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Realistic Mathematics Education's Effect on Students' Performance and Attitudes: A Case of Ellipse Topics Learning

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Abstract: Realistic Mathematics Education (RME) has gained popularity worldwide to teach mathematics using real-world problems. This study investigates the effectiveness of elliptic topics taught to 10th graders in a Vietnamese high school and students' attitudes toward learning. The RME model was used to guide 45 students in an experimental class, while the conventional model was applied to instruct 42 students in the control class. Data collection methods included observation, pre-test, post-test, and a student opinion survey. The experimental results confirm the test results, and the experimental class's learning outcomes were significantly higher than that of the control class's students. Besides, student participation in learning activities and attitudes toward learning were significantly higher in the RME model class than in the control class. Students will construct their mathematical knowledge based on real-life situations. The organization of teaching according to RME is not only a new method of teaching but innovation in thinking about teaching mathematics.

Keywords: Equation of an ellipse, learning outcomes, Realistic Mathematics Education, real-world problems, student feedback.

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Introduction

Students typically have an intuitive grasp of certain mathematical concepts long before being introduced to concepts in a formal classroom (Deniz & Kabael, 2017). This suggests the value of developing mathematics education in which students apply mathematical knowledge to real-world situations (Sumirattana et al., 2017). In 2108, the Vietnamese Ministry of Education and Training enforced the General Education Curriculum in Mathematics, allowing schools and teachers to design math learning curricula. Thus, math teachers can use new and modern teaching methods like Realistic Mathematics Education or RME (Do et al., 2021), which promises things to make a major adjustment in mathematics education in Vietnam (Tran et al., 2020). An analysis of mathematics education reform in the Netherlands by Gravemeijer et al. (2016) finds that RME is appropriate for a broad range of reforms, founded on the belief that students must actively form their knowledge (Gravemeijer et al., 2016).

RME was conceived and developed in the Netherlands (Ardiyani et al., 2018; Van den Heuvel-Panhuizen & Drijvers, 2014). According to Freudenthal, students learn mathematics by interacting with real-world problems and reconstructing their mathematical knowledge with the help of teachers (Freudenthal, 1991, as cited in Bray & Tangney, 2016; Laurens et al., 2017; Yilmaz, 2020). Freudenthal considers mathematics as a human activity (Freudenthal, 1973). Furthermore, mathematics is seen as a human activity that is related to practice (Kusumaningsih et al., 2018; Laurens et al., 2017; Makonye, 2014; Mulbar & Zaki, 2018; Nguyen et al., 2019; Peni, 2019; Sumirattana et al., 2017; Van den Heuvel-Panhuizen & Drijvers, 2014).

Real-world situations (or real-life situations) are prominent in the learning process in 'the light of the RME'. The term

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"realistic" not only reflects a connection with the real world but also refers to problems that most students can imagine (Ardiyani et al., 2018; Karaca & Özkaya, 2017; Scherer, 2020; Van den Heuvel-Panhuizen & Drijvers, 2014). Because students already apply formal, general, and minimal mathematics to real-world problems and situations. These personal experiences serve as the context for the initial development of mathematical concepts, tools, and processes (Sumirattana et al., 2017; Van den Heuvel-Panhuizen & Drijvers, 2014). Through RME, students participate and make real-world decisions based on their prior knowledge and experience under the guidance and organization of the teacher (Scherer, 2020).

RME has five characteristics, including (1) using real-world contexts, (2) developing models to turn original situations into mathematical problems, (3) students reproducing guided formation of mathematical concepts, (4) student-teacher interaction, and (5) view of mathematics as an integrated subject (Bray & Tangney, 2016; Clements & Sarama, 2013; Kusumaningsih et al., 2018; Laurens et al., 2017).

As a result, educators distinguish between horizontal mathematization and vertical mathematization (Drijvers et al., 2019; Laurens et al., 2017; Makonye, 2014; Treffers, 1987, 1991, as cited in Deniz & Kabaal, 2017; Yilmaz, 2020; Zolkower et al., 2020). By converting real-world problems into mathematical problems, students use what is known as horizontal mathematization. Thanks to horizontal mathematization, students are asked to abstract concepts with symbols and solve problems using different models or algorithms (Yilmaz, 2020), thus tapping into students' creativity (Arifin et al., 2021). The process of vertical mathematization involves abstracting the conception in the world of symbols and then solving the problem by using alternative models or algorithms to locate the relevant algorithm (Treffers, 1987, 1991, as cited in Yilmaz, 2020). These two processes are depicted in Figure 1.

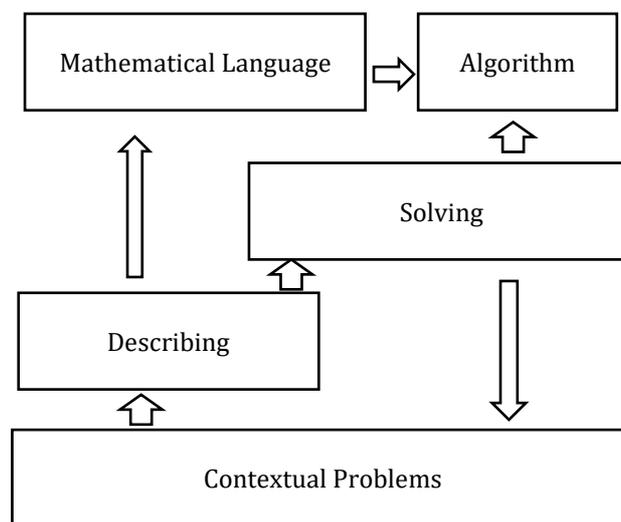


Figure 1. Horizontal Mathematization and Vertical Mathematization (adapted from Gravemeijer, 1994, as cited in Yilmaz, 2020)

When working with mathematical operations, certain characteristics arise. A guided reinvention, didactical phenomenology, and emergent models are three heuristics designed by RME (Gravemeijer, 1999, as cited in Gravemeijer, 2020a, 2020b; Sumirattana et al., 2017; Wahyudi et al., 2017; Yilmaz, 2020). The idea of RME is that mathematics is a human activity; it revolves around supporting the shift from informal knowledge to formal knowledge through hands-on practice with real-world problems. (Yilmaz, 2020). According to Freudenthal, one of the most important characteristics of mathematical operations is the organization, including the subject matter and the mathematical problem. This research seeks to understand how and why phenomena are organized using mathematical concepts, processes, or rules (Freudenthal, 1973, as cited in Gravemeijer, 2020a). Meanwhile, emergent models use a series of submodules towards a general model. The general model of formal mathematical reasoning was developed from ordinary mathematical operations (Gravemeijer, 2020a). The model's appearance does not differ between "model-of" and "model-for," but rather the students' thinking. Thus, "model-of" refers to a practical operation, while "model-for" refers to a mathematical framework or emerging mathematical practice (Gravemeijer, 2020a).

With the transition from "model-of" to "model-for", Gravemeijer distinguishes four levels of activity as follows: (1) Situational activity in the task context and realism in the person's experience; (2) Referential activity, where the model refers to the activity in the task context; (3) General activity, where models refer to the framework of mathematical relations; and (4) Formal mathematical reasoning no longer depends on "models-for" mathematical operations (Deniz & Kabaal, 2017; Gravemeijer, 1999, as cited in Gravemeijer, 2020b; Gravemeijer, 2020a).

