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Zero – an Uncommon Number: Preschoolers' Conceptual Understanding of Zero

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Abstract

The conceptual development of natural number in preschoolers is well-researched. However, less is known about the conceptual development of zero. Recent studies suggest that children develop an understanding of zero after learning to count. It remains unclear, when a conceptual understanding of "zero" as number word for an empty set emerges. This paper integrates numerical and language theories about how, where and when the concept of zero is formed and is integrated into the class of natural numbers. The counting skills of 107 preschoolers were assessed for the number range between zero and eight as well as for their ordinal understanding of zero. The results show that compared to the natural numbers, zero was substantially more difficult. Children are able to list zero in a number word sequence (0, 1, 2, 3 .... or 3, 2, 1, 0), but were unable to describe a set as having zero numbers. This latter conception contradicts findings regarding natural numbers, in that an empty set is counter intuitive. Zero could be correctly placed when consecutive order was required, but addition and subtraction by counting was more difficult. The results suggest that the conceptual development of zero differs qualitatively from the natural numbers. Based on the results, the ordinal understanding of zero as a predecessor to one, together with its matching linguistic concepts is proposed to be the key to the conceptual development of zero.

Keywords:

Numerical Cognition, Zero, Ordinal Representation, Early Numeracy, Conceptual Development

Introduction

Much is known and researched about how children learn the concept of natural number, but when it comes to "zero", there is much not known. We generally use words to describe the nothingness or emptiness in everyday relations and we naturally talk about the lack of something, for example, "There are two apples, but there are no bananas." We reject objects, things or conditions and therefore form relations of nothing. All this seems facile since even children as young as two utter sentences like, "there are no cookies on my plate". But talking about "zero" and referring to it mathematically as an empty set seems to be much more complex. Going back in history it can be seen that to equip zero with an unique symbol and to integrate this into the ordinal sequence of natural number was a long journey that spanned many centuries, and followed different paths. The Babylonian system was one of the earliest to include place value when recording amounts. They had a dot as placeholder sign to indicate, where a specific place value was zero (e.g. in 2022, where there are zero hundreds). However, the idea that a sign could represent zero items was revolutionary, when it came up in ancient India. Bottazzini (2021) states that about the year 628 AD, the function of zero transformed from merely being a placeholder for an empty position in the notation of a number to a natural number with the consequent properties of a natural number. In Europe it was not until Fibonacci's Liber Abaci in 1202 (Sigler, 2002) that "zero" was broadly accepted as part of the number system (Ifrah, 1998).

So, what appears to be difficult is not to talk about nothing, but to mathematically frame a concept for zero and integrate this into the class of natural number. As many as 15% of preservice elementary school teachers do not refer to zero as a natural number (Krajcsi et al., 2021). It is clear that there is a lack of understanding how children develop the concept "zero" as both an "empty set" and as a "placeholder". All we know for sure is that the concept of "zero" seems to be harder to learn than the concepts of "one, two, three...". In the present study we will briefly summarize what is known about the understanding of zero and will frame the problem in theoretical terms specifying the lexical concepts of natural number which then provides the basis for zero as an abstract numerical concept. We will then present data showing the developmental hierarchy of natural number and specify how the progressive understanding of "zero" develops. Finally, we will suggest an outline of how "zero" is handled when it comes to the ordinal dimension of the number line.

Development of Natural Number

From the very beginning children encounter numbers, values and sets of things. Learning to speak means to build references between objects or actions and the corresponding vocabulary. While starting with mostly content words, productive vocabulary is from the start used to describe relationships between objects. At two years of age children no longer seem to have the need to refer to each object in singular form but start to refer to sets of similar things using natural quantifiers (Barner et al., 2007). What is remarkable here is that this ability to use natural quantifiers, forms the foundation to engage verbally with the world of numerical relationships. Soon after, the first concrete denomination of a set of two occurs. Children now refer to two entities as being exactly two whereas earlier they had used a natural quantifier like "many" instead. Using exact number words to describe their surroundings, children refer to lexical concepts which are concrete and abstract at the same time. Whereas the "twoness" of something is a unique, distinct, and therefore concrete feature, it can differ in shape, color, form and size (Wiese, 2007), which gives it a degree of abstraction. Unlike for example "yellow" which refers to the characteristic of the object, number words will always refer to the relation the objects hold with each other. So "two" as a lexical concept will almost always have a different referent while the numerical value forms the linking, stable element.

