# Realistic Mathematics Learning on Students' Ways of Thinking

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*Abstract: This research is motivated by the importance of developing ways of thinking (WoT) in geometry learning at the Sambas District Elementary School, West Kalimantan, Indonesia. In general, this study aims to describe realistic mathematics teaching to WoT elementary school students in completing geometry material related to the design of the Sambas Malay traditional house. The method used in this research is a qualitative method with a hermeneutic phenomenology approach. The subjects of this study were fifth-grade students from one of the public elementary schools in Sambas district with a purposive sampling technique. Data collection techniques used include observations, tests, and interviews. The results showed that students' thinking in solving geometry problems in schools that applied realistic mathematics learning was described in high-ability students, the mental acts displayed were interpreting, problem-solving and inferring. Then, for students with moderate ability, conduct mental behavior explanation and mental behavior reasoning. For students with low mental abilities, the acts displayed are interpreting and problem-solving. After researching the literature on realistic mathematics learning, her goal is to make math learning meaningful and diversify the way students think about solving geometric problems.*

## INTRODUCTION

Realistic mathematics learning is an adaptation of realistic mathematics education adapted to the social environment, culture, and characteristics of the Indonesian people. Realistic learning of mathematics comes from the Dutch language, namely "zich realiseren", which means "imagine" or "imagine" (Van Den Heuvel-Panhuizen, 2003). In this case, the realistic learning of mathematics is based on Freudenthal's view that mathematics is a human activity (Gravemeijer, 1994). Therefore, Gravemeijer (1994) suggests three key principles of realistic mathematical learning, namely, guided reinvention / progressive mathematization, didactic phenomenology, and self-developed models. This is similar to the realistic process of mathematical learning, in general, it has five characteristics (Wahyudi et al., 2017) that include (1) the use of context problems, (2) the use of models, (3) the use of contributions. of students, (4) the occurrence of interactions in the learning process, and (5) the use of different learning theories that are relevant, interrelated, and linked to other learning topics.

Realistic mathematics learning occurs through contextual linking of real and real problems. Realistic mathematics learning refers to problems whose situations are related to the real world and allows the incorporation of mathematical concepts, methods, and results in the solving process known as mathematical context problems (Bliss, et al. 2019; Blum & Niss, 1991; Blum, 2002; Cojocariu, et al. 2014; Niels Jahnke, 2016; Risdiyanti & Prahmana, 2020; Prahmana, et al. 2021). Realistic mathematics learning presents students' reality and real-life experiences in everyday life as a starting point for learning and makes math activity for students. Several previous studies that involved realistic mathematics learning include research (Le, 2006; Van Den Heuvel-Panhuizen, 2003; Tanujaya, et al. 2021) claiming that reality plays an important role in realistic mathematics learning. In realistic mathematics learning, students can apply mathematical concepts to solve mathematics problems. Mathematical concepts are shown in Figure 1.

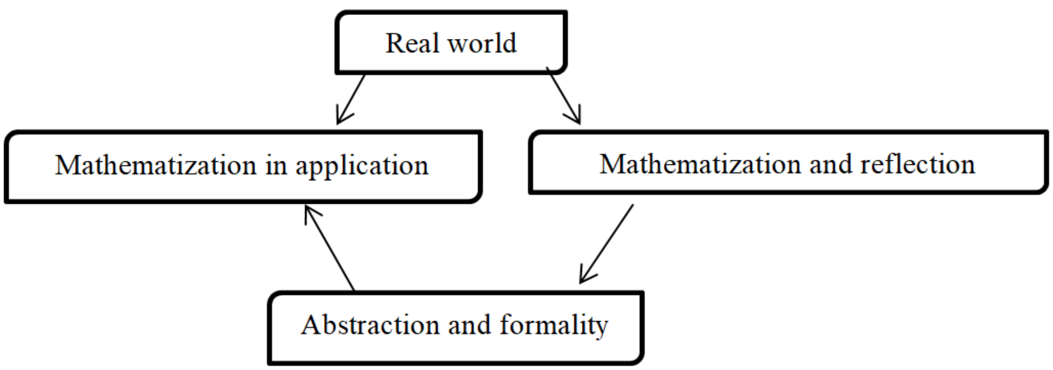



Figure 1: De Lange's Mathematization Concept

In this case, students have the opportunity to rediscover mathematical concepts. According to the research of Gold et al. (2017), good mathematics education must observe how students acquire mathematical concepts.

The integration of culture as a context in the mathematics learning process was examined in the research of Ambrosio (1985). Local cultural elements that can be linked to realistic learning of mathematics are language, knowledge, technology, equipment, art, livelihoods, religion, relatives, customs, traditional buildings, and buildings. community organizations. There are several studies on the integration of local culture in the learning of mathematics (Brandt & Chernoff, 2015; Maryati & Prahmana, 2019; Muhtadi, et al. 2017; Owens, 2012; Rosa & Orey, 2011; Torres-Velasquez & Lobo, 2004). Reinforced by the research of Widada et al. (2018) that mathematics, which takes into account local quantitative, relational, and cultural aspects of society, in this case, is integrated into contextual problems that can be observed or understood by students through a mathematical process. The benefits of the local culture are one of the potentials present in each region that can be used as interesting contextual didactic materials for teaching in schools (Subijanto, 2015).

