Journal of Pedagogical Research
Volume 6, Issue 1, 2022
https://dx.doi.org/10.33902/JPR.2022175841

*Research Article*

# Exploring pre-service mathematics teachers' knowledge of content and students through case reading and discussion

**Nadide Yılmaz**

*Karamanoğlu Mehmetbey University, Turkey (ORCID: 0000-0003-1624-5902)*

The importance of case reading and discussion has been steadily increasing in teacher education. This study attempted to investigate what changes, if any, have been brought about by pre-service mathematics teachers' working on cases on their knowledge of content and students. Nineteen pre-service teachers participated in a 14-week teacher training program and worked on cases related to concepts in the learning area of numbers. The current study employed the mixed method and used the written records constituted by the cases on which the pre-service teachers worked on and the audio recordings of the discussion processes as the data collection tools. In the analysis of the collected data, content analysis was used. The findings revealed that the pre-service teachers' working on cases contributed to the development of their knowledge of content and students. The pre-service teachers were able to comment more on how students think and where they can have difficulties and understand the reasons behind these difficulties in the later stages of the training. The findings also revealed that the cases the pre-service teachers were engaged in helped them feel like a real teacher. The results showed that case reading and discussion can be one of the important contributors to the development of the pre-service teachers' knowledge of content and students.

Keywords: Case reading and discussion; Pre-service teachers; Knowledge of content and students; Numbers

Article History: Submitted 24 July 2021; Revised 15 February 2022; Published online 13 March 2022

## 1. Introduction

Successful mathematics teachers analyze events that occur in the classroom to gain insights into the quality of students' mathematical activities and make decisions about their future teaching (Leong, 2012). Teachers training themselves and adopting an investigative attitude in terms of their effective analysis of the teaching process is important (Ball & Cohen, 1999). This is a bit different for pre-service teachers because pre-service teachers do not usually encounter real classroom incidences. The researchers drew attention to this point and emphasized that the courses taken in undergraduate education are mostly theoretical and the number of courses that direct to application (practice) are insufficient (Darling-Hammond, 2006; Gravett et al., 2011; L. Shulman, 1992). Pre-service teachers who had a gap between theoretical and practical knowledge

**Address of Corresponding Author**

Nadide Yılmaz, PhD, Faculty of Education, Karamanoğlu Mehmetbey University, 70100 Karaman, Turkey.

✉ nadideylmz70@gmail.com

**How to cite:** Yılmaz, N. (2022). Exploring pre-service mathematics teachers' knowledge of content and students through case reading and discussion. *Journal of Pedagogical Research, 6*(1), 171-195. https://dx.doi.org/10.33902/JPR.2022175841

and did not gain enough experience in this sense experienced "practice shock" when they entered the real classroom environment (Ball, 2000; Merseth, 1999; L. Shulman, 1992; Whitelaw et al., 2008). Pre-service teachers who gained knowledge and skills about how the theoretical information they had learned could be reflected in the application environment were able to construct the teaching process more effectively (Ball, 2000; L. Shulman, 1992). When evaluated in this context, the acquisition of knowledge and skills that will enable pre-service teachers to be ready for real classroom environment is directly related to the education they receive during their undergraduate years (J. Shulman, 1992). However, it is not so easy to construct learning environments that will make it possible for pre-service teachers to be ready for the real classroom environment (Harrington, 1999). Recently, researchers have focused on "high-leverage practices" with the aim of maximizing the quality of teacher education (Ball & Forzani, 2009; Carpenter et al., 1996; McDonald et al., 2013). These practices can be defined as the activities conducted to foster the in-depth learning of mathematics (National Council of Teachers of Mathematics [NCTM], 2014). One of the high-leverage mathematics teaching practices that draw attention is elicit and use evidence of student thinking (NCTM, 2014).

Preparing pre-service teachers for a real classroom environment should be one of the primary goals of teacher education programs (L. Shulman, 1992) because when teacher education programs are traditionally conducted, information given about how to transfer theoretical knowledge into practice can remain limited. This may cause pre-service teachers to experience pedagogical difficulties while performing their job when they are appointed to teaching posts (Ball, 2000; Doyle, 1986; L. Shulman, 1992; Merseth, 1999). In this context, traditional approaches are not adequate to build a strong bridge between theory and practice in teacher education. Although the practices (e.g., School of Experience) that pre-service teachers perform to gain real classroom experience are beneficial in many respects, they are still discussed because pre-service teachers can focus on the irrelevant features of students in this process or they can superficially evaluate the points they need to focus on (Santagata et al., 2007). Another criticism is that these practices include limited time and certain teaching processes (Sherin, 2004). All these result in criticism levelled to teacher training programs and drew attention to the need to integrate alternative practices into undergraduate programs (Abell & Cennamo, 2004). Emphasizing this point, researchers have drawn attention to the fact that pre-service teachers can experience a real classroom environment with the help of case reading and discussion (CRD) (Merseth, 1996; L. Shulman, 1992). Another important contribution of pre-service teachers' working on cases is the positive effect induced on the knowledge of content and students (Lin, 2002; Merseth, 1991). The great emphasis put on the mastery of student knowledge for teachers to conduct an effective teaching process (Lampert et al., 2013) indicates the importance of developing this knowledge. In this connection, the current study aimed to reveal the changes brought about by the pre-service mathematics teachers' participation in a CRD-based teacher training program on their knowledge of content and students.

### 1.1. Rationale of the Study

Carpenter et al. (1989) pointed out that teachers who knew more about how students think could create more effective learning environment than those who did not. Carpenter et al. (1988) emphasized that teachers' knowledge about whether their students can solve problems in different ways is directly related to student achievement. Studies drew attention to the importance of teachers' understanding students' thoughts, but define this process as follows: "creating instruction that builds on children's thinking has proven challenging" (Jacobs et al., 2011, p. 98). Thus, pre-service teachers who have less experience with students are likely to have difficulties (Tirosh, 2000). For example, Driel and Berry (2010) revealed in their study that pre-service teachers had difficulties in getting to know students and were not competent enough to understand students' answers. In another study, researchers revealed that pre-service teachers tended to evaluate students' answers as right or wrong instead of trying to understand them (Crespo, 2000;

Jacobs et al., 2010). Both pre-service and in-service teacher training programs that focus on student thinking can increase the quality of teachers and the achievement of students (Franke et al., 2001; Lampert & Ball, 1998; Sowder, 2007; Tirosh, 2000). In this sense, CRD can be an important supporter for pre-service teachers (Masingila & Doerr, 2002; Sowder, 2007; Wilson & Ball, 1996) because knowledge of content and students can be observed better with cases such as student works, materials, videos, etc. (Ball & Cohen, 1999). This can offer pre-service teachers various opportunities for the development of their knowledge of content and students (Wilson & Ball, 1996). While working on cases, pre-service teachers analyze, interpret, and discuss student thinking. This helps them to examine the teaching process more deeply and feel like in a real classroom environment (Masingila & Doerr, 2002). To this end, the answers to the following question were sought.

- What changes occurred in the knowledge of content and students of the pre-service middle school mathematics teachers having participated in a CRD-based teacher education program?

## 2. Theoretical Framework and Related Literature

### 2.1. Case Reading and Discussion (CRD)

A case in classroom context is "…a piece of controllable reality, more vivid and contextual than a textbook discussion, yet more disciplined and manageable than observing or doing work in the world itself" (J. Shulman, 1992, p. xiv). Cases are effective tools to be used in teacher education for pre-service teachers to understand the teaching process and students (Powell, 2000; Sykes & Bird, 1992). Cases can be used for various purposes, these purposes are called stimulants to reflection, opportunities to practice analysis, and made decisions on action (Merseth, 1996). The effectiveness of preparing pre-service teachers for complex learning environments has made case-based teaching important (Butler et al., 2006; Mayo, 2004). In the literature, although different names have been used such as 'case-study instruction' (Heitzmann, 2008); 'case study method' (Kunselman & Johnson, 2004); 'case reading and discussion' (Shulman, 2004); 'case-based teaching' (Gravett et al., 2017); 'case-study methodology/pedagogy' (Heitzmann, 2008); 'case discussion as pedagogical method' (Sato & Rogers, 2010), the term "case reading and discussion (CRD)" was adopted in the current study as it was intended to make the pre-service teachers think and comment on cases. CRD can be defined as a teaching method in which pre-service teachers analyze cases and try to find solutions by discussing about these cases (Ball & Bass, 2003). Researchers argue that using cases in teacher education is one of the effective alternatives in preparing pre-service teachers for the profession (Hashweh, 2004; Heitzmann, 2008; Masingila & Doerr, 2002; Merseth, 1991, 1996; Norquist, 2008; Sato & Rogers, 2010; L. Shulman, 1992; 2004; J. Shulman, 1992).

CRD takes its foundations from the situated perspective of learning and social constructivism. In the situated perspective, learning occurs through participation and interaction in communities of practice with the help of authentic activities (Borko et al., 2007; Lave & Wenger, 1991). Having the opportunity to participate in such an environment, the individual can adapt to changing situations and transfer what she has learned and adapt it to different situations (J. Shulman, 1992). The teacher involved in such a process can improve his/her knowledge and skills (Lundeberg et al., 1999). The use of the situated perspective in professional development programs offers the opportunity to explore the context covered in the learning process from different perspectives as well as the chance to experience the complex nature of teaching through contexts (Little, 2002; Merseth, 1996). In addition, the active participation of individuals in class or group discussions during the CRD process enables them to learn through social interaction (Mayo, 2002). Individuals build new knowledge on their previous knowledge by focusing on problem solving in the CRD process, which is directly related to the constructivist approach (Harrington, 1995; Mayo, 2004). In this way, the learner not only has the opportunity to construct knowledge individually through his/her own experiences, but also enriches his/her learning through social interaction and reflection while interpreting and discussing cases (Mayo, 2002).

Research findings show that CRD supports the development of pre-service teachers in various respects. Researchers revealed that pre-service teachers gain knowledge about how theoretical knowledge is reflected in practice through cases (Ball & Cohen, 1999; Lin, 2002; Masingila & Doerr, 2002; L. Shulman, 1992). In addition, pre-service teachers' working on cases allows them to gain insight into the complexity of the real school environment, which helps them think like a teacher (Heitzmann, 2008; Kunselman & Johnson, 2004; Merseth, 1991, 1996, 2003; Mostert, 2007; J. Shulman, 1992; L. Shulman, 1992, 2004). Pre-service teachers' thinking about the solution while working on cases supports them to develop multiple perspectives (Lin, 2002; Lundeberg & Scheurman, 1997). Pre-service teachers' reflections on cases can be seen as an effective instruction in establishing best practice and making teaching more effective (Kleinfeld, 1992; Lin, 2002). This whole process allows pre-service teachers to discover themselves; therefore, they can make more effective decisions by thinking critically and reflectively on their developing practices (Jay, 2004; Levin, 1995; Masingila & Doerr, 2002; Mayo, 2004; Merseth, 1992). Another contribution of pre-service teachers' working on cases is related to the development of their knowledge (Barnett & Tyson, 1999; Lin, 2002; Merseth, 1991, 2003; J. Shulman, 1992; Smith & Friel, 2008).

