Learning Design on Surface Area and Volume of Cylinder Using Indonesian Ethno-mathematics of Traditional Cookie maker Assisted by GeoGebra

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Abstract: "Kue Putu" is a traditional Indonesian cake that is well-known to the students. Besides, the "Kue Putu" mold is made from cylinder-shaped bamboo culm and possesses the potency for learning geometry in a meaningful way. In addition, Indonesian Realistic Mathematics Education (IRME) is a mathematical learning strategy in which students study using context relevant to learners' life as beginning points. The article examines a mathematics learning design on the area and volume of cylinders using ethnomathematics context carried by traditional cake mold assisted by GeoGebra. This material is designed to help students understand the relationship between surface area and volume by examining the diameter and height of the cylinder. In developing the design, a research method, namely Design research, was applied. Following Design Research from the Gravemeijer model, we tested design research in terms of Hypothetical learning trajectory on area and volume of cylinders in the three phases, namely preliminary design, teaching experiment, and retrospective analysis. The study resulted in the theoretical design and practical instruments based on the method that contributes to instructional theory and supports student learning on the area and volume of the cylinder. Students show excellent reasoning on how increasing cylindrical radius gives a more significant effect than increasing its height. The student also construes the design of the cylinder that provides the most considerable volume by expanding its base or radius.

INTRODUCTION

Since Realistic mathematics education (RME) developed in the Netherlands, it is now widely spread worldwide, including in Indonesia. RME has a context in its characters, and it plays a significant role in learning. A good context can be developed from mathematics as it can be imagined or derived from local situations. In Indonesia, RME has livened up Indonesian teaching and learning activities as an alternating approach (Wijaya, 2012; Prahmana et al., 2020). Even

RME is originally from the Netherlands (Van den Heuvel-Panhuizen & Drijvers, 2014) it becomes Pendidikan Matematika Realistik Indonesia (PMRI) as it Indonesian version that uses local context in it designs (Zulkardi, Putri, & Wijaya, 2020). As context plays a significant role in learning mathematics, a local context close to the student is chosen as part of PMRI design (Putri, Dolk, & Zulkardi, 2015; Nursyahidah, Saputro, & Albab, 2020). Many of them using traditional games (Putri, 2012; Nursyahidah, Putri & Somakim, 2013; Prahmana et al., 2012) and other using Indonesian national heritage like batik (Cici et al., 2014; Widada et al., 2019a; Widada et al., 2019b). One of the local contexts that can also be promoted as a context in mathematics learning is ethnomathematics. Ethno-mathematics itself defines as activities using mathematics that is used by particular ethnic groups such as farmers, anglers, etc. (Nursyahidah, Albab, & Saputro, 2019). Categorizing ethnomathematics as an activity means that ethnomathematics exhibit a human work using mathematics. If it is not an activity, it is excluded from ethnomathematics. In line with this definition, Freudenthal said that mathematics is a human activity. The teacher needs to introduce Mathematics in the same way: using activities (Sembiring, Hadi, & Dolk, 2008).

The philosophy and local heritage are excellent for learning design in the mathematics classroom (Nursyahidah, Saputro, & Albab, 2020). Local heritage offers an eminent starting point in learning mathematics. It serves as an initiator for the student to develop their concept knowledge, algorithm, or tools. Later, when students generalize their informal knowledge to formal knowledge, context can be less used anymore (Van den Heuvel-Panhuizen, 2020). Interesting ethnomathematics can be selected from the activity conducted by traditional food vendors like *Putu Bambu*, a traditional Indonesian cake made from rice and palm sugar. The context is about the mold of the cake in cylindrical form. As the mold is made from bamboo trees, it is frequently ended with non-identical mold. There is a diameters variation in its molds. This situation will be a perfect starting point for the student to construe the relation between the volume and diameter of the cylinder. Learning surface area and volume of the cylinder in this way is lacking in Indonesian textbooks (Khoironisyah, 2007). This topic and other similar topics are taught in a traditional setting where students do not understand mathematics.

Even many activities are identified as ethnomathematics, not all of them can be a good context in learning mathematics. In this issue, Van den Heuvel-Panhuizen (2005) suggested criteria on how context will serve as a proper context: context must be able to make the problem transparent and increase student accessibility to the problem, context exposes all sides of the problem, and context provides an essential strategy for students. In this study, we use the wisdom from Indonesian culture, “Kue Putu”. The mold of *Putu* cake is made from the bamboo culm. However, not all of the bamboo culm is identical. *Putu* cake makers tend to make the mold differ based on the diameter of the bamboo culm. Because the mold is not similar, this case can be an excellent ethnomathematical context to examine the volume of different mold producing the same amount of *Putu*.

After choosing the proper context, one step of using context as a starting point in mathematics learning, developing a model, is crucial. Mathematics in RME will not be taught as formulas that students are ready to keep in their memory. The concept of the surface area and volume of the cylinder is developing using a design heuristic represented in the iceberg hierarchy from informal to formal mathematization (Gravemeijer & Bakker, 2006). Ethno-mathematics in the local community often grows as informal mathematics or just an intuitive phase without any confirmation. Learning sequence designed using ethnomathematics drawn in Hypothetical Learning Trajectory (HLT) helps students understand the situation. Furthermore, they use it in formal mathematization.

HLT is a set of the aim of design, set of activities, and student thinking conjectures. HLT proposed evocation of decisive aspect of designing mathematics lesson. Simon and Tzur (2004) suggest that HLT should fulfill three elements: the aim of student learning activities, mathematical activities that support student learning, and conjecture about how the student will respond to the task.

