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Spatial Working Memory Counts: Evidence for a Specific Association Between Visuo-spatial Working Memory and Arithmetic in Children

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Abstract

We examined the role of visuo-spatial working memory in different types of arithmetic ability in children. Previous research had suggested that arithmetic is not a single entity (Dowker, 2005, 2015), and also that visuo-spatial working memory is specifically involved in mathematical cognition (McKenzie et al., 2003) There has, however, been little research on the relationships between visuo-spatial working memory and different types of arithmetic. We tested 39 children in Year 2 (6 to 7 years) and Year 4 (8 to 9 years), taking measures of written arithmetic, mental oral arithmetic, and derived fact strategy use (the ability to derive unknown arithmetical facts from known facts, by using arithmetical principles). We also measured visuo-spatial working memory, verbal comprehension, and spelling ability. We investigated the relationships between visuo-spatial working memory and our three arithmetic measures, as well as spelling and verbal comprehension, to test whether these effects were specific to mathematical abilities. We found that visuo-spatial working memory was specifically associated with both verbal oral and mental written arithmetic, and but not with spelling or derived fact strategy use.

Keywords:

Arithmetic Ability, Working Memory, Visuo-Spatial, Mathematics

Introduction

Arithmetic is important in many aspects of our lives. It is important in daily practical activities, such as finding the right change for the bus, estimating how long a journey will take, or comparing different special offers in the supermarket. Basic numeracy is also very important in obtaining and keeping a wide variety of jobs, and low numeracy has many negative social and economic consequences to the individual and to society (Gross et al., 2009; Parsons & Bynner, 2005; Rodgers et al., 2019).

In the light of this prevalence, mathematical instruction in school has potential effects reaching far beyond the classroom, particularly with regard to arithmetic procedures, and so it is crucial to lay the educational foundations of these abilities in the best way possible, and to recognize early when children show characteristics that may contribute to difficulties in learning arithmetic and may indicate a need for interventio. In order to achieve this it is important to understand how different cognitive functions contribute to mathematical ability in children. Furthermore, insight into these relationships can increase our theoretical understanding of the nature and development of arithmetic.

The Multi-Component Nature of Arithmetic

Research suggests that arithmetic is not a single entity; it is made up of multiple components (Dowker, 2005; Jordan, Mulhern & Wylie, 2009). It is important to examine the relationship between these components and the cognitive processes which subserve their development to further our understanding of mathematical cognition. It is likely that multiple processes subserve arithmetical development. These may be used differentially depending on the problem (large vs small numbers, adding vs subtracting), and, crucially, depending on the individual child. In fact, considering the finding that within an average British school class of 11 year olds, there is usually a 7 year range in mathematical ability (Askew, Hodgen, Hossain, & Bretscher, 2010), it seems all the more likely that the processes facilitating arithmetical development are not consistent for all children. Thus, understanding mathematical cognition is important from both a practical perspective, to help children succeed at mathematics, as well as a theoretical one, to further our understanding of the processes that subserve arithmetical development, either independently or in combination. In this study, therefore, we are examining performance in two different types of standardized tests of arithmetic: the WISC Arithmetic subtest, which mainly measures arithmetical reasoning in the context of word-problem solving, and the British Abilities Scales Basic Number Skills Test, which mainly measures written calculation. Note that the most recent, third, edition of the latter test includes a significantly larger element of word problem solving. We chose to use the second edition as a purer measure of written arithmetic,

Derived Fact Strategies

One crucial component of arithmetic is the use of derived fact strategies (Baroody, Ginsburg & Waxman, 1983; Canobi, 2005; Canobi, Reeve & Pattison, 1998; Dowker, 2009, 2014; Gilmore & Papadatou-Papastou, 2009; Godau et al., 2014; Jordan et al., 2009; Robinson et al., 2006; Torbeyns et al., 2009). This involves the ability to derive unknown arithmetical facts from known facts, by using arithmetical principles such as commutativity (if 56 + 31 = 87, then 31 + 56 must also be 87) and the addition/ subtraction inverse principle (if 56 + 31 = 87, then 87 – 56 must be 31). The ability to use derived fact strategies is important both as an indicator of children's understanding of arithmetical principles and relationships, and as a basis for going beyond existing knowledge in performing unfamiliar calculations.

The Role of Working Memory in Arithmetic, and Derived Fact Strategies

While there are numerous cognitive processes that have been proposed to be important to arithmetic, one that has been found in some previous studies to be particularly relevant, and which we have chosen to study in this project, is working memory (Bull & Scerif, 2001; Jarvis & Gathercole, 2003). Working memory is widely accepted to refer to the processes by which information is actively held on-line (Baddeley & Hitch, 1974) and to include both phonological and visual-spatial components. Working memory has been implicated as being used differentially in mathematical processing in children of different ages (McKenzie, Bull, & Gray, 2003; Palmer, 2000). McKenzie et al. (2003) showed that younger children (6-7 years) were on the whole unaffected by verbal interference when working out a mental calculation problem presented verbally, suggesting that the phonological loop was not being used in children at this age. Furthermore, the same children were severely impaired when interference was given in the visuo-spatial modality, suggesting that younger children use visuo-spatial strategies in mental arithmetic. Older children (8-9 year olds) tested in the same experiment were equally impaired by both phonological and visuo-spatial interference, though not to the same extent as the younger children. This suggests that older children use both components of working memory in arithmetic processing. Palmer (2000) suggests that this switch in strategy use accompanies the maturation of the central executive, believed to be involved in switching strategies (Baddeley, 1996).

