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Ramazan EROL¹, Elif SAYGI²

¹Afyon Kocatepe University,  0000-0002-2619-9084

²Hacettepe University,  0000-0001-8811-4747

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The Effect of Using Cryptology on Understanding of Function Concept*

Ramazan EROL^{1**}, Elif SAYGI²

¹Afyon Kocatepe University

²Hacettepe University

Abstract

The aim of the study is to teach the concept of function using cryptology and to investigate whether it provides support to students who have difficulty integrating this topic into everyday life. The study's research group consisted of 50 elementary school teachers in the department of mathematics education at a public college in Ankara. Since the study aimed to show the advantages and disadvantages of using cryptology in teaching the basic function concept, qualitative data were collected and analyzed. The data collection instrument of the study is a performance test which consists of 10 questions prepared by an observer, a practicing researcher and an evaluation expert. This performance test was used as both pretest and posttest. For the functions, only function definition, inverse function, one-to-one function, set of values, set of definition, set of images, and cryptography were used where functions can be used in daily life. As a result of the study, it was found that the prospective teachers' awareness of functions has increased and it is beneficial and possible to use cryptology to relate functions to daily life.

Keywords: Function, Cryptology, Encryption, Mathematics teachers' candidates

Introduction

While mathematics has been used in the historical process to satisfy the basic needs of societies and to facilitate certain aspects of life, the rapid development of science and technology in contemporary life has affected social life and increased the importance of teaching and learning mathematics in daily life (Umay, 2007). We can say that we use mathematics in daily life even if we are aware of it or not. The concept of function, which occupies an important position in mathematics, is also very important in terms of how it is perceived by teacher candidates. The sufficiency and quality of the information in the teachers affect the quality of teaching the information. For these reasons, it has been deemed necessary to reveal all aspects of the conceptual knowledge of a teacher trainee and to reveal how much knowledge they have about the concept of function in the minds of teacher candidates (Süzer, 2011).

Functions are one of the fundamental topics of mathematics, unifying in mathematics (NCTM, 1989) and related to many topics of modern mathematics (Malik, 1980; Dreyfus & Eisenberg, 1982; Eisenberg, 1991; Ferrini-Mundy & Graham, 1991; Bowman, 1997; Mishelsen, 2006). Therefore, it can be said that functions are one of the most important concepts in mathematics. In the study of Akkoç (2006), the meaning of the concept of function according to NCTM (1989) is explained as follows:

According to NCTM 1989; *The concept of function is an important unifying concept in mathematics. The function is a special match between the two sets and is spread over the whole curriculum. Functions in arithmetic are the process that equate number pairs to a single number (such as the sum of the numbers in the number pairs); Functions in algebra: are the relations between variables, representer and counted; Functions in geometry: maps the point sets to figures with movements such as shift and rotation; Functions in probability: match events to the probability of occurrence.*

NCTM (2000) emphasizes the need to provide opportunities for all students to understand patterns, relations and functions, from pre-school to secondary education.

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** Corresponding Author: Ramazan Erol, rerol@aku.edu.tr

It is known that the subject of function has an important place in school mathematics. In this context, when the concept of function is examined in general, it provides opportunities for students to understand the relationships between variables, explain coefficient changes, analyze and interpret the graphs. (Clement, 2001). Moreover, as in NCTM (2000) and Common Core State for Mathematics (2011), the concept of function is also included in the secondary mathematics curriculum in Turkey (2018) (i.e. 10th, 11th and 12th grades). According to Ural (2006), functions completely affect the mathematics curriculum. However, it has been included in the college curriculum because it is an integrative and organizing concept. The concept of function is very important for students, mathematics educators, mathematics teachers and preservice mathematics teachers. Students in secondary education face 22 gains in total for 160 lessons during 3 years (MEB, 2018). Therefore, functions can be said to have an effective place in secondary education. In this context, examining teacher candidates' knowledge about the concept of function can give an idea and help them in the subsequent learning and teaching experiences (Süzer, 2011).

