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Mathematics Learning from Concrete to Abstract (1968-2021): A Bibliometric Analysis

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Mathematics learning is illustrated as a developmental progression in the direction of concrete-to-abstract by educational theorists. Various studies rooted in this notion were conducted in the past. This study aimed to profile the landscape of research rooted in this notion which was published from 1968 to 2021. The bibliographic data of 425 related publications were retrieved from the Scopus database for bibliometric analysis. Descriptive analysis and regression analysis were performed to profile the publication trend. Then, author bibliographic coupling analysis was carried out to identify the domains of research related to mathematics learning from concrete to abstract. The findings show an increasing trend of publication following the exponential model. The research was clustered into five research domains: (i) 'manipulatives and arithmetic learning'; (ii) 'mathematics learning of students with learning disabilities'; (iii) 'Concrete-Representational-Abstract sequence in elementary mathematics teaching'; (iv) 'Ideal mathematics teaching'; and (v) 'mathematics problem-solving and mathematics learning of students with autism spectrum disorder'. The two emergent research domains in this research area are (i) 'mathematics learning of students with learning disabilities'; and (ii) 'mathematics problem-solving and mathematics learning of students with autism spectrum disorder', which have the highest proportion of publications since 2015. The findings of this study can help researchers to understand the current landscape of research with the notion of mathematics learning from concrete to abstract, and hence propose pathways for future research.

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Introduction

Mathematics is a gateway discipline to other fields such as science, technology and engineering. Yet, mathematics is often perceived as a difficult subject (Li & Schoenfeld, 2019) due to its abstractness (Ding & Li, 2014; Wang & Cai, 2007; Wong, 2007). Besides, representing the mathematics idea using numbers and symbols [e.g., $2 + 3 = 3 + 2$], the student would have to express the idea in a more general form [e.g., $a + b = b + a$] (Ding & Li, 2014). Learning mathematics concepts in an abstract context could be very challenging for the children because they do not have the capacity for abstract thought (McNeil & Jarvin, 2007). In this regard, educational theorists (e.g., Bruner, 1966; Piaget, 1952) posit mathematics learning as a developmental progression in the direction of concrete-to-abstract. Specifically, mathematics teaching should begin with engaging students with concrete manipulative or real-world examples which represent the mathematical idea, followed by using numbers and symbols to represent the mathematical idea. This is the most widespread assumption which holds in both Eastern (Wang & Cai, 2007, Wong, 2007) and Western contexts (Coles & Sinclairs, 2019; Fyfe et al., 2015).

A wide range of research rooted from the mathematics learning theory proposed by Bruner (1966) and Piaget (1952) has been conducted in the past spanning from types of manipulative used (e.g., Litster et al., 2019), teaching approaches (e.g., Flores et al., 2020a, 2020b), content domain (e.g., Braithwaite et al., 2016; Ching & Wu, 2019), grade level (e.g., O'Meara et al., 2020) and learning context (e.g., Sekeris et al., 2020). To synthesise findings of interventional studies related to the use of manipulatives and teaching approaches rooted from Bruner's Theory (1966) and Piaget's theory (1952) in enhancing mathematics learning of students with mathematics disabilities, meta-analysis and systematic literature review has been conducted by several researchers (i.e., Lafay et al., 2019; Park et al., 2021b; Peltier et al., 2020). Nonetheless, only a small number of articles were included in the analysis. Despite the concrete mathematics learning that has been discussed in a large body of literature, the current research landscape is yet to be studied for suggesting the emerging research foci (Kushairi & Ahmi, 2021).

As such, the bibliometric analysis could be a promising approach for profiling the research landscape. Different from meta-analysis and systematic literature review, bibliometric analysis involves a large collection of publications in the corpus of literature. Thus, it could provide a comprehensive overview of the research field (Fusco et al., 2020; Gümüş et al., 2020). Thus, bibliometric analysis has been widely used to capture the state-of-art of a given subject area, such as the publication trend (Ellegaard & Wallin, 2015) and the current research domain (Zupic & Čater, 2015). With this meaningful information, insights for future research directions could be drawn (Chen et al., 2019). These fruitful findings would also benefit researchers by narrowing down the literature search scope.

Whilst mathematics learning is posited as a developmental progression in the direction of concrete-to-abstract, various related studies have hitherto been conducted. The findings of the previous studies have been synthesised by conducting meta-analysis and systematic literature review (i.e., Lafay et al., 2019; Park et al., 2021b; Peltier et al., 2020). To fill in the methodology gaps, the previous studies on mathematics learning from concrete to abstract were synthesised using a different approach, named bibliometric analysis. This is because it could provide a holistic view of the research (Fusco et al., 2020; Gümüş et al., 2020) and hence benefit the researchers by narrowing down the literature search scope for future research. Specifically, this study sought to profile the landscape of research on mathematics learning from concrete to abstract published in 1968 to 2021 by conducting a bibliometric analysis. The research questions addressed in this study are as follows.

(1) What is the publication status of the research on mathematics learning from concrete to abstract published in 1968 to 2021?
(2) What are the research domains on mathematics learning from concrete to abstract published in 1968 to 2021?
(3) How/In what ways are the contributions of the most representative author in each research domain?

Methodology

Data Collection Method

The data collection process was conducted following the Preferred Reporting Items for Systematic Reviews and Meta-analyses (PRISMA) guidelines (Moher et al., 2009) as illustrated in Figure 1. The data collection process started with identifying relevant documents based on the topic 'Mathematics Learning from Concrete to Abstract' from the Scopus database which serves as the main database for the bibliometric analysis because of its wide interdisciplinary coverage (Mongeon & Paul-Hus, 2016). The document identification was conducted using the 'Title-Abstract-Keyword' search with four different search strings. The search was restricted to the subject area of mathematics and social science so that the search result would be more relevant to the research topic which is covered under the research field of mathematics education.

Since 'abstract' and 'symbolic' are being used interchangeably in the literature corpus of mathematics education, the first research string consisted of the words: 'concrete' and 'abstract' or 'concrete' and 'symbolic'. Meanwhile, the word 'manipulative' is used as the second search string in document identification because students' learning of abstract mathematics concepts is facilitated by manipulatives (McNeil & Jarvin, 2007). According to Leong et al. (2015), 'representational' or 'pictorial' is the intermediate stage in the concrete fading process of mathematics learning. In view of this fact, the words 'concrete' and the variants of the words 'representational' or 'pictorial' are used as the third search string for document identification. While the 'concrete-pictorial-abstract' instructional sequence is widely applied to facilitate students' mathematics learning (Chang et al., 2017; Abdoulaye, 2021), the variants of 'concrete-pictorial-abstract' and the acronyms are used as the fourth search string for document identification.

