Learning Trajectory of Algebraic Expression: Supporting Students' Mathematical Representation Ability

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Abstract: The mathematical representation ability is essential in solving mathematical problems, especially in algebraic expressions problems. Therefore, it is crucial to have a valid design of learning activities to support students' mathematical representation ability. Several previous studies claimed that realistic mathematics education is one of the learning approaches that could support this ability. This study is design research aiming to evaluate the RME-based learning trajectory oriented to enhance students' mathematical representation ability on algebraic expression. The data collected from the documentation of learning trajectories, documentation of students' answer sheets, and video recording of online teaching and reflection sessions were analyzed descriptively. The findings indicated that the designed RME-based learning trajectory (LT) oriented to support students' mathematical representation ability in algebraic expression has been valid and could be implemented in the pilot experiment. The implementation of the LT-1 and issues found during the pilot experiment are discussed in the paper. The finding implies that the learning trajectory could be continued to the teaching experiment phase after some revisions and adjustment.

INTRODUCTION

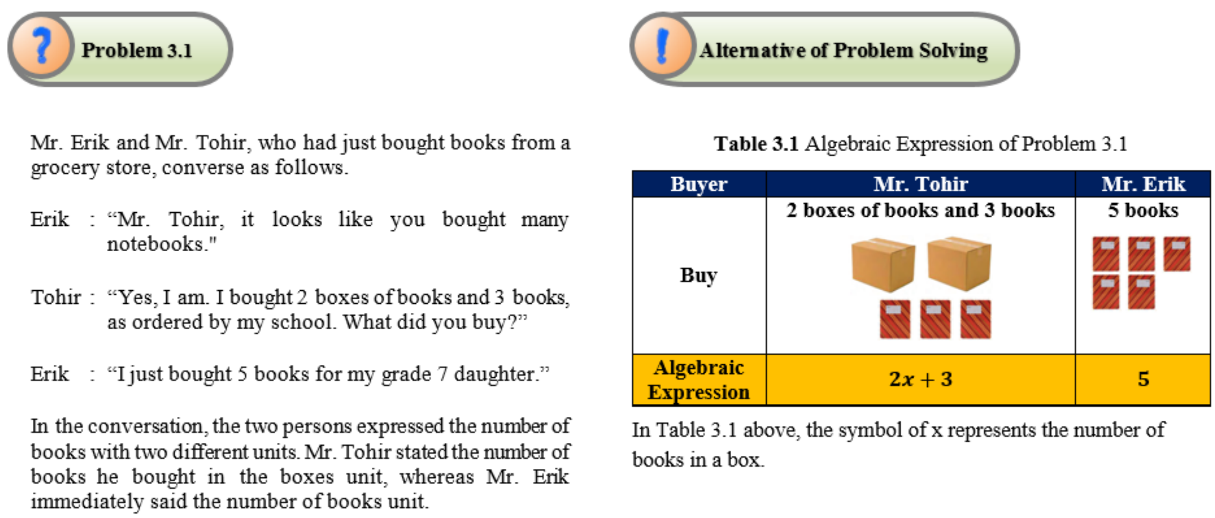
One of the abilities required to learn mathematics is mathematical representation ability (NCTM, 2000), such as simplifying and solving mathematical problems relying heavily on the ability. Representation is the transformation of a problem or idea to a new form, including transforming images or physical models into symbols, words, or sentences (NCTM, 2000). Representation is a means to communicate mathematical problem-solving ideas, and it may be used to facilitate and support conclusions (Pape & Tchoshanov, 2001; Sari & Rosjanuardi, 2018). When learning mathematics, students are suggested to focus more on various forms of mathematical representations to solve mathematical problems well (Afriyani, Sa'dijah, Subandi, & Muksar, 2018). One of the mathematical problems that require mathematical representation skills is algebra.

Algebra is one of the main topics with undeniable importance in mathematics. Comprehension of algebra is important because it is very related and cannot be separated in everyday life, and it is very influential in decision making (Usiskin, 1995). Conceptual comprehension in algebra is defined as identifying functional connections between known and unknown variables, independent and dependent variables, and differentiating and interpreting diverse representations of algebraic concepts (Panasuk & Beyranevand, 2010). Algebraic topics began with basic arithmetics and then advanced to more abstract algebraic operations will be challenging for students (Baroudi, 2006; Sarimanoğlu, 2019). While students might not face difficulties performing arithmetic calculations, Jupri & Drijvers (2016) stated that different things might happen when the calculation involves algebraic expressions.