In addition to the above characteristics, the RME model of mathematics instruction establishes several fundamental principles. Treffers (1978, as cited in Van den Heuvel-Panhuizen & Drijvers, 2014) offers six principles: (1) activity

principle, where students are viewed as active participants in the learning process; (2) reality principle, where education in mathematics should be based on real-life situations that help students develop problem-solving skills; (3) level principle, where students learn mathematics by creating context-relevant scenarios, reducing and generalizing concepts and strategies, and analyzing their relationships; (4) intertwinement principle, where mathematics integrates numbers, geometry, measurement, and data so students can solve problems with a variety of tools and mathematical knowledge; (5) interactivity principle, where students can learn more about a topic by collaborating with each other and exchanging ideas and findings, and (6) guidance principle, where teachers actively engage students in their education, and educational programs emphasize scenarios to help students understand (Nguyen et al., 2019; Nguyen, Trinh & Ngo et al., 2020; Treffers, as cited in Van den Heuvel-Panhuizen & Drijvers, 2014; Van den Heuvel-Panhuizen & Drijvers, 2014; Wahyudi et al., 2017).

Teaching mathematics at various levels has shown many benefits using RME's characteristics. Using meaningful real-life situations and mathematization, RME increases student engagement by increasing student interest and motivation (Bray & Tangney, 2016; Dickinson et al., 2020; Karaca & Özkaya, 2017; Laurens et al., 2017; Mulbar & Zaki, 2018; Putri et al., 2019). According to Makonye (2014), Peni (2019), and Yilmaz (2020), RME improves students' understanding of the relationship between informed and procedural knowledge, thereby improving students' math literacy and academic performance (Ardiyani et al., 2018; Dickinson et al., 2020; Mulbar & Zaki, 2018; Peni, 2019). Also, the RME approach to mathematics education helps students develop mathematical skills (Peni, 2019; Sumirattana et al., 2017). Many studies have shown that RME improves mathematical problem-solving skills (Yilmaz, 2020; Yuanita et al., 2018), mathematical thinking (Kusumaningsih et al., 2018; Laurens et al., 2017; Putri et al., 2019), critical and creative thinking (Laurens et al., 2017; Rudyanto et al., 2019), mathematical reasoning (Saleh et al., 2018; Yilmaz, 2020), and mathematical communication competence (Andriani & Fauzan, 2019). RME helps students apply math in real-world situations by influencing their knowledge, skills, and attitudes (Kusumaningsih et al., 2018; Sumirattana et al., 2017).

Many studies have shown that using RME with other instructional tools can be effective. Studies show that this method improves student engagement, learning attitudes, and creativity (Bray & Tangney, 2016; Rudyanto et al., 2019). RME-based teaching also incorporates multiple representation strategies such as orientation, discovery, association, and evaluation using verbal, visual, mathematical representations (Kusumaningsih et al., 2018; Makonye, 2014) and worksheets (Mulbar & Zaki, 2018).

A hands-on program with traditional assessment encourages students to focus on passing the exam rather than the actual learning process (English, 2000, as cited in Nguyen, Trinh & Ngo et al., 2020). Hence, RME needs to be understood as an input (i.e., program) and output (assessment). Consistent with this point of view, Van den Heuvel Panhuizen emphasizes several requirements for a problem to be considered suitable for evaluation in RME (Van den Heuvel-Panhuizen, 2005, as cited in Dickinson et al., 2020). Taking charge of the situation is solvable and accessible. The problems also allow students to show fundamentals to higher-order mathematical thinking. When faced with unfamiliar situations, students can develop solutions on multiple levels. The problem situation must be suitable for the student to apply prior knowledge and experience (Van den Heuvel-Panhuizen, 2005, as cited in Dickinson et al., 2020). Mulbar and Zaki (2018) suggest that teachers evaluate RME teaching effectiveness based on three factors: student engagement, student achievement, and student responses to RME design.

RME in mathematics education continues to pose many challenges for teachers and schools. According to Yilmaz (2020), although teachers understand RME theory, they do not necessarily connect it to teaching methods. Also, many teachers struggle to connect real-world issues to RME and create authentic activities (Yilmaz, 2020). Vos (2018) argues that in many of the tasks given, the authentic and non-authentic aspects are combined (i.e., with an authentic context, but the questions are artificial and different from the others), what people would ask in practice), while many studies have shown that students are more motivated to learn through authentic questions rather than authentic contexts. Therefore, it is not easy to design lesson plans and especially to find practical examples suitable for the mathematical concepts to be taught and prepare a learning environment that allows the re-formation process to take place is quite complex. Similarly, teachers may struggle with creating lesson plans, and a learning environment that allows for complex re-formation is not easy if they lack an understanding of concepts and their nature (Yilmaz, 2020). Dickinson et al. (2020) say that implementing RME in mathematics classes can take up a lot of class time and cause students and teachers to resist learning methods they deem unnecessary and slow.

Theoretical Background

In Viet Nam, RME has gained increasing attention primarily thanks to research at teacher-training universities (Nguyen et al., 2019). Moreover, the General Education Curriculum in Mathematics (GEMC), promulgated by the Ministry of Education and Training of Viet Nam in 2018, now connects mathematics to real-life situations and other subjects, and in some publications in Viet Nam have introduced (Do et al., 2021; Nguyen, 2018; Nguyen, Trinh & Ngo et al., 2020; Pham & Pham, 2018). Lessons designed using the RME approach can assess students' learning outcomes, interests, and knowledge-building abilities, but research demonstrating RME implementation and student evaluation results are lacking. This study investigates how students learn about an ellipse, a geometrical concept that's commonly taught.

An ellipse applies to many fields of study, including physics, astronomy, architecture, and engineering. Viet Nam's GEMC emphasizes the importance of "solving some practical problems associated with three conic lines (including ellipse), for example, explaining some phenomena in Optics, determining the orbits of the planets in the solar system..." (Viet Nam Ministry of Education and Training [VMoET], 2018). Because the goal of this study was for students' abilities to solve real-world problems and their attitudes toward learning to improve, the RME model was incorporated into the design of elliptic equations teaching situations.

By the criteria outlined in Table 1, the study provides guidelines for determining the level of knowledge that students are expected to attain.