To help the students understand the cylinder material in this study, GeoGebra was used in students' activities. By using GeoGebra can have a positive impact to support students understanding of the properties of a geometry object and the concepts of volume and surface area of 3-D shapes like a cylinder (Putra et al., 2021). In line with this, Dogruer & Akyus (2020) stated that students' understanding of 3D shapes enhanced simultaneously with argumentation and dynamic geometry software of GeoGebra assisted them in visualizing and reasoning in that concept. In addition, using GeoGebra can help stimulate the emergence of several aspects in solving the problem and demonstrate understanding of the concepts being studied (Sukirwan, et al. 2018; Hernández, Díaz, & Machín, 2020).

This study seeks to determine whether or not a mathematics learning design on the area and volume of a cylinder utilizing ethnomathematics context using traditional cake mold assisted by GeoGebra enhances students' knowledge of the relationship between surface area and volume of a cylinder.

METHODOLOGY OF RESEARCH

To examine the instructional design that aids students in problem-solving, we propose the Gravemeijer Design Research Model (Gravemeijer & Cobb, 2006). Design research has five traits: intervention character, process-orientedness, reflecting component, cyclical nature, and theory orientation (Gravemeijer, 2004; Prahmana, 2017). Additionally, design research has been selected for this study since it is a systematic and adaptable strategy for enhancing the quality of classroom instruction through collaboration among researchers and educators to construct a learning design (Gravemeijer, 1994). The design research is divided into three stages: preliminary design, design experiment, and retrospective analysis (Bakker, 2004; Gravemeijer & Cobb, 2006).

Preliminary research

The preliminary design stage focuses on developing learning activities and instruments to assess the learning process. A literature survey was undertaken in this research on the concepts of surface area and volume of a cylinder, PMRI, and analysis of a cylinder material in the Indonesian mathematics curriculum to create a conjecture of students' thinking. Besides, in this phase, Hypothetical Learning Trajectory (HLT) is made as a design and instrument. HLT as design contains activity goals, activities, and conjecture of student response to activity done by them. In this research, there are four activities in understanding the concept of the cylinder. In the second role, HLT guides the direction and focus of research analysis (Alim et al., 2020). HLT is improving during this step using a thought experiment, an experiment in which we guess and check the possibility of student thinking from the experienced teacher's perspective. In this phase also, HLT can be adjusted from the findings at each stage (Nuraida & Amam, 2019).

Design experiment

The design experiment step comprises two cycles. The first is a pilot experiment that attempts to evaluate and enhance the intended learning trajectory in a small group. The second is a teaching experiment in which a learning trajectory is implemented and assessed during the actual learning in the classroom. The students of grade IX SMP were involved in the classroom teaching experiment. Six students participated in the pilot experiment, and 26 students participated in the classroom teaching experiment. This research was assisted by a model teacher and was conducted in the first semester of the 2020/2021 school year at SMPN 38 Semarang, Indonesia.

The data collected in this study were students' strategies in carrying out activities. The research instruments were class observation sheets that included field notes, student activity sheets, and interviews with students and teachers. Student difficulties and their strategy in solving a problem in the activities are used to improve HLT. Techniques pursued in this phase are focus group observation, video-typing, and walkthrough.

Retrospective analysis

The final phase is a retrospective analysis. The data obtained during the design experiment step are assessed by comparing conjecture and HLT to the learning trajectory implementation outcomes during the experiment phase's design (Gravemeijer & Cobb, 2006). Moreover, student difficulties and strategies found from the study are then analyzed findings and notes on how students respond or student strategy on the situation in design. By comparing student thinking conjecture and results in teaching experiments, research questions are solved. The following is an overview of the research framework in this study.

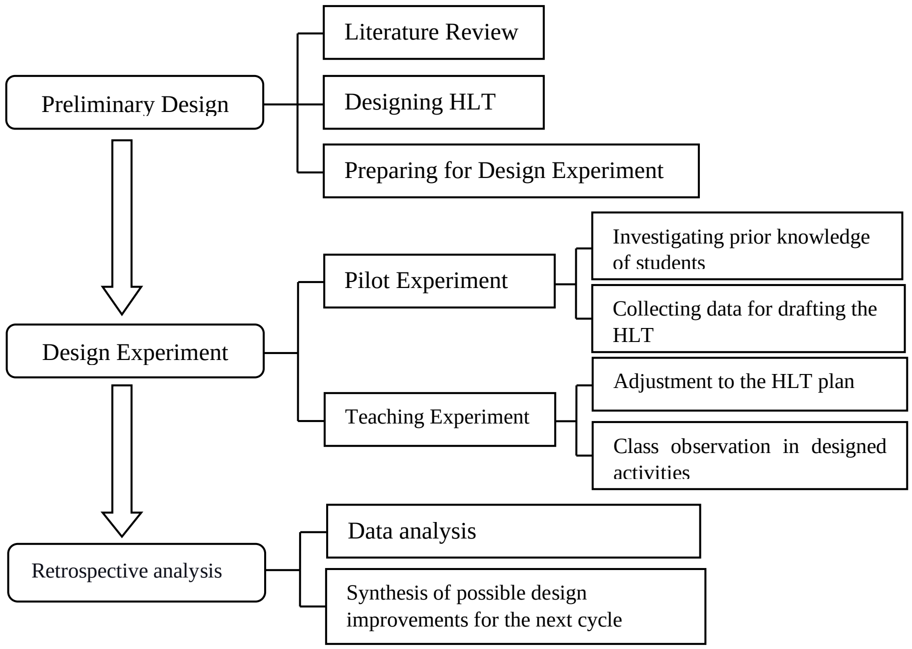


Figure 1: Framework of Research

RESULT AND DISCUSSION

As a result of this research, we acquired a learning trajectory description of the cylinder's surface area and volume utilizing the *Putu* cake mold assisted by GeoGebra. Furthermore, in this chapter, the researchers describe the results obtained throughout each study stage as follows.