The emphasis on computation leads to many misconceptions in students' minds, which will make it more complicated for students (Baroudi, 2006; Sarimanoğlu, 2019). Furthermore, several studies reported various difficulties related to algebraic expression faced by students. Most students have difficulties understanding basic algebraic expressions, especially the meaning of variables in an algebraic expression (Rudyanto, Marsigit, Wangit, & Gembong, 2019). In contrast, the variable in the algebraic expression is the basic concept that must be interpreted to continue learning algebra at a higher level (Booth, McGinn, Barbieri, & Young, 2017). Furthermore, several studies found that most students who still have difficulty understanding the algebraic expression, especially students at the intermediate level, have misconceptions about algebraic prerequisite material (Bush & Karp, 2013) and experience misunderstandings in solving algebraic equations (Sarimanoğlu, 2019).

Often, students' misconception occurs in understanding the meaning of variables in algebraic equations (Knuth, Alibali, McNeil, Weinberg, & Stephen, 2005). Moreover, Egodawatte (2011) stated that misconceptions often occur in four parts of algebra, namely variables, algebraic expressions, algebraic equations, and story problems. Another misconception in algebraic expressions students face is considering the (+) symbol as an invitation to do something; therefore, they simplified $3x + 4$ as $7x$ and $4 + 3x^2$ as $7x^2$ (Chow & Treagust, 2013). Students' misunderstandings in algebraic calculations occur due to a lack of understanding of the expression of a variable that can be a literal symbol as a label for an object; for example, students mistaking the letter "y" in "addition of 3 and y" as something like yogurt and yum or as the alphabet "D" in David's name that can be mixed (Christou, Vosniadou, & Vamvakoussi, 2007). They might expand $(m + n)^2$ as $m^2 + n^2$, distribute $2(x - 5)$ as $2x - 5$, simplify $3y + 2$ as $5y$ and $3a + 2y$ as $5a^2$ (Al-Rababaha, Yew, & Meng, 2020). It might occur because students consider the procedure of simplifying algebraic expressions similar to that of arithmetic problems, where a final answer is a single-digit number (Herutomo & Saputro, 2014).

Interpreting the meaning of a variable as a symbol or a label as a substitute for an object in algebraic problems is the most fundamental step in minimizing students' misunderstanding of algebraic mathematical modeling. One effort to overcome students' misconceptions is to design learning activities accommodating the fundamental step. Starting the learning process by solving contextual problems is considered will help students develop understanding and overcome the misconception. In line with this opinion, the Indonesian mathematics textbook for Year 7 students published by Kemendikbud (2017) also has contextual problems as the starting point in learning. Figure 1 shows the example of a contextual problem provided by the textbook.



Problem 3.1

Mr. Erik and Mr. Tohir, who had just bought books from a grocery store, converse as follows.

Erik : "Mr. Tohir, it looks like you bought many notebooks."

Tohir : "Yes, I am. I bought 2 boxes of books and 3 books, as ordered by my school. What did you buy?"

Erik : "I just bought 5 books for my grade 7 daughter."

In the conversation, the two persons expressed the number of books with two different units. Mr. Tohir stated the number of books he bought in the boxes unit, whereas Mr. Erik immediately said the number of books unit.

Alternative of Problem Solving

Table 3.1 Algebraic Expression of Problem 3.1

Buyer	Mr. Tohir	Mr. Erik
Buy	2 boxes of books and 3 books	5 books
Algebraic Expression	$2x + 3$	5

In Table 3.1 above, the symbol of x represents the number of books in a box.

Figure 1: Contextual Problem and its Problem Solving Alternative Proposed as Introduction to Algebraic Expressions in the Mathematics Textbook (Source: Kemendikbud (2017))

Based on Figure 1, it is visible that after starting the learning activity with contextual problems, the $2x + 3$ symbol is given immediately, without involving students' negotiation and creativity. Students might be wondering why the symbol is needed in the contextual problem and why it must be symbolized by alphabet x. Besides, the introduction of algebraic expressions in the textbooks tends to be represented by the number of objects, although it might be associated with the unknown size of a 2D shape, which is part of geometry learning.