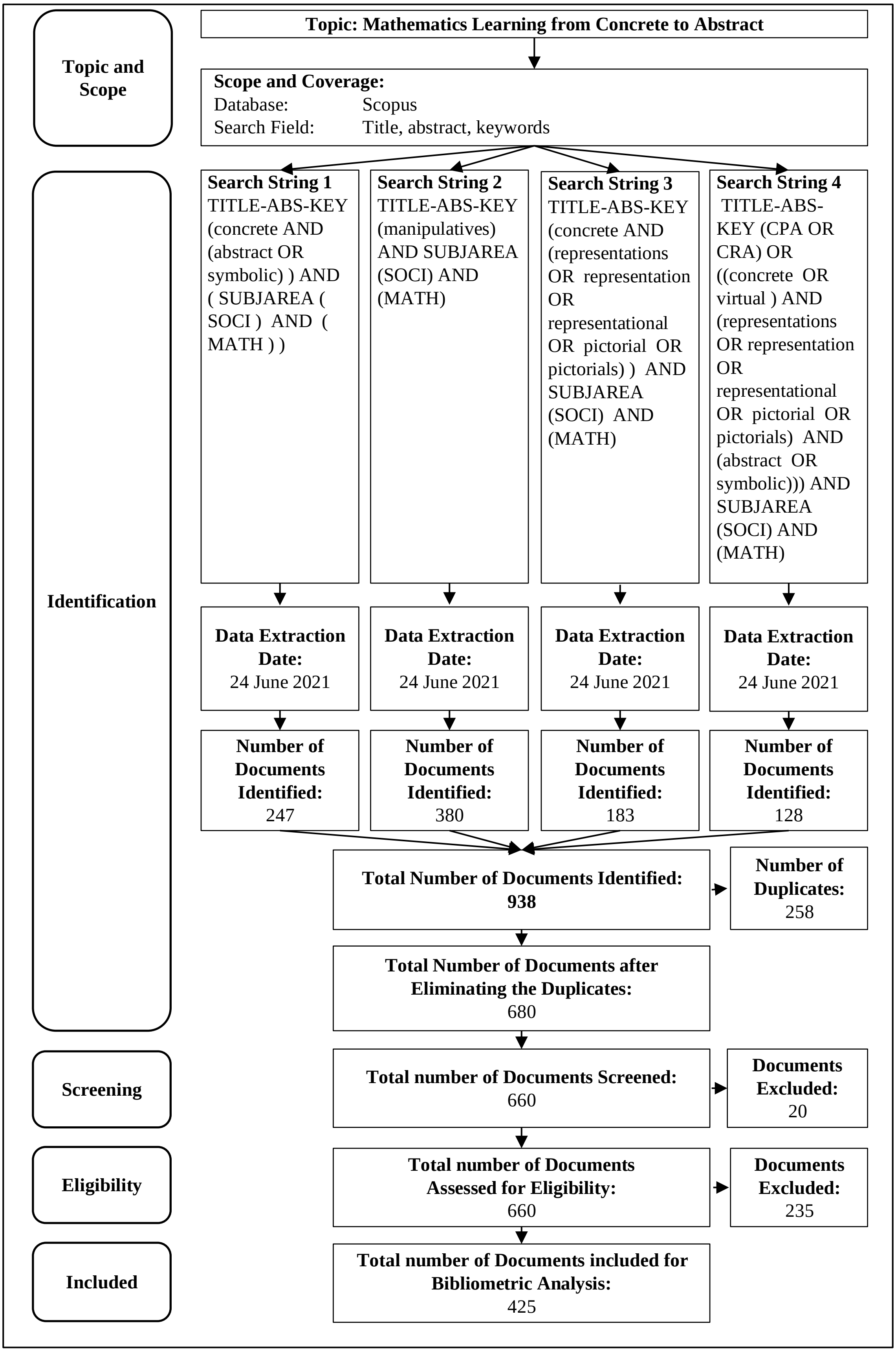

Figure 1. Flow chart of document search

During the document identification stage, a total of 938 publications were identified using the four search strings. After removing the 258 duplicates, the 680 publications were screened based on the language and document type criteria. During the screening stage, a total of 20 documents [2 trade journals, 18 non-English publications] were removed from the list. The remaining 660 documents were further screened for eligibility based on the inclusion and exclusion criterion listed in Table 1.

Table 1. Inclusion and Exclusion Criteria

Inclusion Criteria	Exclusion Criteria
“Concrete” refers to the concrete learning experience, and “Abstract” refers to the symbolic nature of mathematics	“Concrete” is not referring to the concrete learning experience, and “Abstract” is not referring to the symbolic nature of mathematics
The teaching sequence begins from concrete to abstract	The teaching sequence is not beginning from concrete to abstract
Involving teaching and learning of mathematics	Not involving teaching and learning of mathematics
“Manipulative” refers to the teaching aids	“Manipulatives” does not refer to teaching aids
“CPA” refers to “Concrete-Pictorial-Abstract” or “CRA” refers to “Concrete-Representation-Abstract”	“CPA” is not referring to “Concrete-Pictorial-Abstract” or “CRA” is not referring to “Concrete-Representation-Abstract”

During the eligibility stage, a total of 235 documents were excluded with reasons as shown in Table 2. Following this, the number of publications included in the merged list is further reduced to 425 publications. The bibliographic data of these 425 publications were extracted on 24 June 2021 for bibliometric analysis during the last stage of data collection, named inclusion.