### 2.2. Teacher Knowledge of Content and Students

Although teacher knowledge is one of the main factors affecting mathematics teaching (Cengiz et al., 2011), it remains unclear what this knowledge includes (Ball et al., 2001). Although researchers working in this field state that teachers should have many types of knowledge, one of the knowledge types they agreed on is the knowledge of content and students (Ball et al., 2008; Hill, Blunk et al., 2008). Knowledge of content and students can be defined as the knowledge necessary for the teacher to know the points in which students have difficulties and can have mistakes and misconceptions, how they think and how they learn concepts (Ball et al., 2008; Hill, Ball et al., 2008). Researchers also emphasized that student thinking is a critical resource in carrying out effective teaching practices (Lampert et al., 2013). One of the learning areas that teachers should master in relation to the knowledge of content and students is numbers (Van de Walle et al., 2013) because there are numbers and operations in the core of primary and middle school mathematics (Ministry of National Education [MoNE], 2018). NCTM (2000) emphasizes that students should have the following knowledge and skills; "(a) understand numbers, ways of representing numbers, relationships among numbers and number systems, (b) understand the meanings of operations and how they relate to one another, (c) compute fluently and make reasonable estimates (p. 32)". The objectives in the learning areas of numbers and operations in the primary and middle school mathematics curricula in Turkey make up nearly half (49.7%) of all the objectives in these curricula (MoNE, 2018). This clearly indicates that teachers should have the necessary knowledge and skills regarding these learning areas. However, studies emphasize that teachers/pre-service teachers have various difficulties regarding the knowledge of content and students in the learning area of numbers (Ball et al., 2001; Crespo, 2000; Didiş et al., 2014; Moyer & Milewicz, 2002; Tirosh, 2000, Turnuklu & Yesildere, 2007). For example, they experienced difficulties in eliciting students' thoughts (Turnuklu & Yesildere, 2007). In addition, teachers/pre-service teachers evaluated student thinking superficially by asking questions one after another instead of listening to students (Moyer & Milewicz, 2002). Another remarkable finding is that they classified students' answers as true or false only (Crespo, 2000; Didiş et al., 2014; Tirosh, 2000). In addition, they had difficulties in identifying the difficulties, misconceptions, and mistakes that students experienced (Moyer & Milewicz, 2002; Turnuklu & Yesildere, 2007).

## 3. Method

In order to conduct a detailed examination of the changes occurring in the middle school pre-service mathematics teachers' knowledge of content and students, who participated in a CRD-based teacher education program, a mixed method was used. This method is a concurrent

triangulation design, where both qualitative and quantitative data were collected at the same time (Creswell, 2009).

### 3.1. Participants

The participants are third-year students attending the department of elementary school mathematics teaching at a state university in Turkey. When the pre-service mathematics teachers (PMTs) have graduated, they can be appointed as mathematics teachers to 5th-8th graders (aged 11-14). The pre-service teachers take the courses directed to the development of the pedagogical content knowledge in the first two years of their undergraduate education. The study was conducted with 19 pre-service mathematics teachers within the context of the third-year course "Teaching Numbers". The pre-service teachers were asked to get into groups of 3-4 on a volunteer basis. Moreover, the participants were selected to allow maximum variation sampling, which is one of the purposive sampling methods, on the basis of cumulative graduate point averages (CGPAs). This sampling method allows determining whether there is a connection between the pre-service teachers' academic achievement and their knowledge of content and students. The CGPAs of the two groups (upper group1 PMT1, PMT2, PMT3, upper group2 PMT4, PMT5, PMT6) are in the range of 3.50-4.00; the cumulative grade point averages of the two groups (middle group1 PMT7, PMT8, PMT9, middle group2 PMT10, PMT11, PMT12) are in the range of 3.00-3.50; and the cumulative graduate point averages of the two groups (lower group1 PMT13, PMT14, PMT15; lower group2 PMT16, PMT17, PMT18, PMT19) are in the range of 2.50-3.00.

### 3.2. Production of Cases

While constructing the cases, the important points related to the relevant concept and the difficulties that students might experience were investigated. In this way, cases for each week were constructed. In the 10 weeks of the 14-week period (From the 4th week of the course onward), the pre-service teachers were asked to focus on the cases. In Table 1, the subjects addressed and the important points related to these subjects are presented.

Each week, between 3 and 13 cases were created within the scope of the subjects. Cases for some weeks were prepared by the researcher and student answers regarding student difficulties were constructed. In cases for some weeks, the real student answers were used and to this end, the existing research in the literature was used. For example, the study of Eroglu (2012) was used for student answers in three cases of fractions and operations with fractions. The study by Dogruel (2019) was drawn on for two cases of ratios, proportions and percentages, the study by Yapici (2013) was drawn on for four cases of the same subject. Atayev's (2015) study was used in three cases of the subject of integers and operations with integers. Student answers in cases prepared for the subject of rational numbers and operations with rational numbers were created by using Alkan's (2009) study. The study by Gocuk (2019) was drawn on for the student answers of two cases prepared for the subject of exponential expressions while the study by Bayram (2013) was drawn on for the student answers used in four cases prepared in the same subject. Dinc's (2018) study was used for the student answers in two cases used in the subject of square root and irrational numbers. While creating cases, the focus was on two basic types of knowledge of content and students: identifying and interpreting student difficulties, explaining reasons for student difficulties. For example, one of the important points focused on in the subject of the concept of early number and sense of number is cardinal value. The sample case prepared for the component of identifying and interpreting the difficulties of the students regarding this point is presented in Figure 1.

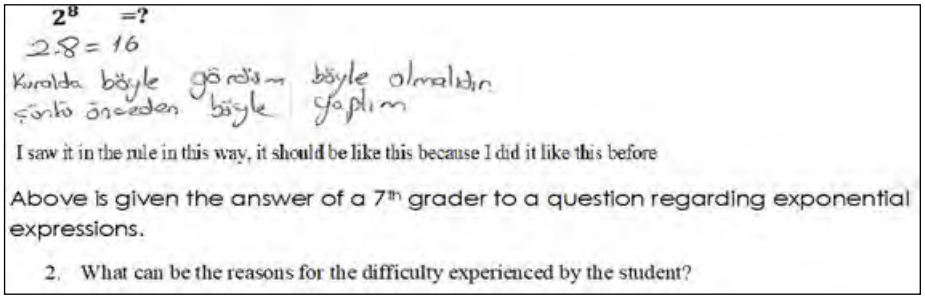
[Insert Table 1 Here]

Figure 1
*The case about identifying and interpreting the difficulties of students*

Bahar teacher, who is a pre-school teacher, went to the school gymnasium to play games with her students. Bahar teacher asked her student Musa to bring 6 balls. When Musa brought the balls, Bahar teacher asked "How many balls did you bring Musa?" She then observes that Musa counted the balls again. Evaluate the knowledge of Musa on the concept of number. What do you think Musa knows/does not know?

Other components of knowledge of content and students focused on explaining reasons for student difficulties. The sample case prepared for these components of knowledge of content and students is presented in Figure 2.

Figure 2
*The Case about Explaining Reasons for Student Difficulties*




The cases were presented to the review of the experts in terms of appropriateness and language-meaning. The cases were examined by four academicians working in the field of mathematics education and two middle school mathematics teachers. On the basis of the feedback provided by the experts, the final form of the cases was given.

### 3.3. Data Sources and Data Collection

The data sources of the current study are the written records consisting of the cases that the pre-service teachers worked on and the audio recordings that contain the pre-service teachers' discussion processes about the cases. A tape recorder was placed on the table of each group and discussion processes were recorded. The researcher also took field notes in the same process. Within the context of the course, information was given on CRD in the first week. In the second week, the place of numbers in the curriculum was discussed and the pre-service teachers were asked to read the article by Erlwanger (1973). In the third week, it was discussed how the pre-service teachers understood the concepts and the article by Erlwanger (1973), a seminal work in the literature. From the fourth week onwards, the subjects were started to be studied and in some part of the lessons, theoretical information was given to the pre-service teachers from the textbook (Van de Walle et al., 2013). In the remaining part of the lesson, the pre-service teachers were asked to work on the subject in groups. Moreover, the researcher walked in the groups to make observations and took field notes, but no intervention was made. Each week, the theoretical part of the lesson lasted for 60-90 minutes and 60-90 minutes were allocated for the pre-service teachers to work on the cases. The cases related to the following content were given to the pre-service teachers, and they were asked to work on and interpret them as a group. The content of the course is presented in Table 2.

### 3.4. Data Analysis

The case papers the students worked on, the transcripts of the audio recordings containing the discussions of the groups on the cases, and the field notes of the researcher were analyzed based

Table 2
*Content of the Course "Teaching Numbers"*

| *Weeks* | *Content* |
|---|---|
| Week 1 | Informing about CRD |
| Week 2 | Place of numbers in the curriculum |
| Week 3 | Importance of understanding student learning |
| Week 4 | The concept of early number and number sense |
| Week 5 | Natural numbers and operations |
| Week 6 | The concept of place value, developing strategies for calculations, estimating |
| Week 7 | Factors and multiples, relatively prime numbers, prime factor, divisibility rules, GCD, LCM |
| Week 8 | Fractions, operations with fractions |
| Week 9 | Decimals |
| Week 10 | Ratio-proportion, percentages |
| Week 11 | Integers, operations with integers |
| Week 12 | Rational numbers, operations with rational numbers |
| Week 13 | Exponential expressions |
| Week 14 | Square root expressions, irrational numbers |

on qualitative and quantitative analysis methods (Creswell, 2009). Knowledge of content and students of the pre-service teachers was evaluated around two basic components. These components are identifying and interpreting student difficulties and explaining reasons for student difficulties (An & Wu, 2012; Ball et al., 2008; Hartman, 2012; Takker & Subramaniam, 2012; Tirosh, 2000). Within the framework of these components, mathematical details were determined and scored while coding the changes occurred in the knowledge of content and students of the pre-service teachers. For example, if the pre-service teachers were able to identify and interpret the student difficulty both operationally and conceptually for each case, it was coded as "robust evidence" and given 3 points. If they were able to identify and interpret the student difficulty only conceptually, it was coded as "medium evidence" and 2 points were given. If they were able to identify and interpret the student difficulty only operatively, it was coded as "low evidence" and 1 point was given. If they failed to identify and interpret the difficulty, it was coded as "no evidence" and 0 point was given. For example, if the pre-service teachers did not make any comment on the student difficulty in the case given in Figure 9, then it was coded as "no evidence" and if they only operationally evaluated by stating "The student does not know that he/she needs to equate the denominators in order to perform the addition operation in fractions.", it was coded as "low evidence". If the pre-service teachers only conceptually evaluated the student difficulty by stating "The student does not know that he/she has to work on the same whole in order to perform the addition operation in fractions.", then it was coded as "medium evidence and if they both operationally and conceptually evaluated the student difficulty by stating "The student does not know that in order to perform the addition operation in fractions, he/she needs to work on the same whole and to do so, the denominators must be equated.", then it was coded as "robust evidence". A similar analysis process was carried out for the other components of knowledge of content and students. The analysis process regarding the components of knowledge of content and students is given in Table 3.