Linking algebra learning with other subjects, e.g., geometry, aligns with one of the characteristics of realistic mathematics education (RME), namely intertwinement (Treffers, 1987). Furthermore, it is essential to provide activities requiring students to solve algebraic expression problems without being overwhelmed by abstract terms such as variables, coefficients, and terms at the beginning of the learning. The abstract terms could be introduced to students after they *were doing mathematics* in solving reasonable or meaningful problems. The utilization of technology, such as online games, might also be inserted in learning algebraic expressions, according to 21st-century learning.

Concerning the issues presented above, this study aims to evaluate the RME-based learning trajectory (LT) oriented to enhance students' mathematical representation ability on algebraic expression. This paper examines the research question: "how is the design of RME-based LT to develop students' mathematical representation abilities on algebraic expressions?" However, this paper is limited to the discussion on the first LT.

RESEARCH METHODS

This study is a design research model proposed by Gravemeijer and Cobb that consists of three phases, namely preparing for the experiment, design experiment, and retrospective analysis (Gravemeijer & Cobb, 2013); the three phases that were implemented in a cyclic process is visualized in Figure 2.

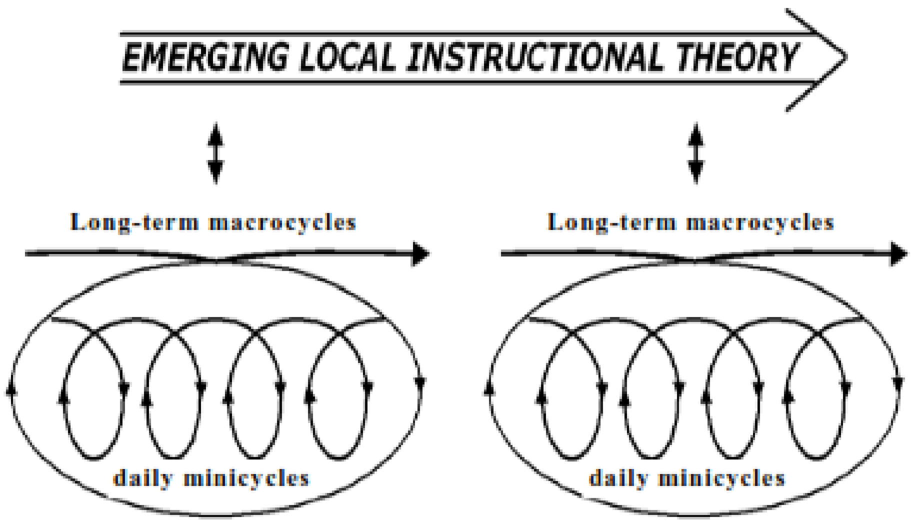


Figure 2: The cyclic process of design research (Fauzan, Musdi, & Afriadi, 2019)

In *preparing for the experiment* phase, we determined the endpoints of the LT in which the students develop their mathematical representation ability through learning algebraic expression with the RME approach. Therefore, we analyzed the literature related to learning trajectory, realistic mathematics education, students' mathematical representation ability, Indonesian national curriculum, grade seven mathematics textbook, and algebraic expression. We then designed LTs based on the result of the analysis. The initial LT of the algebraic expression topic was labeled as Prototype-1. The initial idea for Prototype-1 was obtained through an online workshop involving RME research center teams from several universities in Indonesia, including Universitas Syiah Kuala, and several junior high school mathematics teachers in Banda Aceh. A pilot study involving mathematics teachers from two districts in Aceh province was conducted to assess teachers' responses toward the Prototype-1. Prototype-1 was then revised based on the pilot study result, and we labeled the revised version as Prototype-2.

The data in this study were obtained from the documentation of learning trajectories, documentation of students' answer sheets, and video recording of online teaching and reflection sessions. The online teaching involved a mathematics teacher and 15 grade 7 students at a private school in Banda Aceh, Aceh Province, Indonesia. The data was then analyzed descriptively.

RESULTS

The Design of Learning Trajectory

The RME-based LT of algebraic expression in this study, labeled as Prototype-2, consisted of three learning trajectories, and was designed to accommodate four lessons related to algebraic expressions. The first LT (LT-1) was designed for introducing the concept of algebraic expression, whereas the other two LTs were for addition, subtraction, and multiplication of algebraic expressions. However, as mentioned previously, this paper will only focus on the first LT of the Prototype-2.