Table 1. Measuring Required Knowledge

Required knowledge	Methods
Describing the definition of an ellipse	Using tests
Identifying the elements of the ellipse (lengths of axes, coordinates of vertices, coordinates of focal point, focal length).	
Solving real-world problems about ellipses	

The following are some of the skill requirements to be achieved for the elliptical equation lesson presented in Table 2, specifically:

Table 2. The Scale of Skill Level to Be Achieved for the Ellipse Equation

Skills	Criteria	Levels		
		Level 1	Level 2	Level 3
Identifying the ellipse with visual images (Skill 1)	Defining ellipse shape (Criterion 1)	The experiment could not be performed or was not recognized elliptical shape	Performing experiments or recognizing ellipse shapes	Experimenting and recognizing the shape of an ellipse
	Indicating the elements of the ellipse (Criterion 2)	Unable to identify the names of the elements of the ellipse in the real-world problem	Pointing out the names of the elliptical elements in the real-world problem but cannot match the problem math	Pointing out the names of the elliptical elements in the practical problem and placing them corresponding to the mathematical problem
Determining the elements of the ellipse (major axis, minor axis, vertex coordinates, focal point, focal length) (Skill 2)	Determining the length of the major axis, minor axis based on the given data (Criterion 3)	The length of major and minor axes cannot be determined	Determining the correct length of the major or minor axis	Determining the correct length of the major and minor axes
	Calculating the other elements of the ellipse based on the available information (Criterion 4)	The other factors in the ellipse cannot be calculated	Calculating other elements in the ellipse but the result is not correct	Calculating other elements in the ellipse accurately.
Solving real-world problems (Skill 3)	Bringing from practical problems to math problems (Criterion 5)	Cannot bring from real-world problems to math problems	Bringing from a practical problem to a math problem but cannot determine the way to solve the problem	Bringing from practical problems to math problems and determining the way to solve the problem
	Solving real-world problems, answer questions of the problem (Criterion 6)	Cannot solve a math problem	Solving the maths problem but the result is not correct	Solving math problems and answering real-world problems correctly

Methodology

Research Goal and Questions

This study aimed to determine the feasibility and effectiveness of incorporating the RME model into the design of elliptic equations teaching situations to improve students' abilities to solve real-world problems and positively affect their learning attitudes. The research questions posed are relevant to the research objective stated previously:

1. What difference, if any, does implementing an RME-based teaching process make in terms of students' understanding of ellipse topics at the high school level?
2. Can students' problem-solving skills be improved by using a teaching process based on an RME model?
3. How has the RME model affected students' participation, motivation, and attitude toward mathematics learning?

Sample and Data Collection

The research team organized an RME training course for 20 teachers willing to volunteer in their free time. In selecting a teacher for the experimental class, we looked for someone knowledgeable about and skilled at applying the fundamental principles of implementing the RME model for teaching 45 students. As was usually the case, a teacher that was not trained utilized a conventional model to instruct a control class of 42 students. The students enrolled in this study are 10th graders at a public high school in Cu Lao Dung district, Soc Trang province of Viet Nam (from January 9, 2021, to May 10, 2021). At the time of the study, schools in Soc Trang province were open, and students continued to attend classes because the Covid-19 pandemic had not affected the province. In-person instruction was deemed necessary so that students could learn from one another.

The research looks at classes formed by the school rather than regrouping random samples, so it uses a quasi-experimental approach. The quasi-experiment was conducted similarly to the studies on Yuberti et al. (2019) and Sumirattana et al. (2017) to examine how the collected data might differ from testing a hypothesis. The data collection process is shown in Table 3.

Table 3. The Collection of Experimental Results

Group	Pre-test	Treatment	Post-test
Experimental class (EC)	Average result in Mathematics semester 1	Three lessons with teaching situations applying the RME model	Test results after the experiment Student opinion survey
Control class (CC)	Average result in Mathematics semester 1	-----	Test results after the experiment

(1) Collected data is based on the average results of semester 1 (replacing pre-test), post-test results, and students' survey results. Results from a post-test and exam papers were analyzed to determine how much students had learned about problem-solving. During this study, the test instrument was a description containing three items related to elliptic topics. Data verified to be highly dependable was like the study by Yuberti et al. (2019). This study used a description containing three problems related to elliptic equations as the instrument for the post-test. Design, construction, and medicine are three different fields of study represented by these three real-world problems. It was anticipated that students would require adequate mathematical skills based on the RME approach to complete these math problems successfully. Immediately following the lesson, each of the two classes was given an instrument to complete as a post-test. Testing and validity were required prior to launching the investigation to determine whether or not the experiment was valid and worthwhile. It was believed that the assessments were credible by two well-known mathematics education educators; a similar approach was used in Salsabila's research (2019), which proved to be effective as well. Implementation of the instruments and research was completed following the evaluation and adjustments. The integrity of the instruments was demonstrated by the fact that each individual stated that the instrument was appropriate. After much deliberation, they agreed to carry out the tests because they believed the research subject was important. Aside from that, the research team evaluated the degree to which the curriculum covered academic content and problem-solving skills about mathematics. The validity of a test can be assessed by looking at how well it covers academic content and skills, as suggested by Thao et al. (2020). This method was used to develop the test in this study. This study assessed students' ability to solve real-world problems related to elliptic curves in various fields, including design and construction and medicine. Additionally, to ensure the accuracy of the test results, the Vietnamese national marking process was used, which means that the process was conducted in two separate rounds, one after the other, to ensure that the results were accurate.

The experimental design

In response to the students' learning outcomes in the experimental and control classes, the researcher team and the teacher developed lesson plans covering the ellipse equations in the RME model application. The experimental lesson plan is proposed to be divided into three periods: new lesson, practice-and-consolidation period, and test period.

Finally, students completed a post-test and a survey about their experiences. The effectiveness of the pedagogical experiment was evaluated using both quantitative and qualitative data.

Analyzing of data

The collected data were analyzed quantitatively (with SPSS 25 software) and qualitatively. Qualitative assessments were used to show the efficacy of the treatment using the RME model before and after an intervention. It was postulated that students' average score in the experimental class would differ from the control class's average score, according to the paired t-test method. The qualitative assessment was conducted with the help of the scales in Table 3 to evaluate students' capacities for identifying problems and resolving them in a real-world context. An analysis known as the Shapiro-Wilk test was used to see if scores between the experimental and control groups were normally distributed. As a result of the small number of samples in both the experimental and control classes (less than 50), the Shapiro-Wilk test was used to determine whether the point data was normally distributed. The results indicate that the significance level (Sig.) of the data points is greater than 0.05 (see Table 4), demonstrating that these point data are normally distributed.

Table 4. Shapiro-Wilk Test Normally Distributed Pre-test and Post-test

	Shapiro-Wilk	
	Statistic	Sig.
Pre-test of experimental group	0.966	0.248
Pre-test of control group	0.980	0.677
Post-test of experimental group	0.955	0.100
Post-test of control group	0.962	0.168

The level equivalence of the experimental class and the control class before the experiment and the difference in the mean score value in the post-test of the experimental class and the control class were tested through t-test independent (2-tail). Additionally, the researchers used the Cohen influence level criteria table as well as the Pearson correlation coefficient to determine whether or not there was a correlation between data from the experimental group and the control group.

A total of four items on a Likert scale with five levels are included in the student survey statements to assess attitudes: Totally disagree - Disagree - Neutral - Agree - Totally agree. One item (4) is a multiple-choice statement about the student's favorite learning activities.

Results

Pre-test results

Table 5. Independent T-test of Scores Before Treatment

Group Statistics					
Group	N	Mean	Std. Deviation	Std. Error Mean	
Experimental group	45	6.2067	1.14800	0.17113	
Control group	42	6.1262	1.13890	0.17574	
Levene's Test for Equality of Variances					
F			Sig.		
.098			0.755		
t-test for Equality of Means					
	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference
Equal variances assumed	0.328	85	0.744	0.08048	0.24536

Descriptive statistics in Table 5 show that the mean of the experimental and control classes were 6.2067 and 6.1262, respectively, and there were no significant differences. The Sig value in the Levene test was equal to 0.755 > 0.05, so there was no difference in variance between the two experimental and control groups. The independent t-test results showed the Sig value (2-tailed) is equal to 0.744 > 0.05, so the difference in mean scores between the two classes was not statistically significant (see Table 5). Consequently, the equivalence of mathematical learning levels between the experimental and control classes can be established reasonably.