The pre-service teachers' findings regarding knowledge of content and students during the CRD process were coded within the framework of the themes mentioned above. The process was completed by taking the arithmetic mean of the scores obtained for knowledge of content and students component every week. The resulting arithmetic mean scores were defined as $0 \leq$ low $<1$, $1 \leq$ moderate $< 2$, and $2 \leq$ high $< 3$. After the coding was completed, the process of defining the

Table 3
*The Framework of the Analysis*

| *Evidence types/components of knowledge of content and students* | *Identifying and interpreting student difficulty* | *Explaining reasons for student difficulties* |
|---|---|---|
| Robust evidence (3) | Identifying and interpreting the student difficulty both operationally and conceptually | Explaining the reasons for the student difficulty |
| Medium evidence (2) | Identifying and interpreting the student difficulty only conceptually | Explaining the reasons for the student difficulty partially/incompletely |
| Low evidence (1) | Identifying and interpreting the student problem only operationally | Attributing the reasons for the student difficulty only to the teacher |
| No evidence (0) | Do not identify and interpret the student difficulty | Do not explain the reasons for the student difficulty |

findings was started. Findings were interpreted and presented to the reader with direct quotations where necessary. In order to establish the validity and reliability of the obtained data, the researcher encoded the data in two different times points (2 times at two-month intervals) and the intercoder agreement rate was 92%. Similarly, a mathematics educator working on knowledge of content and students was asked to encode 30% of the data. The intercoder reliability was found to be 88%. This makes it possible to say that the current study is reliable (Miles & Huberman, 1994). The differences of opinion that arose were resolved through discussions.

In the current study, the first role of the researcher was to make the study environment suitable for study and to determine participants. At the beginning of the study, the participants were informed about the research process and then they were selected to be included in the study on a volunteer basis. Moreover, before the study, the participants were informed about the research process and that their names would not be used in the study. In addition to this, the researcher provided guidance throughout the research process for the participants to help them cope with the difficulties encountered and by asking facilitating questions, the researcher helped them overcome these difficulties. The researcher also encouraged them to cooperatively fulfill the responsibilities and implement the decisions made during the process of group discussions. The researcher observed the work of the pre-service teachers but did not provide any direction or did not make any comments on their decisions.

## 4. Findings

In this study, the changes in the knowledge of content and students of the pre-service teachers were evaluated within the framework of the components of identifying and interpreting student difficulties and explaining reasons for student difficulties. The findings were structured in a way to emphasize four points within each knowledge of content and student component. First, the change in the mean scores of the pre-service teachers from the first week they started working on the cases to the last week was examined. Second, the changes between weeks were examined and interpreted by determining the breaking points. Third, the similarities/differences between the grade point averages of the pre-service teachers and the scores they received from the cases were examined. Finally, every week, the points that were thought to be important about the pre-service teachers were presented to the reader by summarizing them in general terms.

### 4.1. Pre-service Teachers' Knowledge about Identifying and Interpreting Student Difficulties

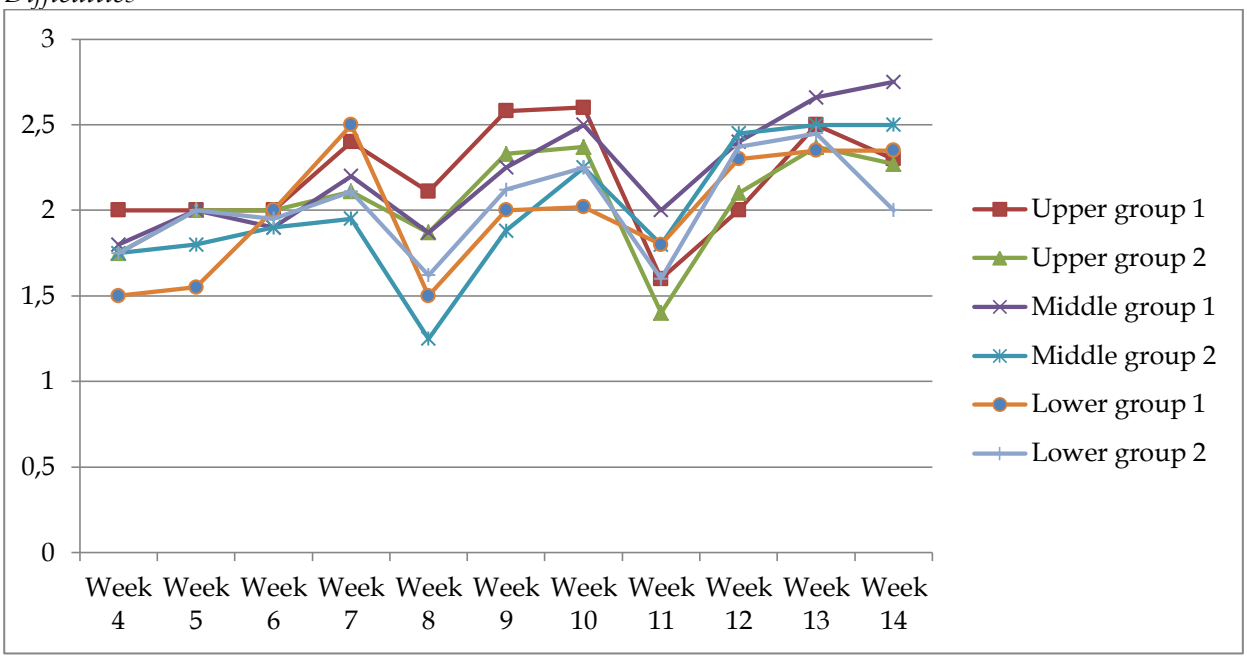
The mean scores obtained as a result of the studies of the pre-service teachers on the cases regarding identifying and interpreting the student difficulties experienced in the focused subjects are presented in Table 4.

Table 4
*Pre-service Teachers' Mean Scores about Identifying and Interpreting Student Difficulties*

| Group/ Week | Week 4 | Week 5 | Week 6 | Week 7 | Week 8 | Week 9 | Week 10 | Week 11 | Week 12 | Week 13 | Week 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Upper group 1 | 2 | 2 | 2 | 2.4 | 2.11 | 2.58 | 2.60 | 1.60 | 2 | 2.50 | 2.30 |
| Upper group 2 | 1.75 | 2 | 2 | 2.11 | 1.87 | 2.33 | 2.37 | 1.40 | 2.1 | 2.37 | 2.27 |
| Middle group 1 | 1.80 | 2 | 1.90 | 2.2 | 1.87 | 2.25 | 2.5 | 2 | 2.4 | 2.66 | 2.75 |
| Middle group 2 | 1.75 | 1.80 | 1.90 | 1.95 | 1.25 | 1.88 | 2.25 | 1.80 | 2.45 | 2.50 | 2.50 |
| Lower group 1 | 1.50 | 1.55 | 2 | 2.5 | 1.50 | 2 | 2.02 | 1.80 | 2.30 | 2.35 | 2.35 |
| Lower group 2 | 1.75 | 2 | 1.95 | 2.11 | 1.62 | 2.12 | 2.25 | 1.60 | 2.37 | 2.45 | 2 |

In addition, a line chart was created to observe the change more easily in the scores of the pre-service teachers for identifying and interpreting student difficulties. In Figure 1, the time-varying change of the arithmetic mean scores of the pre-service teachers for identifying and interpreting student difficulties is given.

Figure 3
*Time-Varying Change of the Pre-service Teachers' Mean Scores for Identifying and Interpreting Student Difficulties*




The mean scores of the pre-service teachers generally increased between the first week when they started working on the cases and the last week. While the upper group 1 had a high mean score at the beginning of the process, they kept their place in the last week. The other groups, on the other hand, had a medium mean score at the beginning, but reached a high mean score at the end. When all the groups were evaluated in general, there was a decrease in their mean scores in the 8th, 11th, and 14th weeks. When the final mean scores of the pre-service teachers were evaluated according to their grade point averages, there was no pattern. The ordering of the increases in the percentages of the final mean scores of the groups in comparison to their first mean scores is as follows: lower group 1> middle group 1> middle group 2> upper group 2 > upper group 1> lower group 2.

Although the pre-service teachers initially made comments on student difficulties, these comments were sometimes wrong or incomplete. For example, in the 4th week, the pre-service teachers were asked to interpret a case related to cardinal value.

Figure 4
*The Case about the Student Difficulty in the Cardinal Value*

Bahar teacher, who is a pre-school teacher, went to the school gymnasium to play games with her students. Bahar teacher asked her student Musa to bring 6 balls. When Musa brought the balls, Bahar teacher asked "How many balls did you bring Musa?" She then observes that Musa counted the balls again. Evaluate the knowledge of Musa on the concept of number. What do you think Musa knows/does not know?

In the case presented in Figure 4, what was expected from the pre-service teachers was to say that Musa could not gain the cardinal number value. However, emphasizing the conservation of numbers, the lower group 1 commented that "*Musa's number conservation has not developed because he thinks that the number of objects changes when they are in different arrangements or depending on their being scattered or amassed.*"; thus, erroneously evaluated the student thinking. The pre-service teachers were not aware of the fact that no comment can be made on the conservation of numbers in this case. A similar situation was observed in the 5th week. The pre-service teachers were asked to interpret the student difficulties regarding the different meanings of the number 0 (Figure 5).