Mathematics learning that makes use of the environment is intended to generate thoughts and give students the greatest possible opportunity to understand mathematical material. Based on the characteristics of realistic mathematics learning, including the use of contextual problems, the use of models or bridges as vertical tools, use of student contributions, interactivity, and integration with other learning topics. This gives students the greatest opportunity to learn math in a real-world context, which is very important in building a solid foundation for a math topic (Hill, et al. 2008). The relevant topic is geometry, especially in the form of flat structures that use the daily life of students, especially for students in rural areas with limited options (Sukirwan, et al. 2018). Using the existing daily context. One of the contexts related to the local culture of the area is the traditional home. In this way, the ability of the student's brain to understand a mathematical problem is formed and mental actions involving ways of thinking (WoT) and ways of understanding (WoU) are well developed. In other words, realistic mathematics learning embedded in the local culture helps students with developmental acts involving ways of thinking and understanding to solve the problems that students experience when solving mathematical problems.

Students start with a real context and then use informal language and symbols to define the problem (Guler & Gurbuz, 2018). Students begin to work with mathematical symbols and can achieve formulas by building relationships between concepts (Gravemeijer, 1994). Furthermore, knowledge is built when students study real-world context problems, and then knowledge is built mathematically after mathematization (Guler & Gurbuz, 2018). When solving a math problem, students have a variety of ways of thinking (Harel, 2008). This fact is reinforced by the research of Jupri and Drijvers (2016) that found differences in students' work when formulating equations, diagrams, or diagrams since mentalities are relevant for student understanding. It is reinforced by several expert studies showing ways of thinking about and understanding mathematics learning (Harel, 2008; 2020; Hunting, 1997; Ikhwanudin, et al. 2019; Ikhwanudin & Suryadi, 2018; Lockkwood & Weber, 2015; Syamsuri, et al. 2016; Widodo, et al. 2019). Furthermore, Harel found that there is a connection between ways of thinking and understanding (Harel, 1998; 2001).

In other words, the way students understand certain concepts affects their way of thinking and vice versa (Çimer & Ursavaş, 2012). This is in the process of building students 'knowledge and students are at the center of the learning process, which aims to develop students' learning skills that focus on developing mindsets and ways of understanding the mental actions of the students. When students learn the desired way of thinking and understanding through repetitive thinking, it allows them to integrate them into the real and academic life of the students (Oflaz & Demircioğlu, 2018). At this point, the dimensions of mathematical didactics are considered; The mental and psychological needs of the students must be taken into account so that the learning objectives can be successfully achieved.

## RESEARCH METHOD

The research method used in this study is a qualitative method with a hermeneutical phenomenological approach. The qualitative method with the hermeneutical approach was chosen because the research carried out is a study carried out to interpret a meaning acquired by someone from experience as well as an understanding of the hermeneutical phenomenology itself (Lindseth & Rn, 2004). Phenomenology and hermeneutics complement each other. This means that a phenomenon cannot be understood without an interpretation of the subject's experiences. The definition of phenomenology was formulated by Grbich (Kafle, 2013) as "An approach to understanding the hidden meanings and essences of an experience collectively". Hermeneutics according to (Kakkori, 2020) is the "art of interpretation".

This research was conducted at 2 Sambas State Primary School, West Kalimantan Province, Indonesia for the 2020/2021 school year. Research locations and objects were determined according to the purpose-specific random sampling procedure. Intentional sampling is a sampling technique performed intentionally by researchers based on the quality of the location or research topic (Tanujaya, et al. 2017). The subjects of this study were students of classes VA, VB, and VC with a total of 92 students. Then, 3 students analyzed the results of student responses regarding the thinking tool based on student abilities, namely 1 high-ability student, 1 moderate-ability student, and 1 low-ability student. The grouping criteria based on the mean and standard deviation are shown in Table 1.

| No | Student ability | Criteria |
|---|---|---|
| 1 | high ability | $X \geq \bar{X} + SD$ |
| 2 | moderate ability | $\bar{X} + SD < X < \bar{X} - SD$ |
| 3 | low ability | $X \leq \bar{X} - SD$ |

Table 1: Criteria for grouping high, moderate and low ability students

Description : $\bar{X}$ is the average value of the initial mathematical ability
SD is the standard deviation of the initial mathematical ability value

This study uses various tools that are used to obtain the necessary data, including observation, tests, interviews. The data analysis stages, instead, used research stages based on the stages of hermeneutical phenomenological analysis according to Ricoeur (Tan, et al. 2009) as follows: explanation, naive understanding, and deep understanding.

## RESULTS AND DISCUSSION

This study aims to describe the understanding of students' ways of thinking in realistic mathematics learning based on local Sambas culture based on students' learning experiences on geometry material. In the primary stage, the researcher discovered the getting to know procedure withinside the 5th grade which was carried out by the teacher in teaching geometry material using realistic mathematics learning. At the start of the lesson, the teacher offers a gap greeting, assesses scholar attendance and scholar readiness in following the lesson, and prepares geometry fabric, particularly flat shape. The trainer offers a miniature of the traditional Malay Sambas house. Then ask students to observe and discuss what flat shapes are discovered withinside the traditional Malay house of Sambas.