It would be ideally desirable to investigate the role of all possible components of working memory in mathematical cognition; but given the constraints of testing children within a limited time-scale, a measure of visuo-spatial working memory seemed appropriate to investigate, as it has been previously shown to be important in both older and younger children. Moreover, arithmetic may be more specifically related to visuo-spatial working memory rather than phonological working memory through the use of the internal 'mental number line' (Dehaene, 2011), which is thought to be a spatial mechanism subserving arithmetic. We included a measure of spelling in our study, to investigate whether the role of visual-spatial working memory was indeed specific to arithmetic. With some exceptions, (e.g. Simmons, Willis & Adams, 2012; Szucs et al., 2014) most studies of working memory and arithmetic have not looked at how working memory relates to different types of arithmetic. In particular, to our knowledge, none have looked directly at the extent to which working memory influences derived fact strategy use.

Attitudes to Arithmetic

Arithmetic depends not only on cognitive processes, but also on emotional factors. There is much evidence (OECD, 2015) that attitudes to mathematics have an important effect on performance. Most studies indicate that primary school children have relatively positive attitudes to arithmetic (e.g. Dowker, Bennett & Smith, 2012; Krinzinger, Kaufmann & Willmes, 2009; Sorvo et al., 2017), but that they often deteriorate later on. However, mathematics anxiety is already a problem for some children in the early years of primary school (Petronzi et al., 2019). These studies also suggest that, whereas in older children and adults, the most crucial attitude predictor of performance is mathematics anxiety, in younger children, it seems to be self-rating Therefore we included brief measures of liking mathematics and of self-rating in arithmetic. We predicted that self-rating in particular would predict performance in standard arithmetic tests, but not in derived fact strategy use, as children's self-ratings may be more associated with tasks resembling typical school tests.

Putting It All Together: The Present Study

This study aimed to investigate the relationship between a selection of domain-general and domain-specific cognitive functions and mathematical ability in children over developmental time. We tested children in Years 2 and 4 of the British schooling system (6-7 year olds and 8-9 year olds respectively). As well as several measures of both written and mental mathematical ability including a measure of derived fact strategy use, measures of visuo-spatial working memory were included. Measures of verbal comprehension and spelling ability were also used to act as proxies for other non-mathematical academic abilities, in order to test how specific any relationships between cognitive and mathematic abilities were. Finally, measures of perceived mathematical ability and attitudes towards mathematics were also recorded, to investigate any effects that these may have on mathematical ability.

Our hypotheses were:

1. *Older children would be better at all arithmetical measures than younger children. However, as standard scores are adjusted for age, they should not have higher standard scores.*
2. *There would be a significant correlation between the two standardized arithmetic tests.*
3. *Derived fact strategy use would be strongly related to other measures of arithmetic, and would correlate more strongly with the written arithmetic test than the mental word problem solving test, as previously found by Dowker (2009).*
4. *Visuo-spatial working memory would be a strong predictor of all arithmetical measures. It would predict mental word problem solving (WISC Arithmetic) and derived fact strategy use more than written calculation (BAS Arithmetic).*
5. *Visuo-spatial working memory would not be a strong predictor of spelling.*
6. *Arithmetical self-rating would predict mental word problem solving (WISC Arithmetic) and written calculation (BAS Arithmetic) but not derived fact strategy use.*

Method

Participants

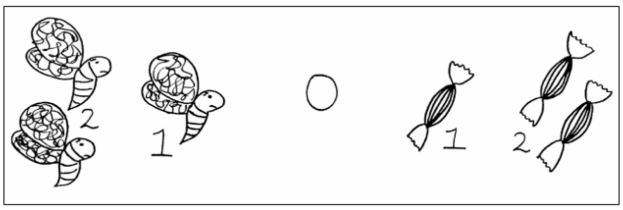
Participants were recruited from two non-selective state primary schools in the Oxford area. The participants consisted of 39 (19 female, 20 male) children in Years 2 and 4 (ages 6-7, and 8-9, respectively). The Year 2 group consisted of 21 children (6 female, 15 male) aged between 6.4 and 7.5 (*M* = 6.9, *SD* = 0.3), and the Year 4 group consisted of 18 children (13 female, 5 male) aged between 8.4 and 9.3 (*M* = 8.10, *SD* = 0.3).

Subtests

Each child completed 7 subtests in total, giving measures of: (a) attitudes towards mathematics and perceived arithmetic ability, (b) arithmetical reasoning and word problem solving (WISC Arithmetic), (c) written calculation (British Abilities Scales Basic Number Skills), (d) use of derived fact strategies, (e) visuo-spatial working memory capacity, (f) verbal comprehension (WISC Comprehension) and (g) spelling ability (British Abilities Scales Spelling).