Figure 5
*The Case about the Different Meanings of the Number 0*

The teacher Ali talks to his students about 0. One of the students, Şeyda, defines 0 as follows:

*"There were three toys on the table a short time ago. I put them all into the drawer. Now I have no toy on the table. That is, I have 0 toy"*

And then the teacher Ali completed the lesson by saying that

*"Yes, isn't it, children? What Şeyda says is correct. Then, 0 means nothing?"*

The teacher in the case only mentioned the nothingness meaning of zero. The pre-service teachers were expected to state that this explanation was lacking and that the different meanings of 0 (being the starting point, the location of 0 in the elevator, 0 $^0$C in the thermometer) should also be mentioned; otherwise, students may have various difficulties. However, some of the pre-service teachers had difficulties in this process. For example, the lower group 1 made the following wrong explanation: *"The teacher Ali wanted to hear that 0 means nothing, yet this explanation was incorrect. Zero does not mean nothing. He misleads the children"*. In another case given about the concept of 0, the pre-service teachers had difficulty in interpreting the student difficulty (Figure 6).

Figure 6
*The Case about the Different Meanings of the Number 0*

*You have seen that most of the students in the classroom answered as 0 when you asked the result of 5/0.*

- ✓ *Please evaluate the answer of the student. What does/doesn't the student know? How does he/she perceive the numbers?*

Here, the middle group 2 made the following comment: "*A middle school student does not know the concepts such as being undefined and eternity. In this case, this undefined statement may cause the student to be confused. In our opinion, it is not right to direct this question to students*." The lower group 1 interpreted student thinking erroneously by stating that "*The student doesn't know that zero is undefined. He/she knows that zero is nothing*". On the other hand, there were some groups correctly interpreting the student thinking by stating that "*He/she does the same thing in the division operation by generalizing the nullifying element property of 0 in the multiplication operation*" (*lower group 2*). As the process progressed, the pre-service teachers were able to interpret students' thoughts more deeply. For example, based on the case given on prime numbers also factors and multiples, which was discussed in the 7th week, the pre-service teachers were asked to interpret student thinking (Figure 7).

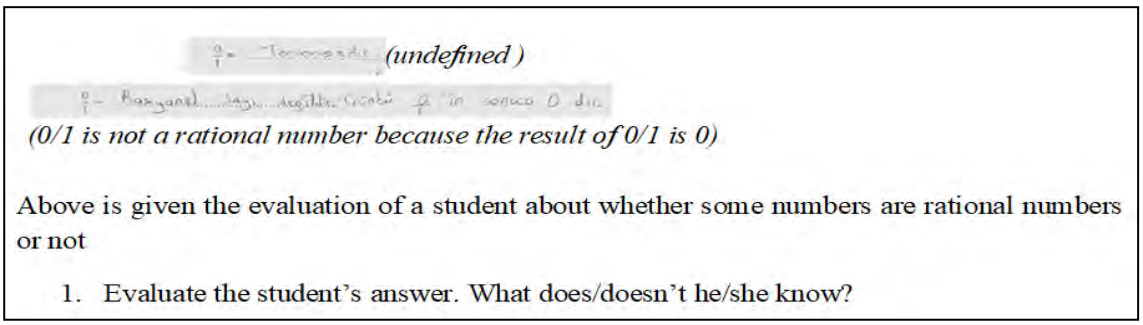
Figure 7
*The Case regarding the Subject of Prime Numbers Also Factors and Multiples*

*The teacher Mehmet asked his students to factor the number 200 into its prime factors. While he was observing what his students were doing, he saw that one of the students expressed the prime factors as 10.5.4.*

1. Evaluate the student's answer. What does/doesn't he/she know?

The middle group 2 were able to interpret the students' thinking as follows: "*The student knows how to factor the given number but confuses the concept of prime number. So, he/she did the factoring wrong. He/she did not know that the prime factors should consist of prime numbers*". The progress in the interpretation of the student thinking manifested itself in the last weeks of the process. For example, this development could be observed concretely in the cases on rational numbers and operations with rational numbers discussed in the 12th week (Figure 8).

Figure 8
*The Case regarding the Determination of Rational Numbers*




*(0/1 is not a rational number because the result of 0/1 is 0)*

Above is given the evaluation of a student about whether some numbers are rational numbers or not

1. Evaluate the student's answer. What does/doesn't he/she know?

In the case given in Figure 8, the pre-service teachers were asked to interpret the difficulties students experienced in determining rational numbers. The groups were successful in this subject and were able to interpret the student responses. For example, the middle group 1 was able to correct the student thinking correctly by stating that "*0/1 is undefined, the student probably memorized the division of a number by zero as undefined, and because he/she thought 0 was nothingness, he/she thought that its division by 1 was undefined. The reason he/she says 0/1 is not rational is that the student does not know that integers are also rational numbers.*"

The striking finding related to this component of knowledge of content and students is that there was a decrease in the mean scores of the pre-service teachers regarding the cases given in the 8th, 11th, and 14th weeks. The pre-service teachers had difficulties in identifying and interpreting student difficulties. For example, they proved inadequate in interpreting the meaning of fractions, the comparison of fractions, and operations with fractions, which were discussed in the 8th week.

For example, the pre-service teachers were asked to interpret the student difficulty regarding addition with fractions (Figure 9).

Figure 9
*The Case regarding Addition with Fractions*

> *The teacher asked the result of the operation 1/3+1/4. Pelin responded as 2/7.*
>
> 1. Evaluate the student's answer. What does/doesn't she know?

Here, what is expected from the pre-service teachers was them to say that it was necessary to work on the same whole to be able to perform the addition with fractions. However, none of the groups mentioned this. For example, the upper group 1 showed that they addressed the student thinking largely operationally by stating that *"The student knows that the numerators need to be added but does not know that the denominators need to be equalized in order to perform the addition operation."* A similar situation was observed in the 11th week when integers were handled. Regarding a case in which the subject of operations in whole numbers was discussed, the pre-service teachers could not interpret student thinking at the desired level (Figure 10).

Figure 10
*The Case regarding Operations with Integers*

> *When a $7^{th}$ grader was asked the result of the operation -4+4, he/she responded as 8*
>
> 1. Evaluate the student's answer. What does/doesn't he/she know?

Here, while the pre-service teachers were expected to interpret the student difficulty by establishing a relationship between negative numbers and the direction of numbers, the pre-service teachers were found to be able to make limited comments with the subtracting operation in natural numbers. For example, the middle group 2 made the following comment: "*The student here does not know that the negative value of the number means reduction in the addition process."* A similar situation was also observed in the last week, and the pre-service teachers had difficulties in interpreting student difficulties regarding irrational numbers and real numbers (Figure 11).

Figure11
*The Case regarding the Determination of Irrational Numbers*

> *The teacher Ayşe asked the $8^{th}$ grade students to think about this question "Are all the rational numbers real numbers?" One of the students responded as follows: "each rational number cannot be a real number. For example, 23/17. 23/17 is a rational number because it is written in fractions. However, when I divide 23 by 17, the resulting number does not come to an end and the decimal part is progressing endlessly. This is an indication of irrationality. Therefore, this number cannot be real."*
>
> 1. Evaluate the student's answer. What does/doesn't he/she know?

In this case, the pre-service teachers themselves had difficulty in defining irrational numbers. For example, the upper group 2 interpreted this sample case as follows: *"The student knows that rational numbers can be written as a/b. He/she considers every decimal number that does not end as an irrational number. He/she knows that numbers infinite in decimals are irrational."* From here, it can be inferred that the pre-service teachers cannot define irrational numbers because every number that

goes to infinity in decimals is not irrational. Therefore, pre-service teachers have their own difficulties regarding the concept.

### 4.2. Pre-service Teachers' Knowledge about Explaining Reasons for Student Difficulties

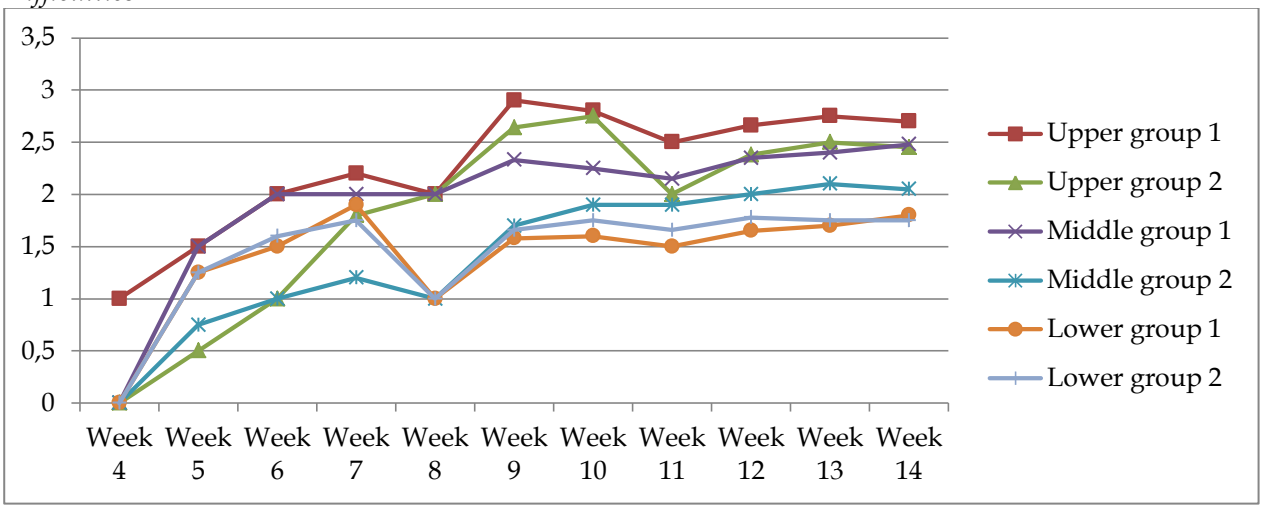
The arithmetic means of the scores obtained as a result of the pre-service teachers' work on the cases to explain the reasons for the student difficulties regarding the focused subjects are presented in Table 5.