Table 2. Number of Documents Excluded with Reason

Reason for Exclusion	No of documents
“Concrete” is not referring to the concrete learning experience, and “Abstract” is not referring to the symbolic nature of mathematics	23
The teaching sequence is not beginning from concrete to abstract	26
Not involving teaching and learning of mathematics	159
“Manipulatives” does not refer to teaching aids	17
“CPA” is not referring to “Concrete-Pictorial-Abstract” or “CRA” is not referring to “Concrete-Representation-Abstract”	10
Total	235

Data Pre-processing

Before performing the bibliometric analysis, the bibliometric data of the publication extracted from the Scopus database was pre-processed by the researchers using Microsoft Excel 2016 because the bibliographic data retrieved might contain ambiguous author names (Sanyal et al., 2021) and keyword variants (Abejón et al., 2018). In the retrieved bibliographic data, the same author might appear with a different name. For example, the author’s name ‘*Moyer-Packenham P.S.*’ is also appeared as ‘*Moyer-Packenham P.*’ and ‘*Moyer-Packenham P.S.*’ in the retrieved bibliographic data. Likewise, the keywords can be presented in different spellings (e.g., ‘problem solving’ and ‘problem-solving’), synonyms (e.g., ‘primary school’ and ‘elementary school’) and also singular or plural form (e.g., 'manipulative' or 'manipulatives') in the retrieved bibliographic data. These variants were merged to enhance the validity of the findings of bibliometric analysis (Gomez-Jauregui et al., 2014).

Data Analysis

For addressing research question one, bibliometric analysis was conducted with year as the main bibliometric indicator. The publication status over the years was profiled based on descriptive statistics computed using Microsoft Excel 2019. For addressing research question two, author bibliographic coupling analysis was conducted to determine the foci of the research. According to Zhao and Strotmann (2008), a bibliographic coupling relationship is established between the two authors if they cite the same documents. The higher the number of overlapping bibliographies in the publications of the two authors, the higher the relevancy of the research conducted by the two authors. Based on the bibliographic coupling count, the author bibliographic coupling matrix was derived, and the author bibliographic coupling network was generated using VOSviewer. Based on the network generated, the authors were clustered together based on the similarity of their research focus. Then, the research focus represented by each cluster was determined based on the keywords with high relevancy weight, named Term Frequency-Inverse Document Frequency (TF-IDF) computed using Microsoft Excel 2019 based on a formula modified by Schiebel et al. (2017) as shown below. Then, the full-length publications with the two highest TF-IDF keywords in each cluster were reviewed systematically.

$$\text{TF-IDF}_{i,j} = tf_{i,j} \times \log \frac{N}{n_i} \tag{1}$$

where $tf_{i,j}$ is the frequency of author keyword i in the publication included in cluster j (assuming there is no repetition of author keywords in each publication), N is the number of documents in the corpus, and n_i is the number of documents containing author keyword *i*.

For addressing research question three, the author with the highest bibliographic coupling strength was identified. This was pivoted on the claim made by Zhao and Strotmann (2008) in which the higher the number of overlapping bibliographies in the publications of the two authors, the higher the relevancy of the research conducted by the two authors. Thus, it was argued that the authors with the highest total bibliographic coupling strength are the most representative author in each cluster as their research has the highest relevancy. After identifying the most representative author in each cluster, their publications were reviewed.

Results

Publication Status

Based on the bibliographic data retrieved from Scopus on 21 June 2021, there were 425 studies on mathematics learning from concrete to abstract published as Articles (84.24%,), Book Chapters (5.65%), Conference Papers (4.24%), Reviews (4.24%), Books (1.41%) or Notes (0.24%). As shown in Figure 2, the research on mathematics learning from concrete to abstract was first published in 1968. However, the publication growth from 1968 to 1996 is slow and thus a flat cumulative curve is shown in Figure 2. With less than five publications per year, there was only 24 research work being published from 1968 to 1996 in total.

The research on mathematics learning from concrete to abstract has started to receive attention from researchers since 1997. As shown in the bar chart of Figure 2, there were five to 10 studies related to mathematics learning from concrete to abstract being published each year from 1997 to 2008, except the years 1999, 2007, and 2003. The number of publications exceeded 10 for the first time in 2009, then the number of publications fluctuated within the range of 15 to 45 from 2010 to 2021. Although there is a slight decrease in the number of publications in 2012, 2015, 2016, and 2019, it does not slow down the publication growth. Rather than concaves downward, the cumulative curve concaves upwards towards a large slope. This indicates the annual publication of research on mathematics learning from concrete to abstract shows an increasing trend during the period of 1997 to 2021.

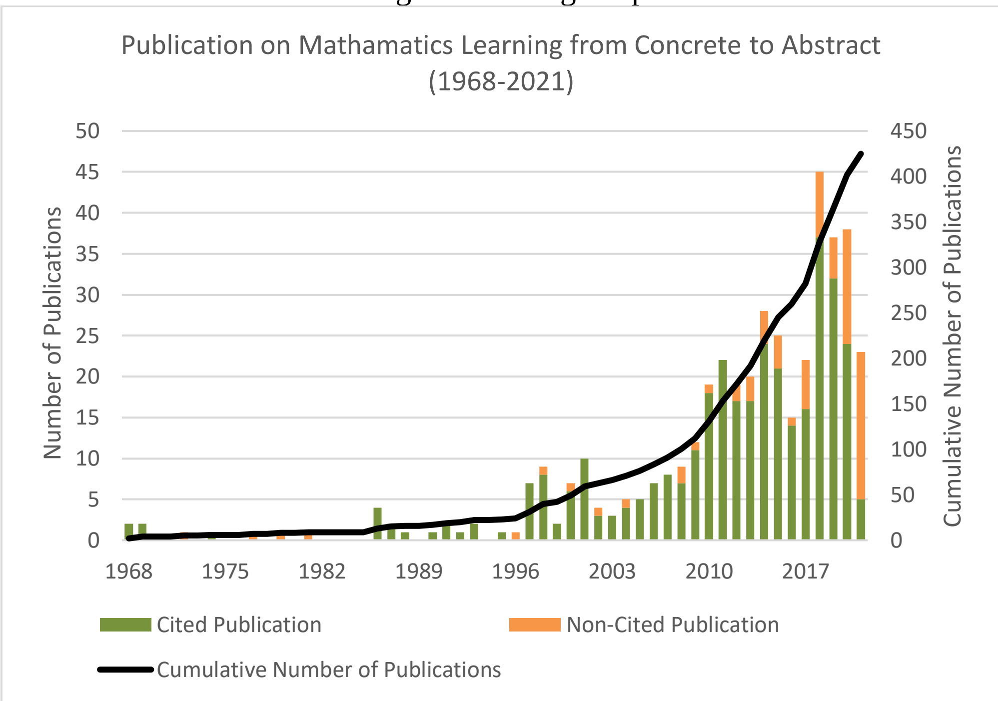

Figure 2. Publications on mathematics learning from concrete to abstract (1968- 2021)

Then, regression analysis was performed on the cumulative publication plot to further examine the publication trend. As shown in Figure 3, the cumulative number of publications from 1968 to 2021 was fitted into the exponential curve, $y = 4.36 \times 10^{-80} e^{0.0933x}$ at the significance level of .05 with variances explained of 98.64 % [R^2=.98, $F(1,52) = 3759.525$, $p < .05$]. This indicates the number of publications of the research on mathematics learning from concrete to abstract increases sharply over time following the exponential growth model.