Functions do not fall under the name of functions in primary and secondary school mathematics, but mathematics subjects include topics that are the basis of the concept of function under numbers and algebra. For this reason, pre-service teachers must master concepts of function (Polat & Şahiner, 2007).

In the studies, it was found that students perceived variables such as x and y found in algebraic expressions and equations as functions. Such trends do not consider the function's definition, but take into account the concept images found in the minds of students (Özmantar, Bingölbali & Akkoç, 2010; Ural, 2006). Despite research on how to learn mathematical concepts, it is still not known how people will learn. Learning indeed occurs, but it is not known exactly which mechanism it is formed (Eisenberg, 1991). The concept of function is difficult to learn. Because, according to Sajka (2003), functions are seen as confusing. The reason for this is that the concept of function has a two-way characteristic.

In other words, students link between different representations indicating the same function and have difficulty in switching between these connections (Sierpinski, 1992). For example, students have difficulty in determining whether an algebraic expression is a function. As an example of this situation, university students have expressed ($x^2 + y^2 = 1$), a circle equation, as a function (Tall & Bakar, 1992). However, some students only know the functions as algebraic equations and they think that the graph is completely different from function (Tall & Bakar, 1991; Vinner 1983; Vinner & Dreyfus, 1989).

According to Ural (2006), students stated that they had some structural difficulties and misconceptions while learning the concept of function. The teacher plays an important role in eliminating these misconceptions and structural difficulties. Şandır (2006) stated that the subject of function should be learned at the conceptual level and more time could be spent for this. Function topics in Turkey are often taught with a memorization approach. The conceptual dimension of this is not mentioned much, but it can be said that there is a teaching method for information processing (Polat & Şahiner, 2007). It is important that the persistence of mathematical knowledge and the fact that conceptual knowledge and knowledge processing support each other in a balanced manner. When the conceptual knowledge is learned, the information processing will gain a meaningful dimension (Hiebert & Lefevre, 1986). However, it is seen that the functions are learned in the level of knowledge processing and not fully integrated into daily life (Ural, 2006; Ural, 2007; Polat & Şahiner, 2007).

Establishing the connection between mathematics and students' daily lives is one of mathematics teaching goals (NCTM, 2000). In this context, in many studies conducted, it is seen that encryption activities are in the teaching of mathematics (Bachman, Ezra & Norton, 2010; Chua, 2006; Chua, 2008; Evered & Gningue, 2001; Özdemir & Erdoğan, 2011; Özdemir & Yıldız, 2012; Özgeldi & Osmanoğlu, 2017; Göktepe Yıldız & Özdemir 2015; Özdemir & Geçim, 2014; Özdemir & Erbay, 2015; Hall, 2003; Hamilton & Yankosky, 2004; Kaur, 2008).

It can be said that the concept of function has an important place in the current mathematics curriculum. In this context, it may be appropriate to learn the structural side of the concept of a function and to adapt it with different mathematical models in daily life. This kind of mathematical modelling is an example of cryptology consisting of mathematical algorithms.

Conceptual Frameworks

The word "Cryptology" is composed of the words "kryptos" and "logos" in ancient Greek. Cryptology today is a mathematical science that uses many disciplines such as electronics, optics and computer science (Saygı & Umay, 2010).

According to Çimen, Akleylek & Akyıldız (2011), cryptology is the branch of mathematics that includes both cryptography (cryptograph) and password analysis (cryptanalysis). The purpose of password science is to ensure the security of the message sent, and the purpose of password analysis is to solve the existing passwords. In cryptology, there are encryption and decryption applications. Encryption is the ability to perform information by using several mathematical operations or by making the information to be displaced according to a particular algorithm. Deciphering is to make the complex message meaningful using some mathematical algorithms (Saygı & Umay, 2010; Çimen, Akleylek & Akyıldız, 2011). According to Menezes (2001), 2000 years ago, the Roman Emperor Julius Caesar used a simple replacement code, known as Caesar Cryptography, a classic example of symmetric key cryptography.