Table 5
*Pre-service Teachers' Mean Scores about Explaining Reasons for Student Difficulties*

| Group/ Week | Week 4 | Week 5 | Week 6 | Week 7 | Week 8 | Week 9 | Week 10 | Week 11 | Week 12 | Week 13 | Week 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Upper group 1 | 1 | 1.5 | 2 | 2.2 | 2 | 2.90 | 2.80 | 2.50 | 2.66 | 2.75 | 2.7 |
| Upper group 2 | 0 | 0.5 | 1 | 1.8 | 2 | 2.64 | 2.75 | 2 | 2.38 | 2.5 | 2.45 |
| Middle group 1 | 0 | 1.5 | 2 | 2 | 2 | 2.33 | 2.25 | 2.15 | 2.35 | 2.4 | 2.48 |
| Middle group 2 | 0 | 0.75 | 1 | 1.2 | 1 | 1.7 | 1.90 | 1.90 | 2 | 2.1 | 2.05 |
| Lower group 1 | 0 | 1.25 | 1.5 | 1.9 | 1 | 1.58 | 1.60 | 1.50 | 1.65 | 1.7 | 1.8 |
| Lower group 2 | 0 | 1.25 | 1.60 | 1.75 | 1 | 1.66 | 1.75 | 1.66 | 1.78 | 1.75 | 1.75 |

In addition, a line chart was created to observe the change more easily in the pre-service teachers' scores about explaining reasons for student difficulties. In Figure 12, the time-varying change in the pre-service teachers' arithmetic mean scores for determining the reasons for the difficulties experienced by the students is given.

Figure 12
*The Time-Varying Change in the Pre-service Teachers' Mean Scores about Explaining Reasons for Student Difficulties*




When the data obtained were evaluated in general, it was noticed that the means of the pre-service teachers increased from the first week they started working on the cases to the last week. However, there are differences in the percentages of the increase. While the upper group 1 had a medium mean score at the beginning of the process, they reached a high mean score at the end of the process. While the upper group 2 and the middle groups 1 and 2 had a low mean score at the beginning of the process, they reached a high mean score at the end of the process. While the lower

groups 1 and 2 had a low mean score at the beginning of the process, they reached a medium mean score at the end of the process. When all the groups were evaluated in general, there was a decrease in their mean scores in the 8th, 11th, and 14th weeks. When the final mean scores of the pre-service teachers were evaluated according to their grade point averages, it was observed that there was no pattern. The ordering of the increases in the percentages of the final mean scores of the groups in comparison to their first mean scores is as follows: Middle group 1> upper group 2> middle group 2> lower group 1> lower group 2> upper group 1.

The pre-service teachers had great difficulty explaining reasons for student difficulties at the beginning of the process. For example, in the 4th week, the pre-service teachers were asked to comment on the reasons for the difficulty of the students regarding the conservation of numbers. The other groups, except for the upper group 1, could not comment on the reasons for the difficulty (Figure 13).

Figure 13
*The Case regarding the Conservation of Numbers*

> The teacher Esra and her students are doing an activity gathered around the table. She puts 8 marbles first close to each other and then distant from each other and asks her students how many marbles were there in each case. Two of the students respond as follows:
>
> *Nilüfer: Teacher, when they are scattered, there are more marbles.*
>
> *Veli: No, I think the numbers of marbles are equal in each case.*
>
> Evaluate the knowledge of Nilüfer and Veli on the concept of conservation of numbers. What do you think Nilüfer and Veli know and do not know? What can be the reasons for the difficulty experienced by the student?

In this case, while all of the groups were able to interpret the student difficulty, except for one group, all the other groups could not comment on the reasons for this difficulty. The upper group 1 attributed this difficulty experienced by the student to the teacher and stated that "*the teacher may not have done many activities on the concept of number and have not devoted much time to the association of it with daily life.*" In the later weeks, the pre-service teachers made more comments on the reasons for the difficulties experienced by the students. For example, in the 5th week, the pre-service teachers were asked to work on the case focusing on the meaning of 0 (Figure 14).

Figure 14
*The Case Regarding the Different Meanings of 0*

> *When you asked the result of the operation 5/0, you saw that most of the students responded as 0.*
>
> What can be the reasons for the difficulty experienced by the student?

While some groups attributed this difficulty experienced by the student to the teacher, some others pointed out that students' misconceptions may have caused this difficulty. For example, the upper group 1 attributed the reason for the difficulty to the teacher by stating that *"It may have caused by the teacher using an incomplete expression while defining 0. For example, if he/she defined 0 only as nothingness, this concept was understood incompletely by the student."* Another group, the upper group 2, interpreted the difficulty by associating it with misconceptions as follows: *"He/she may have overgeneralized that zero is the nullifying element. He/she may have extended the nullifying element property of 0 in the multiplication operation to the division operation."* In addition, as the weeks progressed, the pre-service teachers could diversify the reasons for student difficulties. For

example, in the 7th week, which focused on the rules of division with multiples, the pre-service teachers were asked to work on the following case.

Figure 15
*The Case regarding the Concept of Relatively Prime*

When eighth grade students were asked whether the numbers 8 and 9 are relatively prime or not between them, it was observed that one of the students said that these numbers were not prime between them.

c) What can be the reasons for the difficulty experienced by the student?

The groups were able to comment on this case on the student difficulty and even diversify the reasons. For example, the group 1 diversified the reasons for the student difficulty by stating that "*Failure to associate the relatively prime with the concept of common factor may cause this difficulty. Besides, the definition of the concept of relatively prime might have been directly memorized. He/she may know the concept of relatively prime, but she may have responded without examining consecutive numbers thinking that they are not prime between them.*" Also, group comments that attribute the reasons for the difficulties experienced by students to their misconceptions were also noticed. For example, in the 9th week, when decimals were discussed, the pre-service teachers were asked to work on the following case (Figure 16).

Figure 16
*The Cases regarding Four Operations in Decimals*

*Students were asked to estimate the results of the operations 9 x 0.358 and 8 / 0.458. Some of the students stated that the result of the first operation is higher than 9 and that the result of the second operation is lower than 8.*

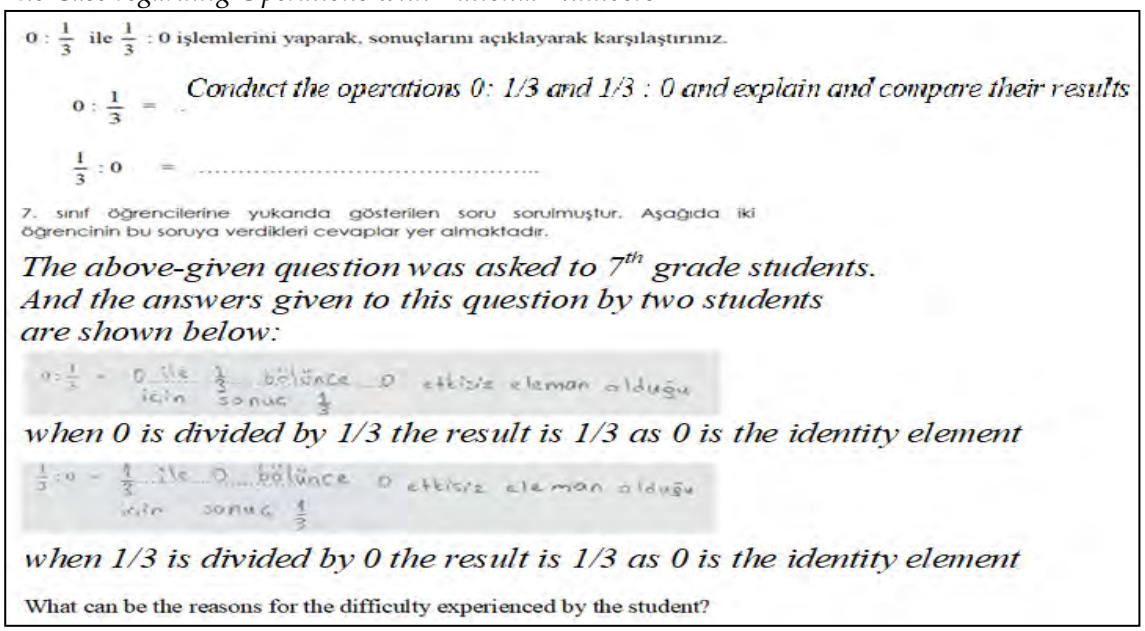
1. What can be the reasons for the difficulty experienced by the students?

The lower group 2 were able to comment on the reasons for the difficulties experienced by the students by associating them with their misconceptions as follows: "*Students can give this answer if they have the misconception that multiplication always increases the number and division always decreases the number.*" A similar situation was observed in the 12th week. They were asked to work on the following case on rational numbers and operations with rational numbers (Figure 17).

Here, they made explanations focused on misconceptions in relation to the reasons for student difficulties. The upper group 2 made the following comment: "These two students may have extended the identity element property of 0 in the addition and subtraction operations to the division operation. Therefore, they may have thought that 0 would have no effect in the division operation."

The pre-service teachers had difficulty in commenting on the reasons for student difficulties related to the subjects focused on in the 8th, 11th, and 14th weeks. In general, there were efforts to attribute the reasons for student difficulty only to the teacher. For example, in the 8th week when fractions and operations with fractions were discussed, the pre-service teachers were asked to work on the following case (Figure 18).

Figure 17
*The Case regarding Operations with Rational Numbers*


$0 : \frac{1}{3}$ ile $\frac{1}{3} : 0$ işlemlerini yaparak, sonuçlarını açıklayarak karşılaştırınız.

*Conduct the operations 0: 1/3 and 1/3 : 0 and explain and compare their results*

$0 : \frac{1}{3} =$ ...

$\frac{1}{3} : 0 =$ .....

7. sınıf öğrencilerine yukarıda gösterilen soru sorulmuştur. Aşağıda iki öğrencinin bu soruya verdikleri cevaplar yer almaktadır.

*The above-given question was asked to 7th grade students. And the answers given to this question by two students are shown below:*

$0 : \frac{1}{3}$ - 0 ile $\frac{1}{3}$ bölünce 0 etkisiz eleman olduğu için sonuç $\frac{1}{3}$

*when 0 is divided by 1/3 the result is 1/3 as 0 is the identity element*

$\frac{1}{3} : 0$ - $\frac{1}{3}$ ile 0 bölünce 0 etkisiz eleman olduğu için sonuç $\frac{1}{3}$

*when 1/3 is divided by 0 the result is 1/3 as 0 is the identity element*

What can be the reasons for the difficulty experienced by the student?

Figure 18
*The Case regarding Operations With Fractions*

*The teacher asked the result of the operation 1/3+1/4. Pelin responded as 2/7.*

1. Evaluate the student's answer. What does/doesn't she know?

In relation to this case, the lower group 1 attributed the reason for the difficulty experienced to only the teacher by stating that *"It might not have been taught the student through discovery."* The same is true for the lower group 2 that commented as follows: *"It might have been taught the student erroneously or incorrectly. The student might not have been provided with opportunities to internalize the subject thoroughly."* The pre-service teachers had difficulties in explaining the reasons for student difficulties experienced in the cases given on the subjects of irrational-real numbers in the last week (Figure 19).