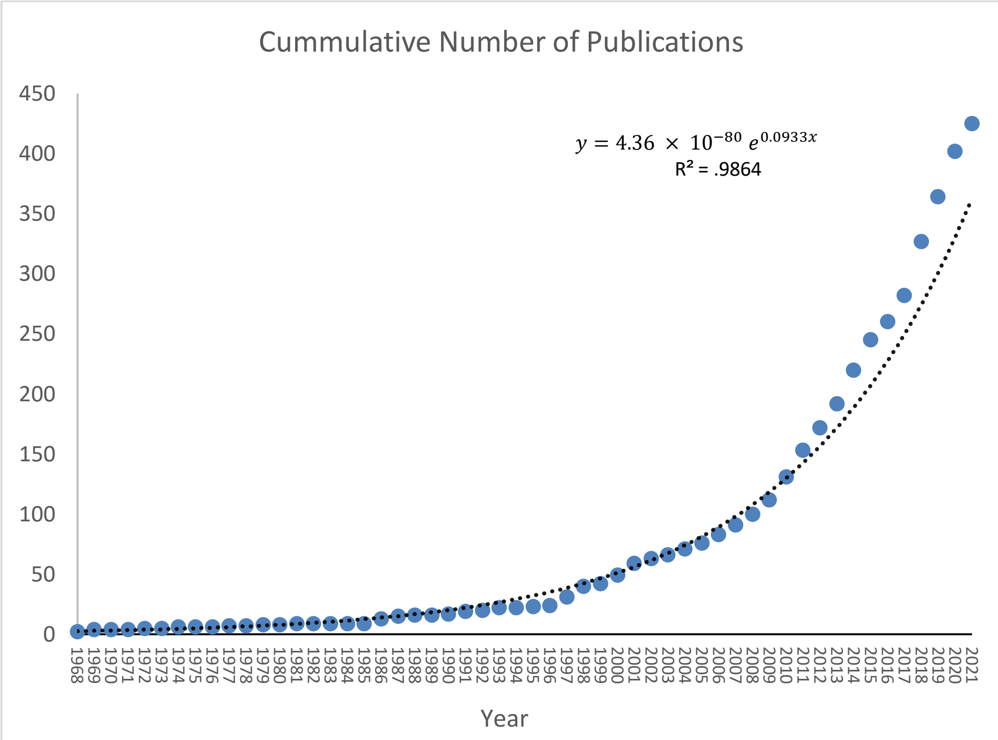

Figure 3. Cumulative number of publications on mathematics learning from concrete to abstract (1968-2021)

Research Domains

To determine the domain of research on mathematics learning from concrete to abstract, author bibliographic coupling analysis was conducted with the threshold of at least two publications. Out of 836 authors, there were only 108 authors who surpassed the threshold and were included in the author bibliographic coupling analysis. While the total link strength indicates total bibliographic coupling frequency, the author with zero total link strength would present as an isolated node in the network. To obtain a clearer clustering result, a total of two authors with zero total link strength were removed from the list. Thus, only 106 authors remained for author bibliographic coupling analysis. The author bibliographic coupling network generated is presented in Figure 4.

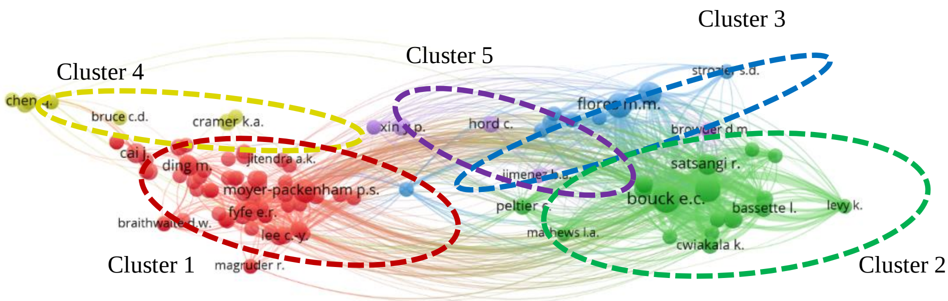

Figure 4. Author bibliographic coupling network [pubication ≥ 2, total link strength>0]

The author bibliographic network generated is a complete network with 106 nodes with 2863 edges. Each node represents the authors, while each edge represents the bibliographic coupling relationship established. In the author bibliographic coupling network, each edge is weighted with bibliographic coupling strength, which refers to the number of overlapping references in the research work published by the two authors. Thus, a high bibliographic coupling strength indicates that the research work published by the two authors were closely related (Gazni & Didegah, 2016). Following this, the 106 authors in the network were grouped into five clusters based on the similarity of the research conducted. Each cluster represents a domain of research on mathematics learning from concrete to abstract and was labelled with the most relevant keywords which were identified based on the TF-IDF weight. The most relevant keywords of each research domain are listed in Table 3.

Table 3. The Most Relevant Keywords of Each Research Domain

Cluster [Colour]	Top Keywords [TF-DIF]	No of Authors	Total No. of Publications	Total Citation per Cluster	Average Citation per Cluster	Range of Publication years	Number of Publications since 2015 [%]
Cluster 1 [Red]	Manipulative [16.37] Arithmetic [12.91]	64	111	2998	46.84	1990-2021	41 [36.94%]
Cluster 2 [Green]	Mathematics [10.41] Learning Disability [8.77]	17	30	1009	59.35	2014-2021	29 [96.67%]
Cluster 3 [Blue]	Concrete-Representational-Abstract Sequence [5.40] Elementary Education [4.85]	9	17	353	39.22	2010-2020	11 [64,71%]
Cluster 4 [Yellow]	Ideal Mathematics Teaching [3.90] Mathematics Teaching [3.63]	8	10	220	27.50	2002-2020	6 [60.00%]
Cluster 5 [Purple]	Mathematics Problem-Solving [5.53] Autism Spectrum Disorder [5.30]	8	10	371	46.38	2012-2021	9 [90.00%]

As shown in Table 3, the red cluster (Cluster 1) refers to the research domain related to ‘manipulatives’ and ‘arithmetic’. The green cluster (Cluster 2) refers to the research domain related to ‘mathematics’ and ‘learning disabilities’. The blue cluster (Cluster 3) refers to the research domain related to ‘Concrete-Representational-Abstract Sequence (CRA)’ and ‘Elementary Education’. The yellow cluster (Cluster 4) refers to the research domain related to ‘ideal mathematics teaching’ and ‘mathematics teaching’. The purple cluster (Cluster 5) refers to the research domain related to ‘mathematics problem-solving’ and ‘Autism Spectrum Disorder (ASD)’.