An often-cited theory identifies innate knowledge of number and magnitude as well as language features as underling this developmental process. Innate knowledge of number and magnitude is described through two evolutionary old systems which together form the core systems (Dehaene, 1999). The approximate number system, being the first of the two, holds information of magnitude. It represents a physical magnitude cognitively by a roughly proportional cardinal value. It is stable over different dimensions like brightness, loudness, and temporal duration, and could be shown in children as young as six months. It underlies Weber`s law, meaning it is increasingly harder to discriminate the absolute distance of two entities of greater magnitudes (Sarnecka & Carey, 2006). Discrimination starts with a ratio of 1:2 and can sharpen up to 9:10 (Halberda & Feigenson, 2008).

The second core system processes mental representations up to a limit of three. With the object tracking system, discrete objects are stored in individual object-files holding one up to three elements. Being nonverbal, object files are compared as being equal or unequal to their match in the world. It has been shown that children use these files to distinguish entities according to quantity (Wynn, 1992) and that it does not work for entities higher than four (Feigenson et al., 2002). Going onwards children rely on counting to form concepts of natural number. Counting principles, introduced by Gelman and Gallistel (1978), form a hierarchy of how counting helps children to get a better insight into ordinal and cardinal aspects of natural number. One of the principles states that the last number word in a counting process represents the magnitude. This principle implies knowledge that going onward in the number line means increasing magnitude.

Language has been presented by Carey (2009) as a third indispensable system for the development of natural number. As stated briefly in the introduction, language has the power to discriminate between singular and plural. Moreover, language, or more precisely the class of number words, forms the scaffolding to which numerical information is attached. Thereby, surface concepts of natural number are formed and will then be specified throughout development (Hartmann & Fritz, 2021). The number word sequence up to ten is learned and memorized in stable order shortly after the second birthday. Being just a string of words at this point in time, it does not hold very deep numerical knowledge, but each number name provides a hook, for the specific lexical concepts of natural number (Negen & Sarnecka, 2012). Ordinal and cardinal aspects of natural number are over a long period sequentially integrated into the string of words, number names, originally learnt.

To build these concepts children make use of the conceptual function bootstrapping, a term coined by Carey (2009). Bootstrapping describes the necessity to combine all three systems and construct completely new concepts. To actually possess understanding of all features means more than simply match a number word to its corresponding individual object file. In fact, Le Corre and Carey (2007) describe the laborious and slow process children go through to map the number words one up to four to the corresponding mental representations. Forming these new lexical concepts takes about one year to develop. And even though cardinal development of natural number seems to move faster after constructing the concept four, the precise semantic mapping of a number word larger than four to the corresponding magnitude still needs about six more months to develop (Le Corre and Carey, 2007). Not until then, children will answer with an approximately close number word when presented with a random magnitude. Prior to this development, their answers are arbitrary. It almost seems as if the approximate number system needs to sharpen, that means to map closely matching sets and number words automatically.

These mappings of a number word to its corresponding magnitude sequentially fills the sequence of number words with numerical information. Based on counting and the stable order of the number word sequence a change in the representation of numbers takes place and numbers become associated with the order of successive quantities. In this mental representation, the successive number words align gradually to increasing quantities. A kind of "mental number line" is constructed this way and forms an ordinal representation (Fritz et al., 2018; Le Corre, 2014). With this knowledge, numbers can be compared to each other according to their position on the number word line, ("which number is bigger 7 or 8"?) and children are able to identify preceding and succeeding numbers ("which number comes before 3, and after 3"?).

The representation of the mental number line allows children to solve basic addition and subtraction tasks by counting. "The rabbit has two carrots and gets two more. How many does it have now?" Tasks like these can be completed by counting forward, always beginning from one and identifying the name of the number they found out as a result. Ordinal concepts do not yet include knowledge of cardinality.

Development of "Zero"

In none of these findings and principles, discussed above can zero be integrated. There seems to be no matching object file for "zero" in the object tracking system, its "magnitude" cannot be embodied by the approximate number system and it does not play any part in the early mental number line. In addition, the mathematical term, "zero", is not in the common vocabulary of young infancy. There are just very few and often contradictory findings about the understanding of zero. However, all of the studies prove pointers to the actual problem of understanding zero.