Since cryptology in general is a mathematical science based on number theory, the algorithms of cryptology consist entirely of mathematical functions. For this reason, cryptology is also used to encrypt one-to-one functions and encode the messages and decode the reverse functions (Saygı & Umay, 2010). As can be seen from the related literature, functions can be a difficult subject to learn (Tall & Bakar, 1991; Vinner 1983; Vinner & Dreyfus, 1989; Sajka, 2003; Ural, 2006; Ural, 2007; Polat & Şahiner, 2007). In this context, cryptology can be an important tool to be used to teach basic function concepts. Cryptology is thought to be an interesting way to gain the concept of function and an interesting and popular science. In this study, it is aimed to teach preservice teachers basic functions using cryptology.

Purpose and Importance of Research

The aim of this study is to teach basic functional concepts using cryptology and to help students who have difficulty integrating this subject into their daily lives. It also seeks to answer the question of which of the sub-concepts of function is effective in the classroom.

Research Problem and Its Sub-Problems

Within the scope of this research: "How is the learning process of mathematics teacher candidates studying in the 1st grade of the university develop in an application where basic function concepts are taught using cryptology?" The sub-problems for the problem as mentioned above sentence are as follows:

1. Is there a statistically significant difference between the pre-test and post-test scores in the achievement test that measures basic function concepts of mathematics preservice teachers studying at the university?
2. How did the preservice mathematics teacher in the 1st grade describe the concept of function throughout the application?
3. How did the first-grade mathematics teacher candidates define the concept of inverse function?
4. How is the development of the function's usefulness in daily life in terms of preservice mathematics teachers studying at the university 1st class?

Method

This study is an empirical investigation aimed at showing the advantages and disadvantages of using cryptology in teaching the concept of function to the first grade mathematics students in a public university in Ankara. In the study, the same test was administered twice at different times to the same group. In other words, since the effect of the study is investigated on a single group, the research design is a single group, pre-test and post-test, which is a weak experimental study. Weak experimental studies have no selectivity (Fraenkel and Wallen, 2009; Büyüköztürk 2012). The experimental study was supported by collecting qualitative data to investigate the conceptual development of using cryptology in teaching basic functional subjects and the conceptual development of mathematics teachers in the university.

Group	Pre-test	Process	Post -test
G	O₁	X	O₂

Figure 1. Single group, pre-test, and post-test pattern according to Fraenkel and Wallen

Data Collection Tools

Three instruments were used to collect data. A performance test, working papers, and records. The achievement test was developed by the researcher and observer in the presence of an assessment expert. The test consists of 10 questions. This achievement test was used with the participants as both a pre-test and a final test. The working papers contain 4 different activities prepared by the researcher and the practitioner. The working

papers are explanations in envelopes within each activity. The purpose of these activities is to teach basic functional concepts using cryptology. The audio recordings were recorded by the researcher during the four week course.

Data Collection Process

The implementation process took a total of four weeks. In each week of the implementation process, the concept of basic function and the concept of cryptology were explained by the practice researcher. The organization of the applications, recording of the audio, and the necessary explanations during the activities were done by the observer. The data collection instruments such as performance test, worksheets and voice recordings were used as follows. The achievement test was administered for two weeks, the first week and the last week. The pre-test was administered before the first week and the post-test was administered after the end of the last week. The speech recordings were recorded every week during the course. And the speech recordings were recorded during the group discussion.

After these recordings were transcribed by the researcher, the opinions of the prospective teachers on the topic of function were analyzed. The answer sheets that were part of the activities were put into the envelopes that were distributed to the groups for 4 weeks to explain them. It is planned that the activities will be carried out in practice before and each week by an observer and a practicing researcher. In addition, the lecturer worked for four weeks. The observer was involved in the organization and implementation of the activities. In this study, the pretest and posttest responses of the prospective teachers were analyzed. The following is a detailed description of each application.