To compare the significance and the emergence of the research domain, the statistical indicators such as the number of authors, total citation per cluster, average citation per cluster, range of publication years and the number of studies published since 2015. The statistical indicators of each cluster are presented in Table 3. Most of the authors conducted studies related to the research domain on ‘manipulatives and arithmetic’ (n=64), followed by ‘mathematics and learning disabilities’ (n=17), and ‘Concrete-Representational-Abstract Sequence (CRA) in Elementary Education’ (n=9). With the highest average citation per cluster, 'mathematics and learning disabilities' is the most prominent research domain related to mathematics learning from concrete to abstract. Each author who conducted studies related to this research domain received 59.35 citation counts on average. While some studies were published in 2020 and 2021, the five research domains were still considered active. With the highest proportion of publications since 2015, the research domains ‘mathematics and learning disabilities’ as well as ‘mathematics problem-solving and autism spectrum disorder’ were the two emerging research domains.

Most Relevant Authors in each Research Domain

The publications of the most representative author in each research domain are listed in Table 4. With the highest total bibliographic coupling strength (Total BCS = 1988), Moyer-Packenham P.S. is the most relevant researcher for the research domain of 'manipulatives and arithmetic learning'. Moyer- Packenham P.S. established a bibliographic coupling relationship with 94 authors with a total reference-overlapping frequency of 1988. Moyer-Packenham P.S. has published nine studies from 2011 to 2020. Bouck E.C. is the most relevant researcher for the research domain of 'mathematics learning of students with learning disabilities'. Bouck E.C. established a bibliographic coupling relationship with 82 authors with a total reference-overlapping frequency of 12989. She published 21 studies from 2015 to 2021.

Flores M.M. is the most relevant researcher for the research domain of 'Concrete-Representational-Abstract sequence in elementary mathematics teaching'. Flores M.M. established a bibliographic coupling relationship with 78 authors with a total reference-overlapping frequency of 3767. Flores M.M. has published 13 research from 2010 to 2020. Hsieh F.-J. is the most relevant researcher for the research domain of 'Ideal mathematics teaching'. Hsieh F.-J. established a bibliographic coupling relationship with 28 authors with a total reference-overlapping frequency of 404. Hsieh F.-J. has published 2 studies in 2018 and 2020 respectively. Root, J. R. is the most relevant researcher for the research domain of 'mathematics problem-solving and mathematics learning of students with autism spectrum disorder'. Root, J. R. established a bibliographic coupling relationship with 41 authors with a total reference-overlapping frequency of 609. Root, J. R. has published 2 studies in 2019 and 2017 respectively.

Table 4. Publications of the most representative author for each research domain

Cluster [Colour]	Research Domain	Most Relevant Author and the corresponding research work [TP, Edge, Total BCS, Average BCS]
Cluster 1 [Red]	Manipulatives and arithmetic learning'	Moyer-Packenham P.S. [9, 94, 1988, 21.15] 1. Are we having fun yet? How teachers use manipulatives to teach mathematics (Moyer-Packenham, 2001) 2. Effects of virtual manipulatives on student achievement and mathematics learning (Moyer-Packenham & Westenskow, 2013) 3. Examining pictorial models and virtual manipulatives for third-grade fraction instruction (Moyer-Packenham et al., 2012) 4. Revisiting the effects and affordances of virtual manipulatives for mathematics learning (Moyer-Packenham & Westenskow, 2016) 5. Kindergarten children's interactions with touchscreen mathematics virtual manipulatives: An innovative mixed-methods analysis (Tucker et al., 2017) 6. Characterizing the growth of one student's mathematical understanding in a multi-representational learning environment (Gulkilik et al., 2020) 7. Using an iceberg intervention model to understand equivalent fraction learning when students with mathematical learning difficulties use different manipulatives (Westenskow & Moyer-Packenham, 2016) 8. Base-10 blocks: A study of iPad virtual manipulative affordances across primary-grade levels (Litster et al., 2019) Learning logic: Examining the effects of context ordering on reasoning about conditionals (Lommatsch & Moyer-Packenham, 2020)

Cluster [Colour]	Research Domain	Most Relevant Author and the corresponding research work [TP, Edge, Total BCS, Average BCS]
Cluster 2 [Green]	Mathematics learning of students with learning disabilities	Bouck E.C. [21, 82, 12989, 158,40] 1. Using virtual manipulative instruction to teach the concepts of area and perimeter to secondary students with learning disabilities (Satsangi & Bouck, 2015) 2. Comparing the effectiveness of virtual and concrete manipulatives to teach algebra to secondary students with learning disabilities (Satsangi et al., 2016) 3. The concrete–representational–abstract approach for students with learning disabilities: an evidence-based practice synthesis (Bouck et al., 2018a) 4. Concrete and app-based manipulatives to support students with disabilities with subtraction (Bouck et al., 2017b) 5. A systematic review of the literature on mathematics manipulatives to support students with disabilities (Bouck & Park, 2018) 6. Teaching equivalent fractions to secondary students with disabilities via the virtual–representational–abstract instructional sequence (Bouck et al, 2017a) 7. Manipulative apps to support students with disabilities in mathematics (Bouck et al., 2018c) 8. A meta-analysis of single-case research using mathematics manipulatives with students at risk or identified with a disability (Peltier et al., 2020) 9. Adding it up: comparing concrete and app-based manipulatives to support students with disabilities with adding fractions (Bouck et al., 2018b) 10. Using the virtual-abstract instructional sequence to support the acquisition of algebra (Bouck et al., 2019) 11. The virtual-representational-abstract framework to support students with disabilities in mathematics (Bouck & Sprick, 2019b) 12. App-based manipulatives and explicit instruction to support division with remainders (Bouck et al., 2020d) 13. Using the virtual–representational–abstract with overlearning instructional sequence to students with disabilities in mathematics (Park et al., 2021a) 14. Virtual manipulatives: a tool to support access and achievement with middle school students with disabilities (Bouck et al., 2020b) 15. App-based manipulatives and the system of least prompts to support the acquisition, maintenance, and generalization of adding integers (Bouck & Park, 2020) 16. Comparison of concrete and app-based manipulatives to teach subtraction skills to elementary students with autism (Bassette et al., 2019) 17. Learning fraction concepts through the virtual-abstract instructional sequence (Bouck et al., 2020c) 18. Learning fractions with a virtual manipulative based graduated instructional sequence (Bouck et al., 2020a) 19. Manipulating algebra: comparing concrete and virtual algebra tiles for students with intellectual and developmental disabilities (Long et al., 2021) 20. Manipulative-based instructional sequences in mathematics for students with disabilities (Bouck et al., 2021) 21. Using the VA framework to teach algebra to middle school students with high-incidence disabilities (Bone et al., 2021)
Cluster 3 [Blue]	Concrete-Representational-Abstract	Flores M.M. [13, 78, 3767, 48.29] 1. Using the concrete-representational-abstract sequence to teach subtraction with regrouping to students at risk for failure (Flores,