Wellman and Miller (1986) worked with Arabic notation and verbal count items ranging from 0 to 5. They found a delay in the use of zero compared to the rest of the natural numbers. They stated that children could name the symbol "0" around the fourth birthday and that children six years of age could describe zero as being the smallest number and could compare numbers. The findings of Bialystok and Codd (2000) contradicted these observations. They worked with a "Give-me" task to investigate children's knowledge of natural number including zero. They conclude, that preschoolers understand the concept of zero and can solve "give-zero" tasks. It is important to note here, that they did not ask to "give zero cookies" but rather to "give no cookies". Merrit and Brannon (2013) state that zero is handled differently by children and might not even be considered to be a number since it is not part of the counting list. And even though children could state that zero is smaller than one they did not naturally categorize it to be a number (Krajcsi et al., 2021).

One main problem seems to derive from linguistics, more precisely, the vocabulary. Spoken language usually does not refer to empty sets as being zero but uses a variety of different words or phrases to describe the characteristic of an empty set. Zero is characterized and referred to as no apples, nothing to eat, empty glass, vacant chairs, blank spaces. One problem in addressing zero might therefore be its low frequency use and the different realizations in spoken language. In contrast to natural numbers which in everyday life is referred to by precise number words, "zero" is usually referred to semantically indirect references, e.g. "no", "empty" or "nothing".

Nonetheless, young children are capable of working with empty sets in everyday life. To draw an analogy, all these "empty-set-words" do hold numerical content the way natural quantifiers do. But unlike other natural quantifiers they do not naturally find the corresponding mathematical denomination. So, the number word "zero" might have the difficulty of being doubly abstract. During development there is not one prominent number word with which children can match the distinctness of emptiness. The second abstract feature is, that when referring to "no entities", these entities do not have a match in the real word. There are no apples, sheep, cars, marbles or whatever to be seen. From a linguistic perspective, children need to create a concept for the lack of something instead of a concept of the magnitude of something.

In summary, the origin of the more difficult and later emerging understanding of zero might be explained in the double abstraction of linguistically no matching referent in the real world and not one specific number word combined with numerically no anchor point in the numerical core systems since neither of the core systems are laid-out to represent the lack of something but rather to present magnitudes. This explanation does not preclude that comparisons of empty-sets and magnitudes are possible. These are possible even in very early infancy since everyday language provides vocabulary for the lack of something.

Research questions

Common theories postulate a hierarchy in the development of precise concepts of natural numbers: Children develop the concept of "one" before they develop the concept of "two", and "three" is developed after "two". A growing body of studies bolster this assumption based on empirical data (e.g. Le Corre et al., 2006; Sarnecka & Lee, 2009; Negen & Sarnecka, 2012). Obviously, children do not develop a concept of "zero" before they have developed the concept of "one". Quite the opposite - since the acquisition of the concept of zero is much more abstract, recent studies suggest that children do not acquire the concept of "zero" until they have learned some number words, at least the number words 1 – 4, indicating that they are cardinal principle knowers (Krajcsi et al., 2021; Pixner et al., 2018).

Based on the concept of cardinal principle knowledge, an initial understanding of the relations between numbers develops. Children start to construct an ordinal number line, in which numbers are aligned as gradually increasing quantities. Empirical evidence shows that such an ordinal understanding of the natural numbers implies an understanding of the meaning of number words greater than 4 (Fritz et al., 2018; Le Corre, 2014). But even if, according to the findings of Krajcsi et al., children perceive 0 as smaller than 1, the question of what previous knowledge is required in order to integrate 0 into the mental number line of increasing quantities has not been answered.

These findings raise two main research questions that we aim to address in this study:

1. When in the process of acquiring the meaning of the natural numbers one to eight does the understanding of the natural number 0 develop? Given the high level of abstraction of the number zero, it is expected that the understanding of the number zero will emerge only when the children have mastered at least the meaning of the numbers one to four.

2. Does children's development of an ordinal concept of "zero" require a cardinal concept of "zero"? Does the integration of the number zero into the ordinal number line only happen after the meaning of the number words zero to eight have been grasped - in other words, does the integration of 0 into this list require an understanding of the natural numbers zero to eight?