The first week of the study was conducted in three course sessions. The achievement test, which was developed by the observer and the researcher in the presence of an expert in measurement and assessment, was used as a pretest for the pre-service teachers before the first week of the course. The knowledge of the pre-service teachers about the basic functional topics was measured by this achievement test (pretest). The achievement test was developed by the observer and the researcher from the field with reference to the relevant literature. The achievement test consists of 10 questions. The validity and reliability of the test was ensured by expert opinions of 3 mathematics educators.

In the second week, pre-service teachers were asked to write cryptic text as a function and find the definition and value sets of the function in which the cryptic text is written. The activity, lecture and group discussions were conducted in three lessons.

In the first lesson, the concepts of cryptology, password, key and the history of cryptography were explained. In the second lesson, a new activity was introduced to the topic by recalling the concepts from a week ago.

In the third week, the course started with the question-answer technique on the topic of functions without going into the first lesson. A brief reminder of the previous two lessons was included. The last two lessons were on surjective functions and one-to-one functions.

This week, one of the basic function concepts: one-to-one and surjective functions were set as the topic for a lecture. In the fourth week, one-to-one function (1-1), surjective function, and inverse function with closed texts were distributed to pre-service teachers when the last week of teaching basic function concepts using cryptology had come.

Data Analysis

The responses of pre-service teachers to the functional test were qualitatively transferred into the computer. Responses that were close were collated and converted into a table. The aim is to determine the frequency of responses that are close to each other. The closeness of the responses to each other was determined by observing the responses of the pre-service teachers to the basic concepts of function by the observer and the researching practitioner. In this way, the pretest and posttest responses were examined and the pre-service teachers' development of their definition of basic concepts of function was explored. For questions (1, 2, 3, 6, 7, 8, 9, and 10) that measure conceptual information found in the pretest and posttest, "1" represents correct responses and "0" represents incorrect responses. In addition, question 2 also measures operational information. Questions 4 and 5 test information about function and cryptology. SPSS 22 software was used to analyze the scores obtained. Since this is the only group in the study, t-test was used to see if there was a significant difference between the means of the pre-test and post-test scores. The statistical significance of the difference (p) was tested at the 0.05 level.

In addition to statistical analysis, qualitative content analysis was also used in this study. Büyüköztürk(2012: 236) defines the purpose of qualitative research as "trying to understand the participants from their point of view". In other words, such studies focus on the discourses of different participants on a topic (Büyüköztürk, 2012). By analyzing the collected data, descriptive facts were revealed by revealing the themes and patterns related to the situation under study (Yıldırım & Şimşek, 2016; Creswell, 2013).

According to Fraenkel and Wallen (2009), the internal validity of a study means that the relationship or effect between two or more variables that one is trying to observe exists only between these variables without the effect of any other variable. (2012) According to Büyüköztürk, planning, conducting research, collecting data, analyzing given data and all similar stages are suitable for experts' opinions.

Findings

First Sub-Problem Findings and Comments

The first sub-problem of the inquiry is as follows: "Is there a statistically significant difference between pre-test and post-test scores on the achievement test that measures the basic functioning concepts of first grade mathematics teachers? In the study, before explaining the basic function concept using cryptology, participants were asked questions with the same content after applying the basic function concept using cryptology to measure students' function knowledge. The T-test was used to determine if there was a significant difference between the means of the students' scores before and after the test. The Kolmogorov Smirnov p-value of the difference between the pre-test and post-test total scores is 0.079 and it shows that the score differences are suitable for normal distribution. The calculated value of n^2 is .81. Accordingly, it can be said that 81% of the variance in post-test scores after application was caused by cryptology. This effect size reflects a broad effect.