The instructional activities in the LTs were designed based on RME principles and organized according to the level of *emergent modeling* (Gravemeijer, 1994; Gravemeijer, 2007; Gravemeijer, Lehrer, van Oers, & Verschaffel, 2013; Bos, Doorman, & Piroi, 2021), namely *situation* level, *model-of* level, *model-for* level, and *formal* level. The instructional activities of the LT-1 is presented in Table 1.

As presented in Table 1, on the situational level, we proposed the context of predicting the maximum number of balls fit inside some closed different-size baskets and dividing rectangular-land based on the Islamic way of distributing inheritance. The context of dividing rectangular land with unknown sizes was intended to develop students' ability to transform word problems into drawing, which is one type of representation. The following three activities in the LT-1 were also designed to promote students' ability in mathematical representations. Through the third and the fourth activities, the students were expected to represent the problem into the mathematical symbol and algebraic expression, while the fifth activity required students to use their own language to describe the meaning of a variable, a coefficient, and a constant.

Prototype-2 was then assessed and validated by validators. As previously mentioned, Prototype-2 was validated by eight validators consisting of 5 lecturers from the mathematics education department and three secondary school mathematics teachers. The lecturers are mathematics education experts who have been involved in realistic mathematics projects for more than ten years. The teachers were selected because of their willingness to participate as validators in this study. Furthermore, they are willing to be teachers in the teaching experiment phase.

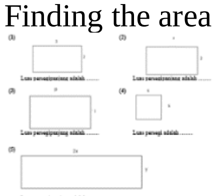
Level of Activities	Activities
	Lesson 1 Topic: Introduction to Algebraic Expression
Situation	1. Finding the possible total number of balls inside closed different-size baskets. 
	2. Solving problem related to dividing rectangular land with unknown size
Model Of	3. Finding the possible total number of balls inside closed different-size baskets, if x represents the number of balls in a small basket and y represents the number of balls in a large basket. 
Model For	4. Finding the area of rectangles in which variables predetermine sizes. 
Formal	5. Writing the meaning of variables with students' own words and giving other examples of coefficient and constant.

Table 1: Instructional Activities of LT-1

The analysis of the validation sheets resulted in the average validity score reaching 4.54, indicating that Prototype-2 was in the *valid* criteria. Furthermore, each of the four aspects of the validity, namely content, format, language, and display, also reach *valid* criteria. The eight validators agreed that the LT-1 of Prototype-2 could be used with minor revisions, as presented in Table 2.

LT	Activity	Validator	Suggestions
1	1	SW	The size of balls in every basket should be the same.
	1	PJ	Change the figure with scale pictures.
	1	TZ	Need to add one activity before activity 1
	4	ST, SW, FH, EM	Change the variables used in the worksheets as a or m or n to avoid confusion with the multiplication symbol, ×.

Table 2: Summaries of Validators' Suggestions to the Prototype-2

The LT-1 was then revised based on the validator's suggestions. Generally, there were not many modifications have been made to the activities in the LT-1. We just added one activity in the beginning and made minor revisions to typos and the symbols used. The revised version of the LT-1 is presented in Table 3.

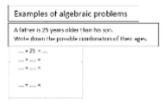
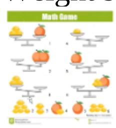
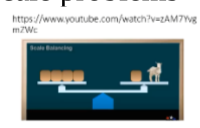
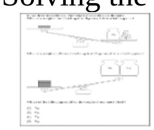
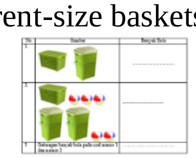
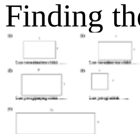
Level of Activities	Activities
Situation	Lesson 1 Topic: Introduction to Algebraic Expression *Activities in Classroom Discussion* 1. Find the possible pairs of father's and son's ages.  2. Weight scale problems  
	Activities in Group Discussion 1. a. Solving the problem related to weighed scale.  b. Find the possible pairs of whole numbers that add up to 10 .... + ... = 10 .... + ... = 10 .... + ... = 10 x + ... = 10 2. Finding the possible total number of balls inside closed different-size baskets.  3. Solving problem related to dividing rectangular land with unknown size
Model Of	4. Finding the possible total number of balls inside closed different-size baskets, if x represents the number of balls in a small basket and y represents the number of balls in a large basket. 
Model For	5. Finding the area of rectangles in which variables predetermine sizes. 
Formal	6. Writing the meaning of variables with students' own words and giving other examples of coefficient and constant.