Methods

Sample

In this study, a total of *N* = 107 kindergarteners (62 female, 45 male) participated. The children's mean age was M_{age}=57.61 months (SD_{age}=7.88 months), ranging from 44 months to 71 months. 30 children spoke German and an additional language at home, while 7 children did not speak German, but another language at home. Most common foreign home languages were English (*n* = 15), Arabic (*n* = 4), Turkish (*n* = 3), and Polish (*n* = 3). Children were recruited in 11 kindergartens from mostly urban backgrounds. Kindergartens were selected with the aim to represent upper-class (3 kindergartens), middle-class (4 kindergartens), and lower-class (4 kindergartens) backgrounds.

In advance, parents and children were informed about the procedures and aims of the study. Written consent was obtained from the parents beforehand. All national research standards were met during this study. The data collection was done by three experienced graduate university students that were trained by the researchers responsible for the study.

Instruments

Give-N: Children's counting skills were assessed with the Give-N task. In the Give-N task, children were given 15 counters and asked to give a specific number of counters (e.g. "Give me five counters, please"). The requested numbers covered 1 to 8 and 0. Zero was always administered as a number word not as a linguistic term describing zero. All numbers were requested in three trials each as suggested by Sarnecka and Lee (2009). Numbers were randomized in the three trials to avoid position effects. In the three trials, counters were changed (e.g. stones, candy, toys). The internal consistency of the Give-N tasks was good both for the natural numbers only (Cronbach's α = .856) and including zero (Cronbach's α = .854).

Ordinal Concepts of Zero (OCZ): The ordinal number concept for zero was assessed with eight tasks (Fritz et al., 2018). Two of the tasks each were assigned to one of the four aspects of the ordinal concepts: ordering (e.g. "Which number comes before one?"), comparing (e.g. "Which number is smaller – five or zero?"), addition (e.g. "You have three and get zero more – how many do you have now?"), and subtraction (e.g. "You have seven and give away zero – how many do you have now?"). The instructions of these tasks were given verbally with the usage of the term zero in all cases. Internal consistency for the eight items was good (Cronbach's α = .77).

The tasks were derived from more detailed tests for early arithmetic concepts of natural numbers (Ricken et al., 2013). In the original version, the items refer to natural numbers (e.g. "Which number comes after three?"). The Rasch model underlying the original test confirmed that the original items constitute a unidimensional scale, that is describe one arithmetic concept (Fritz et al., 2018; Ricken et al., 2013).

Results

Analysis strategy

According to Sarnecka and Lee (2009), children's counting skills can be classified by the highest number that they can reliably produce in the Give-N task. A number is produced reliably when (a) the child produces the correct number at least in two out of three trials, (b) the lower numbers are also produced correctly at the same benchmark, and (c) if the number is not produced when asked for higher numbers. Based on the children's answers in the Give-N task, they were categorized into a Knower-level that corresponds to the highest number they could reliably produce. Analogously, children's knowledge of zero was determined ("Zero-knowers"). Children, whose knower-level was bigger than three were also categorized as being cardinal-principle-knowers (CP-knowers), whereas children with a lower knower-level were categorized as subset-knowers.

Based on theoretical and empirical findings, children's understanding of zero and natural numbers can be assumed to develop in the form of overlapping waves (Clements & Sarama, 2014; Siegler & Alibali, 2005). The model of overlapping waves assumes that numerical competence does develop in phases that can be described by specific strategies or response patterns. However, these phases do not separate into distinct steps, but overlap. Thus, a child at a specific phase (e.g. two-knower) is characterized by giving exact two items when asked to, but random items when asked for a number bigger than two. Nonetheless, this specific child occasionally might be able to give three or four items when asked to, or fail when asked for one or two items.

As the current study aims at investigating children's understanding of zero in relation to their understanding of natural numbers, the overlapping waves model appears appropriate for data analysis. Previous studies have successfully employed one-dimensional Rasch-models to measure competence development within the overlapping waves framework in different contexts (Clements et al., 2008; Fritz et al., 2018; Herzog et al., 2019; Schulz et al., 2020). Here, two one-dimensional polytomous Rasch-models – one including the Give-N tasks for numbers 0-8 and one including the Give-N tasks and the ordinal concept of zero – will be used to address both research questions. All analyses were conducted using R (R Core Team, 2018) and the package TAM (Robitzsch et al., 2021).