Post-test results

Table 6. Independent t-test of Scores After Treatment

Group Statistics					
Group	N	Mean	Std. Deviation	Std. Error Mean	
Experimental group	45	7.0289	1.09640	0.16344	
Control group	42	6.2619	0.96574	0.14902	
Levene's Test for Equality of Variances					
F	Sig.				
0.089	0.766				
t-test for Equality of Means					
	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference
Equal variances assumed	3.452	85	0.001	0.76698	0.22215

Descriptive statistics in Table 6 show that the mean values of the experimental and control classes were 7.0289 and 6.2619, respectively, reflecting significant differences. The Sig value in the Levene test is equal to 0.766 > 0.05, so there was no difference in variance between the two groups. The independent t-test results show the Sig value. (2-tailed) equals 0.001 < 0.05, so the difference in mean score between the two classes was statistically significant. Accordingly, the null hypothesis was rejected, and the alternative hypothesis was accepted. Additionally, based on the average scores of the two classes, it appears that the students in the experimental class outperformed the students in the control class in terms of academic performance. Furthermore, the standard mean deviation has been calculated to be 0.8. Because this value falls between 0.8 and 1.0, it is reasonable to conclude that the effect size is significant according to Cohen's criteria. In addition, researchers investigated whether or not the results of the two tests given to the experimental class correlated. It was also possible to look at the relationship between the post-test and the pre-test results in Table 7.

Table 7. Data Correlation of the Two Tests Distributed to the Experimental Class

		Pretest	Posttest
Pretest	Pearson Correlation	1	0.984**
	Sig. (2-tailed)		0.000
	N	45	45
Posttest	Pearson Correlation	0.984**	1
	Sig. (2-tailed)	0.000	
	N	45	45

** . Correlation is significant at the 0.01 level (2-tailed).

With the Sig significance level (2-tailed) less than 0.05, the correlation test results revealed that the scores of the experimental class in the two tests taken before and after the experiment are correlated with one another. Therefore, the Pearson correlation coefficient of 0.984 indicates a very strong relationship, as demonstrated by the data.

In general, the students in the experimental group were more engaged, enthusiastic, and interested in solving real-world problems than those in the control group. The allure of real-world problems was also associated with the attraction of mathematical knowledge with a wide range of practical applications.

Results of Skills Obtained in the Study Sheet

Students also participated in a warm-up activity, completing individual worksheets and group activities incorporated into class assignments. The following activity presented a problem situation to students to encourage them to acquire new knowledge and develop an interest in learning. The problem was stated as follows: "A piece of paper is wrapped around a cylindrical bottle, and then a compass is used to draw a circle on the paper. When the paper is laid flat, is the shape drawn on the paper an ellipse?" (Stewart et al., 2015). The results of the worksheet analysis revealed that the majority of the students in this situation met the requirements of criterion 1 (see Figure 2).



Figure 2. The Worksheet of Student S41

(Figure 2 in English: Wrap a piece of paper around a cylindrical bottle, then use a compass to draw a circle on the paper as shown. Is the figure drawn on it a circle? Figure 3.2 Is this a circle? Answer: When the paper is flattened, the figure drawn is a circle).

Example 1: The whispering gallery in the Museum of Science and Industry in Chicago is 47.3 feet long. The distance from the center of the room to the foci is 20.3 feet. Find an equation that describes the shape of the room. How high is the room at its center? (1 feet \approx 0.3048 m) (Sullivan, 2013)



This example was intended to guide students in writing an ellipse's canonical equation when knowing the length of the major axis, the focal point, and finding the highest point in the room. The quality of the students' work was evaluated by the skill criteria listed in Table 3. Table 8 contains the results of the study.

Table 8. Statistical Results of the Criteria of Skills Achieved in Example 1

Levels	Example 1									
	Criterion 2		Criterion 3		Criterion 4		Criterion 5		Criterion 6	
	f	%	f	%	f	%	f	%	f	%
1	2	4.44	2	4.44	6	13.33	8	17.78	12	26.67
2	2	4.44	9	20	13	28.89	15	33.33	19	42.22
3	41	91.12	34	75.56	26	57.78	22	48.89	14	31.11

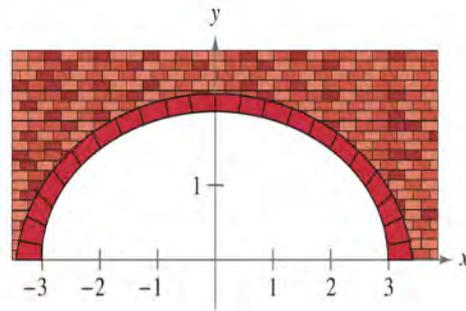
For criterion 2 on recognizing elements of ellipses, most students (91.12%) pointed out the names of elliptical elements in practical problems and could match them with mathematical problems (levels of degree 3). In criterion 3, on determining the length of the major and minor axes based on the given data, up to 75.56% of students correctly identified the lengths of the major and minor axes. Criterion 4 required students to calculate the other ellipse elements using the available information, and 57.78% of students could do so correctly. Students who transitioned from real-world to mathematical problems were nearly half (48.89%) of those who determined how to solve the problem based on criterion 5. According to criterion 6, up to 31.11% of students could address math problems and correctly answer questions about real-life problems.



Figure 3. The Worksheet of Student S37 (Figure 3 in English: $2a = 47.3 \Rightarrow a = 23.65$, $c = 20.3$. We have: $b^2 = (20.65)^2 - (20.3)^2 = 147.2325$. The canonical equation to find is $x^2 / 559.3235 + y^2 / 147.2325 = 1$).

Some students recognized the canonical equation but misidentified the focus. The mistake shown in Figure 3 shows that the student could not determine the height at the center of the room but still wrote the correct equation (see Figure 3). A large number of students were able to master the canonical equation of the ellipse, even though there were some errors when it came to determining the ellipse. In this case, it indicates that the activity has met its objective.

Example 2: A fireplace arch is to be constructed in the shape of a semi-ellipse. The opening is to have a height of 2 feet at the center and a width of 6 feet along the base. The contractor outlines the ellipse on the wall by the method "thumbtack and string." Give the required positions of the tacks and the length of the string (Larson, 2012).



The goal of Example 2 was to identify the elements of the ellipse (major axis, minor axis, and focal length). In order to complete the loop, students must determine where to pin two nails from the fireplace's edge and how long it should be. Students' work was evaluated following the criteria 2-6 on the scale in Table 9 to determine their overall performance.

Table 9. Statistical Results of the Criteria of Skills Achieved in Example 2

Levels	Example 2									
	Criterion 2		Criterion 3		Criterion 4		Criterion 5		Criterion 6	
	f	%	f	%	f	%	f	%	f	%
1	1	2.22	3	6.67	3	6.67	7	15.56	7	15.56
2	1	2.22	5	11.11	9	20.00	10	22.22	21	46.67
3	43	95.56	37	82.22	33	73.33	28	62.22	17	37.77

For criterion 5, from real-world problems to mathematical problems, up to 62.22% of students brought from practical problems to mathematical problems and determined the direction to solve the problem. For criterion 6 on solving practical problems, answering questions of practical problems has up to 37.77% of students solving math problems and correctly answering questions of practical problems. For criterion 6 on solving real-world problems, answering real-world problems, and solving (pure) math problems correctly, 37.77% of students correctly answered practical problems and resolved math problems correctly, respectively. As a result, when comparing criterion 6 to criterion 5, level 3 was significantly lower (24.45%).