Table 2. *T-test results of pre-service teacher*

Evaluation	N	\bar{X}	S	Sd	t	p
Post test scores	50	9.46	1.528	49	14.676	0.00
Pre test scores	50	5.46	1.717			

Examining Table 2, it is clear that the points of a study that teaches the concept of function through cryptology are given to pre-service teachers. After the study, there was a statistically significant increase in the total score of pre-service teachers, $t(49) = 14.676$; $p < .01$. The mean score of pre-service teachers was 5.46 and increased to 9.46 after explaining the function of cryptology. This result shows that the concept of basic function of cryptology has a significant impact on the success of prospective teachers (Büyüköztürk, 2012).

Results and comments on the second sub-problem.

The second sub-problem of the study was posed as "How did first grade mathematics teachers define the concept of function during application?". For this reason, the questions of the achievement test were examined before and after the test 1. Also, the incorrect answers and then the frequency of correct answers are presented in the tables. Pre-test and post-test question 1: What is the function? Please describe it. Give an example of a function. The first question of the pretest and posttest was examined in two parts. First part: what is the function? Please describe it. The second part is: Give an example of a function.

Table 3. *Pre-Test 1. Question, First Part Findings and Comments*

Pre-service Teachers' Answers	The Number of pre-service Teachers' (f)
This is function in definition with expressions like $f(x)$.	17
I don't know, I can't explain.	15
This is the Equation System.	13
Those who make the right definition.	5

In the pretest, 45 pre-service teachers answered the first part of Question 1 incorrectly, while five pre-service teachers answered correctly. In the study of Table 3, pre-service teachers are expected to answer the question "What is the function?" with Erbaş (2013: 45), namely, "Function: is a special mapping between the two sets, and x and y are called a function corresponding to a single element of y , where each element of x corresponds to two clusters." However, in pretest 1. it was found that pre-service teachers could not give a clear answer on the definition of function.

Pre-service teachers: In a process defined in the set of values from the definition set of E, the empty element must not be left in the set and each element I select from the definition set must correspond to only one element in the set of values and this process is called a function. Teacher E's answer was correctly accepted. One of the other answers is that the expression " $f(x) = mx + n$ " is called a function. In this answer, pre-service teacher R thought that the expression " $f(x)$ " could be a function. In the pretest part 1, it can be seen that most of the pre-service teachers cannot define the function.

Table 4. Post Test 1. Question First Part's Answers Findings and Comments

pre-service Teachers' Answers	The Number of pre-service Teachers' (f)
Those who make the wrong definition.	6
This is a custom mapping with the Definition Set and the Value Set.	18
Those who use the question of "from where and to" in the definition.	14
Those who use "two non-empty sets..." in the definition.	12

In the first part of the pre-test, 44 of the 50 teacher candidates responded correctly. When Table 4 is examined, it can be said that teacher candidates have positively defined the concept of function. According to the information given in Table 3, the teacher candidates did not answer "I do not know, do not remember" in the last test. It is a desirable development that preservice teachers use two sets of non-empty statements in their definitions. The teacher candidate B's answer is: "A non-null set of A from a non-null set of A will have no unmapped elements in the set A, and each element in A will match a single element from B, for example: $f: \mathbb{N} \rightarrow \mathbb{N} f(x) = 2x$."

Table 5. Pre-test 1. Question, second part finding and comments

Pre-service Teachers' Answers	The Number of pre-service Teachers (n)
No examples.	30
Ones that do not specify definitions and sets of values (as indicated by $f(x)$).	8
Ones that give true answers like $(f: \mathbb{R} \rightarrow \mathbb{R} f(x) = ax + b)$	7
Figure drawing by responding correctly. (with Venn diagram)	5

Pre-test 1. question In the second part of the test, 12 of 50 preservice teachers were accepted as correct. When Table 5 is examined, preservice teachers were asked to specify sets of definitions and values when asked "give an example of a function". So $(f: \mathbb{R} \rightarrow \mathbb{R} \text{ and } a, b \in \mathbb{R} = f(x) = ax + b)$ in the form of mathematical symbols and expressions were expected them to give these as examples. Accordingly, it was observed that only 7 preservice teachers gave their examples as desired. "Example: $f: \mathbb{R} \rightarrow \mathbb{R} f(x) = 2x + 1$ " is the answer of the teacher candidate D, who gave one of the correct examples. What is highlighted in Table 5 is that the majority of teacher candidates cannot give an example.