The teacher then distributes a student worksheet containing contextual questions related to local traditional samba culture, especially geometric materials. In the process of learning realistic mathematics, teachers provide students with a foothold in the form of exploratory questions to build their knowledge to find answers to geometric problems. In a realistic math learning process, the teacher acts as a moderator, and students actively participate in discussions, process questions, communicate the results of discussions, exchange ideas, and complete friends' answers. At the end of the lesson, the student completes the lesson and the teacher refines the outcome of the conclusion. A three-question written exam was then conducted, including indicators of interpreting, problem-solving, explaining, and inferring.

This is the result of an analysis that tracks how students think about geometry. The researchers asked 92, 5th-grade elementary school students questions about the essay test and traced them back to different ideas based on student answers. Figure 2 shows the distribution of student reactions to the idea of geometric materials.

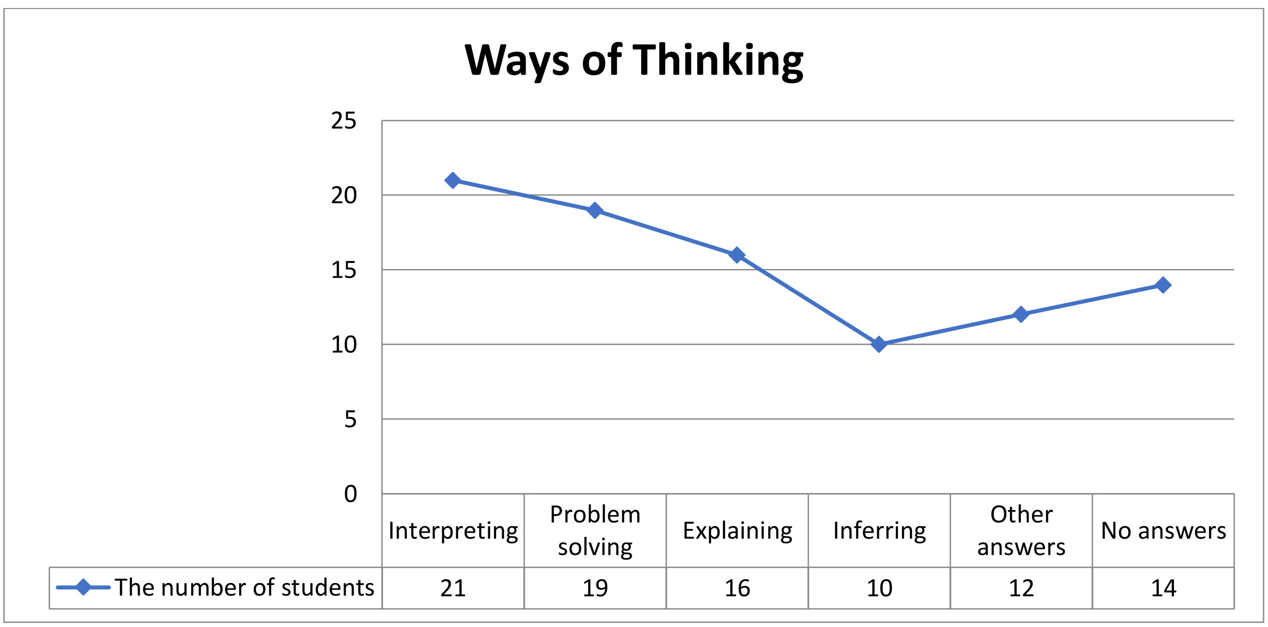



Figure 2: The distribution of student responses related to ways of thinking

In Figure 2, it can be seen that in working on ways of thinking questions on geometry material, overall students can solve problems on interpreting indicators, namely as many as 21 students. Furthermore, on the problem-solving indicator, 19 students can do well. Then as many as 16 students were able to solve the problem of solving ways of thinking on geometry material by explaining indicators. Meanwhile, as many as 10 students were able to solve the questions of ways of thinking with inferring indicators. In addition, from the results of the analysis of students' ways of thinking test questions with indicators, it was found that 14 students did not answer the questions given, and as many as 12 students answered with other answers.

Students can explore and develop the knowledge of students related to the local culture of the Sambas region on the problems posed by a combination of realistic mathematics learning based on the local culture. Table 2 shows the percentage of student ways of thinking test tool results based on the indicators.

| No | Indicator | Percentage | Category |
|---|---|---|---|
| 1 | Interpreting | 76.8 | Good |
| 2 | Problem solving | 53.67 | Sufficient |
| 3 | Explaining | 52.19 | Sufficient |
| 4 | Inferring | 54.6 | Sufficient |

Table 2: Recapitulation of the percentage of students' Ways of thinking scores obtained from the test results made based on the indicators

Table 2 shows that students who obtained the highest percentage of 76.8% longed to the good category in making meaning from symbols or images with various interpretations. Furthermore, 53.67% of students who can solve the given problem by paying attention to alternative or geometric problem-solving strategies are classified into sufficient categories. In addition, the percentage of student scores of 52.19% longs to the sufficient category, students can make explanations of geometric problems related to real contexts. While a percentage score of 54.6% with sufficient category, students can make conclusions from solutions to solving geometric problems.