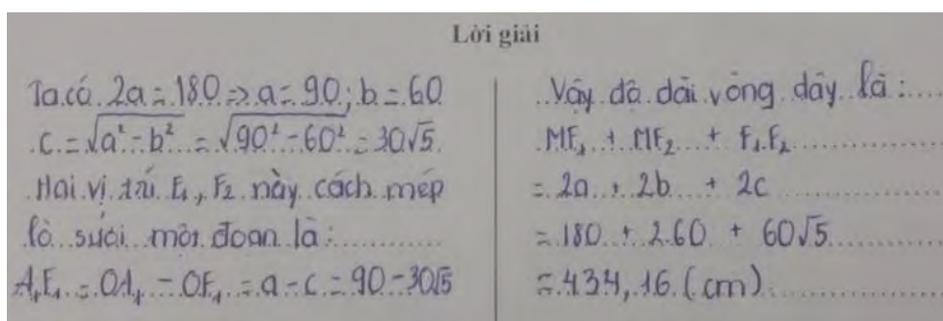


Figure 4. The Worksheet of Student S5 (Figure 4 in English: $2a = 180 \Rightarrow a = 90; b = 60, c = \sqrt{a^2 - b^2} = \sqrt{90^2 - 60^2} = 30\sqrt{5}$.

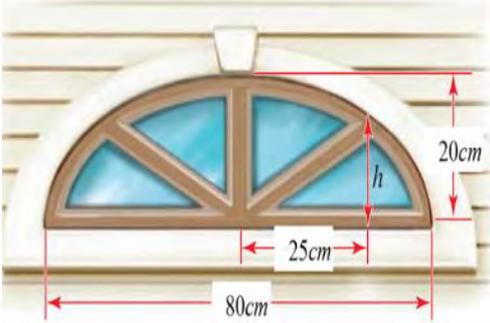
These two positions F_1, F_2 are at a distance from the edge of the heater: $A_1F_1 = OA_1 - OF_1 = a - c = 90 - 30\sqrt{5}$. So the length of the loop is: $MF_1 + MF_2 + F_1F_2 = 2a + 2b + 2c = 180 + 260 + 60\sqrt{5} \approx 434.16 (cm)$)

One typical mistake made by some students involved determining the length of the small shaft; they (apparently) did not understand the request for the assignment to identify the "position of two nails at a distance from the edge of the fireplace carefully". Nevertheless, students were able to identify the problem-solving direction of practical problems. The student mistake shown in Figure 4 used an incorrect formula ($MF_1 + MF_2 = 2a + 2b$; the correct formula is $MF_1 + MF_2 = 2a$), but the student resolved the math problem nevertheless (Figure 4).

As a result, most students were already familiar with the identification and calculation of the elements of an ellipse and

were able to provide an accurate solution. Furthermore, compared to Example 1, the percentage of students who achieved level 3 for the criteria increased generally. In part, this demonstrates the beneficial effects of the pedagogical measures that were implemented.

Example 3: A "Sunburst" window above a doorway is constructed in the shape of the top half of an ellipse, as shown. The window is 20 cm tall at its highest point and 80 cm wide at the bottom. Find the height of the window 25 cm from the center of the base (Stewart et al., 2015).



In Example 3 (above), the student's responsibility was to determine how long the major and minor axes were while determining how tall an ellipse was at a specific point in the problem.

Table 10. Statistical Results of the Criteria of Skills Achieved in Example 3

Levels	Example 3									
	Criterion 2		Criterion 3		Criterion 4		Criterion 5		Criterion 6	
	f	%	f	%	f	%	f	%	f	%
1	0	0.00	0	0.00	1	2.22	2	4.44	21	46.67
2	1	2.22	5	11.11	7	15.56	7	15.56	16	35.56
3	44	97.78	40	88.89	37	82.22	36	80.00	8	17.77

Regarding criterion 5, 80% of students used real-world problems to determine how to handle the math problem (Table 10). Nevertheless, for criterion 6 on solving real-world problems, only 17.77% of students could solve math problems and answer real-world problems correctly.

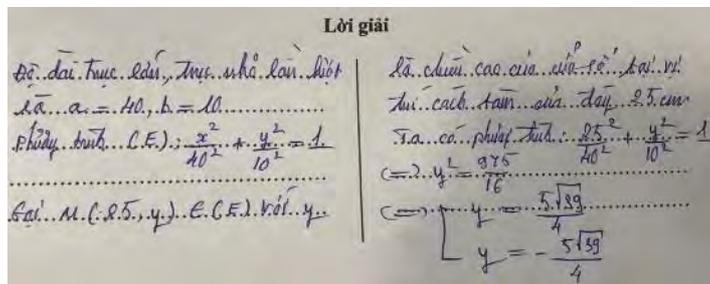


Figure 5. The Worksheet of Student S29 (Figure 5 in English: The lengths of the major and minor axes are $a = 50; b = 10$.

So the equation of the ellipse is: $x^2 / 40^2 + y^2 / 10^2 = 1$. Let $M(25,y)$ where y is the window's height at a position with

distance away from the floor of the bottom 25cm. We have the equation: $x^2 / 40^2 + y^2 / 10^2 = 1$

$$y^2 = 975 / 16 \Rightarrow y = 5\sqrt{39} / 4; y = -5\sqrt{39} / 4$$

Student worksheets contained errors such as incorrectly determining the length of the major axis and failing to answer the real-world problem question, while the student work in Figure 5 shows a similar error both in determining the length of the minor axis and failing to answer the real-world question (see Figure 5). This is unfortunate because the students identified a problem-solving approach but failed to find an exact solution.

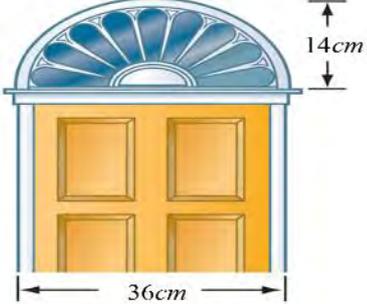
Examination of student worksheets reveals the following observations, which are worth noting. Students' most common errors included incorrectly calculating the length of the major or minor axes and solving the math problem but failing to complete the real-world problem. Aside from that, students were still stumbling over their calculations and lacked the habit of analyzing real-world problems. Student progression through each teaching situation was generally positive, though the worksheets reveal that students' ability to solve real-world problems was not particularly strong. The reason may be that the teacher has not mobilized all students to participate in the task; they are less exposed to problems with real-life content. For this reason, educators need to engage in active study of RME documents and the functional design of teaching situations that incorporate the RME model into classroom instruction. The expectation is that students will gradually become familiar with the steps of the RME activities in this way, and they will begin to

recognize that there is always a connection between math and reality.

Post-test Results of Skills Obtained

Following that, students were given three real-world problems to cope with, which required them to apply the knowledge they gained about ellipses and ellipse equations that they had learned in class.