(a) Attitudes towards mathematics and perceived mathematic ability were assessed using a pictorial rating scale, from Thomas and Dowker (2000) (see Fig. 1).

Figure 1
Pictorial scale used to measure attitudes to mathematics and perceived mathematic ability. From Thomas & Dowker (2000).



Children were shown the scale, and asked to point to the picture representing how much they liked maths, ranging from two sweets (like maths very much) to two wasps (dislike maths very much) with a neutral circle in the middle (neither like nor dislike maths). Children were then asked to use the scale to show how good they thought they were at maths, ranging from two sweets (very good at maths) to two wasps (not very good at maths at all) with a neutral circle in the middle (neither good nor bad at maths).

Responses were recorded by the examiner onto paper.

(b) The Arithmetic subtest of the Wechsler Intelligence Scale for Children – Fourth Edition (Wechsler, 2004) was administered. Children were given a series of verbally presented arithmetic problems to solve mentally.

(c) Written calculation ability was assessed using the Number Skills subtest of the BAS II (BAS Arithmetic).

(d) Use of derived fact strategies was tested with Dowker et al.'s (2005) Derived Fact Strategy Addition test. Children were first assessed with 20 mental addition questions, which became progressively more difficult, from 4+5 to 235+349 (see Appendix A). Questions were presented visually and read aloud by the examiner, and responses were verbal. Testing continued until the child gave an incorrect response to six items in a row, or completed the test. From this, they were assigned to one of 5 sets for the Derived Fact Strategies test. Each set consisted of 28 pairs of addition questions, designed to test the child's use of six arithmetic principles:

1. *The identity principle (if 8+2 = 10, then 8+2 = 10)*
2. *The commutativity principle (if 8+2=10, then 2+8 = 10)*
3. *The N+1 principle (if 8+2 = 10, then 8+3 = 11)*
4. *The N-1 principle (if 8+2=10, then 8+1=9)*
5. *The N+2 principle (if 8+2=10, then 8+4=12)*
6. *The addition/subtraction inverse principle (if 8+2=10, then 10-2=8)*

The answer to the first question of each pair was given to the child, along with the second question of the pair, which could be solved relatively quickly if the child used the answer to the first question plus the principle being tested. The questions were designed to be slightly too difficult for the child to solve unaided. For example, if the pair of questions was: [349 + 234 = 583, 349 + 236 =], the answer 585 could be given if the child used the answer to the first question of the pair, and the N+2 principle. The children received three addition problems per principle, as well as three addition problems preceded by numerically unrelated problems as controls. If any answers were ambiguous, a fourth question was given for that principle.

(e) Visuo-spatial working memory (VSWM) capacity was tested using the DotMatrix subtest of the Automated Working Memory Assessment (AWMA; Alloway, 2008). This was administered on a laptop screen. Children were asked to recall the location of a series of red dots on a white grid. The dots appeared one at a time, and the sequences of dots became increasingly longer until the child was unable to report their correct locations in the correct order. The sequence length began at 1 dot and increased to a maximum of 9 dots. The child indicated where the dots had been by pointing to locations on the blank grid after each sequence of dots, and the examiner used the keyboard to report whether the child had been correct or incorrect. The test automatically stopped once a certain number of incorrect answers had been recorded.

(f) Verbal comprehension was assessed using the Comprehension subtest of the WISC-III (Wechsler, 2004).

(g) Spelling ability was assessed using the Spelling subtest of the British Ability Scales Second Edition (BAS II; Elliott, Smith, & McCulloch, 1996).

Procedure

Written consent was obtained from a parent or guardian for each child to be included in the study. In addition to this, children were informed that they could choose to stop participating in the study at any time. Before beginning the study, approval was sought and granted from the Oxford University Central University Research Ethics Committee (CUREC).

All children were tested individually in a quiet area of their school. In order to control for practice and fatigue effects, the order in which the components of the study were administered was randomised for each child, using the online software Research Randomizer 4.0 (Urbaniak & Plous, 2013). At the start of their test session, each child was presented with an image of a trophy cabinet printed onto A5 paper (see Appendix B). They were told that they would be able to add one sticker to their cabinet upon completing each subtest, and that they would be able to take it home once the session was over.

The subtests were then administered in a randomised order, with children resting for approximately two minutes in between each subtest.

Results

The data were analysed using SPSS Statistics.

Standard/ Scaled Scores on Standardized Tests

The mean standard score for Working Memory was 119.13 (*sd* = 13.3). The mean scaled score for the WISC Arithmetic subtest was 13.36 (*sd* = 2.88) and the mean scaled score for the WISC Comprehension subtest was 12.82 (*sd* = 3.84) The mean scaled score for the British Ability Scales Basic Number Skills subtest was 121.28 (*sd* = 14.14) and the mean scaled score for the British Ability Scales Spelling subtest was 117.38 (*sd* = 14.14).