Preliminary Design

In this phase, the researchers implemented the initial idea of learning the surface area and volume of the cylinder utilizing the context of the *Putu* cake mold. *Putu's* cake mold was picked as the context in this study since its shape represents a cylinder and is familiar for students, enabling students to grasp the ideas of surface area and volume of a cylinder. Additionally, there is an exciting aspect to the situation of creating *Putu* cake molds from bamboo trees. Because the sizes of the bamboo culms are not identical, the variation that arises is that the molds have different diameters. To keep each mold holding the same filling, *Putu's* cake-cutter craftsman makes a few manipulations. The variations involve making short culms of bamboo with a large diameter and making long culms of bamboo small in diameter. It will be an intriguing topic to discuss with pupils.

Figure 2. *Putu's* cake in the bamboo mold

Moreover, the development of HLT in every learning activity is the most crucial part of designing student learning activities. The design is inseparable from the learning trajectory, which contains a lesson plan for teaching the material. In this case, the learning trajectory is a concept map that students will pass during the learning process. In addition, the HLT planning process was done by conducting a literature review, conducting observations, and designing a learning trajectory for the surface area and volume of the cylinder as a series of instructional learning. Also, a curriculum review to ensure that lessons based on the mathematical standards were appropriate for students in the ninth grade. The analytical process involves establishing instructional materials, objectives of the lesson, and learning indicators. The learning process developed by HLT includes four activities for three meetings. Each activity is simple, engaging, and delightful. For detail, Table 1 summarizes the activities and the student's conjectures in this study.

Name of activity	Aim	Conjecture of student response
Creating the "Kue Putu" mold	To identify the components of the cylinder	- Students are familiar with and capable of describing the shape of the "Kue Putu" mold; afterward, they create the model using paper - Students make different types of cylinder nets - Students are puzzled when attempting to determine the elements of a cylinder - Students attach the cylinder model's net recklessly and without regard for the proper size to create a cylinder model's net
Determining the formula for a cylinder's surface area	To devise a formula for the cylinder's surface area through cylinder nets that find in the previous activity	- Students investigate determining the cylinders' surface area by adding the surface area of the cylinder net - Students locate if the cylinder's surface area equals the sum of the base area multiplied by two and the curved surface area

		- Students find the area of cylinder nets thru employing their knowledge of circle and rectangular areas - Students discover the curved surface is equal to a rectangle with a length is the cylinder base circumference ($2\pi r$), and width is t (height of cylinder)
Discovering the formula of cylinder volume	To discover a formula for the volume of the cylinder using the assistance of students' worksheets by identifying the formula principle of the prism's volume	- Students can recall the volume formula of the prism - Students might conclude by determining the prism volume formula if the volume formula of the cylinder equals the base area times height
Resolve problems related to volume and surface area of cylinders	- To investigate whether the different dimensions of cylinders have the same or almost the same volume or not - To explain why changes in height give a more negligible impact on the cylinder volume than changes in volume - To assess that same surface area provides variation in volume - To maximize the volume of a cylinder using fixed surface area	- Students can discern the effect of each height and radius on the volume - Students think that the *Putu* mold is unfairly filled - Students can comprehend the instructions - Students believe that both designs can produce different amounts of volume

Table 1: The outline of the learning process's activities and hypotheses

Design Experiment

At this stage, the researcher implemented a learning trajectory designed for ninth-grade junior high school students on the cylinders' surface area and volume using the *Putu* cake mold. However, in the mids of the covid-19 pandemic, learning takes place online. We efficiently track student work progress through the use of Geogebra Classroom and Google Meet conferencing. Figure 3 illustrates the GeoGebra Classroom activity panel.

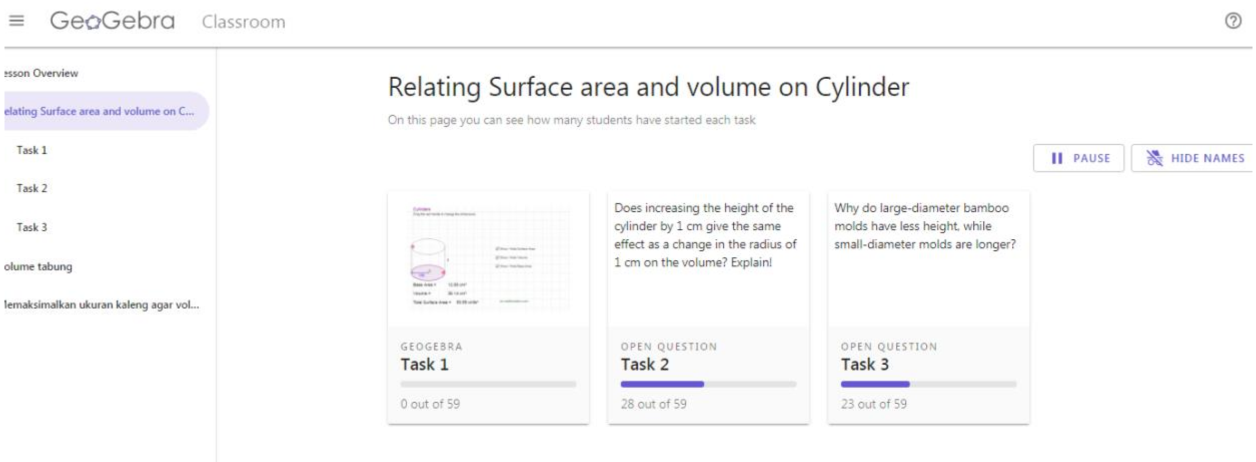


Figure 3: GeoGebra Classroom activity panel

Besides, there are four activities carried out in the design experiment stage, including:

Activity 1: Creating the "Kue Putu" mold to identify the components of the cylinder

Before beginning the class, the teacher activated the audio of the cake putu vendor, which sounded like the shrill sound typical with the steam boiler "Ngiiiingggg". When the teacher turned on the audio, the students appeared eager to learn about the material that would be discussed. Following that, the teacher offered a salutation and questioned the pupils, "Do you know what sound I played earlier?" Most students said that the sound was that of a *Putu* cake vendor. In contrast, others responded that it was the sound of chimneys, steam boilers, and broken machines. The teacher then confirmed that the voice was indeed that of a cake *Putu* vendor.

Additionally, the instructor inquired about the *Putu* cake. The teacher addressed clarifying questions to ascertain pupils' understanding of the *Putu* cake mold, which will serve as the context for the learning process. Students can make several references to the *Putu* cake mold, as demonstrated in Dialogue 1.

Dialogue 1.
Teacher : *Have you bought a putu cake from a peddler?*
Students : *Yes, I had.*
Teacher : *What do you know about Putu cake?*
Students : *Putu cake form a small cylinder with a brown sugar filling and a coconut topping. This made me starve.*
Teacher : *When ordering Putu cake, did you consider Putu's cake mold? Could you elaborate on what you've observed?*
Students : *The mold is made from bamboo, the shape of the mold is a cylinder, the mold is like a small drum.*

Dialogue 1 demonstrates that students were acquainted with the *Putu* cake mold and could determine if the mold represents a cylinder. It will make the learning process more fascinating for pupils to learn the surface area and cylinder volume since the context is relevant. The instructor explains the context of the *Putu* cake mold, which serves as a jumping-off point for the learning process. Following that, the teacher notifies pupils of the learning objective, which is to determine the cylinder's elements by making a *Putu* cake mold model using paper. Additionally, the teacher informs pupils about classroom activities with discussions and presentations.

The teacher then divides the pupils into several small groups of 4-5 students using the Goggle Meet breakout room to facilitate student discussion. The teacher instructs learners to construct a paper model of *Putu's* cake mold. When the teacher observed student discussion activities in the Google Meet breakout room, the students appeared engaged and attempted to create various versions of cylinder nets as *Putu's* cake molds. After successfully creating a putu cake mold model out of tube nets, students were asked to identify the elements of the cylinder. In this activity, students may quickly determine that the tube element has a circular top and bottom side and a rectangular curved side.

After the students have completed Activity 1, the teacher asks one group to share the outcomes of their discussion. One group employed Goggle Meet to present the discussion results, demonstrating the cylinder nets successfully, made for the *Putu* mold model and mentioning the cylinder elements. The teacher subsequently invites other groups to raise questions or request clarification on the results that have been presented. Following that, the teacher reinforces material topics related to nets and elements of the cylinder. Additionally, it could be concluded that students perceive the idea of the cylinder and its parts appropriately in light of the first activity's objectives.

Activity 2: Determining the formula for a cylinder's surface area

In activity 2, students are provided student worksheets and GeoGebra applets that are organized to assist them in grasping the notion of a cylinder's surface area. Students are instructed to determine the surface area of the cylinder using the cylinder nets made in the preceding activity to solve questions in the student worksheet. Then, students have drawn a net of the cylinder to identify the elements. Additionally, students can infer that the cylinder surface area equals the area of the cylinder nets. When the teacher validates the students' replies, students explain that the surface area of the cylinder is the same as the nets area because when the cylinder is opened, it forms a cylinder net and vice versa.

Students can understand that to get the formula for a cylinder's surface area, and they must first determine the area of each element. Students can readily calculate the area of the top and bottom sides of the cylinder when discussing in groups through the Goggle Meet breakout room. On the other hand, students frequently struggle to find the length and width of a curved side. Students thought that the length of the rectangle, which is the curved side was the diameter of the side of the base. Following that, the teacher asks students to try to prove it by using cylinder nets that have been made. The student then realized that their statement was wrong. Next, the teacher instructs pupils to determine the area of the curved side using the GeoGebra applet (see Figure). Students use the Geogebra applet to open the cylinder into a net of cylinders, allowing them to visualize that the length of the rectangle on the curved side of the cylinder equals the circumference of the base side of the cylinder.

