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A STEAM Practice Approach to Integrate Architecture, Culture and History to Facilitate Mathematical Problem-Solving

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Abstract: In this paper we propose STEAM practices that would foster mathematics learning through modelling architecture while connecting to culture and history. The architectural modelling process is applied by the teachers as participants of these practices from different countries allowing a broad cultural and historical connection to mathematics education. The modelling is implemented in GeoGebra platform as it is an open-source platform to allow teachers to model on a mathematics basis. The architectural modelling process does not provide participants with steps to follow but rather allows them to explore the architectural models' components and construct them with various approaches which may foster problem solving techniques. We aim to investigate how different phases of this approach (such as motivation, modeling, and printing process) reflect on opportunities of learning in STEAM education, with a particular lens in mathematical development from open tasks. This paper will show two use cases that took place in Upper Austria and the MENA region.

Keywords: architecture; STEAM practices; problem solving; 3D mathematical modelling; mathematics education; augmented reality

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1. Introduction

We can identify mathematical concepts (e.g., symmetries, algorithms, patterns or shapes) in various architectural constructions worldwide [1,2]. Moreover, typologies and their relationship to geometries are considered architecture features [3]. For example, the golden ratio or geometric shape patterns are widely present in architecture, cultural heritage, and the history of humans. Thus, architecture, such as the Eiffel Tower in France or the Taj Mahal in India, are built based on mathematical and foremost STEAM (science, technology, engineering, arts, and mathematics) contents and skills [4].

We will present and discuss STEAM tasks and settings that integrate architecture, culture, and history to mathematics learning to promote reasoning and problem solving while using the participant's mathematical knowledge. Architectural modelling allows the intersection between disciplines of study such as mathematics, culture, and history. The tasks and practices connect mathematical knowledge to build architectural constructions that the participants know, visit, or even locate in their own countries. The cultural and historical connection appears when the participants collect data related to the architectures they choose. The purpose behind proposing these STEAM tasks and settings that combine culture, history, and architecture to mathematics education is to foster modelling skills, imaginary skills, problem-solving skills while using the participant's mathematical know-how. El Bedewy et al. [5] proposed practices that can allow participants to use their mathematical knowledge while modelling, which may contribute to mathematics education.

The proposed practices encapsulate various technologies that allow participants to connect the real world and the virtual world to allow new perspectives for teachers and students to delve into active learning on STEAM contents and skills connected to architecture, history, and cultural heritage. The architectural modelling is implemented using GeoGebra (https://www.geogebra.org/, accessed on 30 November 2021), because it offers mathematical tools and functions to model in 2D or 3D views to construct the main components and layout of the architectural models. This architectural modelling simulation requires mathematical operations, such as symmetry, transformations, extrusions, and many others. These mathematical concepts can be afforded using GeoGebra through basic simple steps that are implemented through logical steps to simulate the real architectures. Through the journey of performing the logical steps in architectural mathematical modelling, problem solving strategies become obvious. Contents can be made visible and manipulated by the user, which can enrich the learning by exploration. The user's exploration can be encountered mathematically during the modelling process, fostering problem solving and inquiry. The participant's problem-solving behaviors are analyzed before and during the interventions [6,7].

Practices that encapsulate new technologies that bridge the gap between the two worlds, physical and digital, can pave the way for teachers to introduce more IBL learning approaches to their students. One of these purposes we are encapsulating in this research is mathematical modelling and visualization of architectural models in many forms, such as physical or digital. According to Lieban et al. [8], 3D printing may foster the modelling process because its application would enhance the student-centered learning approach; which may have a significant effect on students' motivation when representing things that they created themselves. Some of these examples will be introduced to show the connection between technologies, mathematical modelling and their uses in educational environments.

2. Literature Review

The proposed practice's nature, which is learning tasks, fosters problem solving due to the fact that it is of an unguided task nature and would leave room for the participants for exploring and approaching these modelling tasks in many ways. Therefore, these practices may contribute to the problem-solving principles as Blum and Niss [9] defined problem solving, as "*simply refers to the entire process of dealing with a problem in attempting to solve it*"; they also raised the point that they tackle the problem-solving nature from the perspective of its relation to "*mathematics instruction*", which deals with adopting the issues related to the practices that actually implement the problem-solving process in mathematics education.