Problem 1: Interior design
 The rounded top of the windows is the top half of an ellipse.
 Write an equation for the ellipse if the origin is at the midpoint of the bottom edge of the window (Gilbert et al., 2014).



The objective of Problem 1 was to assess students' ability to determine the major and minor axes and write their canonical equations, both of which were required skills. Students were expected to have gained practical experience in determining the ellipse center and problem-solving orientation when given at the understanding level.

Table 11. Statistical Results of the Criteria of Skills Achieved in Problem 1

Levels	Problem 1							
	Criterion 2		Criterion 3		Criterion 5		Criterion 6	
	EC	CC	EC	CC	EC	CC	EC	CC
1	0	0	0	3	0	4	3	4
	0%	0%	0%	7.14%	0%	9.52%	6.67%	9.52%
2	0	0	2	4	3	4	3	5
	0%	0%	4.76%	9.52%	6.67%	9.52%	6.67%	11.90%
3	45	42	43	35	42	34	39	33
	100%	100%	95.24%	83.34%	93.33%	80.96%	86.66%	78.58%

The results of Problem 1 were extremely positive, as shown in Table 11, with most students in both the experimental class (86.6%) and the control class (78.5%) completing the task correctly. However, in the control class, some students were still doing the wrong test, such as: unable to identify the major or minor axis; not mastering the form of the canonical equation, leading to wrong conclusions with the requirements set out. Findings suggest that some students may be unfamiliar with the practice of applying real-world issues to mathematical problems.

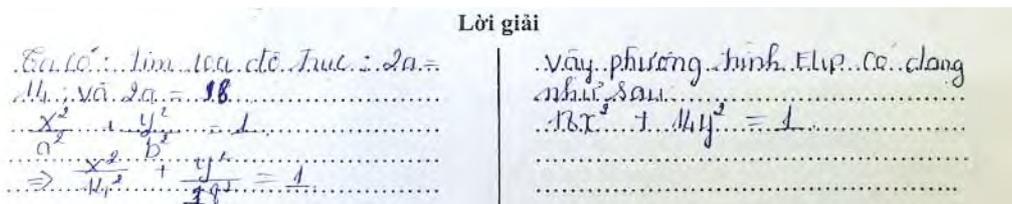
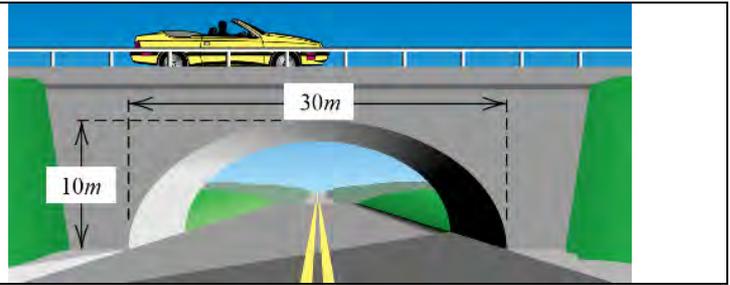


Figure 6. The Worksheet of Student S09 (Figure 6 in English: We have $2a = 14$ and $2b = 18$; $x^2 / a^2 + y^2 / b^2 = 1$; So the ellipse equation has the following form: $18x^2 + 14y^2 = 1$).

Student work shown in Figure 6 contains errors caused by an inability to determine the ellipse elements. The student substituted the lengths of the major and minor axes into the canonical equation and confused the requirements to write the equation of the circle, which resulted in the student writing the equation of the circle incorrectly. Some students were familiar with an ellipse's canonical equation but made mistakes when determining the major and minor axes and probably were perplexed when they tried to substitute them back into the equation. The student's worksheets correctly determined the length of the major axis, but the length of the minor axis was incorrect. Additionally, some students generated incorrect canonical equations, possibly because they did not substitute results correctly. Several students frequently encountered these errors. Some students confused the equation of the ellipse with the equation of the circle.

Problem 2: Dimensions of an arch

An arch of a bridge is semielliptical, with a major horizontal axis. The arch's base is 30 m across, and the highest part of the arch is 10 m above the horizontal roadway, as shown. Find the height of the arch 6 m from the center of the base (Swokowski & Cole, 2009).



This problem was designed to assess students' abilities in determining the center of the ellipse, the major axis, and the minor axis, writing their canonical equations and calculating the height at a distance from the center that was predetermined. Problem 2's task was similar to Problem 1 but necessitated a higher skill level. The knowledge used was the "point on the curve" of this content, which students have learned, so they were completely able to solve the problem. Students may have difficulty locating the center of the arch and solving quadratic equations. The reason may be that students have not identified some mathematical factors in the problem.

Table 12. Statistical Results of the Criteria of Skills Achieved in Problem 2

Levels	Problem 2									
	Criterion 2		Criterion 3		Criterion 4		Criterion 5		Criterion 6	
	EC	CC	EC	CC	EC	CC	EC	CC	EC	CC
	f	f	f	f	f	f	f	f	f	f
	%	%	%	%	%	%	%	%	%	%
1	0	0	0	0	2	4	2	5	4	7
	0	0	0	0	4.44	9.52	4.44	11.90	8.88	16.67
2	0	0	1	2	3	2	5	1	5	6
	0	0	2.22	4.76	6.67	4.76	11.11	2.38%	11.11	14.29
3	45	42	44	40	40	36	38	36	36	29
	100	100	97.78	95.24	88.89	85.72	84.45	85.72	80.01	69.04

Table 12 shows that the results of Problem 2 were extremely positive in terms of criterion 6 on solving real-world problems and responding to the questions of others. 80.01% of the students in the experimental class and 69.04% of the students in the control class were able to solve the math problem and correctly answer the real-world problem question. As a group, most of those who participated in the control class was successful in determining the direction to take in order to resolve the problem, but they were unsuccessful in transitioning from the real-world problem to the math problem, and they continued to make mistakes when determining the length of major or minor axes.

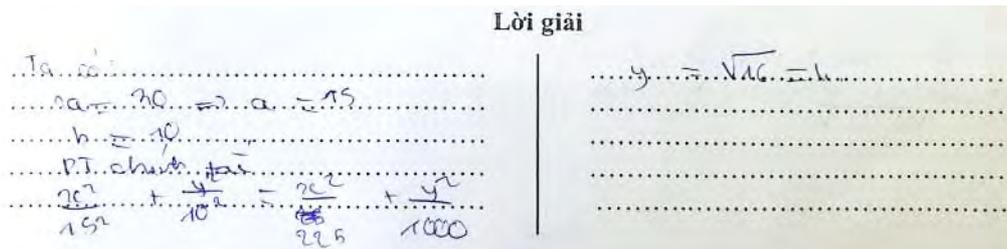


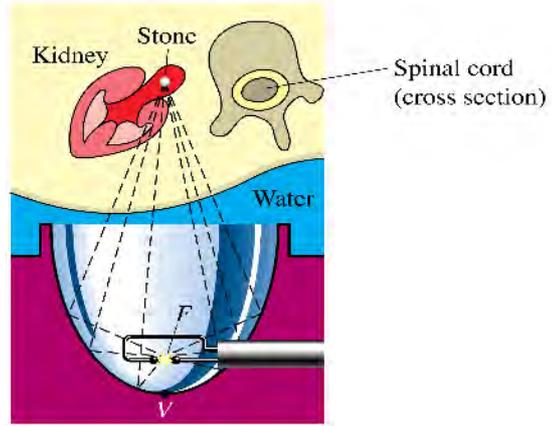
Figure 7. The Worksheet of Student S34 (Figure 7 in English: We have $2a = 30 \Rightarrow a = 15$; $a = 10$. The canonical equation is: $x^2 / 15^2 + y^2 / 10^2 = x^2 / 225 + y^2 / 1000$; $y = \sqrt{16} = 4$).