Cluster [Colour]	Research Domain	Most Relevant Author and the corresponding research work [TP, Edge, Total BCS, Average BCS]
	sequence in elementary mathematics teaching	2010) 2. Teaching subtraction and multiplication with regrouping using the concrete-representational-abstract sequence and strategic instruction model (Flores et al., 2014c) 3. An investigation of the effects of CRA instruction and students with autism spectrum disorder (Stroizer et al, 2015) 4. Using the concrete-representational-abstract sequence and the strategic instruction model to teach computation to students with autism spectrum disorders and developmental disabilities (Flores et al., 2014b) 5. Teaching multiplication with regrouping to students with learning disabilities (Flores et al., 2014a) 6. Teaching problem solving to students receiving tiered interventions using the concrete-representational-abstract sequence and schema-based instruction (Flores et al., 2016) 7. Using the concrete–representational–abstract sequence to teach conceptual understanding of basic multiplication and division (Milton et al., 2019) 8. Teaching fraction concepts using the concrete-representational-abstract sequence (Flores et al., 2020a) 9. The effects of the concrete-representational-abstract sequence for students at risk for mathematics failure (Hinton & Flores, 2019) 10. Teaching the partial products algorithm with the concrete representational abstract sequence and the strategic instruction model (Flores et al., 2020b) 11. Improvement in elementary students' multiplication skills and understanding after learning through the combination of the concrete-representational-abstract sequence and strategic instruction (Flores & Hinton, 2019) 12. A case study using CRA to teach students with disabilities to count using flexible numbers: applying skip counting to multiplication (Gibbs et al., 2018) 13. Teaching the partial products algorithm using the concrete-representational-abstract sequence (Flores & Milton, 2020)
Cluster 4 [Yellow]	Ideal mathematics teaching	Hsieh F.-J. [2, 28, 404, 14.43] 1. Ideal mathematics teaching behaviours: A comparison between the perspectives of senior high school students and their teachers in Taiwan and mainland China (Hsieh et al., 2020) 2. Exploring profiles of ideal high school mathematical teaching behaviours: perceptions of in-service and pre-service teachers in Taiwan (Hsieh et al., 2018)
Cluster 5 [Purple]	Mathematics problem-solving and mathematics learning of students with autism spectrum disorder	Root J.R. [2, 41, 609, 14.85] 1. An updated evidence-based practice review on teaching mathematics to students with moderate and severe developmental disabilities (Spooner et al., 2019) 2. Schema-based instruction with concrete and virtual manipulatives to teach problem solving to students with autism (Root et al., 2017)

Notes. TP = Total Publication, BCS = Bibliographic Coupling Strength

Discussion

What is the publication status of the research on mathematics learning from concrete to abstract?

In this study, the research on mathematics learning from concrete to abstract shows a slow publication growth from 1968 to 1996. However, the publications increase rapidly following the exponential model since 1997. These findings are similar to the study conducted by Ramirez and Rodriguez Devesa (2019) in which the publications in mathematics education is predicted to increase exponentially. While mathematics learning from concrete to abstract is rooted in the constructivist epistemology of Jean Piaget (1952), the increasing trend of the research is supported by Greenes (1995) in which constructivism theory dominated the research in mathematics education since the 1990s.

What are the research domains on mathematics learning from concrete to abstract published in 1968 to 2021?

The research on mathematics learning from concrete to abstract published in 1968 to 2021 was clustered into five research domains: (i) 'manipulatives and arithmetic learning'; (ii) 'mathematics learning of students with learning disabilities'; (iii) 'Concrete-Representational-Abstract sequence in elementary mathematics teaching'; (iv) 'ideal mathematics teaching'; and (v) 'mathematics problem-solving and mathematics learning of students with autism spectrum disorder'. The findings indicated that 'mathematics and learning disabilities' as well as 'mathematics problem-solving and autism spectrum disorder' were the two emerging research domains. This is in line with the current focus of mathematics education which emphasises the teaching approaches for supporting students with mathematics learning disabilities (Gökçe & Guner, 2021) following the introduction of the Sustainable Development Goal 4 'ensure inclusive and equitable quality education and promote lifelong learning for all' by UNESCO.

Research Domain 1: Manipulatives and Arithmetic Learning

'Manipulatives and arithmetic learning' is the most dominant research domain. As defined by Moyer-Packenham (2001), manipulatives are the teaching aids that serve for representing abstract mathematical ideas explicitly and concretely. While arithmetic operations are genuinely abstract, the children are usually being engaged with manipulatives to support the construction of conceptual understanding (Uttal et al., 1997). With adequate conceptual understanding, the students' arithmetic strategies would be changed from using manipulatives to using cognitive strategies (Carpenter & Moser, 1994), such as verbal counting, regrouping or decomposition, and columnar retrieval or the standard algorithm (Geary, 1994). The teachers' use of concrete manipulatives in mathematics lessons has been studied by Moyer-Packenham (2001). Various concrete manipulatives (e.g., base-10 blocks, colour tiles, geoboards, dice, pattern blocks, hundred boards, fractions bars, tangrams, Cuisenaire rods, etc.) have been used in the mathematics classroom. The most commonly used concrete manipulative is the hundred board, which is used to introduce the place-value concept (Moyer-Packenham, 2001). With the advancement of technology, several virtual manipulatives have been developed to support the mathematics teaching and learning process. The use of virtual manipulatives has been reported by Moyer-Packenham and Westenskow (2012, 2016), Moyer-Packenham et al. (2012), Tucker et al. (2017) and Lister et al. (2019).