Significant Differences between Genders and Year Groups

A two-way multivariate analysis of covariance (MANCOVA; $\alpha = .05$) was conducted with Year Group and Gender as the grouping factors and BAS Spelling Raw Score, BAS Spelling Standard Score, BAS Arithmetic Raw Score, BAS Arithmetic Standard Score,

WISC Arithmetic Scaled Score, WISC Arithmetic Raw Score, WISC Comprehension Scaled Score, WISC Comprehension Scaled Score, VSWM Raw Score, and VSWM Standard Score as the dependent variables. Age (in months) was included as a covariate.

It was found that, after controlling for Age, Year Group still had a significant main effect on VSWM Raw Score, $F(1, 34) = 4.90$, $p = .034$, $\eta^2 = .13$; and BAS Spelling Raw Score, $F(1, 35) = 6.70$, $p = .014$, $\eta^2 = .16$.

For each of these dependent variables on which Year Group had a significant main effect, scores were significantly higher for children in the Year 4 group than the Year 2 group (see Table 1.; see Figure 2.).

Figure 2

Comparison of significantly different mean scores for Year 2 and Year 4 when controlling for Age. Error bars show the Standard Deviation.

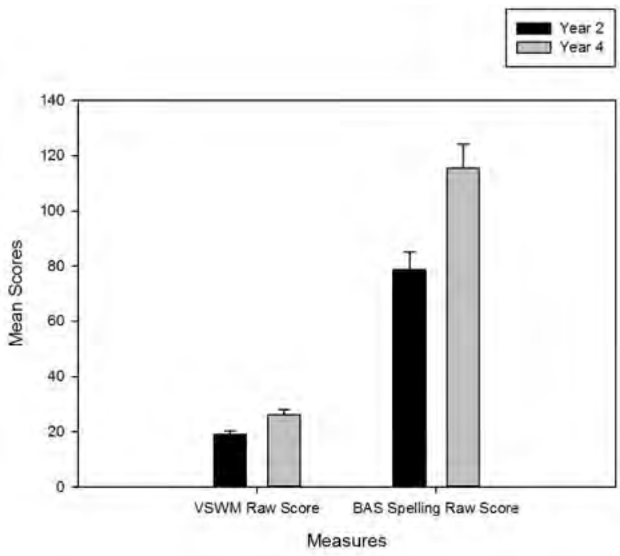


Table 1

Comparison of significantly different mean scores for Year 2 and Year 4 when controlling for Age.

	Year 2		Year 4	
Measure	Mean	*SD*	Mean	*SD*
BAS Spelling Raw Score	83.22	17.69	108.31	10.85
Raw Working Memory	19.61	3.68	26.19	3.33

Gender had a significant main effect on BAS Arithmetic Raw Score, $F(1, 35) = 5.34$, $p = .027$, $\eta^2 = .14$. Furthermore, there was also a significant main effect of Gender on BAS Arithmetic Standard Score, $F(1, 35) = 4.77$, $p = .036$, $\eta^2 = .12$. A comparison of means showed that BAS Arithmetic Raw and Standard Scores were significantly higher for males than for females (see Table 2; see Figure 3.). There were no significant interactions between Year Group and Gender.

Figure 3

Comparison of significantly different mean scores between Genders when controlling for Age. Error bars show the Standard Deviation.

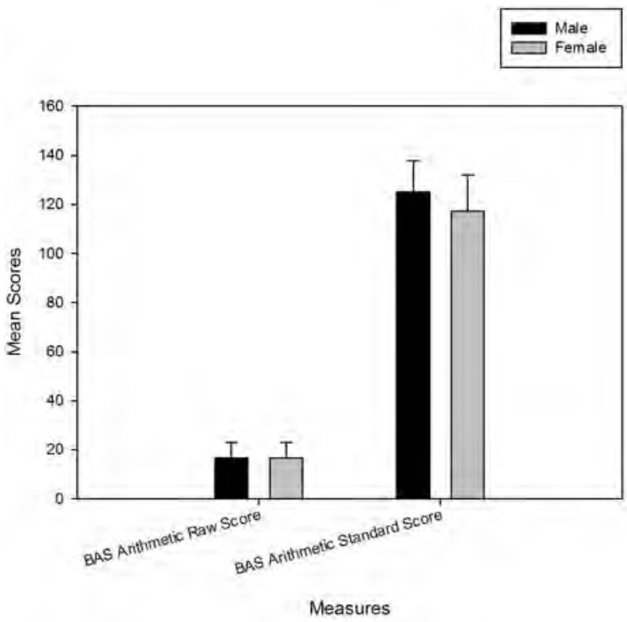


Table 2

Comparison of significantly different mean scores between Genders when controlling for Age.

	Year 2		Year 4	
Measure	Mean	*SD*	Mean	*SD*
BAS Spelling Raw Score	83.22	17.69	108.31	10.85
VSWM Raw Score	19.61	3.68	26.19	3.33

Derived Fact Strategy Use

Of the 23 Year 2 children, 2 were assigned to Set 2, 13 to Set 3, 4 to Set 4 and 4 to Set 5. All 16 of the Year 4 children were assigned to Set 5.

The mean number of types of derived fact strategy used (out of a potential 6) was 4.1 (*sd* = 1.45), In Year 2, it was 3.7 (*sd* = 1.52) and in Year 4, it was 4.69 (s..d. 1.14). As found in other studies (e.g. Dowker, 2014), there were wide differences in frequency of different strategy types. All children used Identity and nearly all (36 out of 39) used Commutativity.