Many students could not determine the length of the minor axis, even though they had learned about the math problem from the real-world problem and had determined the direction to solve the problem successfully from the real-world problem. The student whose worksheet is shown in Figure 7 could not determine which way to go with the problem or the form of the canonical equation of the ellipse, leading to an incorrect solution). Some students were able to identify the solution to the problem, but they failed to solve the quadratic equation and did not conclude the question of the real-world problem. A few students correctly identified the ellipse's major axis, minor axis, and canonical equations. Because the students could not identify the solution to the problem, an incorrect answer was given. A few students' worksheets demonstrate that they could provide real-world problems but were incorrect when solving quadratic equations. Many control-class students made the same mistake.

Problem 3: Lithotripter

A lithotripter of height 15 centimeters and a diameter of 18 centimeters is to be constructed. High-energy underwater shock waves will be emitted from the focus F that is closest to the vertex V.

- a. Find the distance from V to F.
- b. How far from V (in the vertical direction) should a kidney stone be located? (Swokowski & Cole, 2009)



In Problem 3, the goal was to assess students' ability to determine where the center of the ellipse, the major axis, and the minor axis are located, calculate the distance from V to F, and determine how far the kidney stone should be placed (vertically). This highly applied problem required students to recognize the relationship between mathematics and practice and use elliptical skills and knowledge when solving real-world issues. Students may have difficulty identifying major and minor axes and bringing real-world problems to math problems and problem-solving.

Table 13. Statistical Results of the Criteria of Skills Achieved in Problem 3

Levels	Problem 3									
	Criterion 2		Criterion 3		Criterion 4		Criterion 5		Criterion 6	
	EC	CC	EC	CC	EC	CC	EC	CC	EC	CC
	f	f	f	f	f	f	f	f	f	f
	%	%	%	%	%	%	%	%	%	%
1	0	2	1	2	2	9	6	12	14	21
	0	4.76	2.22	4.76	4.44	21.43	13.33	28.57	31.11	50
2	1	0	2	3	5	8	8	8	9	13
	2.22	0	4.44	7.14	11.11	19.05	17.78	19.05	20	30.95
3	44	40	42	37	38	25	31	22	22	8
	97.78	95.24	93.34	88.1	84.45	59.52	68.89	52.38	48.89	19.05

Not surprisingly, the percentage of students achieving level 3 in both groups decreased significantly as the complexity of the criteria increased (Table 13). Although most student work in the experimental class was good (achieving levels 2 and 3), students in the control class were significantly less likely than students in the experimental class to achieve levels 2 and 3 on criteria 4, 5, and 6.

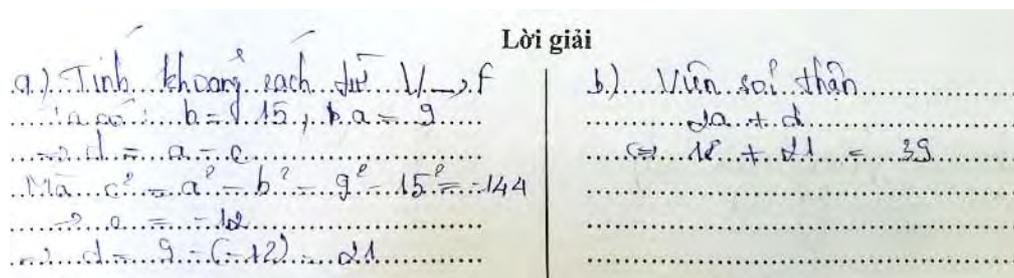


Figure 8. The Worksheet of Student S41 (Figure 8 in English: a) Calculate the distance from V to F. We have: $b = 15$; $a = 9$; $d = a - c$. But $c^2 = a^2 - b^2 = 9^2 - 15^2 = -144 \Rightarrow c = -12 \Rightarrow d = 9 - (-12) = 21$; b) Kidney stone $2a + d \Leftrightarrow 18 + 21 = 39$.)

As the student worksheet in Figure 8 shows, the student could not distinguish the major and minor ellipse axes. One of the mistakes in the students' worksheets was that they did not pay enough attention to the condition $a > b > 0$. The length was a positive number rather than a negative number. In practice, this demonstrates that students were unable to make the connection between math problems and understanding. Some students successfully solved the math problem; however, one student made the mistake of identifying the location of the kidney stone, which should have been located at a distance from V equal to $2c + 3$.

Student opinion survey results

Upon completing the lesson, 45 students in the experimental class completed a questionnaire to express their thoughts. The survey consisted of five statements built on the Likert scale with five levels to collect students' perceptions of learning efficiency and interest in the courses. Results are provided in Tables 14-18.

Table 14. Results of Student Feedback on Item 1

Item 1. I like elliptical equation lessons					
Levels	Totally disagree	Disagree	Neutral	Agree	Totally agree
%	0.0%	0.0%	11.1%	55.6%	33.3%

Students in the experimental class strongly preferred and enjoyed the lessons on elliptic equations associated with practical situations, with 88.9% expressing a preference or enjoyment (Table 14). In comparison, a small percentage of students (11.1%) expressed neutral views, and no students expressed dissatisfaction with the lessons.

Table 15. Results of Student Feedback on Item 2

Item 2. I found that the practical situations included in the ellipse equation help to study effectively and better understand the relationship between learned knowledge and real-world problems					
Levels	Totally disagree	Disagree	Neutral	Agree	Totally agree
%	2.2%	0.0%	8.9%	22.2%	66.7%

With Item 2, the researchers hoped to determine the extent to which students had learned and their perception of the relationship between learned knowledge and real-world situations. Because practical situations were the main content in teaching elliptic equations, the statistical results in Table 15 show that 22.2% agreed and 66.7% completely agreed that the situations presented helped them learn and practice effectively and better understand the relationship between their learned knowledge and the real-world problems. Aside from that, a small percentage of 8.9% held a neutral opinion, and 2.2% said they disagreed.

Table 16. Results of Student Feedback on Item 3

Item 3. I found that group activities and study cards stimulated my interest in learning and helped me study more actively.					
Levels	Totally disagree	Disagree	Neutral	Agree	Totally agree
%	0.0%	0.0%	4.4%	46.7%	48.9%

Findings in Table 16 demonstrate that the effectiveness of group work with study sheets was highly appreciated by 95.6% of students (46.7% agree, 48.9% strongly agree), only two students (4.4%) perceived as normal.