Table 6. Post Test 1. Question, Second Part Answers Findings and Comments

Pre-service Teachers' Answers	The Number of pre-service Teachers (f)
No examples.	7
Ones that do not specify definitions and sets of values (as indicated by $f(x)$).	5
Ones that give true answers like $(f: \mathbb{R} \rightarrow \mathbb{R} f(x) = ax + b)$	26
Figure drawing by responding correctly. (with Venn diagram)	12

In the second part of the post-test 1. The answer of 38 people in the second part was considered correct. When Table 6 is examined, preservice teachers' sample of mathematical symbols and expressions showed a positive development according to Table 5 ($f = 7$). In addition, the number of teacher candidates who do not give any examples in Table 5 is 30 and this number has decreased to 7 in the post-test.

Findings and Comments Related to the Third Sub-Problem

The third sub-problem of the study was defined as "How did the first-grade mathematics teacher candidates define the concept of inverse function?". The findings of this sub-problem were reached by analyzing the 3rd and 10th questions of the achievement test. Pre-test and post-test 3. question: find the inverse of the function given by this rule ($3 f: \mathbb{Z} \rightarrow \mathbb{Z}, f(x) = 3x - 2$).

Table 7. Pre-Test 3. Question Findings and Comments

pre-service Teachers' Answers	The Number of pre-service Teachers (f)
$f^{-1}(x) = (x+2)/3$	49
He wrote this $((y+2)/3)$ statement and the result was missing.	1

Looking at the above table, we find that no one can give the correct answer to the 3rd question in the achievement test, which we have described as the question that measures the conceptual knowledge of teacher candidates. In this question, it was found that the prospective teachers did not consider the definition and value sets in this function. It can be said that what Özyıldırım (2015) defined as mechanical knowledge was solved by Hiebert and Lefevre (1986) with operational information. The correct answer to the 3rd question is: $(f: Z \rightarrow Z, f(x) = 3x - 2)$ the inverse of the function is undefined in the set Z .

Table 8. Post-Test 3. Question Findings and Comments

Pre-service Teachers' Answers	The Number of pre-service Teachers (f)
$f^{-1}(x) = (x+2)/3$	5
Those who wrote that the function f is not defined in this $(f: Z \rightarrow Z)$ range.	18
Those who, by giving value to "x", wrote that the function is not defined in Z .	18
Those who wrote that "Because "f(x) is not one-on-one and a surjective, it does not have a its reverse".	9

When Table 8 is examined, it was observed that only 5 people gave the wrong answer and 45 of the teacher candidates gave correct answers. According to Table 7, preservice teachers paid attention to whether the function was defined in the definition range. Teacher's answer M: "I cannot find the inverse of the function given by this rule and I can't say that "f: Z → Z f(x) = (x + 2) / 3" and "f(2) = 4/3" but not "4/3" integer. But if I change its whereabouts, I can find the opposite. For example, if: R → R, I have a range."

The other question analyzed to identify the inverse of the function is the 10. question of pre-test and post-test: Is there an inverse of each function? Please explain.

In the analysis, it was observed that 23 teacher candidates gave the desired answer and explanation in the last question of the pre-test. However, only 15 people who wrote the "no" answer did not comment. In the last test, it was observed that all preservice teachers gave the correct answer and made an explanation. It was observed that teacher candidates' answers have conditions of one-to-one and surjective in the explanation.

Findings and Comments Related to the Fourth Sub-Problem

The fourth subproblem of the study: how do first grade mathematics teacher candidates develop their knowledge of the utility of function in daily life. The results of this sub-problem were determined using the 4th and 5th questions of the pretest and the posttest.