Figure 19
*The Case regarding the Features of Irrational Numbers*

The teacher Bahar asked her 8th grade students to evaluate the statement *"There are both rational and irrational numbers"*. Below are given the evaluations made by two students on this statement.

*Student 1 "Yes, there are, for example the number π is both irrational and can be written as rational"*

*Student 2 "Yes, I think root expressions that are perfect squares are both rational and irrational, like* ? 16*."*

1. What can be the reasons for this difficulty experienced by students?

In relation to this case, the middle group 1 made the following comment *"Using different values of the number π led to misconception in the student. Since he/she evaluates the squared expressions by*

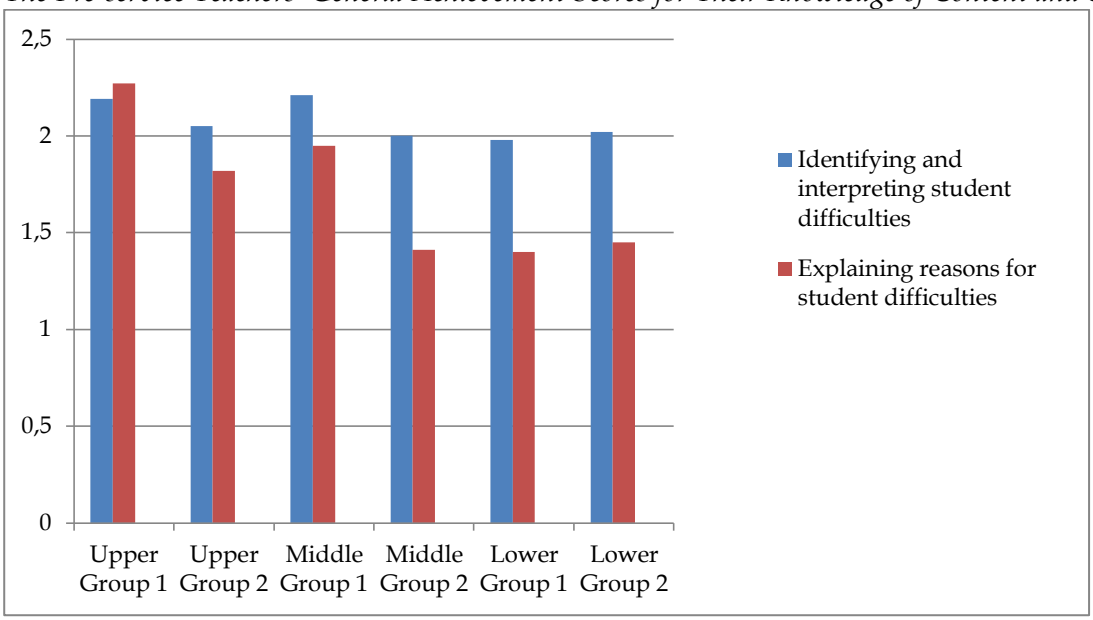
*generalizing, he/she calls the perfect square numbers as irrational numbers."* The number π actually has no different values here. The pre-service teachers revealed their own misconceptions by thinking that taking the values such as 3 or 3.14 for ease of operation as if the number π had different values.

The mean of the scores obtained by the pre-service teachers for their student information during the 11-week period was taken as their general achievement score, and Table 7 and Figure 20 was obtained by evaluating them on the basis of the groups and the components of knowledge of content and students.

Table 7
*The General Achievement Mean Scores Obtained From the Pre-service Teachers' Knowledge of Content and Students*

| The components of knowledge of content and students | Upper group 1 | Upper group 2 | Middle Group 1 | Middle Group 2 | Lower Group 1 | Lower Group 2 |
|---|---|---|---|---|---|---|
| Identifying and interpreting student difficulties | 2.19 | 2.05 | 2.21 | 2 | 1.98 | 2.02 |
| Explaining reasons for student difficulties | 2.27 | 1.82 | 1.95 | 1.41 | 1.4 | 1.45 |

Figure 20
*The Pre-service Teachers' General Achievement Scores for Their Knowledge of Content and Students*




According to the Figure 20, except for the upper group 1, all the groups had the highest mean score from the component of identifying and interpreting student difficulties, followed by the mean scores related to the component of explaining the reasons for student difficulties. The mean score for the component of explaining reasons for student difficulties was found to be the highest for the upper group 1, followed by the mean scores for the components of identifying and interpreting student difficulties.

## 5. Discussion and Conclusion

In this study, researchers investigated how pre-service teachers' work on cases affected their knowledge of content and students. The pre-service teachers were given cases on various concepts in the learning area of numbers for 11 weeks and were asked to work on cases in groups of three or four.

The findings showed that the pre-service teachers had difficulties in identifying and interpreting student difficulties, explaining the reasons for these difficulties at the beginning of the

process. While the pre-service teachers remained inadequate in identifying and interpreting student difficulties, none of the groups, except for one group, expressed opinions about the reasons for these difficulties. The reasons for these difficulties experienced by the pre-service teachers can be evaluated under two headings. The first reason can be stated as the pre-service teachers' lack of content knowledge because when there is a shortage in the content knowledge, this will negatively affect the pedagogical content knowledge of pre-service teachers. This emphasis has been made in various studies, and attention has been drawn to the connection between content knowledge and pedagogical content knowledge (Ball et al., 2008; Carpenter et al., 1996). Another reason for the difficulties experienced by the pre-service teachers can be considered as the lack of experience. Considering the content of undergraduate programs, although pre-service teachers take various courses related to content education, the content of these courses does not focus directly on student thinking. Although there is an implicit emphasis on knowledge of content and students in the content of the courses, pre-service teachers not focusing on student thinking may have triggered the emergence of such a result. In the literature, studies also stated that pre-service teachers not gaining experience regarding knowledge of content and students may cause a lack of knowledge (Bjuland & Mosvold, 2015; Heaton & Mickelson, 2002). Some research results also show that pre-service teachers who do not have sufficient practical or theoretical experience may have difficulty in analyzing cases and associating cases with the theoretical knowledge they have learned (Mostert, 2007).

In the later stages of the process, the pre-service teachers were able to evaluate the reasons for these difficulties as well as how the students would think and where they would have difficulties. They generated more ideas about interpreting student difficulties, and they were able to diversify the reasons for these difficulties. When the reasons for this development in the pre-service teachers' knowledge of content and students are considered, a few points should be emphasized. The first of these is that the cases were structured in such a way as to support the pre-service teachers to feel like they are in a real classroom environment. In this way, the pre-service teachers felt like a real teacher in the problem situations they encountered and tried to solve these problems. In addition, they also learned about how the theoretical knowledge they learned reflected in the application process; in other words, they were in a real classroom environment. This can enable them to gain experience. In the literature, studies stated that pre-service teachers and teachers who are new to the profession have difficulty in adapting to real classroom environments (Dolk et al., 2002; Quek & Wang, 2010). At this point, it can be concluded that with the current study, pre-service teachers can be supported to overcome this adaptation problem that they may experience. Another point that fostered the development of the pre-service teachers regarding knowledge of content and students can be stated to be them working as a group in this process. The pre-service teachers' focusing on the problem involved in the case and revealing their thoughts led them towards the solution of the problem. This fosters their understanding of the complex structure of the mathematics teaching process and the formation of multiple perspectives on this process (Lin, 2005). Particular attention was paid for the groups to be consisted of 3-4 people because some studies have shown that case discussions conducted in small groups yielded more successful results (Norquist, 2008). When the literature is reviewed, studies stated that analyzing and discussing cases in social environments triggers constructivist learning to occur and this can contribute to the development of knowledge of content and students (Guskey, 2003; J. Shulman, 1992). Another point can be considered that to promote the development of knowledge of content and students in pre-service teachers is thought to be the duration of the teacher education program. The current study lasted for 14 weeks, and the pre-service teachers carried out applications that would allow them personally to employ their knowledge of content and students for 11 weeks. These applications supported the pre-service teachers to internalize their knowledge of content and students. As Smith (2001) emphasized, this effort and time devoted help learning.

Although there are improvements in terms of the points mentioned above, in some weeks, a decrease was observed in the scores taken by the pre-service teachers for all the components of

knowledge of content and students regarding the cases. the subjects that the pre-service teachers had difficulty in commenting on the components of knowledge of content and students were fractions and operations with fractions, integers, and operations with integers, and square root and irrational numbers. The reasons for these difficulties experienced by the pre-service teachers can be evaluated under several headings. The first of these is that the subjects mentioned contain some epistemological difficulties. In the history of mathematics, studies stated that it was not easy for people to accept irrational numbers and negative numbers. The fact that these number sets are different from other frequently used number sets made it difficult to accept them (Guedj, 2006; Sertöz, 2002). Therefore, it is not surprising that the pre-service teachers also experienced similar difficulties in these subjects, which have been found to be challenging by people throughout the history of mathematics. Another reason for these difficulties experienced by the pre-service teachers can be considered as their lack of content knowledge. For example, their perception of equating denominators as a rule in adding fractions and not knowing why may have caused them to remain incapable of commenting on such difficulties of students and thinking about their reasons. In the literature, attention is drawn to the issue that teachers/pre-service teachers' deficiencies in their content knowledge may negatively affect their pedagogical content knowledge (Charalambous, 2010).

When the development of the pre-service teachers' knowledge of content and students is holistically evaluated, at the end of the process, their mean score taken from the component of explaining reasons for student difficulties is lower than the mean scores taken from the identifying and interpreting student difficulties. This allows the interpretation that not all the components of knowledge of content and students are improving at the same time. Learning is a complex process and changes in the teacher's knowledge may be complex and may not often proceed in a linear manner (Fullan, 1991; Van Es & Sherin, 2008).

Knowing and understanding students in increasing the effectiveness of the teaching process is very important (Reynolds, 1992). In this regard, the current study contributed to the development of the pre-service teachers' knowledge of content and students. Similar points are emphasized in the literature, and the importance of pre-service teachers' working on cases is pointed out (Hernandez-Serrano & Jonassen, 2003; Loughran, 2002; Putnam & Borko, 2000). The results obtained show that CRD can be one of the important alternatives for the development of knowledge of content and students in pre-service teachers. In this connection, results of this study suggest that the courses in undergraduate programs should be structured in a way that they include cases and designed in such a way as to make students realize the importance of knowledge of content and students. Moreover, the current study focused on the learning area of numbers. In future research, cases directed to the development of other learning areas (e.g., algebra, geometry) related to pre-service teachers' knowledge of content and students can be designed and implemented.