Table 17. Results of Student Feedback on Item 4

Item 4. In the process of the teacher teaching the ellipse equation, are you interested in learning activities? (Respondents could choose more than one answer)	
Activities	%
Practice learning the shape of an ellipse	55.6%
Interact, discuss and come up with the best solution to solve the problem	68.9%
Present solutions and draw conclusions about the problem	42.2%
Solve new real-world problems related to the lesson content	57.8%
Knowledge consolidation exercises	48.9%

In Item 4, statistical findings revealed that all activities assisted the students in learning elliptic equations (42.2% or more). Up to 68.9% of students said they appreciated participating in interactive activities and discussing the problem. Table 17 also shows that the participation rates of students in practical activities to learn the shape of the ellipse and to resolve new practical problems related to the lesson content were 55.6% and 57.8%, respectively. A smaller but still sizeable percentage of students also said they participated in exercises to consolidate knowledge, present solutions, and resolve the problem (48.9% and 42.2%).

Table 18. Results of Student Feedback on Item 5

Item 5. I look forward to participating in similar classes in other lessons					
Levels	Totally disagree	Disagree	Neutral	Agree	Totally agree
%	0.0%	0.0%	4.5%	44.4%	51.1%

Students were asked to complete this item to determine the appeal of this new learning model. As Table 18 indicates, students' appreciation for the lessons was high, with 95.5% of students completely agreeing or agreeing to take similar lessons in other lessons. Only two students (4.4%) held a neutral opinion. This suggests the feasibility of using similarly designed lessons in the future.

Discussion

Student mathematical problem-solving abilities have improved due to the teaching process, which included learning activities based on the RME approach. Additionally, students have provided positive feedback on the process. The findings of this study are consistent with the findings of a large body of previous research in the field of mathematics education (Bray & Tangney, 2016; Deniz & Kabael, 2017; Karaca & Özkaya, 2017; Makonye, 2014). Experiments show that putting students in meaningful, real-world situations and using the RME model to teach ellipse equations increased participation and interest. Research by Bray and Tangney (2016); Karaca and Özkaya (2017); Laurens et al. (2017); Mulbar and Zaki (2018); Putri et al. (2019); and Dickinson et al. (2020) reached similar conclusions. Researchers could also track students' mistakes and difficulties forming and applying knowledge to new situations using worksheets throughout the learning process. It is possible that students were unfamiliar with the types of tasks that were associated with real-world problems that are often not addressed in textbooks that deal with elliptic curves and other related topics. Also, it was originally assumed that These students demonstrate a lack of problem-solving abilities when confronted with real-world situations.

According to the worksheet analysis, lessons learned about ellipse equations improved from the beginning to the end of the experiment. The effectiveness of worksheets in RME-oriented teaching was also confirmed by Mulbar and Zaki (2018). Many students in the experimental class met all criteria, including correctly determining the major and minor axes' lengths (Criteria 3 and 4). They transitioned from real-world to mathematical problems and correctly answered the question of the real-world problem (Criteria 5 and 6) that gradually increased over the activities and performed well on the post-test.

This study concludes that, given an opportunity, the RME-oriented teaching design can yield greater benefits in helping the students in the experiment class achieve better understanding and practical application of mathematical knowledge about the ellipse when it comes to solving real-world problems (Vos, 2018). A significant difference was found between the outcomes of mathematics learning and the level of achievement of criteria for skills in solving real-world problems related to the subject matter of mathematics, according to the results of data processing and qualitative analysis conducted. Similar conclusions were also reached by Ardiyani et al. (2018); Kusumaningsih et al. (2018); Mulbar and Zaki (2018); Sumirattana et al. (2017); Yuanita et al. (2018); Peni (2019); Dickinson et al. (2020); Yilmaz (2020). Also, students' modeling competencies are formed and developed due to their involvement in real-world problem-solving exercises (Lu & Kaiser, 2021).

Most students were interested in the fact that teachers used practical situations to teach new knowledge and practice and used interesting lessons, which helped students absorb information more quickly and deepen their understanding. As a result of having the opportunity to practice and learn topics related to the relationship between math knowledge in school and practice, this may be a contributing factor. Studies have shown a relationship between attitudes and performance (Dowker et al., 2012; Mata et al., 2012). However, according to Mazana et al. (2019), students may initially have a favorable attitude toward mathematics, but their attitudes change as they progress to higher challenge levels. One possible explanation is that the students have observed the relationship between mathematics and practice, and as a result, they have grasped the significance of mathematics in everyday life. Hence, it is too soon to know how attitudes might change in the future.

Conclusion

This study found that the experimental class's real-world problem-solving skills improved over time. This seems to contradict Nguyen et al. (2019) findings in their study of grade 12 students in the northern mountainous area of Vietnam. Students might require some adjustment time to a new way of learning before teaching with an RME approach will positively impact their learning, which could explain this.

Also, findings help answer the previously stated research questions. First, students demonstrated mastery of the knowledge and skills expected of them to tackle the ellipse topics. They understood how to apply their knowledge and skills to solve real-world problems related to their chosen study topic. Next, students demonstrated problem-solving skills by recognizing and identifying math problems. They also proposed solutions to problems they encountered, using mathematical knowledge and skills compatible with their solutions. The study's findings also show a two-way

relationship between students' positive attitudes and math achievement. Real-world problems motivated students to participate in RME-based teaching activities.

The students in the experimental class were more engaged than those in the control class. They predicted, discovered, and learned new information faster than their peers. Teachers created situations for students to think independently, solve problems, communicate mathematically, and build teamwork skills. Through self-discovery and problem-solving techniques, many students gained confidence and took responsibility for their learning.

Recommendations

These conclusions pose new challenges for the research team and teachers with an RME-oriented mathematics curriculum. According to Webb and Peck (2020), the role of teachers in the application of RME in mathematics education is extremely important. To address the issues raised above, teachers must understand the existing knowledge base and the students' learning level before designing learning activities, including selecting real-world problems, teaching and learning methods, and creating appropriate guiding questions. Teachers' patience in guiding students toward the long-term benefits of RME application is also an important factor. Similar views have been reached by Laurens et al. (2017), Meika et al. (2018), Sumirattana et al. (2017), and Yuanita et al. (2018).

Future studies could propose plans to restructure math learning content at high schools in the direction of the RME to take full advantage of the positive benefits from this and build an assessment framework suitable to the curriculum oriented to RME application. Moreover, the framework for evaluating the effectiveness of teachers' teaching with RME should be clarified to assist them in adjusting how they use the RME model in mathematics teaching and learning. In Viet Nam, these recommendations echo those of Nguyen et al. (2019), Nguyen, Trinh & Pham (2020), and Do et al. (2021). In addition, future studies could apply the RME approach in other areas of mathematics education, such as statistics.

Limitations

The experimental results also have certain limitations. Despite their ability to visualize the situation's development, some students were confused when given practical context tasks. For this reason, the experimental period had a little positive impact on some students. Some students could still apply math knowledge to everyday situations when solving real-world problems. Concurrently, they lacked effective problem-solving mathematical language skills. Without checking their work or considering the reasonableness of the answer, students may have missed opportunities to correct themselves.

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Authorship Contribution Statement

Duong Huu Tong: Conceptualization, design, writing, editing/review. Tien-Trung Nguyen: critical revision of the manuscript, final approval. Bui Phuong Uyen: Drafting manuscript, supervision. Lu Kim Ngan: Data acquisition, data analysis, statistical analysis. Lam Truong Khanh: technical or material support. Phan Thi Tinh: editing and writing.

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