To summarise the effectiveness of manipulative use in supporting students' learning of mathematics, Carbonneau et al. (2013) had reviewed 55 relevant empirical evidence. The meta-analyses reported statistically significant results in favour of the use of manipulatives with small to moderate effect sizes. Nonetheless, researchers (i.e, Moyer-Packenham, 2001; Uttal et al., 1997) argued that students would not be able to understand the abstract and symbolic mathematical idea solely based on the manipulatives because manipulatives do not bring any mathematical insights. The students might not be able to link their actions with manipulatives to abstract symbols without proper instruction. In fact, teachers play an important role to support the concrete fading process. After contextualising the mathematical concepts using concrete manipulatives, the instruction should be "fading" away to the more abstract mathematical idea (Bruner, 1996). Thus, researchers (i.e., Fyfe et al., 2015; Fyfe & Nathan, 2019; McNeil & Fyfe, 2012) suggested that the learning of mathematics from concrete to abstract should be focused on "concrete fading" in the future.

Research Domain 2: Mathematics learning of students with learning disabilities

Students with mathematics disabilities are often characterised by the deficits such as having poor working memory and facing various difficulties in comparing the magnitude, understanding the place-value, learning mathematical facts, performing computation, organizing information presented in problems, and solving mathematics problems. (Bouck et al., 2018a; Soares et al., 2018). To support mathematics learning of the students with learning disabilities, studies involving mathematics instruction with concrete and virtual manipulatives have been conducted by several researchers (i.e., Bouck et al., 2017, 2021; Satsangi & Bouck, 2015; Satsangi et al., 2016) in the past. To synthesise the findings of the effectiveness of mathematics instruction involving manipulatives, Bouck and Park (2018) have reviewed 36 past studies. Various manipulatives have been used in the teaching and learning of arithmetic basic operations (n=20), place value (n=5), fractions (n=4), area and perimeter (n=3), algebra (n=5), and money (n=2). All 36 past studies reported a significant difference in favours the mathematics instruction with manipulatives. This is because the use of manipulatives would provide a concrete mathematical learning experience to the students by allowing them to work with visual representations of mathematical ideas (Soares et al., 2018). Following this, the cognitive load of students would be reduced (Bouck & Park, 2018) and hence enhance their learning. While technology advancement changes the landscape of instruction delivery, future research should continue to explore the use of virtual manipulatives in supporting students with disabilities using more various research designs.

Research Domain 3: Concrete-Representational-Abstract Sequence in Elementary Mathematics Teaching'

According to Leong et al. (2015), CRA instructional sequence was rooted in the three modes of mathematical representations [i.e., (i) enactive; (ii) iconic; and (iii) symbolic] proposed by Bruner (1966). The CRA instructional sequence involved three phases: (i) concrete phase; (ii) representational phase; and (iii) abstract phase. The instruction at the first phase involved using various prompts and cues to guide the students to construct their conceptual understanding of mathematics through interacting with the concrete manipulatives, followed by allowing students to demonstrate the mathematics skills independently using manipulatives. At the second phase of instruction, the concrete manipulatives were replaced by pictures and/or drawings. Students are guided to draw pictures to represent the mathematical concepts and followed by performing the computation. The third stage of instruction emphasised procedural fluency. The students were guided to perform the computation following the algorithm, rather than based on visual aids such as pictures and drawings.

CRA instructional sequence has been proven as an effective approach in supporting elementary students' mathematics learning (Hinton & Flores, 2019) across various topics, such as algebra, place value, basic arithmetic operations, fractions, area and perimeter, as well as word problems (Flores, 2010). The use of CRA has also been extended to the field of special education. Several researchers (i.e., Flores, 2010; Flores et al., 2014a, 2014b; Mancl et al., 2012; Stroizer et al., 2015) have conducted studies on CRA to support the learning of students with learning disabilities as well as the autism spectrum disorder. To synthesise the findings of the past studies on CRA involving students with learning disabilities, Bouck et al. (2018a) reviewed 20 relevant articles. Bouck et al. (2018a) concluded that CRA is a useful instructional sequence that supports students with learning disabilities in mathematics learning. While studies on CRA were mainly conducted in the elementary school context, future studies are suggested to focus on more complex mathematics topics such as algebra.

Research Domain 4: Ideal Mathematics Teaching

Teachers' instructional practice shapes students' mathematics learning experience. An effective instructional practice could promote students' mathematical understanding (Wang & Hsieh, 2017). Perceptions of ideal mathematics teaching have been reported in the studies conducted by Hsieh et al. (2018, 2020). Both pre-service teachers and in-service teachers in Taiwan considered concrete representation as an ideal instructional strategy (Hsieh et al, 2018). For the comparison of perception between teachers and students, Hsieh et al. (2020) highlighted mathematics teaching involving concrete representation were valued more by the high school students from both Taiwan and China, rather than their teachers. This is because conceptual development is often evolving from concrete experience to abstract understanding (Cramer & Wyberg, 2009). The students would grasp the meanings of mathematical ideas by manipulating the concrete objects. Thus, concrete representation would support students' cognitive engagement in mathematics. In short, concrete representation is regarded as an ideal instructional strategy by teachers and students. However, Cramer and Wyberg (2009) echoed the concern on the disconnection between the action of manipulating the concrete objects and the underlying mathematical procedure represented. This would deteriorate the value of manipulatives in mathematics learning. Thus, future studies should focus on the instruction which could link the manipulation of concrete objects explicitly with the mathematical procedure represented to enhance students' conceptual understanding and support the concrete fading process.