The Rasch-model is a probabilistic model that measures both ability of the participants and difficulty of the items on one scale. Based on the item difficulty measures, a hierarchy in their development can be investigated. In this study, a polytomous Rasch-model was employed that gathered the responses in the Give-N tasks for each number including zero, as well as the ordinal concept of zero. In a polytomous model, several answer categories are combined. In this case, the categories were characterized by the number of correctly answered trials. The overall difficulty of the tasks is expressed by the beta-value. The discrimination between children with few and many correct answers in the trials is expressed by the alpha-value. The degree of fit of the data to the model is (besides others) expressed by the MNSQ-infit values. Infit values less than 1 indicate a redundancy in the items, infit values bigger than 1 indicate that the items do not measure the same construct. Wright and Linacre (1994) defined a range of .7 to 1.3 as sufficient.

Children's understanding of zero

In this study, 39 children were categorized as subset-knowers and 68 children as CP-knowers. A total of 38 children were zero-knowers, of which the vast majority of 35 were also CP-knowers. The relation of counting skills and knowledge of zero gets even more visible when considering the percentages: Only 8.3% of the subset-knowers were zero-knowers, but 51.5% of the CP-knowers. A Chi-square test confirmed the statistical significance of the difference in distribution ($\chi^2(1)=20.742$, $p < .001$). Focusing on the CP-knowers, seven-knowers and eight-knowers had the highest percentages of zero-knowers. More than 73.7% of the zero-knowers were at least seven-knowers. However, one third of the seven- and eight-knowers in this study had not yet developed an understanding of zero.

Mean age of the children increased with increasing knower-level. However, the age increase across knower-levels is not constant. A one-factorial ANOVA confirmed general age differences between the knower-levels ($F(8, 98) = 3.885$, $p < .001$, $\eta^2 = .241$), but

Bonferroni-corrected post-hoc tests only confirmed age differences between two-knowers and seven-knowers as well as eight-knowers. Zero-knowers were eight months older on the average than non-zero-knowers ($F(1, 105) = 34.698$, $p < .001$, $\eta^2 = .248$). Mean age of the zero-knowers was 62.9 months (SD = 7.57 months, range: 48.93-78.30) and thus even higher than the mean age of the eight-knowers. Children's classification into knower-levels and the corresponding mean ages are summarized in Table 1.

Table 1

Knower-levels and mean ages

Knower-Level	Children	Zero-knowers	Age
	n	*n (%)*	*M (SD)*
Subset	39	3 (8.3%)	53.49 (6.65)
0	3	0 (0%)	50.99 (2.25)
1	5	0 (0%)	55.47 (9.57)
2	19	1 (5.3%)	51.32 (4.87)
3	12	2 (20%)	56.71 (7.54)
CP	68	35 (51.5%)	59.97 (7.65)
4	7	2 (40%)	55.85 (3.92)
5	2	0 (0%)	58.79 (5.54)
6	13	5 (38.5%)	57.64 (6.07)
7	14	9 (64.3%)	60.08 (9.90)
8	32	19 (59.4%)	61.85 (7.64)

**Note: Subset=Subset-knowers; CP=Cardinal principle-knowers; n=subsample size; M = mean; SD = standard deviation.*

The Relation of Zero to Natural Numbers up to Eight

To address the first research question, a first Graded Partial Credit Model (GPCM) was employed based on the responses of the Give-N task for numbers one to eight and zero. Each number was asked in three trials, which leads to four response categories ranging from 0 to 3 correctly answered trials. The MNSQ-infit values of the first GPCM ranged between .73 and 1.29 for all items and categories, which is considered acceptable (Wright & Linacre, 1994). The EAP reliability of the model was .832 and therefore good.

Item parameters of model 1 are summarized in table 2. For numbers 1 to 8, beta values increased successively, indicating that bigger numbers were more difficult in the Give-N task. Differences between the numbers were bigger for numbers 1 to 4 (minimum = .235 logits, range=10.817 logits) and smaller for numbers 5 to 8 (minimum = .012 logits, range=.350 logits). All items discriminated relatively strongly between children with high and low counting ability. This finding suggests that the natural numbers up to eight form consistent competencies.