Table 3: Revised Version of LT-1 Instructional Activities

The Pilot Experiments

A pilot experiment involving 15 grade 7 students of one private school in Banda Aceh, Indonesia, was carried out to examine how the designed LT works. Considering the Covid-19 pandemic, we conducted the pilot teaching experiment online via Zoom Meeting. As the apperception, the teacher asked students about some mathematics formulas they had learned to remind students that mathematics uses symbols (see Figure 3). Through the classroom discussion, the students said that the symbols were used to simplify mathematical problems.

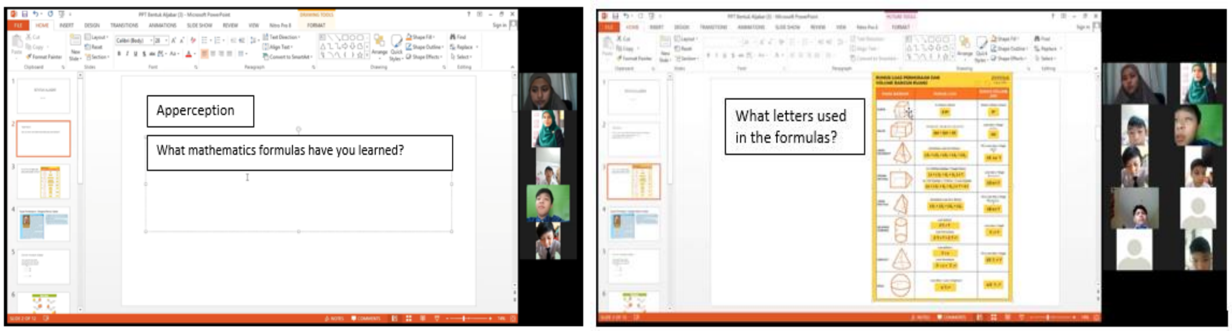


Figure 3: Recalling Mathematics Formulas as Apperception Activity

After communicating the learning motivation and objective, the activity continued to classroom discussion about some problems as presented in Figure 4.

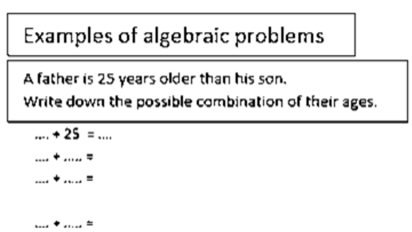
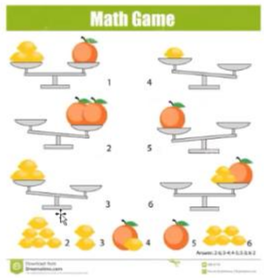


Figure 4: Problem of Possible Ages of Father and Son

It was observed that during the classroom discussion, the students did not have difficulties finding the likely ages of father and son. Students could quickly answer that, for example, if the son is five years old, then the father is 30 years old, which came from 5 + 25. Similarly, when the question was reversed, the students also did not have trouble determining how old the son is if the father's age is 35 years. However, different cases happen when the teacher asks students questions involving variables. When the teacher asked, "How old is the son if the father's age is x?" the students needed more time and intensive assistance from the teacher before they got the answer $(x - 25)$.

The classroom discussion was then continued with problems presented in Figure 5. It was observed that the students did not face difficulty in solving problems in Figure 5a and Figure 5b.