30 out of 39 (16 out of 23 in Year 2 and 14 out of 16 in Year 4) used Addend + 1. 28 out of 39 (13 out of 23 in Year 2 and 15 out of 16 in Year 4) used Addend – 1. 21 out of 39 (10 out of 23 in Year 2 and 11 out of 16 in Year 4) used Addend + 2.

By contrast with all the above, only 6 out of 39 used Inverse (two out of 23 in Year 2 amd four out of 16 in Year 4.. This was even less common than the production of correct answers to control problems (produced to at least one problem by 9 out of 39 children,)

Correlations between Standardised Measures and Derived Fact Strategy Set, Age, and Number of Strategies Used

Pearson Correlation Coefficients were computed between the standardised measures (VSWM Standard Score, WISC Arithmetic Scaled Score, WISC Comprehension Scaled Score, BAS Arithmetic Standard Score and BAS Spelling Standard Score) and Derived Fact Strategy Set, Age, and Number of Strategies Used. The results are summarised in the correlational matrix below (see Table 3).

Predictors of Number of Strategies Used

A simultaneous entry multiple linear regression was conducted to test the best predictors for the Number of Strategies Used. The predictor variables were Age, BAS Arithmetic Standard Score, BAS Arithmetic Standard Score, Derived Fact Strategy Set and VSWM Standard Score. Number of Strategies Used was the dependent variable. The overall regression was significant, with the four predictor variables explaining 37% of the variance in the Number of Strategies Used, adjusted R^2 = .37, F(5, 33) = 5.42, p = .001. Derived Fact Strategy Set was a significant predictor of Number of Fact Strategies Used, β = .75, t(33) = 3.14, p = .004; and so was BAS Arithmetic Standard Score, β = .37, t(33) = 2.12, p = .042. The remaining predictor variables were not significant at predicting Derived Fact Strategy Set. Thus, BAS Spelling Standard Score, β = -.29, t(33) = -1.82, p = .078; and VSWM Standard Score, β = -.27, t(33) = -1.47, p = .151; and Age, β = .22, t(33) = -0.91, p = .368, did not predict Derived Fact Strategy Set.

Another regression analysis tested how well a different set of predictor variables predicted Number of Strategies Used. The predictor variables were Age, WISC Arithmetic Scaled Score, WISC Comprehension Scaled Score, VSWM Standard Score, and

Derived Fact Strategy Set. Number of Strategies Used remained the dependent variable. The overall regression was significant, with the five predictor variables explaining 26% of the variance in the Number of Strategies Used, adjusted R^2 = .26, F(5, 33)

Table 3
Correlations between standardised measures and Age, Derived Fact Strategies Set, and Number of Strategies Used.

	Measures	1	2	3	4	5	6	7	8
1	VSWM Standard Score	-	.38*	.56**	-0.11	.60**	.45**	0.27	0.21
2	WISC Comprehension Scaled Score		-	0.27	-.33*	0.19	0.31	0.27	.38*
3	WISC Arithmetic Scaled Score			-	0.15	.63**	.46**	0.28	0.21
4	BAS Spelling Standard Score				-	0.17	-0.07	-0.16	-.42*
5	BAS Arithmetic Standard Score					-	0.24	.35*	-0.08
6	Derived Fact Strategy Set						-	.57**	.73**
7	Number of Strategies Used							-	.36*
8	Age								-

* p < .05 ; ** p < .01

= 3.61, p = .010. Of the five predictor variables, Derived Fact Strategy Set was the only significant predictor of Number of Fact Strategies Used, β = .67, $t(33)$ = 2.87, p = .007. None of the other variables was a significant predictor.

Predictors of BAS Arithmetic Standard Score

A simultaneous-entry multiple linear regression was conducted to test the best predictors of BAS Arithmetic Standard Score. BAS Spelling Standard Score, VSWM

Standard Score, Age, WISC Comprehension Scaled Score, and WISC Arithmetic Scaled Score were the predictor variables, and BAS Arithmetic Standard Score was the dependent variable. The overall regression was significant, with the five predictor variables explaining 48% of the variance in BAS Arithmetic Standard Score, adjusted R^2 = .48, $F(5, 33)$ = 8.09, p < .000. Of the five predictor variables, VSWM Standard Score was a significant predictor of BAS Arithmetic Standard Score, β = .41, $t(33)$ = 2.71, p = .011; and so was WISC Arithmetic Scaled Score, β = .43, $t(33)$ = 2.83, p = .008. The remaining predictor variables were not significant at predicting BAS Arithmetic Standard Score.

Effects of Year Group and Gender on Number of Strategies Used when Controlling for Age and Derived Fact Strategy Set

To test whether Gender or Year Group had a significant effect on Number of Strategies Used, after controlling for Age and Derived Fact Strategy Set, a two-way univariate analysis of covariance (ANCOVA; = .05) was conducted with Year Group and Gender as the two independent variables, and Number of Strategies Used as the dependent variable. Age and Derived Fact Strategy Set were included as covariates.