Then an analysis of the results of students' answers to the tests of students' ways of thinking will be described in realistic mathematics learning. Questions 1a and 1b are questions with indicators of interpretation presented in Figure 3.

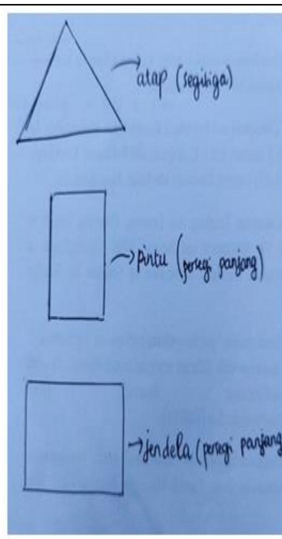
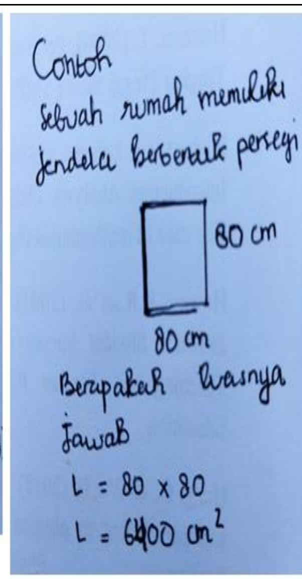
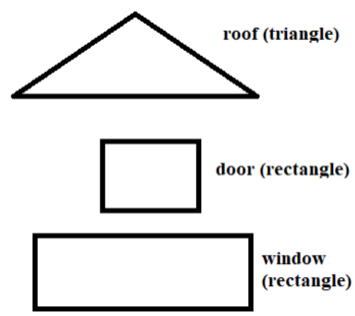
| No | Indicator | Test questions |
|---|---|---|
| 1a | Interpreting | Write and sketch (draw) what types of flat shapes you know from your observations on the sambas Malay house! |
| 1b | | Do you remember how to find the area of a square and give an example! |

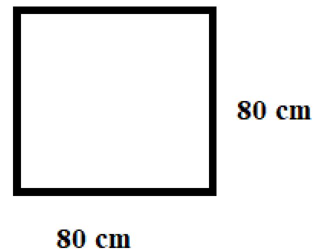
Figure 3: Questions number 1a and 1b interpreting indicator

Observe the shape of the roof, windows, and doors of the Malay Sambas house in Figure 4.

Figure 4: Sambas Malay traditional House

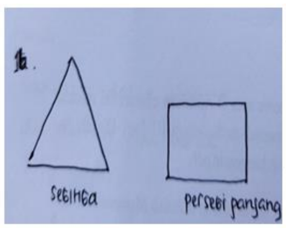
The question of ways of thinking that has been made by the researcher in Figure 4 aims to describe in depth how students make meaning from symbols or images with various interpretations. The results of the analysis of students' answers with high, moderate, and low abilities related to interpreting indicators are presented in Table 3.

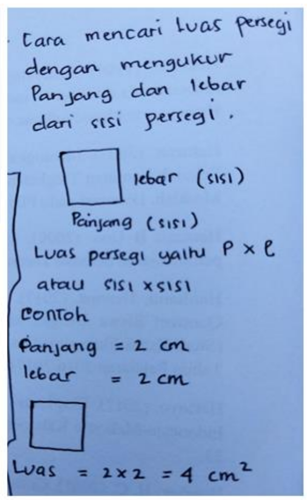
| Student ability | Answers to test questions | Translate |
|---|---|---|
| High ability student (ST04) |   | <br>1b. Example<br>A house has square windows |



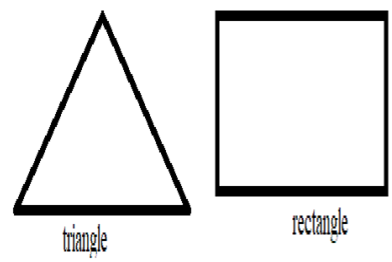

What is the area of the square
Answer
L = 80 X 80
L = 6400 $cm^2$

Moderate ability student (SS63)

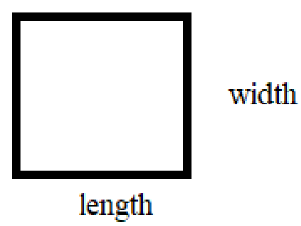


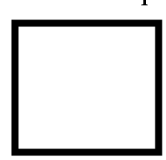

1a




1b. how to find the area of a square by measuring the length and width of the sides of the square




The area of a square is length x width or side x side
example
length of square = 2 cm
width of square = 2 cm