Question 4 of the pretest and posttest: How do you explain the meaning of the function to another person or to a student who has never seen the function before?

Table 9. Pre-test 4. Question's answers, findings and comments

Pre-service Teachers' Answers	The Number of pre-service Teachers (f)
I don't know, I don't remember, I have no idea	16
I explain with the subject of equations.	7
I explain it through the function examples.	5
I illustrate these with clusters.	11
I give examples from daily life (machine example, factory sample, mother-child relationship, puzzle)	11

In this question, the answer of 22 people who gave the following answer (I give examples from daily life and draw a figure and explain it with clusters) was accepted as correct. It is seen that 28 preservice teachers "do not know" the function as they are understood. According to Table 9, it can be said that preservice teachers are incomplete in explaining the function. The response of teacher candidate Y, who was one of the 11 people who gave examples from everyday life: I described this using visuals, gave examples from daily life and explained the definition under the mother's name and the value under the child's name with examples. I would say that there are no 2 elements

in the set of values in the value set. For the 4th question in the pre-test, it can be said that pre-service teachers prefer to explain functions through shapes.

Table 10. Post Test 4. Question, Findings and Comments

Pre-service Teachers' Answers	The Number of pre-service Teachers (n)
I describe it as a conversion process.	4
I draw a figure.	4
I explain it with the sets and matching.	19
I give examples of everyday life.	14
With encryption by using the alphabet.	9

It was observed that preservice teachers preferred to explain the subject of function with the subject of clusters and pairing. The answer of 42 people in this question can be considered right, according to Erbaş (2013). The answer "I do not know, cannot tell" in Table 9 is not given here. Considering cryptology-based activities, those who prefer to tell by encryption are limited to 9 people have shown that this answer is not preferred. For example the teacher candidate O's answer: "We have an element and we do this by doing various operations on another element. It refers to the function as a conversion process."

Another question related to the fourth sub-problem was reached by examining the pre-test and post-test question 5. Question 5 of the pre-test and post -test: Give an example of where the function can be used in everyday life.

Table 11. Pre-Test 5. Findings and Comments Related to the Question

Pre-service Teachers' Answers	The Number of pre-service Teachers (n)
I do not know.	25
Used in problem examples.	8
Those who say that it is used in integral, derivative, civil engineering, architecture.	3
Those who say that it is used in calculator, electronic tools, Excel and computer programs.	10
Those who say that it is used in encrypted doors and encryption methods.	4

A remarkable point, if we examine Table 11: In the 5th question of the pretest, students predict that the function can be used in daily life for encryption methods. Another notable case is that the answer of 36 students is considered wrong. One of the teacher candidates gave the following example: For example, we can use this in the relation between salary and rent, $f(x) = 3x + 700$, $f(x)$ = salary and x = rent of the house. He also explained that the algebraic expression given in the function can be used in everyday life.

In the fifth question of the last test, 38 teacher candidates about the use of function in daily life say that functions can be used in cryptology. On the other hand, 10 students repeated in both the pretest and the last test that these functions are used in calculators, Excel and computer programs.

When the answers of pre-service teachers to questions 1, 3 and 10 were examined in the pretest, it was found that these answers were based on memorization. However, it can be said that they have difficulty in defining the concept of function and giving examples. It was observed that algebraic expressions were defined as function or even expressions of $f(x)$ as direct function. When examining the answer sheets, it can be seen that 45 participants are unable to define the function in question 1 and therefore the information provided by these participants is incomplete. However, when examining the answers to question 1 in the last test, it was found that 6 people could not define a function. Therefore, it can be said that the activities carried out in the last period have reduced this lack of knowledge. In particular, question 3 shows that pre-service teachers do not pay attention to the definition domain of the function and that the inverse function and the function are learned by the method of memorization according to Polat & Şahiner (2007). In the results on the fourth subproblem, the results of the pretest showed that the teacher candidates could not integrate the concept of function into daily life. As a result, it can be said that the pretest answers generally do not contain information about the topic of functions and the operational information is based on the memorization method. The results of the pretest on teacher candidates' ideas about where the function is used in daily life are incomplete and inaccurate. However, this situation changes when the posttest responses are examined.