According to Niss [10], modelling is defined as a mathematically related ability task because it requires the following abilities: "*reading and communicating, designing and applying problem solving strategies, or working mathematically (reasoning, calculating, . . .)*". Blum and Borromeo [11] believe that the mathematization process in the modelling cycle in Figure 1 can transform the real model introduced in the task into a mathematical model formulated out of specific equations and formulas. The mathematical ability to calculate or solve equations can result in mathematical outputs that can contribute to real results. As we reflect on the modelling cycle in Figure 1 in our practices, the problem situations consist of the architectural modelling process, and for that, participants come up with situation models as a result of their modelling process. During the modelling process, participants are expected to go through the mathematization phase to apply mathematics in an attempt to come up with a similar model already existing in the real world. With the modelling cycle focus, we are interested in the mathematization process because we believe this is our practice's contribution to teaching/learning to allow participants to model architecture based on mathematical knowledge they have and may gain through the application of these practices.

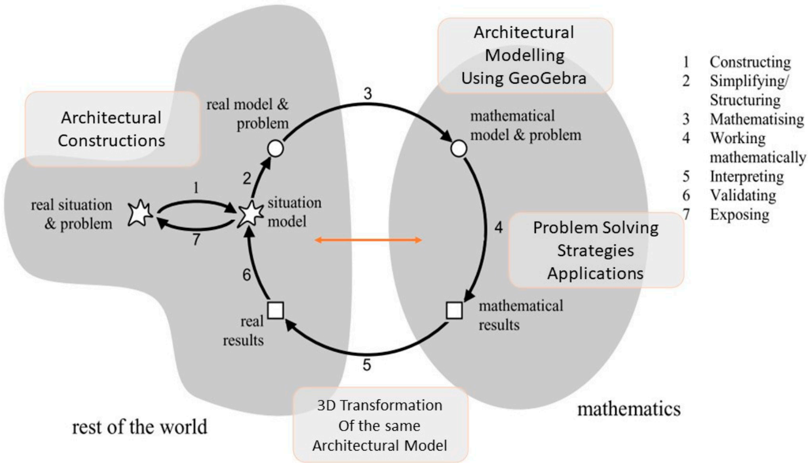


Figure 1. Modelling cycle, 7-step modelling cycle referred to Blum and Leiss [12] with an added connection to the proposed practices phases.

According to Ferri [13], the mathematically-based terminology is different for each participant because each participant has a unique problem-solving attitude that depends on their mathematical knowledge and way of thinking.

Lieban and Lavicza [14], in their work, considered that the challenge or the problem is to create physical and digital representations as equal to each other as possible, and they tried to complement their findings with Polya's [15] work on problem-solving. This was related to their work with seesaws' digital modelling and the problem-solving methods as they mentioned: "*guess and check, use of symmetry, draw a picture and be creative to mention some of them*." The authors also highlighted generalization and specialization as problem solving ways for the students to figure out the modelling tasks. They tried to link the physical exploration with digital modelling, which helped the students to understand the relationship between the two representations of the modelled task. "*The combined use of both physical and digital resources seems to bring a relevant contribution for refining students' thinking and enhancing their mental schemes or strategies during their geometrical modelling.*" Moreover, problem solving skills (e.g., evaluation or representation of mathematical models) were identified by Haas [16] as key skills in connecting real world objects, mathematics, and mathematical modelling.

In our proposed practices we believe that the definition of our problem is similar to Lieban and Lavicza [14], an approach which attempts to come up with a similar representation of the models in physical and digital forms. The models we refer to are architectural constructions that exist in our real world, and we allow participants of these practices to replicate them with GeoGebra modelling, and later with a technological possibility for various representations using 3D printing, 3D scanning, augmented reality (AR), origami, and 4D frames.

2.1. Hypothetical Learning Trajectories Supporting STEAM Practices

We supported teachers and practitioners with a dynamic lesson plan (DLP) to apply these notions to STEAM practices as architectural modelling connected to culture and history. Preparing STEAM practices, and their integration, could be challenging to teachers according to Falloon et al. [17]; where lesson preparation, sometimes the integration or adoption of some STEM practices in the education field, sometimes result in various opinion confusion for those who will implement it and adopt it as teachers and will result in questions arising as how to plan for it, teach it, and finally evaluate it regardless of its great impact on the self, society, and economics. As stated by Cameron, L [18], the lesson plan should include the lesson flow to serve as documentation of how the teachers reached the decision-making process. This will help in the design and establish self-confidence from the teacher's side to be able to continue with the actual implementation of the lesson.