Research Domain 5: Mathematics Problem-Solving and Mathematics Learning of Students with Autism Spectrum Disorder

Problem-solving is the heart of mathematics learning because it exposes students to when and why to apply the mathematical knowledge learned rather than how to perform the computation following the algorithm learned (Root et al., 2017). Solving mathematics word problems could be difficult for many students because it involves literary skills besides numerical skills (Daroczy et al., 2015). To support the at-risk students' learning of solving word problems, Xin (2013) introduced a potentially effective instructional technique named Conceptual Model-based Problem-Solving (COMPS). This technique involves mapping the real-world situational models onto mathematical symbolic models such as formulae and algorithms (Xin, 2013). Thus, the use of COMPS would promote students' understanding of the underlying quantitative relationship presented in the word problems and hence support them to represent the mathematical relationship using the mathematics symbolic model equations (e.g., Referent Unit × Number of Units = Product) (Xin 2013).

As highlighted by Root et al. (2017), teaching word problem-solving to students with ASD could be two-fold as challenging as teaching students without any learning disability. This is because most of the students with ASD have been diagnosed with both intellectual disabilities and low intelligent quotient (IQ) (Yakubova et al., 2019). Thus, Root et al. (2017) suggested teaching problem-solving with students with ASD using modified schema-based instruction and also various types of manipulative as well as graphic organisers. The graphic organisers could support students with ASD in organizing the numerical information presented in the word problems and hence support them in developing the understanding of the mathematical relationship between the numerical information. As such, the students with ASD could devise a plan to solve the word problems. Since students with ASD have poor mastery of basic mathematics facts, they would rely on manipulatives to complete the computations in the context of word problems. To have a better understanding of mathematics instruction involving students with ASD, Spooner et al. (2019) have reviewed 36 relevant studies conducted from 2005 to 2016. Besides manipulatives and graphic organisers, the findings recommended the use of systematic instruction, technology-aided instruction, and explicit instruction in teaching mathematics to this population. While various studies have been conducted to identify the effective instructional strategy in teaching students with ASD, future studies are suggested to focus on developing the learning progression for this specific population to explain students learning of mathematics concepts.

What are the contributions of the most relevant author in each research domain?

Moyer-Packenham P.S. is the most relevant author in the research domain ‘manipulatives and arithmetic learning’. The studies conducted by Moyer-Packenham P.S. is mainly related to the effectiveness of using virtual manipulatives in teaching early years mathematics and elementary mathematics across various topics (Litster et al., 2019; Moyer-Packenham et al., 2012; Moyer-Packenham & Westenskow, 2013, 2016; Tucker et al., 2017) Thus, the studies are highly relevant to the top keywords in the research domain, that are 'manipulatives' and 'arithmetic' as the top keywords.

Bouck E.C. is the most relevant researcher in the research domain named ‘mathematics learning of students with learning disabilities’. The studies conducted by Bouck E.C. mainly related to the effectiveness of using concrete and virtual manipulatives in teaching various mathematics topics to students with learning disabilities (i.e., Bassette et al., 2019; Bouck & Park, 2020, Bouck et al., 2017b, 2018b, 2018d, 2020a, 2020b; Long et al., 2021; Satsangi & Bouck, 2015; Satsangi et al., 2016). Thusly, the studies are highly relevant in the research domain with 'mathematics' and 'learning disabilities' as the top keywords.

Flores M.M. is the most relevant author for the research domain named ‘Concrete-Representational-Abstract sequence in elementary mathematics teaching’. The studies conducted by Flores M.M. were mainly related to the effectiveness of teaching elementary mathematics to students with special needs using a concrete-representational-abstract sequence (i.e., Flores, 2010; Flores & Hinton, 2019, Flores & Milton, 2020; Flores et al, 2014b, 2014c, 2016, 2020a, 2020b; Hinton & Flores, 2019; Milton et al., 2019; Stroizer et al., 2015). Thereupon, the studies are highly relevant in the related research domain with ‘concrete-representational-abstract sequence’ and ‘elementary education’ as the top keywords.

Hsieh F.-J. is the most relevant researcher for the research domain ‘Ideal mathematics teaching’. Hsieh et al. (2018, 2020) conducted studies to compare the perceptions of teachers and students on ideal mathematics teaching behaviours in Taiwan and China. Thence, the studies found are highly relevant to two top keywords of the research domain in question, which are 'ideal mathematics teaching' and 'mathematics teaching'.

Root, J. R. is the most relevant researcher for the research domain ‘mathematics problem-solving and mathematics learning of students with autism spectrum disorder’. Besides synthesising the past-related studies on teaching mathematics to students with moderate and severe developmental disabilities such as ASD (Spooner et al., 2019), Root et al (2017) also conducted a study to compare the effectiveness of schema-based instruction with concrete and virtual manipulatives to teach problem-solving to students with ASD. On this account, the studies are highly relevant to the top keywords in the said research domain, namely ‘mathematics problem-solving’ and ‘autism spectrum disorder’.

Conclusion

Implications of the Studies

This study was conducted to profile the landscape of research on mathematics learning from concrete to abstract published in 1968 to 2021. The findings of this study would help researchers to understand the current landscape of research with the notion of mathematics learning from concrete to abstract, and hence propose the pathways for future research. Specifically, the researchers are encouraged to conduct research focusing on mathematics learning from concrete to abstract because the findings on publication growth suggested that this relevant research will continue to serve as the major research scope in mathematics education. Among the five research domains, the researchers are recommended to focus on the two emerging research domains, namely (i) ‘mathematics learning of students with learning disabilities’; and (ii) ‘mathematics problem solving and mathematics learning of students with autism spectrum disorder’. This calls upon the cross-disciplinary collaboration among the researchers from the field of mathematics education and special education for conducting research based on the notion ‘mathematics learning from concrete to abstract’ to support students with mathematics learning disabilities and an autism spectrum disorder.

Limitations of the Studies

There were several limitations in this study. First, this study only included a partial sample of the global scientific output with the notion ‘mathematics and learning disabilities’ because the data presented are limited to the Scopus database. Besides that, studies published after the retrieval date of 21 June 2021 were not taken into account in this study. Hence, it is suggested to replicate the study by merging the data retrieved from several databases.

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Conflicts of interest/Competing interests

The authors declare that there is no conflict of interest.

Ethics statements

This study does not involve any human participants and/or animals.

Authors' contributions

All authors contributed to the study conception and design. Literature search and data analysis were performed by Chin Huan, Menaga Suseelan and Chew Cheng Meng. The first draft of the manuscript was written by Chin Huan. Chew Cheng Meng and Menaga Suseelan commented on previous versions of the manuscript. All authors read and approved the final manuscript.

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