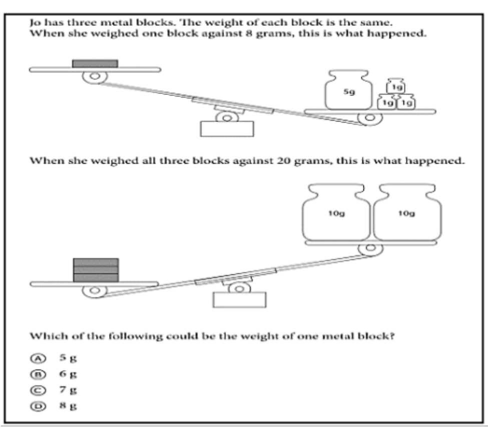


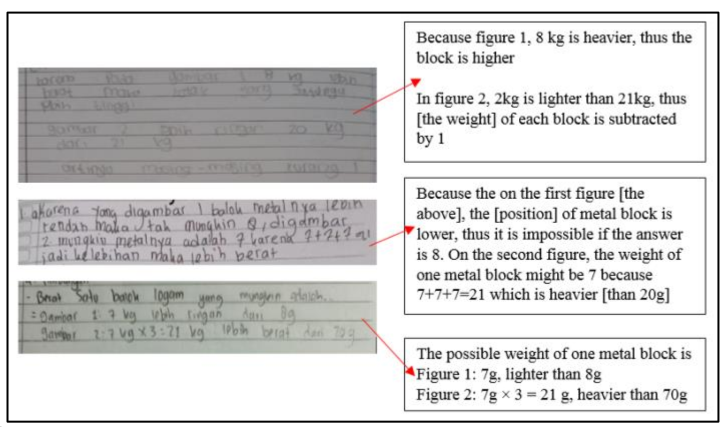
(a) (b)

Figure 5: Weigh Scale Problems

After the classroom discussion, the students were required to work in a group of two students. The teachers assigned the students worksheet through WhatsApp Group. While still being online in the zoom meeting, the students were asked to discuss the problems with their partners through WhatsApp. As written in Table 3, there were five activities to be solved in the student worksheet.

The first activity was almost like the activity presented during the classroom discussion about the scale balancing problem (see Figure 6a) and possible numbers added to 10. Related to the problem in Activity 1a, based on the students' written answers, all groups gave the correct answer that is 7 kg. However, only two groups wrote the reason for the answer, as displayed in Figure 6b.



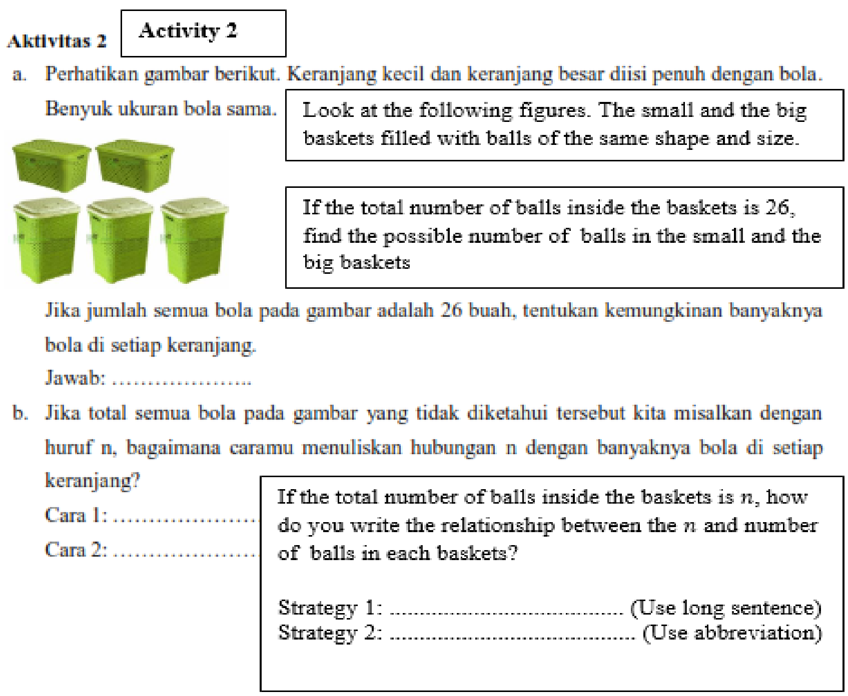


a b

Figure 6: Problem and Students' Answers of Activity 1a

Related to the questions in Activity 1b, only three groups could provide the correct answer. Even though the other students could answer the first three questions with different pairs of numbers, they could not answer the fourth question containing variable x. While some students did not answer it, some assumed the x with a number, e.g., 7, then wrote $7 + (10 - 7) = 10$. The teacher then brought this students' answer to the class discussion until the students understood it.

Activity 2 dealt with the total number of balls placed in two closed baskets of different sizes. Through this activity, after exploring some possibilities of pairs of numbers, students were expected to use symbols to represent the number of balls in a small basket and a large basket. The problem and students' answers are displayed in Figure 7.


Aktivitas 2 **Activity 2**

a. Perhatikan gambar berikut. Keranjang kecil dan keranjang besar diisi penuh dengan bola. Benyuk ukuran bola sama.

Look at the following figures. The small and the big baskets filled with balls of the same shape and size.