There were no significant main effects of Gender or Year Group on Number of Strategies Used, However, Derived Fact Strategy Set did have a significant effect, $F(1, 33)$ = 11.46, p = .002, η^2 = .26.

To investigate this effect further, a new variable was computed, which grouped those in Derived Fact Strategy Set 5 in one group, and all other Derived Fact Strategy Sets in another. This was called Set Type. To test whether Set Type may account for the effect that Derived Fact Strategy Set had on Number of Strategies Used, a multivariate ANCOVA (α = .05) was conducted. Set Type was the grouping factor, and BAS Spelling Standard Score, BAS Arithmetic Standard Score, WISC Arithmetic Scaled Score, WISC Comprehension Scaled Score, BAS Arithmetic Raw Score, VSWM Standard Score, and Number of Strategies Used were the dependent variables. Age was entered as a covariate. Set Type had a significant main effect on BAS Spelling Standard Score, $F(1, 36)$ = 9.39, p = .004, η^2 = .21; BAS Arithmetic Standard Score, $F(1, 36)$ = 6.84, p = .013, η^2 = .16; WISC Arithmetic Scaled Score, $F(1, 36)$ = 7.65, p = .009, η^2 = .18; VSWM Standard Score, $F(1, 36)$ = 7.93, p = .008, η^2 = .18; and BAS Arithmetic Raw Score, $F(1, 36)$ = 8.48, p = .006, η^2 = .19. However, there was no significant effect of Set Type on Number of Strategies Used.

Predictors of the Use of Each Derived Fact Strategy Principle

To investigate the best predictors of the use of each Derived Fact Strategy principle, a series of simultaneous-entry binary logistic regressions were conducted. Since the use of the Identity and Commutativity Principles were at ceiling level, these were not investigated. The first binary logistic regression included Use of the Addend +1 Principle ('did use' vs. 'did not use') as the dependent variable, and Age, BAS Spelling Standard Score, BAS Arithmetic Standard Score, and VSWM Standard Score as the covariates. The effect of Age was significant, χ^2 = 4.05, df = 1, p = .044; and so was the effect of BAS Arithmetic Standard Score, χ^2 = 5.61, df = 1, p = .018.

The second binary logistic regression included Use of the Addend -1 Principle ('did use' vs. 'did not use') as the dependent variable, and kept the same covariates. The effect of Age was significant, χ^2 = 4.47, df = 1, p = .034. The final binary logistic regression included Use of the Inverse Principle ('did use' vs. 'did not use') as the dependent variable, and kept the same covariates. There were no significant effects.

Testing the Effects of Self-Rating and Liking for Maths

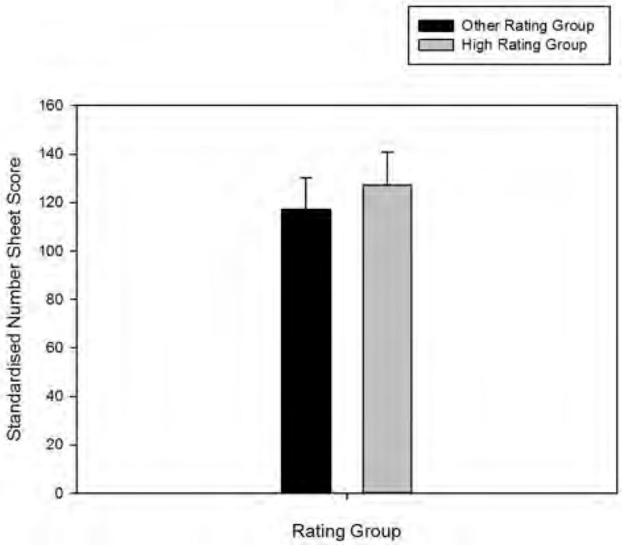
Maths rating scores were split into two groups: those who had reported the highest maths rating in one group (N = 16), and those who had reported less than the highest rating in the other (N = 23). A multivariate ANOVA (α = .05) was run, with Rating Group as the grouping factor, and BAS Spelling Standard Score, BAS Arithmetic Standard Score, BAS Arithmetic Raw Score, WISC Arithmetic Scaled Score, VSWM Standard Score, WISC Comprehension Scaled Score, Number of Strategies Used, and Age as the dependent variables. Rating Group had a significant main effect on BAS Arithmetic Standard Score, $F(1, 37)$ = 5.14, p = .029, η^2 = .12. By comparing means (see Fig. 4), we can see that BAS Arithmetic Standard Score were significantly higher for those in the High Rating Group (M = 127.13, SD = 13.82) than for those in the Other Rating Group (M = 117.22, SD = 13.16).

Maths liking scores were split into two groups: those who had reported the highest rating in one group (N = 16), and those who had reported less than the highest rating the other (N = 23). A multivariate ANOVA (α = .05) was run, with Liking Group as the independent variable, and BAS Spelling Standard Score, BAS

Arithmetic Standard Score, BAS Arithmetic Raw Score, WISC Arithmetic Scaled Score, VSWM Standard Score, WISC Comprehension Scaled Score, Number of Strategies Used, and Age as the dependent variables. Liking Group did not have a significant main effect on any of the dependent variables.