Compared to the natural numbers 1 – 8, zero was substantially more difficult. Moreover, zero differentiated less between children with high and low counting ability. Thus, zero seems to be not as consistent as the natural numbers up to eight.

Table 2

Parameters of the GPCM models.

	Model 1		Model 2	
Item	Alpha	Beta	Alpha	Beta
One	2.218	-11.493	2.109	-2.548
Two	1.960	-1.963	2.120	-1.842
Three	2.922	-.911	omitted	omitted
Four	2.038	-.676	2.668	-.406
Five	2.823	-.200	3.018	-.053
Six	1.745	-.185	1.936	.063
Seven	2.993	.054	2.363	.232
Eight	2.151	.150	2.451	.348
Zero	.579	.731	1.047	.617
OCZ_ord	-	-	.678	.698
OCZ_com	-	-	.688	-.023
OCZ_add	-	-	.599	1.163
OCZ_sub		-	.790	1.249

**Note: OCZ_ord = ordinal concept of zero, subskill ordering; OCZ_com = ordinal concept of zero, subskill comparing; OCZ_add=ordinal concept of zero, subskill addition; OCZ_sub = ordinal concept of zero, subskill subtraction*

The Relation of the Ordinal Concept of Zero to the Meaning of Zero

To address the second research question regarding the relation of the ordinal concept of zero and an understanding of the meaning of zero, the subskills ordering, comparing, addition, and subtraction were added to a second GPCM. To avoid distortions in the GPCM caused by varying category numbers, categories of the Give-N task were adapted to three categories as provided by the OCZ tasks. For this reason, the categories for 0 and 1 correctly answered trials were collapsed to one category.

With one exception, the MNSQ-infit values of the initial second GPCM ranged between .79 and 1.25 for all items and categories. Only item "Three" showed insufficient infit values (.58) and was therefore omitted. The remaining items in the final second GPCM had good MNSQ-infit values ranging from .81 to 1.22 for all categories. The EAP reliability of the final model was .827.

Item parameters of the final second model are summarized in table 2, too. As in model 1, numbers 1 to 8 increased in difficulty. While numbers 1 to 4 were more distinct in difficulty, numbers 5 to 8 had closer difficulty measures. Again, the natural numbers strongly discriminated between children with high and low ability as expressed in the values of alpha. In line with the results from the first GPCM, zero was more difficult and discriminated less regarding children's ability than the natural numbers.

Regarding the OCZ, ordering was slightly more difficult than the understanding of zero. Addition and subtraction as subskills of the OCZ were substantially more difficult than the understanding of the meaning of zero. Against expectancies, comparing numbers was relatively easy and not substantially more difficult than the counting competency of the CP. Especially was the comparing facet of the OCZ less difficult than the understanding of the meaning of zero. All items measuring the OCZ had very low alpha-values, indicating that the development is less consistent than that of the natural numbers or zero.

Discussion

The aim of this study was to investigate how the understanding of zero in counting and ordinality is related to the understanding of the natural numbers. The results of the current study show that an understanding of zero as a counting reference to an empty set is harder to understand than the natural numbers up to eight. Both the parameters and the age differences between zero- and non-zero-knowers support this notion. This finding is in line with previous studies that found that Cardinal Principle-knowers (CP) are more proficient in the understanding of zero than children, who only know the numbers one, two or three (subset-knowers) (Krajsci et al., 2021; Pixner et al., 2018). In contrast to Pixner et al. (2018), we found substantial age differences between zero-knowers and non-zero knowers. The findings go even beyond: Obviously, the CP-knowledge is not sufficient, as illustrated by the substantial difference in difficulty between zero and four in the Rasch models and the fundamental skewness in the distribution of zero-knowers across subset-knowers and CP-knowers. More experiences with even more numbers are needed to consider zero as a number. This raises the question, to which extent the cardinal principle is the adequate framework for zero. Or, in other words, is the cardinal principle the only relevant knowledge children need to understand zero?