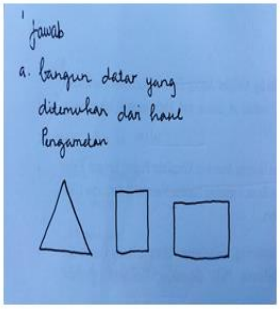
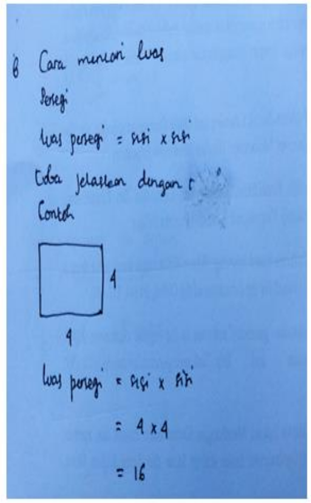
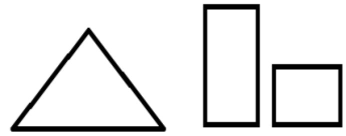
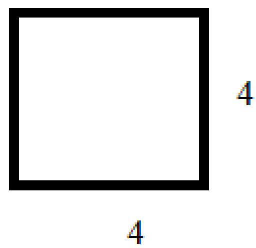
| | | |
|---|---|---|
| Low ability students (SR25) |   | area of square = 2 x 2 = 4cm$^2$<br>1a. answer<br>flat shape found from observations of the traditional Malay house of sambas<br><br>1b. How to find the area of a square. area of square = side x side<br>Please explain with an example<br><br>area of square = side x side<br>area of square = 4 x 4<br>area of square = 16 |

Table 3: Recapitulation of student answers on the interpreting indicator

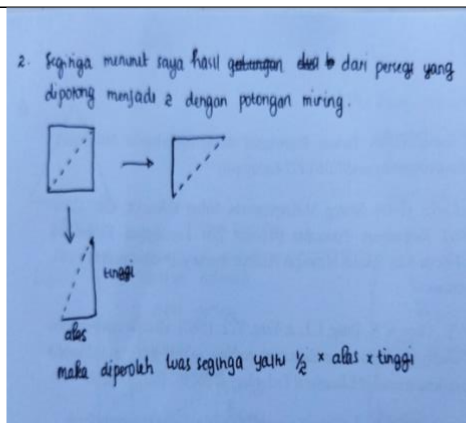
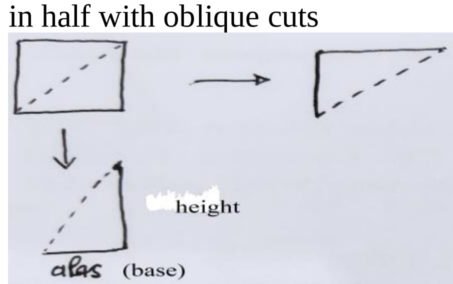
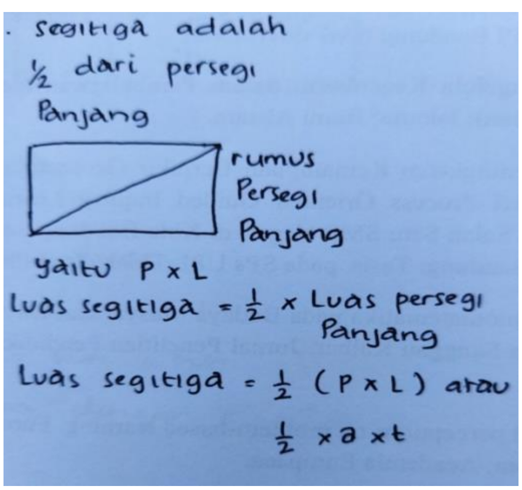
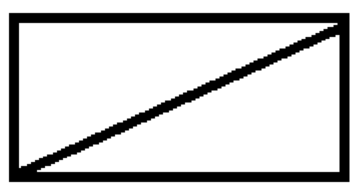
Based on Table 3, the results of the analysis of the answers with interpreting indicators are presented. in students with high, moderate, and low abilities, including subjects are high ability students, SS63 subjects are moderate ability, and SR25 subjects are low ability students. Using the test analysis of Question 1a, the three students were able to interpret the shape of the roof, windows, and doors of the house in a flat shape. Even though the SR25 theme doesn't write down the names of all the flat shapes you're working on. Regarding question 1b, all three students can correctly give an example related to the area of a square together. Both high ability students (ST04) and low ability students (SR25) carefully solve the given problem to obtain the square area, that is, the side x side formula. This shows that students can understand and find the geometric concept of flat shapes and the broad concept of flat shapes. Next, a moderate ability student (SS63) does not write down how to get the square area formula, but immediately gives an example of a question related to the square area. In this case, the SS63 topic is only limited to memorizing formulas and using them.

Next, the answer to question number 2 will be discussed with an indicator explaining, presented in Figure 5.

| No | Indicator | test questions |
|---|---|---|
| 2 | explaining | Explain the reason why the formula for the flat shape of the traditional house is ½ x base x height? |

Figure 5: Questions number 2 are explaining indicator

The questions of ways of thinking that have been made by the researcher shown in Figure 6 aims to describe in depth how students explain the relationship related to geometric material, especially the area of a flat triangle with a square or rectangular shape. Furthermore, the results of the analysis of student answers with high mathematical ability, medium and low regarding indicators explaining are presented in Table 4.