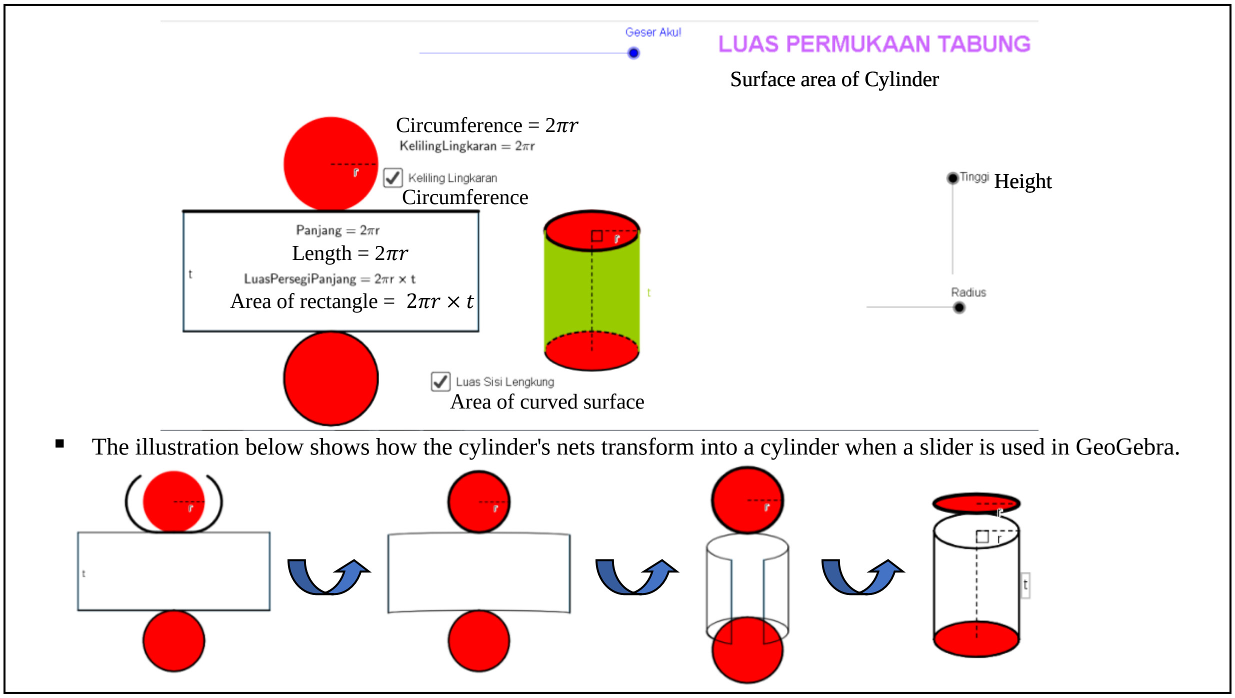


Figure 4: GeoGebra Applets to assist students in finding the surface area of the cylinder

Furthermore, based on this activity established aided by student worksheets, the cylinder nets of the *Putu* cake mold model, and the GeoGebra applet, students can deduce that the formula for a cylinder's surface area equals the sum of its base, top side, and the curved side.

Activity 3: Discovering the formula of cylinder volume

The activity carried out at this stage is the activity to discover the formula of cylinder volume. The teacher shared the student worksheet link via the Google Meet chatbox. Next, students work in groups to solve questions on the student worksheet. Students were instructed to determine the volume of a cylinder by comparing it to the volumes of a triangular prism and a cuboid. However, some students struggle to discover the connection between the volume of a cuboid, triangular prism, and a cylinder. Hence, the teacher poses several questions to the students to elicit assistance in determining the connection related to three of it, as illustrated in Dialogue 2. However, after discussing with the teacher, students could discover that the cylinder volume formula is the base area multiplied by height, where the base is a circle.

Dialogue 2.

Teacher : *What is the correlation between the (p) length and (l) width in the cube volume formula?*

Students : *It is a rectangle area formula that is on the base cuboid.*
Teacher : *So, how is the cuboid volume formula formed?*
Students : *Base area (length times width) times height of a cuboid.*
Teacher : *Ok, could you consider what a connection between the volume of a cuboid, a triangular prism, and a cylinder is?*
Students : *Ohhh, I know Mam, the volume equals base area times height*

Activity 4: Resolve problems related to volume and surface area of cylinders

To give students a progressive understanding of the surface area and volume of a cylinder, in the last activity teacher gave two problems related to the effect of variation in height and radius to *Putu's* mold volume and examined different volumes from the same surface.

The first problem is related to the local wisdom of *Putu* bamboo makers. This problem would assist students in investigating whether the different dimensions of cylinders have the same or almost the same volume and explain why changes in height have less impact on the cylinder volume than changes in radius. Students face situations on Putu bamboo mold, traditional food made from rice flour and palm sugar steamed in Bamboo culm mold. The molds are not identical. Students need to investigate whether the molds will give the same volume. This investigation was carried out using a manipulative applet adapted from GeoGebra sources, consisting of a cylinder model with surface area and volume and buttons to help students to be able to change the height and radius of the cylinder (see Figure 5). Students can explore the impact of height and radius variation on volume by dragging red buttons. Information related to surface area and volume of cylinder appears beneath the cylinder model. In these activities, students need to find the relation between height and radius variation to the volume. Then, students need to conclude that Putu makers are fair enough in making the volume the same or almost the same filling.