### 5.1. Limitations

As in any study, the current study has some limitations. As the current study is limited to a university in Turkey, it is thought that there is a need to investigate the knowledge of content and students of pre-service teachers from different universities in Turkey as well as from universities abroad through cases. Moreover, the current study is limited to third-year pre-service teachers and the study was conducted over a semester period. Longitudinal studies to investigate the development of pre-service teachers' knowledge of content and students when they are involved in real classroom applications (e.g., teaching practicum) can be conducted.

**Acknowledgements.** A part of this study was presented as an oral presentation at the III. International Teacher Education Accreditation Congress in Ankara between 30 November – 1 December 2019.

## References


Abell, S. K., & Cennamo, K. S. (2004). Videocases in elementary science teacher preparation. In J. Brophy (Eds.), *Using video in teacher education* (pp. 103– 130). Elsevier Ltd.

Alkan, R. (2009). *Analysis of mistakes and concept regarding the topic of rational numbers in mathematics lesson of the seventh class in the primary education* [Unpublished master's thesis]. Gazi University, Ankara.

An, S., &Wu, Z. (2012). Enhancing mathematics teachers' knowledge of students' thinking from accessing and analyzing misconceptions in homework. *International Journal of Science and Mathematics Education*, *10*(3), 717-753. https://doi.org/10.1007/s10763-011-9324-x

Atayev, G. S. (2015). *Sixth grade students' achievement levels, errors, and underlying reasons of the errors regarding comprehension and ordering of integers* [Unpublished master's thesis]. Middle East Technical University, Ankara.

Ball, D. (2000). Bridging practices: Intertwining content and pedagogy in teaching and learning to teach. *Journal of Teacher Education*, *51*(3), 241-247. https://doi.org/10.1177/002248710051003013

Ball, D. L., & Bass, H. (2003). Toward a practice-based theory of mathematical knowledge for teaching. In B. Davis & E. Simmt (Eds.), *Proceedings of the 2002 Annual Meeting of the Canadian Mathematics Education Study Group*, (pp. 3–14). CMESG/GCEDM.

Ball, D. L., & Cohen, D. K. (1999). Developing practice, developing practitioners: Toward a practice-based theory of professional education. In G. Sykes, & L. Darling-Hammond (Eds.), *Teaching as the learning profession: Handbook of policy and practice* (pp. 3-32). Jossey Bass.

Ball, D. L., & Forzani, F. M. (2009). The work of teaching and the challenge for teacher education. *Journal of Teacher Education, 60*(5), 497–511. https://doi.org/10.1177/0022487109348479

Ball, D. L., Lubienski, S., & Mewborn, D. (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. In V. Richardson (Ed.), *Handbook of research on teaching* (pp. 433-456). Macmillan.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389-407. https://doi.org/10.1177/0022487108324554

Barnett, C., & Tyson, P. (1999). Case methods and teacher change: Shifting authority to build autonomy. In M. Lundeberg, B. Levin, & H. Harrington (Eds.), *Who learns what from cases and how? The research base for teaching and learning with cases* (pp. 53–69). Erlbaum.

Bayram, G. (2013). *The relationship between 8th grade students' number sense and achievement related to exponentials* [Unpublished master's thesis]. Pamukkale University, Denizli.

Bjuland, R., & Mosvold, R. (2015). Lesson study in teacher education: Learning from a challenging case. *Teaching and Teacher Education, 52,* 83–90. https://doi.org/10.1016/j.tate.2015.09.005

Borko, H., Jacobs, J., Eiteljorg, E., & Pittman, M. E. (2007). Video as a tool for fostering productive discussions in mathematics professional development. *Teaching and Teacher Education, 24*(2), 417-436. https://doi.org/10.1016/j.tate.2006.11.012

Butler, M. B., Lee, S., & Tippins, D. J. (2006). Case-based methodology as an instructional strategy for understanding diversity: Preservice teachers' perceptions. *Multicultural Education, 13*(3), 20-26.

Carpenter, T. P., Fennema, E., & Franke, M. L. (1996). Cognitively guided instruction: A knowledge base for reform in primary mathematics instruction. *The Elementary School Journal, 97*(1), 3-20. https://doi.org/10.1086/461846

Carpenter, T. P., Fennema, E., Peterson, P. L., & Carey, D. A. (1988). Teachers' pedagogical content knowledge of students' problem solving in elementary arithmetic. *Journal for Research in Mathematics Education, 19*(5), 385–401. https://doi.org/10.5951/jresematheduc.19.5.0385

Carpenter, T. P., Fennema, E., Peterson, P. L., Chiang, C., & Loef, M. (1989). Using knowledge of children's mathematics thinking in classroom teaching: An experimental study. *American Educational Research Journal*, *26*(4), 499–531. https://doi.org/10.3102/00028312026004499

Cengiz, N., Kline, K., & Grant, T. (2011). Extending students' mathematical thinking during whole-group discussions. *Journal of Mathematics Teacher Education, 15*(5), 1–20. https://doi.org/10.1007/s10857-011-9179-7

Charalambous, C. Y. (2010). Mathematical knowledge for teaching and task unfolding: An exploratory study. *The Elementary School Journal*, *770*, 247-278. https://doi.org/10.1086/648978

Crespo, S. (2000). Seeing more than right and wrong answers: Prospective teachers' interpretations of students' mathematical work. *Journal of Mathematics Teacher Education, 3,* 155–181. https://doi.org/10.1023/A:1009999016764

Creswell, J. W. (2009). *Research design: Qualitative, quantitative, and mixed methods approaches* (3rd ed.). Sage.
Darling-Hammond, L. (2006). Constructing 21st-century teacher education. *Journal of Teacher Education, 57*, 300-314.https://doi.org/10.1177/0022487105285962
Didiş, M.G., Erbaş, A. K., Çetinkaya, B., & Çakıroğlu E. (2014). Prospective secondary mathematics teachers interpretations of students thinking, *Research in Mathematics Education, 16*(1), 77-78. https://doi.org/10.1080/14794802.2013.849863
Dinç, Y. (2018). *An investigation of eighth grade students' knowledge construction process of square root numbers* [Unpublished master's thesis]. Osmangazi University, Eskişehir.
Dogruel, A. B. (2019). *An examination of elementary school mathematics teachers' pedagogical content knowledge on ratio and proportion* [Unpublished master's thesis]. Afyon Kocatepe University, Afyon.
Dolk, M., Hertog, J. D., & Gravemeijer, K. (2002). Using multimedia cases for educating the primary school mathematics teacher educator: A design study. *International Journal of Educational Research, 37*, 161–178. 10.1016/S0883-0355(02)00058-7
Doyle, W. (1980). *Classroom management*. Kappa Delta Pi.
Driel, J. H., & Berry, A. (2010). The teacher education knowledge base: Pedagogical content knowledge. In B. McGraw, P. L. Peterson, E. Baker, (Eds.). *International encyclopedia of education* (pp. 656-661). Elsevier.
Erlwanger, S. H. (1973). Benny's conception of rules and answers in IPI mathematics. *The Journal of Children's Mathematical Behavior, 1*(2), 7-26.
Eroglu, D. (2012). *Examining prospective elementary mathematics teachers' knowledge about students' mistakes related to fractions* [Unpublished master's thesis]. Middle East Technical University, Ankara.
Franke, M. L., Carpenter, T. P., Levi, L., & Fennema, E. (2001). Capturing teachers' generative growth: A follow-up study of professional development in mathematics. *American Educational Research Journal, 38*, 653-689.https://doi.org/10.3102/00028312038003653
Fullan, M. G. (1991). *The new meaning of educational change*. Cassell Educational Limited Wellington House.
Gocuk, Ş. (2019). *Determination of the concept errors of secondary school eighth grade students about themal numbers and rooted numbers* [Unpublished master's thesis]. Fırat University, Elazığ.
Gravett, S.,Henning, E., & Eiselen, R. (2011). New teachers look back on their university education: Prepared for teaching, but not for life in the classroom. *Education as Change, 15*(1), 123-142. https://doi.org/10.1080/16823206.2011.643636
Gravett,S; Beer, J., Odendaal-Kroon, R., & Merseth, K. K. (2017) The affordances of case-based teaching for the professional learning of student-teachers. *Journal of Curriculum Studies, 49*(3), 369-390. https://doi.org/10.1080/00220272.2016.1149224
Guedj, D., (2006). *Numbers: The universal language.* Thames & Hudson.
Guskey, T. R. (2003). What makes professional development effective? *Phi Delta Kappan, 84*(10), 748-750. https://doi.org/10.1177/003172170308401007
Harrington, H. L. (1995). Fostering reasoned decisions: Case-based pedagogy and the professional development of teachers. *Teaching and Teacher Education, 11*(3), 203-241. https://doi.org/10.1016/0742-051X(94)00027-4
Harrington, H. L. (1999). Case analyses as a performance of thought. In M. A. Lundeberg, B. B. Levin, & H. L. Harrington (Eds.), *Who learns what from cases and how: The research base for teaching and learning with cases* (pp. 29–48). Erlbaum.
Hartman, G. L. (2012). *Helping prospective teachers to understand children's mathematical thinking* [Unpublished doctoral dissertation]. Columbia University, USA.
Hashweh, M. Z. (2004). Case-writing as border-crossing: Describing, explaining and promoting teacher change. *Teachers and Teaching: Theory and Practice, 10*, 229–246.https://doi.org/10.1080/1354060042000204423
Heaton, R. M., & Mickelson, W. T. (2002). The learning and teaching of statistical investigationing teaching and teacher education. *Journal of Mathematics Teacher Education, 5*(1), 35–59. https://doi.org/10.1023/A:1013886730487
Heitzmann, R. (2008). Case study instruction in teacher education: Opportunity to develop students' critical thinking, school smarts and decision making. *Education, 128*, 523–541.
Hernandez-Serrano, J., & Jonassen, D. H. (2003). The effects of case libraries on problem solving. *Journal of Computer Assisted Learning, 19*(1), 103–114.https://doi.org/10.1046/j.0266-4909.2002.00010.x
Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education, 39,* 372-400. https://doi.org/10.5951/jresematheduc.39.4.0372

Hill, H. C., Blunk, M. L., Charalambous, C. Y., Lewis, J. M., Phelps, G. C., Sleep, L., & Ball, D. L. (2008). Mathematical knowledge for teaching and the mathematical quality of instruction: An exploratory study. *Cognition and Instruction, 26*(4), 430–511. https://doi.org/10.1080/07370000802177235

Jacobs, V. R., Lamb, L. L. C., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education, 41*(2), 169-202. https://doi.org/10.5951/jresematheduc.41.2.0169

Jacobs, V. R., Lamb, L. L. C., Philipp, R. A., & Schappelle, B. P. (2011). Deciding how to respond on the basis of children's understandings. In M. G. Sherin, V. R., Jacobs, & R. A. Philipp (Eds.), *Mathematics teacher noticing: Seeing through teachers' eyes* (pp. 97–116). Routledge.

