measured in academic intervention studies. Moreover, the inclusion criteria that we outlined for this systematic review excluded studies with a qualitative design. Qualitative studies may provide researchers and practitioners with information about treatment acceptability measures that we were not able to capture from single case and group design studies.

Additional Future Research Considerations

Throughout this discussion, we provided readers with considerations for future research. However, given the limitations of this systematic review, as well as of the literature base on social validity and treatment acceptability in mathematics interventions, there are additional recommendations for future research. Regarding empirical investigations of the effectiveness of mathematics interventions for students with LD, we recommend more authors consider including measures of social validity in their intervention studies. A larger literature base will allow researchers to draw more robust conclusions regarding student and teacher perceptions of mathematics interventions, and a larger sample of studies would allow researchers of systematic reviews to identify patterns in social validity results regarding study features such as grade level and mathematics content area.

Finally, we originally intended to identify the relationship between treatment acceptability scores and intervention effectiveness (e.g., a meta-analysis of the relationship); however, we were unable to complete this analysis as intended. The included studies reported both (a) scores for treatment acceptability and (b) intervention effects (e.g., percent of non-overlapping data, percent increase in correct scores on outcome measures, Tau U, anecdotal statements of the effectiveness of the intervention based only on visual analysis) in a variety of different ways. The majority of studies reported social validity in a qualitative manner and the results for treatment acceptability were overwhelmingly positive. Considering there was little variation in treatment acceptability and too few studies reported quantitative results, we were unable to identify patterns of the relation between treatment acceptability and intervention outcomes. Future research syntheses may explore this relation further.

Conclusion

There are decades of research on social validity and treatment acceptability and educational experts agree on the important role social validity plays in intervention research. Yet, the results of our systematic review indicate a need for more researchers to consider administering and reporting on participants' perspectives of mathematics interventions in order to determine the relationship between treatment acceptability and intervention outcomes. Given social validity and treatment acceptability may impact the degree to which interventions are implemented, it is imperative that future researchers collect treatment acceptability data to inform efforts to enhance interventions for students with LD.

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