If the total number of balls inside the baskets is 26, find the possible number of balls in the small and the big baskets

Jika jumlah semua bola pada gambar adalah 26 buah, tentukan kemungkinan banyaknya bola di setiap keranjang.

Jawab: ………………..

b. Jika total semua bola pada gambar yang tidak diketahui tersebut kita misalkan dengan huruf n, bagaimana caramu menuliskan hubungan n dengan banyaknya bola di setiap keranjang?

Cara 1: …………………

Cara 2: …………………

If the total number of balls inside the baskets is n, how do you write the relationship between the n and number of balls in each baskets?

Strategy 1: .... (Use long sentence)
Strategy 2: .... (Use abbreviation)

Figure 7: Problems in Activity 2 of Student Worksheet

Students should solve the problems of Activity 2 through discussion with their friends via WhatsApp chats. However, it was observed that the group discussion did not go well, and only a few students did the discussion, while the other students worked individually. After some time, the students asked for the teacher's assistance. Thus, the Activity 2 was solved through classroom discussion, as revealed in the excerpt below.

AMS : The [total] number of balls is 26, then it was asked how many balls were in each basket. That means 26 divided by 5.

Teacher : No, no. Take a look at the baskets. Are they the same size?

NZS : Miss, may I answer?

Teacher : Of course. There were two small baskets and three large baskets. If the sizes are different, how about the number of balls in each basket? Give it a try.

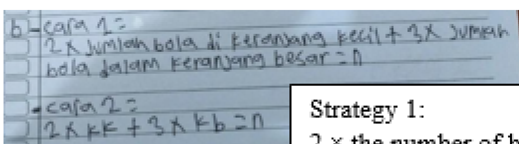
SI : Miss, the large basket [contain] six balls, the small one [contain] four balls.

Teacher : Ok, how did you get the answer?

SI : Because the large basket automatically has more balls. 26 divided by 8 resulted in a decimal number, 26 divided by 7 resulted in a decimal number, 26 divided by 5 also resulted in a decimal number. However, if, for example, 26 divided by 6 balls, it equals 18 balls, thus the small baskets if 4 balls multiplied by 2, then the remainder is 8.

Based on the above excerpt, the strategy used by SI was trial and error. He tried to find pairs of numbers from the distribution of 26 balls which resulted in integers because it was impossible if the number of balls is a decimal number. So, he found that the number of balls in the three large baskets is 18; thus, each large basket contains 6 balls. Therefore, the number of balls in the two small baskets is 8, so that each small basket contains 4 balls.

The learning continued to group discussion about Activity 2b. However, after some time of no response from the students, the teachers started to give a clue by proposing abbreviations to represent the number of balls in a small basket and a large basket until the students could solve the problems. Figure 8 shows a student's answer to the problem in Activity 2.

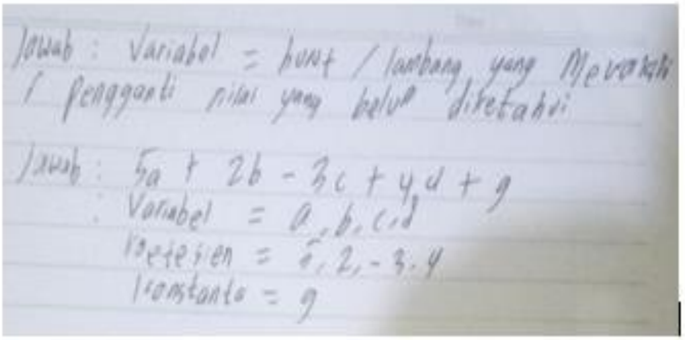


Strategy 1:
2 × the number of balls in a small basket + 3 × the number of balls in large basket = n

Strategy 2:
2 × kk + 3 × kb = n

Figure 8: Example of Students’ Answer of Activity 2 Problems

The teacher then assigned students to complete the next activities at home because the time had been up. Figure 9 displays the example of students' conclusions about variables, coefficients, and constants.



Answer: Variable = alphabet/symbol representing/subtituting unknown value.

Answer: 5a + 2b – 3c + 4d + 9
Variables: a, b, c, d
Coefficients: 1, 2, 3, 4
Constant: 9

Figure 9: Example of Students' Conclusion about Algebraic Expression

DISCUSSION

The RME-based LT of algebraic expression in this study was designed following the Gravemeijer and Cobb model consisting of preparing for the experiment, design, and retrospective analysis phases (Gravemeijer & Cobb, 2013). We did not accommodate the division of algebraic expression in the designed Prototype-2 because the activities proposed in the Indonesian mathematics textbook have involved vertical mathematization, which was in line with our idea of teaching division of algebraic expressions.