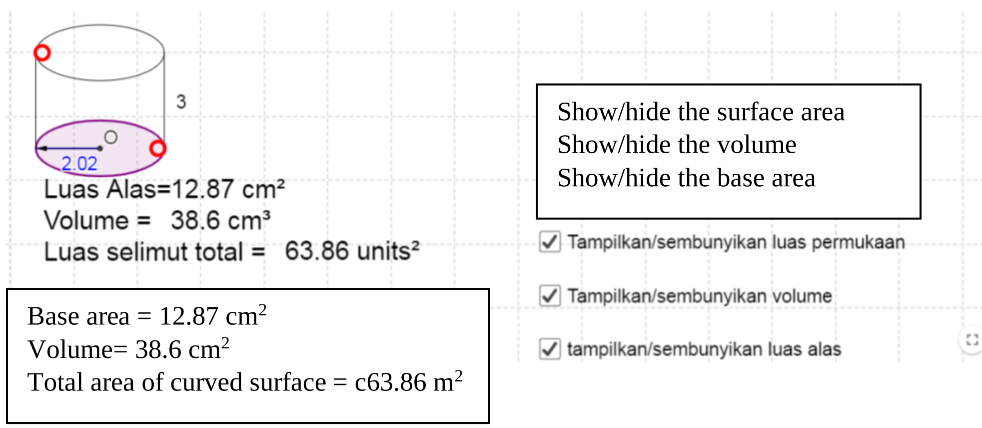
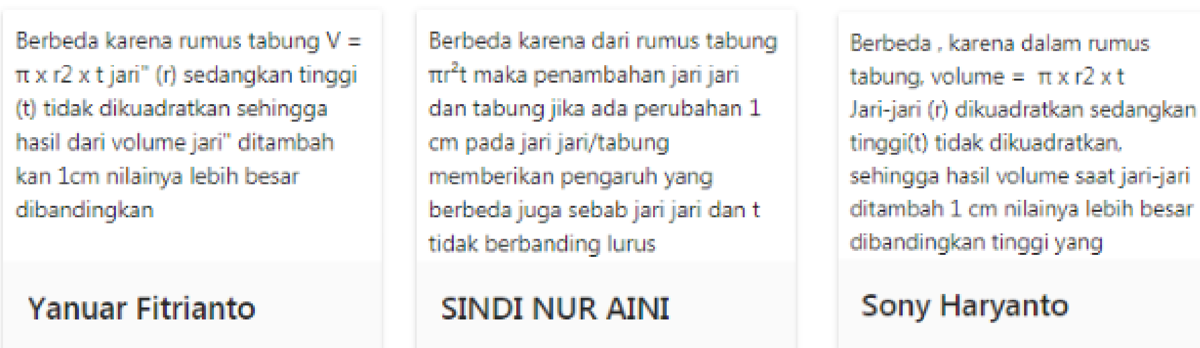


Figure 5: Problem-related to height and radius variation effect on volume

Common Issues	Suggested question to prompt
Students cannot distinguish the effect of each height and radius on the volume	"Try to change 1 unit in height. How is the volume? Now try to change 1 unit in radius. How is the volume? Which changes give much effect?
Students think that the *Putu* mold is unfair in filling	"Assume height and radius of "thin" and "fat" mold, calculate the volume, and compare the volume! Is it significantly different or almost the same?

Tabel 2: Question to prompt the students to conjecture in the first problem

Responses to this activity vary, but most share ideas that the impact of variation in height and radius gives a different result. Figure 6 explains the student's argument that radius variation gives a more significant effect on the volume.

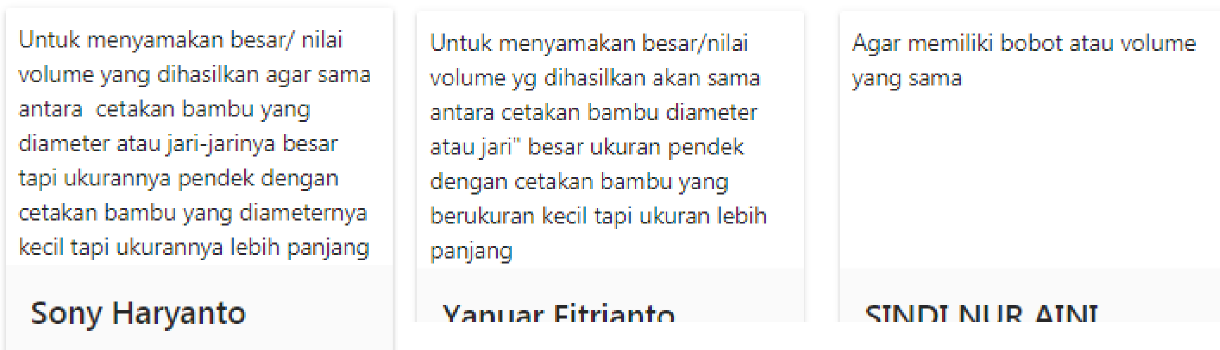


variation in height and radius give different (impact) because formula of cylinder volume is $V = \pi r^2 t$. Radius of cylinder is squared, while t (height) does not. Adding 1 cm in radius gives greater that adding 1 cm in cylinder height	variation in height and radius give different (impact) because formula of cylinder volume is $\pi r^2 t$. Both r and t are not linear correlated	variation in height and radius give different (impact), because formula of cylinder volume is $V = \pi r^2 t$. Radius of cylinder is squared, while t does not.
Yanuar Fitrianto	Sindi Nur Aini	Sony Haryanto

Figure 6: Sharing idea of student related to the variation of height and radius of cylinder to the volume

Based on Figure 6, students are reasoning in a progressive mathematics way. It is fascinating, in any case. They compare 'r' and 't' forms in the cylinder formula, which is 'r' is squared, so 'r' addition gives quadratic growth in volume. Different from 'r', adding 1 unit in 't' only gives linear volume growth.

From this reasoning, students start to understand Putu's mold height differentiation. They agree that artisans make short culms of bamboo with a large diameter and long culms of bamboo with a small diameter to maintain the same filling volume. Furthermore, Figure 7 tells us how Putu's mold craftsman idea supports student thinking.



To equalize the size of the volume produced by bamboo molds with a larger diameter or radius with a shorter and bamboo prints with a smaller diameter or radius but higher	To equalize volume or filling of putu cake, bamboo mold with large diameter cut into short size, whether bamboo mold with narrow radius cut in long size	To produce the same volume
Sony Haryanto	Yanuar Fitrianto	Sindi Nur Aini

Figure 7: Student thinking Putu's Bamboo mold manipulation

Furthermore, the second issue linked the aluminum plate. This challenge could help students understand why the same surface area produces different volumes and maximize the volume of a cylinder with a fixed surface area. In this problem, students face situations on two different canned beverage packaging made of aluminum plate sheets. The packaging is slim and fat even though it is made of the same size plate. Students need to explore and investigate which packaging gives the most significant volume. This investigation was carried out using a manipulative applet adapted from GeoGebra Sources, consisting of an aluminum plat model with surface area and volume and buttons to help students to be able to change the height and radius of the cylinder (see Figure 8). Then, the students need to answer a question "to make a packaging that can have the most considerable volume, which side should be coincided?"