<table>
<tr><th>Student ability</th><th>Answers to test questions</th><th>Translate</th></tr>
<tr><td>High ability student (ST04)</td><td>
</td><td>Triangle in my opinion the result is cut in half with oblique cuts<br>
<br>Then the area of the triangle is 1/2 x base x height</td></tr>
<tr><td>Moderate ability student (SS63)</td><td>
</td><td>A triangle is 1/2 of a rectangle.<br><br>The formula for a rectangle is length x width<br>area of triangle = 1/2 x area of a rectangle<br>area of triangle = 1/2 x (length x width)</td></tr>
</table>

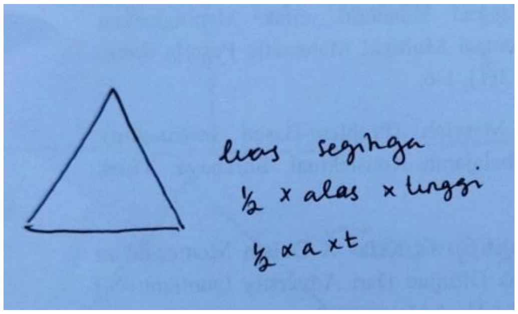
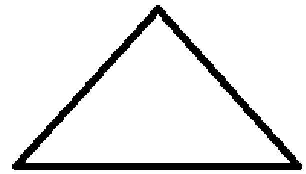
| | | |
|---|---|---|
| Low ability students (SR25) |  | or<br>area of triangle = 1/2 x base x height<br><br>Area of triangle 1/2 x base x height<br>½ x a x t |

Table 4: Recapitulation of student answers on the explaining indicators

Based on table 4 obtained the results of the analysis of student answers show that students are high abilities (ST04) even though they have written the area of the triangle correctly, but the process for obtaining the area of the triangle is not correct. Students are wrong in explaining the process to obtain the area of a triangle. This is because students do not understand the meaning of the triangle relationship in a square. Furthermore, students with moderate abilities (SS63) can understand questions and answers using mental act explaining through pictures and writing and provide systematic and precise explanations. This means that students can explain the relationship between rectangular flat shapes and triangular flat shapes. The low-ability student (SR25) cannot explain the relationship between the area of a triangle and the area of a square. The students quickly drew the triangle and the area of the triangle but did not follow the correct procedure to find the area of the triangle. This means that students do not correctly understand and recognize the concept of geometric materials. This is because students learn by learning formulas rather than understanding concepts. Finally, Question 3 is a question about problem-solving indicators and inferring indicators. The student responses analyzed were those of high, moderate, and low abilities shown in Figure 6

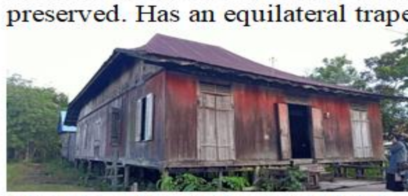
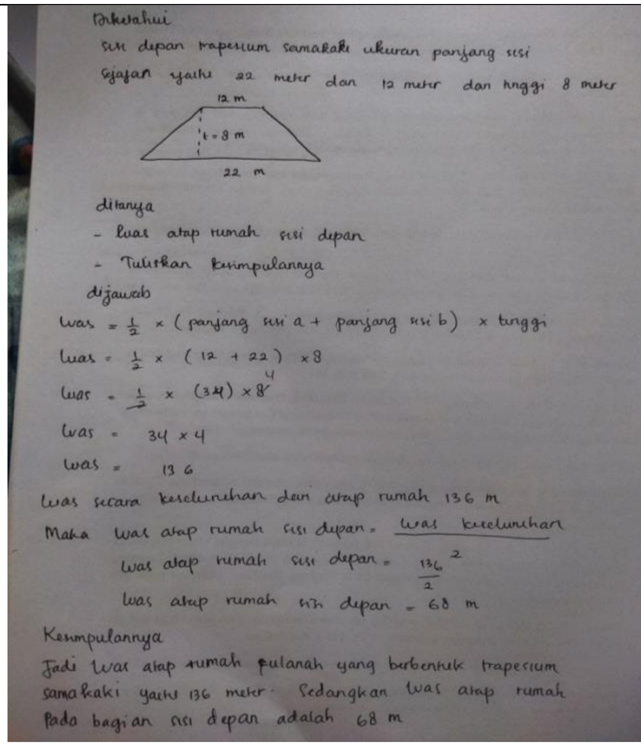
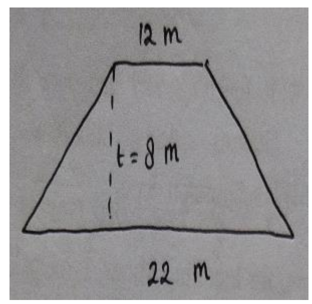
| No | Indicator | Test questions |
|---|---|---|
| 3 | Problem-solving<br>inferring | Fulanah's house is one of the traditional houses that is still preserved. Has an equilateral trapezoidal roof<br><br>The front side of the roof of Fulanah's house has a parallel side length of 22 meters and 12 meters respectively and a height of 8 meters. Determine the area of the roof of the front side of the house and write the conclusion! |