Based on the pilot experiment of LT-1, it was observed that at the beginning of the learning process, in line with the statement of Rudyanto et al. (2019), the students in this study also have difficulty understanding the meaning of variables. While they could easily find pairs of numbers added up to a number, e.g., to 10 as in Activity 2b in the worksheet, they faced difficulty when the question involved a variable. Rather than answering $(10 - x)$, two students assumed the x with a number, for example, with 7, then wrote $7 + (10 - 7) = 10$. This case became additional evidence to the statement of Jupri & Drivers (2016) that students might face difficulties when a calculation involves algebraic expression.

It took more time for the students to understand the meaning of the variable. The classroom discussion talked about the solution to Activity 2b that consists of a variable represented by n, which also confused the students. Therefore, rather than directly use the variable n, the teacher proposed abbreviations such as kk (from *keranjang kecil*) and kb (from *keranjang besar*), respectively represented Indonesian terms for the small basket and large basket. We argued that using abbreviation to represent the number of balls inside the baskets help the students to understand problems consisting of variables because the students could answer the problems in Activity 3b using either general variables such as x and y or abbreviation such as kk and kb. Furthermore, in Activity 4, the students used either x and y or F and R to represent Farhan and Rusna.

During the learning process, it was observed that the students need more time to understand the symbolic representation rather than verbal and pictorial representation. This case is in line with Novitasari, Usodo, & Fitriana's (2021) study, which stated that most of the students who participated in their study did not reach a good ability in symbolic representation. However, the students' answers to problems in Activity 3 to 5 and the conclusion presented in Figure 9 indicated that, to some extent, the students have been able to achieve the learning objective of LT-1; they have been able to understand the meaning of variables, coefficients, and constants in algebraic expressions.

There were several obstacles encountered related to the implementation of online learning. Firstly, it was challenging to engage students to participate in group discussion actively, whereas one of the principles of RME is the interactivity between every learning component and subject (Treffers, 1978; van den Heuvel-Panhuizen & Drijvers, 2014). The observation of the online learning process indicated that students were more active in interacting with their teachers in class discussions than interactions with their group members. During the reflection session after the learning involving the teacher and the researchers, the teacher stated that the direct approach she often used to teach could be one reason for the students' being less active in discussion, especially in group discussion. This statement is in line with Webb & Peck (2020), stating that "teachers' decisions and actions are influenced a milieu of personal and contextual factors that include teachers' prior experiences (including the apprenticeship of observation), teachers' beliefs about mathematics and teaching and learning, local curricular policies, available resources, the expectations of the community, and other factors." The next obstacle was in the time allocation used by the teacher. The time allocation for checking students' attendance, assigning group members, and the introductory activities should be managed well so all designed activities may be done together in class with teacher assistance. These obstacles become one remark to revise the starting activity in the LT-2 to ensure that all students had really achieved the learning objective.

CONCLUSIONS

The current study aims at evaluating the RME-based learning trajectory oriented to enhance students' mathematical representation ability on algebraic expression. The results suggest that the LT-1 designed in this study meets the valid criteria and, to some extent, could support students' mathematical representation ability on algebraic expression. The result and discussion indicate that the students who participated in this study were more familiar with verbal and pictorial representations than symbolic representations. It was more challenging for students to express the word problems into symbols than to images. Therefore, some minor activities should be included in the LT so that each level of the emergent model flows more smoothly. This indication became one remark to revise the starting activity in the LT-2, to make sure that all students had really achieved the learning objective.

Despite the conclusion above, we acknowledge some limitations of this study. First, rather than just depending on documentation of learning trajectories, documentation of students' answer sheets, and video recording of online teaching and reflection sessions, the data of interviews should be included in this study to enhance the data triangulation. Second, as the data in this study was gathered from one group of students, thus we acknowledge that the result could not be generalized. Additionally, we did not conduct a pre-test before the teaching experiment, and we just studied students' understanding of the mathematical representation of algebraic expression through the literature. Consequently, we could not be very sure about how increase was the level of students' understanding. Last, the implication of this study suggests that teachers and educational researchers keep completing and revising learning trajectories related to the mathematical representation abilities.

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