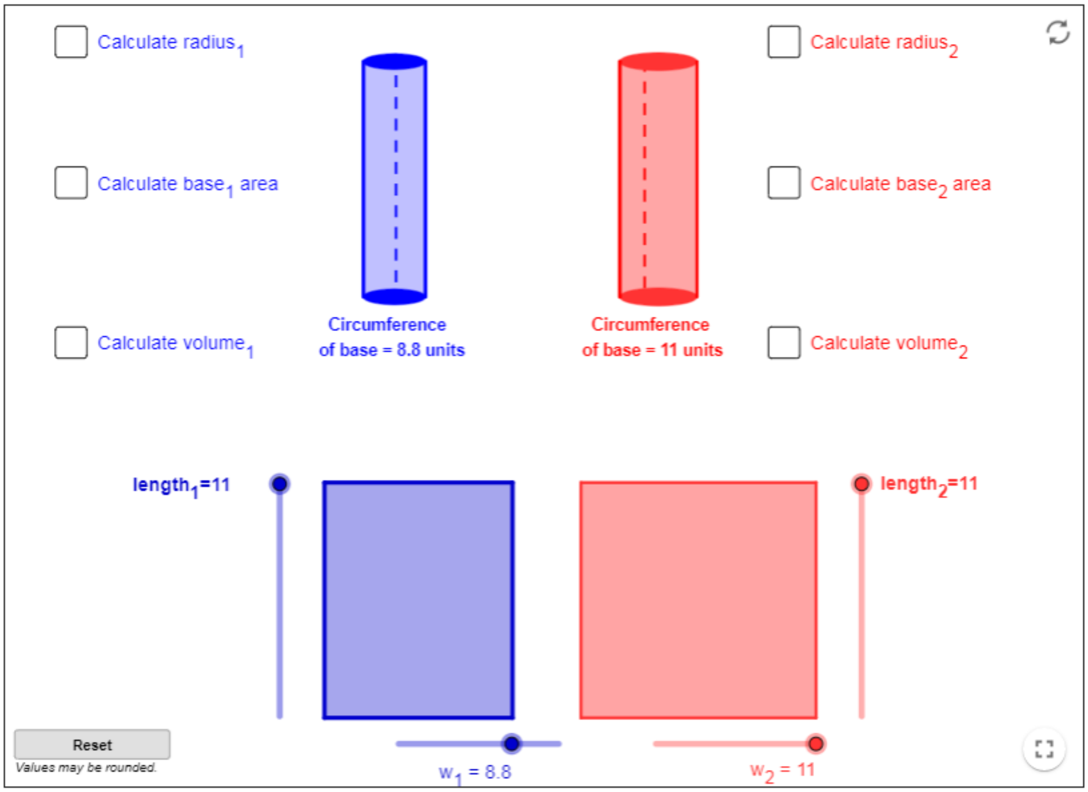


Figure 8: Activities examining different volumes from the same surface area

Common Issues	Suggested question to prompt
Students don't understand the instruction	"Take a sheet of paper. Coincide short side. What cylinder do you have? Now consider coinciding the long side of your paper. What cylinder do you have? Are both cylinders identical? How do you compare which design gives the most significant volume?"
Students think that the two designs can give the same volume	"Set the plate in swapped position—the height of the fat can serve as the perimeter of cylinder base. Compare the volume?"

Tabel 3: Question to prompt the students to conjecture in the first problem

Students are asked to deepen their understanding of the relationship between radius, height, and surface to cylinder volume in the second problem. How if the mold of Putu is made from the aluminum board. With the same surface area (identic size board), does a cylinder with the base made by coinciding the longest side give a more significant volume than a cylinder with the base made by coinciding the narrow side of the plate. Figure 9 shows the variation of cylinder design. This task facilitates students to examine different volumes that may be produced from the same surface area. The question to this task is which design of cylinder (coinciding narrow or longest) the most significant volume gives?

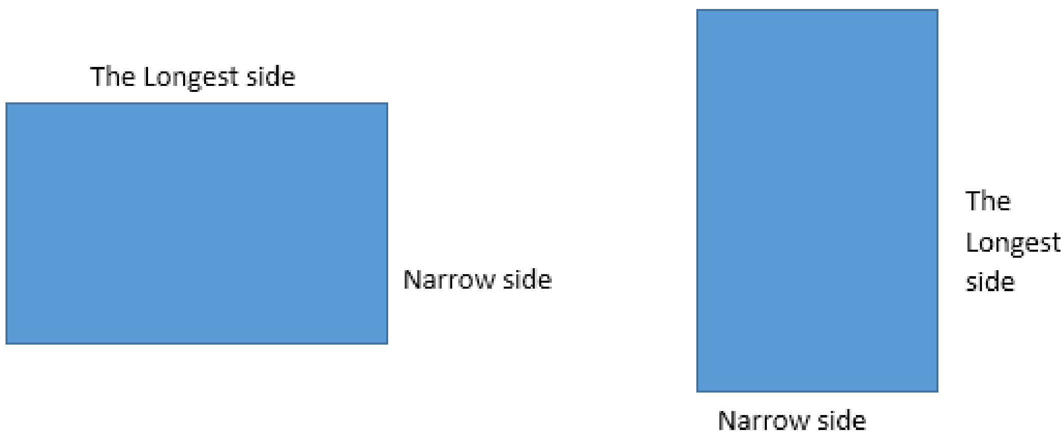


Figure 9: Designs of two cylinders, coinciding longest side and narrow side

Students respond to this task by taking an example of measure 8 × 10. Figure 10 tells how students explore applets using the exact size of the aluminum board to make the different designs of cylinders.