Doubts regarding the relevance of the cardinal principle for the understanding of zero may be grounded in the different mechanisms underlying the learning processes of counting in natural numbers and zero: Whereas natural numbers have a referent (the number word) and a reference (the corresponding set) that can be mapped: "Four" refers to a set of four items. However, in the case of zero, there is a referent (the word "zero"), but no visible reference, since there is no item. But how can an empty set be represented? Thus, there might be a qualitatively different process responsible for the development of understanding zero. Empirical evidence in support of this notion can be found in the differences in the discrimination between more or less able children of understanding the natural numbers and zero in the Rasch model, which might indicate qualitatively different learning processes. We therefore assume that the main developmental driver for understanding zero results on the one hand from the concept of ordinal representation of numbers and on the other hand from the matching linguistic concepts. Since the semantic concept of zero is double abstract, meaning no visible reference point, and no anchor in the core systems, it must be constructed via multiple avenues of access. In other words, zero does not seem to be a "natural" number, if "natural" is determined the way, that the relation of the number word, its magnitude and its visible reference can be mapped onto each other.

Regarding the relation of the OCZ and the understanding of zero, results were inconsistent. The operations addition and subtraction were substantially more difficult than the understanding of zero in this study. These findings suggest that the operation aspect of the ordinal number concept is based on counting knowledge both for natural numbers and zero. A closer look at the processes involved reveals that operations require an understanding of numbers in the context of counting. Addition by counting does not work differently for natural numbers and zero.

In contrast to the operation aspect of the ordinal number concept, comparison was less difficult, and even easier than the understanding of zero. This means that children were more likely to locate zero within the number word sequence than to give zero items. This finding contradicts the findings for natural numbers that number comparison is based on counting proficiency (Le Corre, 2014). In this sense, zero seems to be different from the natural numbers. Against the background of a potentially qualitatively different developmental path to understanding zero, the ordinal understanding of zero as a predecessor of the number 1 might be a driver of development.

Based on the theory of Carey`s bootstrapping process there is a need to actively construct zero as the starting point of the number word sequence. Since the number word sequence is not learned starting with zero but always goes from one up to ten, the concept of zero does not start with a placeholder function like all other natural numbers. Thus, perhaps semantic – numerical information is first constructed via bootstrapping to all placeholding number words up to 4. After that, counting processes take over for numbers greater than 4. As these surface concepts develop, ordinal aspects form. Here, ordering and comparing come first.

Now the problem is to find a suitable place in the number line for the number zero. Semantic terms that express nothing are helpful here because they indicate that zero is even smaller than one. Comparisons of all kinds of linguistic expressions for empty sets with one or more objects lead children to place zero still before one. Perhaps children first need to understand the successor and predecessor functions to develop an understanding that zero is the predecessor of one.

At this point, children can apply the predecessor function to the counting routines: If zero comes before one, it has exactly one item less, resulting in an empty set. Such a developmental path would mimic the assumptions of the successor function as a developmental driver for the cardinal principle in reverse (Carey, 2009). The proposed development of the understanding of zero is in line with the results of the Rasch model, in which the ordering aspect of zero was only slightly more difficult than the counting knowledge of four, identified with CP knowledge. This could lead to the interpretation that the representation of zero may be tied to its ordinal position rather than to the very abstract cardinal representation of an empty set.

Beyond the research questions, the increasing and pronounced difficulties of numbers one through four, as located on the difficulty continuum shown by the application of the Rasch model provides additional evidence for the assumption that the natural numbers up to four are successively developed (Negen & Sarnecka, 2012). However, with respect to numbers five through eight, the results can be interpreted in two ways: First, the smaller difficulty gaps between numbers five through eight could indicate that counting knowledge of these numbers is associated with increasing conceptual knowledge. The slightly increasing difficulties between five and eight are due to the longer counting processes, which are more prone to random errors. On the other hand, the increasing difficulties as reflected by the Rasch model show that these numbers, like numbers one through four, are learned hierarchically and successively. However, the development of numbers five through eight could be accelerated by more routine, which would explain the decreasing differences between the difficulties of the numbers. Accelerated development with increasing numbers could be the reason why previous studies have not found significant differences in counting skills between these numbers: Children who have understood the meaning of the number four are likely to know larger numbers, as understanding of the numbers five through eight can be very rapid.

The first interpretation supports the construct of the CP-knowledge. The second interpretation suggests that numbers bigger than four are not conceptually embedded in the CP-knowledge, but that these numbers are also learned successively. Further research – especially longitudinal studies –might inform the proposed interpretations. However, the first interpretation can be better brought in line with the literature at the moment.

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