Jay, J. (2004). Variations on the use of cases in social work and teacher education. *Journal of Curriculum Studies*, *36*(1), 35–52. https://doi.org/10.1080/0022027032000135067

Kleinfeld, J. (1992). Learning to think like a teacher: The study of cases. In J. Shulman (Ed.), *Case methods in teacher education* (pp. 33-49). Teachers College Press.

Kunselman, J., & Johnson, K. (2004). Using the case method to facilitate learning. *College Teaching, 52*, 87–92.https://doi.org/10.3200/CTCH.52.3.87-92

Lampert, M., & Ball, D. L. (1998). *Teaching, multimedia, and mathematics: Investigations of real practice.* Teachers College Press.

Lampert, M., Franke, M. L., Kazemi, E., Ghousseini, H., Turrou, A. C., Beasley, H., Cunard, A., & Crowe, K. (2013). Keeping it complex: Using rehearsals to support novice teacher learning of ambitious teaching. *Journal of Teacher Education*, *64*(3), 226–243. https://doi.org/10.1177/0022487112473837

Lave, J., & Wenger, E. (1991) *Situated learning: Legitimate peripheral participation.* Cambridge University Press. http://dx.doi.org/10.1017/CBO9780511815355

Leong, K. E. (2012). *Good mathematics teaching: Perspectives of beginning secondary teachers,* [Unpublished doctoral thesis]. Columbia University.

Levin, B. B. (1995). Using the case method in teacher education: The role of discussion and experience in teachers' thinking about cases. *Teaching and Teacher Education, 11*, 63-79. https://doi.org/10.1016/0742-051X(94)00013-V

Lin, P. J. (2002). On enhancing teachers' knowledge by constructing cases in classroom. *Journal of Mathematics Teacher Education, 5*, 317–349. https://doi.org/10.1023/A:1021282918124

Lin, P. (2005). Using research-based video cases to help pre-service primary teachersconceptualize a comtemporary view of mathematics teaching, *International Journal of Science and Mathematics Education, 3*, 351–377. https://doi.org/10.1007/s10763-004-8369-5

Little, J. W. (2002). Locating learning in teachers' communities of practice: Opening up problems of analysis in records of every- day practice. *Teaching and Teacher Education*, *18*(8), 917-946. https://doi.org/10.1016/S0742-051X(02)00052-5

Loughran, J. J. (2002). Effective reflective practice: In search of meaning in learning about teaching. *Journal of Teacher Education, 53*(1), 33-43. https://doi.org/10.1177/0022487102053001004

Lundeberg, M. A., & Scheurman, G. (1997). Looking twice means seeing more: Developing pedagogical knowledge through case analysis. *Teaching and Teacher Education, 13*(8), 783-797. https://doi.org/10.1016/S0742-051X(97)00020-6

Lundeberg, M.A., Levin, B. B., & Harrington, H. L. (1999). *Who learns what from cases and how: The research base for teaching and learning with cases.* Erlbaum.

Masingila, J. & Doerr, H. (2002). Understanding pre-service teachers' emerging practices through their analysis of a multimedia case study. *Journal of Mathematics Teacher Education*, *5*(3), 235-263. https://doi.org/10.1023/A:1019847825912

Mayo, J. A. (2002). Case-based instruction: a technique for increasing conceptual application in introductory psychology. *Journal of Constructivist Psychology*, *15*(1), 65–74.https://doi.org/10.1080/107205302753305728

Mayo, J. A. (2004). Using case-based instruction to bridge the gap between theory and practice in psychology of adjustment. *Journal of Constructivist Psychology*, *17*, 137–146. https://doi.org/10.1080/10720530490273917

McDonald, M., Kazemi, E., & Kavanagh, S. S. (2013). Core practices and pedagogies of teacher education: A call for a common language and collective activity. *Journal of Teacher Education, 64*(5), 378–386. https://doi.org/10.1177/0022487113493807

Merseth, K. K. (1991). The early history of case-based instruction: Insights for teacher education today. *Journal of Teacher Education*, *42*(4), 243–249. https://doi.org/10.1177/002248719104200402

Merseth, K. K. (1992). Cases for decision making in teacher education. In J. H. Shulman (Eds.), *Case methods in teacher education* (pp. 50–63). Teachers College Press.

Merseth, K. K. (1996). Cases and case methods in teacher education. In J. Sikula, T. J. Buttery, & E. Guyton (Eds.), *Handbook of research on teacher education* (p. 722–743). Simon & Schuster Macmillan.

Merseth, K. K. (1999). Foreword: A rationale for case- based pedagogy in teacher education. In M. A. Lundeberg, B. B. Levin, & H. Harrington (Eds.), *Who learns what from cases and how: The research base for teaching and learning* with cases (pp. 19-25). Erlbaum.

Merseth, K. K. (2003). *Windows on teaching math: Cases of middle and secondary classrooms*. Teachers College Press.

Miles, M, B., & Huberman, A. M. (1994). *Qualitative data analysis: An expanded Sourcebook.* Sage.

Ministry of National Education [MoNE]. (2018). Matematik dersi öğretim programı (İlkokul ve ortaokul 1,2,3,4,5,6,7 ve 8. sınıflar) [Elementary and middle school mathematics education program (Grades 1,2,3,4,5,6,7 and 8)]. Author.

Mostert, M. P. (2007). Challenges of case-based teaching. *The Behavior Analyst Today, 8*(4), 434-442. http://dx.doi.org/10.1037/h0100632

Moyer, P. S., & Milewicz, E. (2002). Learning to question: Categories of questioning used by preservice teachers during diagnostic mathematics interviews. *Journal of Mathematics Teacher Education*, *5*, 293-315. 10.1023/A:1021251912775

National Council of Teachers of Mathematics [NCTM]. (2000). *Principles and standards for school mathematics.* Author.

National Council of Teachers of Mathematics [NCTM]. (2014). *Principles to actions: Ensuring mathematical success for all students*. Author.

Norquist, J. (2008). *Implementing the case method: An educational analysis required*. Karolinska Institutet.

Powell, R. (2000). Case-based teaching in homogeneous teacher education contexts: A study of preservice teachers' situative cognition. *Teaching and Teacher Education, 16*, 389-410. https://doi.org/10.1016/S0742-051X(99)00068-2

Putnam, R. T. & Borko, H. (2000). What do new views of knowledge and thinking have to say about research on teacher learning? *Educational Researcher, 29*(1), 4-15. https://doi.org/10.3102/0013189X029001004

Quek, C. L. G. &Wang, Q. (2010). Supporting beginning teachers' case–based learning in a technology-mediated learning environment. In Steel, C., Keppell, M., Gerbic, P., & Housego, S. (Eds.) *ASCILITE 2010* (pp. 1-4). The University of Queensland.

Reynolds, J. A. (1992). What is competent beginning teaching? A review of the literature, *Review of Educational Research, 62,* 1-35. https://doi.org/10.3102/00346543062001001

Santagata, R., Zannoni, C., & Stigler, J. W. (2007). The role of lesson analysis in pre-service teacher education: An empirical investigation of teacher learning from a virtual video-based field experience. *Journal of Mathematics Teacher Education, 10*(2), 123-140. https://doi.org/10.1007/s10857-007-9029-9

Sato, M., & Rogers, C. (2010). *Case methods in teacher education*. Elsevier.

Sertöz, S. (2002). *Matematiğin aydınlık dünyası* [The bright world of mathematics].Tübitak.

Sherin, M. G. (2004). New perspectives on the role of video in teacher education. In J. Brophy (Ed.), *Using video in teacher education: Advances in research on teaching* (Vol. 10, pp. 1–27). Oxford: Elsevier Press.

Shulman, J. H. (1992). Teacher-written cases with commentaries: A teacher- researcher collaboration. In J. H. Shulman, (Eds.), *Case methods in teacher education* (pp. 131-155). Teacher College Press.

Shulman, L. S. (1992). Toward a pedagogy of cases. In J. H. Shulman (Eds.), *Case methods in teacher education* (pp. 1–30). Teachers College Press.

Shulman, L. S. (2004). *The wisdom of practice*. Jossey-Bass.

Smith, M. S. (2001). *Practice-based Professional development for teachers of mathematics*. National Council of Teachers of Mathematics.

Smith, M. S. &Friel, S. N. (2008). Cases in mathematics teacher education, In M. S. Smith & S. N. Friel (Eds.), *Cases in mathematics teacher education: Tools for developing knowledge needed for teaching* (p. 1-8). The AMTE Monograph Series.

Sowder, J. T. (2007). The mathematics education and development of teachers. In F. K. Lester Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 157-223). Information Age.

Sykes, G., & Bird, T. (1992). Teacher education and the case idea. *Review of Research in Education, 18*, 457-521. https://doi.org/10.3102/0091732X018001457

Takker, S., & Subramaniam, K. (2012). Understanding teacher's knowledge of and responses to students' mathematical thinking. In *Proceedings of the 12th international congress on mathematical education (ICME-12), Topic Study Group 4,* (pp. 4906-4915). ICME.

Tirosh, D. (2000). Enhancing prospective teachers' knowledge of children's conceptions: The case of division of fractions. *Journal for Research in Mathematics Education, 31*(1), 5-25. https://doi.org/10.2307/749817

Turnuklu, E. B., & Yesildere, S. (2007). The pedagogical content knowledge in mathematics: Pre-service primary mathematics teachers' perspectives in Turkey. *Issues in the Undergraduate Mathematics Preparation of School Teachers: The Journal*, *1,* 1-13.

Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2013). *Elementary and middle school mathematics: Teaching developmentally*. Pearson.

Van Es, E. A. & Sherin, M. G. (2008). Mathematics teachers' "learning to notice" in the context of a video club. *Teaching and Teacher Education, 24*(2), 244-276. https://doi.org/10.1016/j.tate.2006.11.005

Whitelaw, S., De Beer, J., & Henning, E. (2008). New teachers in a pseudocommunity of practitioners. *Education as Change, 12*, 25–40. https://doi.org/10.1080/16823200809487205

Wilson, S. M., & Ball, D. L. (1996). Helping teachers meet the standards: New challenges for teacher educators. *The Elementary School Journal*, *97*(2), 121-138. https://doi.org/10.1086/461858

Yapıcı, A. (2013). *An investigation of 5, 6 and 7th grade students' number sense related to percent* [Unpublished master's thesis]. Hacettepe University, Ankara.