Figure 4

Comparison of significantly different mean scores in Standardised Number Sheet Score for High and Other Rating Group. Error bars show the Standard Deviation.



Discussion

Children in Year 4 obtained significantly higher scores than those in Year 2 for the raw arithmetical measures (BAS Arithmetic Score, and WISC Arithmetic Score), and there was no significant difference in their standardised scores for these measures. These results were in line with our first hypothesis, that older children would be better than younger children at all arithmetic measures, though there would be no significant differences in their standardised scores since these control for age. Furthermore, there was a significant correlation between the two standardised arithmetic measures, suggesting that these tests tap some overlapping abilities. This finding was in line with our second hypothesis. Together, these results seem to show that our measures of arithmetic ability were testing similar skills, and that these skills improve with age, thus provide a valid basis for investigating the relationships between arithmetic and other cognitive functions.

Our third hypothesis was that use of derived fact strategies would be related to other arithmetic measures, though this relationship would be stronger for written arithmetic than for mental arithmetic. We found that use of derived fact strategies, denoted by the variable Number of Strategies Used, was significantly predicted by BAS Arithmetic Standard Score, and by Derived Fact Strategy Set, though not by WISC Arithmetic Scaled Scores, thus offering partial support for this hypothesis, since use of derived fact strategies was significantly related to written, but not mental, arithmetic scores. The lack of significant relationship between mental arithmetic and derived fact strategy use is further shown in the results of the correlation analysis: use of derived fact strategies correlates with written arithmetic, but not significantly with mental arithmetic

This set of findings gives a useful insight into the relationship between children's ability to go beyond existing knowledge and perform unfamiliar calculations (as assessed by the number of derived fact strategies used) and written arithmetic ability. It could be that, although in the present study it was the mental arithmetic problems that required more reasoning, children have more experience with written arithmetic problems that include the need for numerical reasoning, while mental arithmetic tasks may more commonly emphasize fluency and fast responses An alternative explanation could be that both the Derived Fact Strategies test and the BAS Number Skills Worksheet were administered with visually presented problems, whereas the WISC Arithmetic subtest was presented verbally, with children having to abstract the mathematical problem from a 'word problem', where the sum was not always transparent. Such processes may place different demands on cognitive abilities such as executive function, thus explaining why derived fact strategy use may be related to written but not mental arithmetic. These explanations could be tested by administering a verbal analog of the Derived Fact Strategies addition test. If verbal derived fact strategy use remained related to verbal arithmetic scores, then we could perhaps conclude that the modality of the tests caused the results we observed.

Our subsequent hypotheses were related to visuo-spatial working memory. Specifically, we hypothesised that visuo-spatial working memory (VSWM) would be a strong predictor of all arithmetical measures, and would be related more to mental arithmetic (WISC Arithmetic) and use of derived fact strategies than written arithmetic (BAS Arithmetic). Furthermore, we hypothesised that VSWM would not be a strong predictor of spelling, suggesting a domain-specific input to mathematics. The results from the Pearson Correlation Coefficient analysis show that VSWM is strongly positively correlated with WISC Arithmetic Scaled Score, BAS Arithmetic Standard Score, Derived Fact Strategy Set, but not with Derived Fact Strategy Use or BAS Spelling Standard Score. Furthermore, the regression analyses revealed VSWM to be a significant predictor of BAS Arithmetic Standard Score, but not of Derived Fact Strategy Use. These findings partially support our hypotheses: VSWM is related to all arithmetic measures apart from derived fact strategy use, and is not related to spelling scores. However, it was not more strongly related to mental arithmetic than written arithmetic, since these were both still significant at the 1% level.

Our results suggest that the involvement of VSWM is specific to arithmetic, since it was not related to spelling performance. It may be that spelling is related to phonological working memory instead, since it requires manipulating phonological units of word into orthographic code. The results suggest that the domain-general measure of working memory has a specific role in arithmetic in children. Our investigation of the role of working memory in derived fact strategy use revealed no significant relationship, either in terms of Pearson Correlation Coefficient analysis, or regression analysis. This result may suggests that VSWM capacity does not influence the rate at which new derived fact strategies are acquired and used, but rather the dexterity with which they can be applied once acquired. Future research could investigate this hypothesis, by studying the effect of VSWM on the rate of acquisition of derived fact strategies, how quickly they are selected when presented with a novel arithmetic problem, and whether interrupting VSWM with a concurrent spatial memory task inhibits the use of derived fact strategies.

It may also be that derived fact strategy use requires more verbal rather than spatial working memory resources. This is brought into question by Puvandendran et al's (2016) study of an individual with Broca's aphasia and very limited verbal working memory, who nevertheless was highly successful in using derived fact strategies to compensate for retrieval difficulties. However, this patient may have been unusual in many ways, and in any case there is a difference between children who are still developing their arithmetical concepts and skills, and adults who are compensating for brain injuries that cause disruption to aspects of their existing arithmetical concepts and skills.