Figure 6: Questions number 3 are a problem-solving and inferring indicator

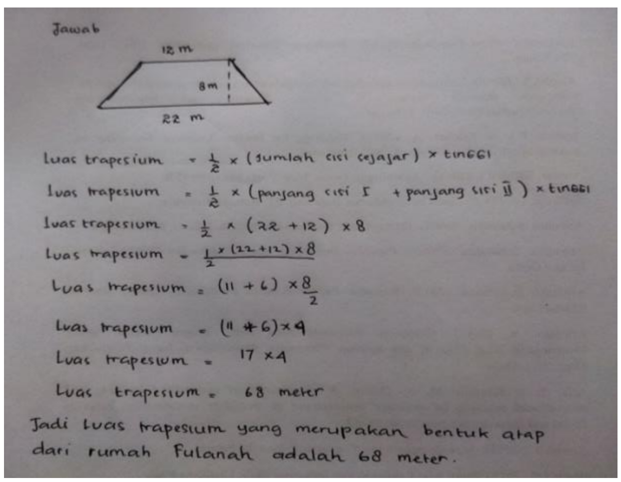
Figure 6, discusses the ways of thinking test questions with problem-solving indicators and inferring indicators. Furthermore, the results of the analysis of student's answers with high, moderate, and low abilities related to problem-solving indicators and inferring indicators are presented in Table 5.

| Student ability | Answers to test questions | Translate |
|---|---|---|
| High ability student (ST04) | <br>Diketahui<br>sisi depan trapesium samakaki ukuran panjang sisi sejajar yaitu 22 meter dan 12 meter dan tinggi 8 meter<br>12 m<br>t = 8 m<br>22 m<br>ditanya<br>- luas atap rumah sisi depan<br>- Tuliskan kesimpulannya<br>dijawab<br>luas = $\frac{1}{2}$ x (panjang sisi a + panjang sisi b) x tinggi<br>luas = $\frac{1}{2}$ x (12 + 22) x 8<br>luas = $\frac{1}{2}$ x (34) x 8 4<br>luas = 34 x 4<br>luas = 13 6<br>luas secara keseluruhan dari atap rumah 136 m<br>Maka luas atap rumah sisi depan = $\frac{\text{luas keseluruhan}}{}$<br>luas atap rumah sisi depan = $\frac{136}{2}$ 2<br>luas atap rumah sisi depan = 68 m<br>Kesimpulannya<br>Jadi luas atap rumah pulanah yang berbentuk trapesium samakaki yaitu 136 meter. Sedangkan luas atap rumah pada bagian sisi depan adalah 68 m | Is known to<br>the front side of an isosceles trapezoid, the length of the parallel sides is 22 meters and 12 meters, and a height of 8 meters<br><br><br>asked<br>the front side of the roof<br>Write the conclusion<br>answered<br>Area of trapezoid = 1/2 x (length of side a + length of side b) x height<br>area of trapezoid = 1/2 x ( 12 + 22) x 8<br>area of trapezoid = 34 x 4<br>area of trapezoid = 136<br>The total area of the roof of the house is 136 meters<br>Then the roof area of the front side of the house = 1/2 x the area of the trapezoidal roof of the house as a whole |

the front side of the roof = 1/2 x 136
the front side of the roof area = 68 meters

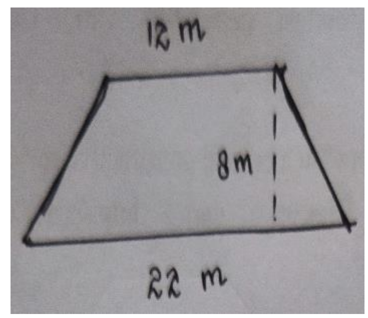
Conclusion
So, the roof area of the Fulanah's house which is in the form of an isosceles trapezoid is 136 meters. While the roof area of the house on the front side is 68 meters

Moderate ability student (SS63)


Jawab

12 m, 8 m, 22 m

Luas trapesium = 1/2 x (jumlah sisi sejajar) x tinggi
Luas trapesium = 1/2 x (panjang sisi I + panjang sisi II) x tinggi
Luas trapesium = 1/2 x (22 + 12) x 8
Luas trapesium = (1 x (22 + 12) x 8)/2
Luas trapesium = (11 + 6) x 8/2
Luas trapesium = (11 + 6) x 4
Luas trapesium = 17 x 4
Luas trapesium = 68 meter
Jadi Luas trapesium yang merupakan bentuk atap dari rumah Fulanah adalah 68 meter.




Area of trapezoid = 1/2 x (number of parallel sides) x height
area of trapezoid = 1/2 x (length of side 1 + length of side 2) x height
area of trapezoid = 1/2 x (22 + 12) x 8
area of trapezoid = $\frac{1\,x\,(22+12\,)x\,8}{2}$
area of trapezoid = (11 + 6 ) x 8/2
area of trapezoid = 17 x 4
area of trapezoid = 68 meters
So the area of the trapezoid which is the shape of the roof of the fulanah's house is 68 meters