Discussion

This study aims to examine the changes in the learning of the basic function concepts of primary school mathematics preservice teachers using cryptology. For this purpose, 50 teacher candidates were compared with the pre-test and last -test results of the development of basic function concepts (definition of function, one-to-one function, inverse function, set of values, set of definitions) by implementing cryptology activities. In the findings obtained from the study, it was observed that the preservice teachers did not remember the concept of function according to the pre-test results and the majority of the class remained incomplete. In this context, as Hitt (1998) states, the definition of function associated with various concepts is preferred by preservice teachers.

One of the biggest problems in terms of function is that the number of representations of this concept is very high (Eisenberg, 1991). When the teacher candidates were asked to give an example of a function, it was concluded that they did not give the correct example, and the examples given were incomplete. In this seemingly simple question (the question asked for the function example), there were troubles. According to a study by Akkoç (2005), mathematics teacher candidates preferred to define the function verbally. However, they tried to give an example by using a cluster matching diagram.

In the first week of the application, teacher candidates solved the encrypted texts in the activities, but it was found that they could not reconcile this with the concept of a function. Many authors, such as Sierpinska (1992), Carlson (1998) and Clement (2001), found that students who succeeded in algebra courses at the university could define a function with a simple algebraic formula (Dubinsky, 2013). Towards the end of the implementation process, it was observed that the teacher candidates were able to make the function definition at the desired level in the activities and the final test results. In addition, it was seen that preservice teachers had written a function example in a correct and meaningful way.

As Hitt, (1998) stated in his study, preservice teachers prefer to create a matching rule or a sequential binary when defining the function. However, it was seen in this study of the basic function concepts based on cryptology, the teacher candidates mentioned that they had a mapping rule in defining the one-to-one function. The functions in cryptology applications are exact but they have not made any explanations. Moreover, as a result of the application, as in the pretest, they defined the one-to-one function from the definition set to the value set as a set matching graph. It can be concluded that the applications of cryptology activities that represent the concept of one-to-one function, the basic concept of function, do not produce changes in the development of pre-service teachers. Hiebert & Lefevre (1986), Özyıldırım (2015) defined operational information, mathematical symbols and representations as follows: mechanical knowledge capable of defining formulas and performing process steps appropriately. In this context, pre-service teachers defined the inverse function from the basic function concepts and stated that there is no inverse function for any function. However, they also stated that there is no inversion of one-to-one and non-surjective functions. However, in pretest question 3, it was observed that they tried to take the answer to the question without considering this information. In this case, it was observed that the teachers presented a memorised solution. Carlson (1998) in his study found that university students have difficulty in producing functions that correspond to real life. When examining the results of the pretest on the use of the concept of function in daily life, it is found that most of them have no knowledge of this topic. Nevertheless, 4 teacher candidates stated that they used encoding methods in the pretest. According to the results of the posttest and the answers about the activities, it can be said that the teacher candidates know the concept of function in daily life.

Recommendations

It can be said that the study is a new application in the literature. It was found that cryptology has entered the new educational research. In this sense, other mathematical topics can be integrated with cryptology in education. The study is a descriptive study with a pre-test load test in a group, which is one of the weak experimental designs. The design of the study can be changed to a quasi-experimental design and a new study can be conducted by forming control and experimental groups.

The study can be conducted not only for the first year mathematics teacher candidates but also for the secondary, high school and elementary students. The research is based on teaching the basic functions of cryptology activities. In addition, a research on cryptology and matrices can be developed. However, it is also possible to offer various researches on patterns that can form the basis of functions for primary and secondary school students. In addition to these subjects, the subject of equations can also be taught to primary and secondary school students with the help of cryptology.

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