Figure 10: Applet of comparing two designs of cylinders using the same aluminum board

Most students answered that cylinder of mold that gives the most significant volume is the fat one. This design coincides with the longest side of the aluminum plate. Figure 11 shows an example of a student's response.

Answer

jika ukuran selimut tabung sama yaitu (8x10) dan (10x8) maka akan memberikan luas selimut yang sama. Namun, akan didapat volume yang berbeda. jika dilihat dari perhitungan diatas, yang memberi volume terbesar pada saat tabung dengan ukuran plat(selimut) (10x8)

If the plate area is the same (8×10) and (10×8), the surface area of the cylinder is the same. However, the volumes differ. From the calculation above, (design) that gives the biggest volume is a cylinder that coinciding the longest side of the board (10×8)

Figure 11: Student argues that fat model gives the most significant volume

In this response, students take 10×8 and 8×10 as measurements of the cylinder. The most significant volume comes from a cylinder coinciding with the longest side, making it the base of the cylinder. The most extensive mold design is the fat model.

Retrospective Analysis

The HLT shown in Table 1 serves as a guideline for responding to study objectives. HLT is contrasted about what occurs during the learning process to research and describes how learners might transfer informal methods such as surface area and volume to formal strategies. Moreover, the HLT was compared to the data gained in the design experiment to explain learners' methods and thinking processes in comprehending surface areas and the volume of a cylinder using the context given.

As predicted by the student conjecture, the first problem given demonstrates that students used excellent reasoning. Students get that the purpose of *Putu* mold height differentiation is to achieve the same volume. The learner recognizes that a large mold diameter equals the volume of long bamboo culms with a tiny diameter. All of the students' thoughts are predicted in the learning conjecture and no confusing question. It can be seen from the student's answer. They did well on the question.

The second problem in Activity 4 indicates that the student realizes that surface area gives different volume amounts. Students inferred that even though these plates have the same size, the volume produced was different. However, in this activity, we lack information that the base of the cylinder made from the position was not measured. For the second cycle, we need to improve the size of plates and the base made after construction. We gave them more support because students lacked in assuming plate size. In the next cycle, the plate should be fixed-sized and placed in a portrait-like and landscape-like position.

Finding from this study indicates that ethnomathematics plays a good starting point to understanding mathematical ideas. The situation in ethnomathematics close to the student is compatible with Freudhental (1991) views. The student also realizes and respects the wisdom of local communities. Mathematics also exists in the traditional cake. Even mathematics in this community is not practiced as formal mathematics, and they struggle to solve the problem using mathematics (Nursyahidah, Albab, & Saputro, 2020). It is recommended to explore more local wisdom to be a context in mathematics. This idea is compatible with the Van den Heuvel-Panhuizen (2005) thought that the proper context could make the problem transparent and increase student accessibility to the problem; context exposes all sides of the issue. Context provides an essential strategy for students. It is supported from the finding in Figure 6 that students realizing bamboo mold differ in height is to anticipate reasonably in the volume of cake.

Besides, based on activities 1 to 4, all RME characteristics applied throughout the learning process, such as learners' involvement and interaction, have been depicted. This includes pupils finding the surface area and volume of the cylinder by their ideas, as well as pupils interacting with each other during discussion groups and demonstrations. Furthermore, intertwining occurs when students utilize the circle and rectangular area concepts to get the cylinder's surface area and the prism volume idea to determine the cylinder's volume. Additionally, the traits of model-based learning are demonstrated. At the same time, learners use paper as a medium for modeling the Putu cake mold to locate the cylinder net and the elements, and when students used GeoGebra applet to solve problems related to surface area and volume of a cylinder. Additionally, the use of context was incorporated by utilizing a *Putu* cake mold to represent a cylinder.

It is essential to teach students using RME because it makes students construe the high ability of reasoning. Student deliberately uses their inferences using the connection of the formula they get from the previous grade and the fact they found from the applet. Learning mathematics using the RME approach also arms students with mathematical modeling to the situation and develop student mentally to generalize pattern using advanced modeling call model. This mathematization process brings ethnomathematics far beyond its origins. Mathematics is well known and taught in school because it is presented formally proven theory. If HLT of many ethnomathematics of Indonesian origin is established, it will give an outstanding contribution to teaching and learning of RME, or PMRI in Indonesia. It also helps culture and national heritage be safe from extinction.

CONCLUSIONS

The design learning developed in this study consists of four activities, namely: creating the "Kue Putu" mold to identify the components of the cylinder, determining the formula for a cylinder's surface area, discovering the formula of cylinder volume, and resolve problems related to volume and surface areas of cylinders. Besides, using ethnomathematics as a context for learning mathematics support student understand of the relation between surface areas, height, and radius to the volume of a cylinder. The cake mold of *Putu* Bamboo helps the student to be able to make the problem transparent and increase student accessibility to the problem, context exposes all sides of the issue, and context provides an essential strategy for students. Students show excellent methods and reasoning during the classes and how increasing the cylindrical radius will give a more significant effect than increasing its height than that height does. The student also construes the design of the cylinder that gives the biggest volume by expanding its base or radius.

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