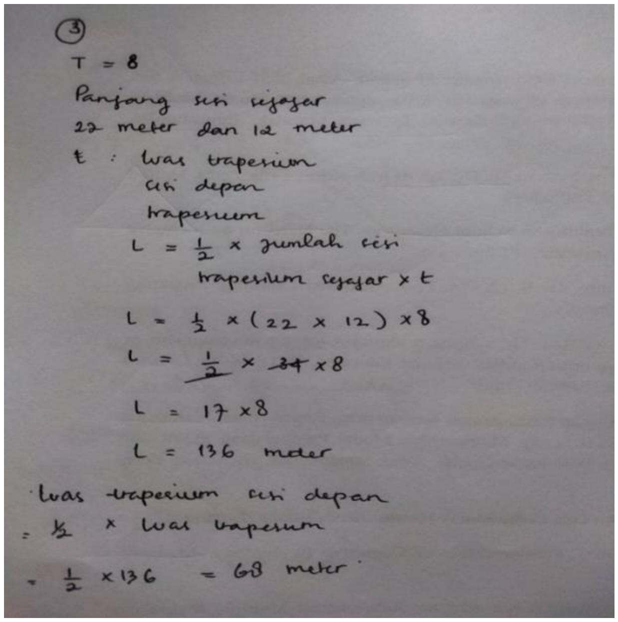
| | | |
|---|---|---|
| Low ability students (SR25) | <br>(3)<br>T = 8<br>Panjang sisi sejajar<br>22 meter dan 12 meter<br>t : luas trapesium<br>sisi depan<br>trapesium<br>L = ½ x jumlah sisi<br>trapesium sejajar x t<br>L = ½ x (22 x 12) x8<br>L = ½ x ~~34~~ x 8<br>L = 17 x8<br>L = 136 meter<br>luas trapesium sisi depan<br>= ½ x luas trapesium<br>= ½ x136 = 68 meter | height = 8<br>The lengths of the parallel sides are 22 meters and 12 meters<br>area of trapezoid front side of trapezoid = 1/2 x number of parallel sides of trapezoid x height<br>area of trapezoid = 1/2 x (22 + 12 ) x 8<br>area of trapezoid = 1/2 x 34 x 8<br>area of trapezoid = 17 x 8<br>area of trapezoid = 136 meters<br>area of the front side of the trapezoid = 1/2 x the total area of the trapezoid<br>area of trapezoid = 1/2 x 136<br>area of trapezoid = 68meters |

Table 5: Recapitulation of student answers on the problem solving and inferring indicator

Based on Table 5, the results of the related analysis are obtained from the problem-solving strategy used by students with high ability (ST04) in solving problems where the process to determine the area of the trapezium applies the right procedure. Students understand each stage in the work to get a solution related to the area of the trapezium. In addition, the ST04 subject wrote the correct conclusion to find the answer to the geometry problem. Furthermore, students with the moderate ability (SS63) are already able to carry out strategies in the process of solving the trapezoid area problem, even though the answers obtained are correct. However, the calculation process was wrong and made an error in the division operation so that an error occurred in the student's answer. This is because students do not pay much attention to the calculation process for obtaining the area of the trapezoid. Using the results of the low-ability student's answer (SR25), the student uses solution strategies and procedures to solve the correct trapezoidal area, but in the SR25 topic, the conclusion is as an answer to question number 3 not described.

Student results analysis ways of thinking about applying realistic mathematics learning to solve geometric problems at school tend to help high abilities students understand the indicators of interpretation, problem-solving, and inferring. This indicates that there is. In addition, students with moderate abilities tend to understand indicators ways of thinking, namely explanation, and inferring. On the other hand, low abilities students tend to understand interpreters and problem-solving indicators. Demetriou (2004) states that students have more diverse ways of thinking (WoT), more strategies to solve problems, and the opportunity to become flexible thinkers in dealing with new situations and problems. Consistent with the study of.

In addition, an analysis of the student's answers to the three questions revealed that the material for the flat-shaped area was not understood, the procedure was applied incorrectly, and the prerequisite material was not mastered. The above findings are in line with the results of the research by Ikhwanudin et al. (2019) which states that the patterns of student errors in learning mathematics include lack of understanding of fractions, insufficient understanding of fractions, and the denominator equation and the method of adding fractions are mistakenly applied. Reinforced by the results of the study of Mazzocco et al. (2011) which states that students with mathematics learning disabilities state that they make mistakes in comparing and estimating numbers.

The results of this study can be used by teachers of mathematic subjects who apply realistic mathematics learning to achieve their learning goals. Teachers need to be confident that they can guide students to learn and practice mathematics, change their attitudes from passive recipients to active individuals, and develop mathematical reasoning. In addition, realistic mathematics learning will be the mainstay for helping low abilities students develop and improve their mathematical skills while answering mathematic problems.

## CONCLUSIONS

Realistic mathematics learning makes mathematics learning meaningful and students' ways of thinking in solving geometric problems are increasingly diverse. The student's ways of thinking found were various visual interpretations and symbols, ways of explaining (explaining), approaches or strategies in problem-solving, and ways of concluding. The more diverse ways of thinking that are raised by students, the implications for the higher the student's ability score. This shows that the more diverse students' ways of thinking, the higher their geometric problem-solving ability. Furthermore, several errors were found in the students' work, including lack of understanding of the area of flat shapes, incorrectly applying the procedure (concept error), and lack of proficiency in the required materials.

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