Our eighth and final hypothesis predicted that arithmetical self-rating would predict mental word problem solving (WISC Arithmetic) and written calculations (BAS Arithmetic), but not derived fact strategy use, since rating is more likely to be based on activities encountered in school. Our results showed that arithmetical self-rating predicted written calculation (BAS Arithmetic), and not Derived Fact Strategy Use. However, arithmetical self-rating also did not predict mental word problem solving (WISC Arithmetic). Whilst this finding is not in line with our hypothesis, since we expected self-rating to accurately reflect both written and mental aspects of arithmetic, this effect could be due to the feedback children receive whilst at school. Since most of their maths work in the classroom (as well as maths homework) is written, they are perhaps more likely to hold an accurate perception of their written arithmetic ability.

Since mental arithmetic is perhaps assessed less frequently, they may have a less clear idea about their ability in this aspect of arithmetic. Furthermore, since the perceived mathematical skill rating involved asking children explicitly about "maths", they may have interpreted this to mean specifically "written mathematics" since that is the predominant mode of mathematical teaching at school.

The children in the present study performed surprisingly well on the derived fact strategies tasks: rather better than would have been predicted from other studies (e.g. Dowker, 2014). The striking exception is that very few of them used the addition/subtraction principle, suggesting that this is particularly difficult. Most studies do concur with the present one in suggesting that the addition/ subtraction inverse relationship is acquired quite late; and is rarely used by children under the age of about 9 or 10 (Bisanz et al., 2009; Demby, 1993; Dowker, 2014; Dube, 2014). However, some studies have given more positive results. Torbeyns, Peters, DeSmedt et al (2016) found fairly frequent use of the principle by 9-to 10-year-old. Baroody et al (1983) found, unsually, that many 6-and 7-year-olds did use the addition/ subtraction inverse principle, and that indeed it appeared to precede the Addend + 1 principle. Gilmore and her colleagues (Gilmore & Bryant, 2006; Gilmore & Papadatou-Pastou, 2009; Gilmore & Spelke, 2008) found that there were considerable individual differences in primary school children's use of this principle, and that some used it successfully. It also appears that the exact nature of the problem presentation can influence performance. In Gilmore & Bryant's (2006) study, for example, included addition and subtraction in the same equations (e.g. 15 + 12 – 12), which may be easier for young children than dealing with an subtraction problem following a separate addition problem, as in the present study.

The study suggests that different aspects of arithmetic are differentially related to different contributory factors. Overall, it suggests that visuo-spatial working memory is related to both written and mental arithmetic, but not to use of derived fact strategies, although use of derived fact strategies is related to ability in written arithmetic. Furthermore, we have found that children's perceptions of their mathematics ability are associated only with their actual written arithmetic abilities, and not with their mental arithmetic abilities.

The finding that visuo-spatial working memory was significantly and independently related to both written and mental arithmetic concur with those of several studies that have suggested a significant relationship between visuo-spatial working memory and arithmetic (Ashkenazi et al., 2013; D'Amico & Guarnera, 2015; De Smedt et al., 2009; McKenzie et al., 2003; Szucs et al., 2014), and emphasize the potential importance of diagnosing and if possible intervening with visual spatial working memory difficulties at an early stage. Future studies should, however, compare the relative importance of verbal and visual-spatial working memory to the tasks discussed here. It appears from most of the studies described above that visual-spatial working memory plays a greater role in arithmetic than verbal working memory, but some studies show the opposite (Keeler & Swanson, 2001; Passolunghi & Mammarrella, 2010). The relative importance of visual-spatial and verbal working memory will depend on the aspect of arithmetic in question (Simmons et al., 2011) and is likely also to vary with age: several studies suggest that younger children's arithmetic depends more on visual-spatial working memory and older children's on verbal working memory (McKenzie et al., 2003), while Soltanou et al (2015) obtained the opposite results. Also, it may be that some studies may have failed to distinguish between the effects of visual-spatial working memory and of spatial ability as such (see Cornu et al., 2018). Clearly, a lot more research in this area is necessary.

The most significant limitation to the present study is of course the relatively small sample. Most findings were either clearly significant or non-significant, and there were few of the borderline and near-significant results that can result from underpowering, but it is important to use this study as just the springboard for studies with a much larger sample, especially as we have been studying relationships between a rather large number of variables, and wish to study still more. There is also still the potential problem that the participants could have been unusual in certain respects. Although the children in this study were from non-selective state schools, they obtained above-average standard/scaled score on most standardized tests, and performed better on the derived fact strategy tests than children in some other studies (e.g. Dowker, 2014). Also, boys performed somewhat better than girls on the mathematics tests, which is nowadays unusual at primary school level. It would be desirable to replicate the study with a larger and more varied sample.

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Appendix

Addition pre-test for Dowker et al.'s (2005) Derived Fact Strategy Addition subtest:

(1)	6 + 3	(11) 31 + 57
(2)	4 + 5	(12) 68 + 21
(3)	8 + 2	(13) 52 + 39
(4)	7 + 1	(14) 45 + 28
(5)	4 + 9	(15) 33 + 49
(6)	7 + 5	(16) 26 + 67
(7)	8 + 6	(17) 235 + 142
(8)	9 + 8	(18) 613 + 324
(9)	26 + 72	(19) 523 + 168
(10)	23 + 44	(20) 349 + 234