Winsløw et al. [19] stated the unpleasant fact that most of the lesson planning is implicit to cultural boundaries, meaning they are not universal and able to be applied across many cultures and places. That is due to the fact that teachers consider lesson planning an individual task or action that cannot be shared and even if shared it would be hard to be understood outside a certain department, school, district, or even country. Winsløw et al. [19] stated "*It also seems to be a bit paradoxical, given that one of the often-cited features of lesson study is to create shared and documented knowledge, rather than (just) private experience and wisdom*". Winsløw et al. [19] believes that there is no defined structure for lesson planning, which makes it a current situation that contributors to the lesson planning develop practices that are only well perceived by them. Unfortunately, these practices will remain unclear to any interlopers.

The proposed lesson planning module for the introduced STEAM practices starts with defining the lesson components through a dedicated web interface to allow teachers to construct their lesson plans. The designed lesson components include the definition of the student's age, the architectural models, the environment and finally, the technology to be used. Once the teacher defines the main components, a generated link will guide them through each lesson component. Figure 2 shows a map that simulates the participant's journey in the lesson planning module to summarize all the modules with the available criteria and include all the building blocks of this research in one practice.

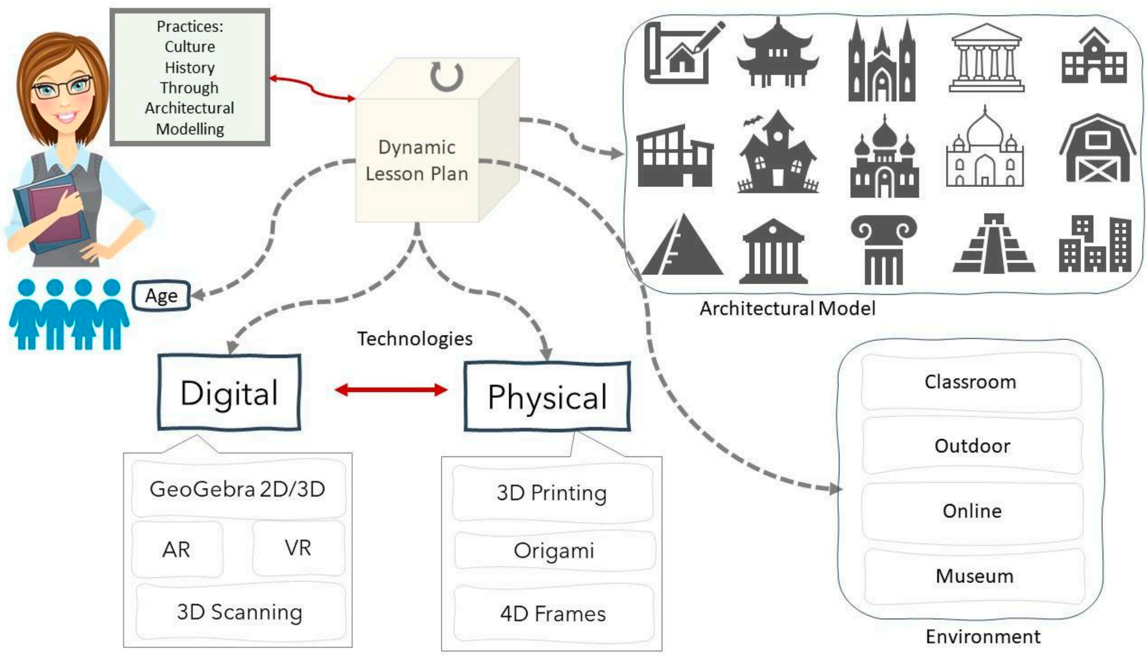


Figure 2. A map showing the lesson planning module from this research scope for teachers to choose from.

This lesson planning module we refer to as the dynamic lesson plan (DLP) is meant to be flexible and give the teachers the freedom of choice in designing their lesson plans. Therefore, the map shown in Figure 2 is mapped with the same order, choices of modules with the criteria available under each module to the DLP shown in Figure 3. Thus, under each module, there are various options where the teacher defines their criteria of choice using the DLP. For example, in Figure 3, the DLP interface shows the first module specifying the student's age. The second module of the lesson criteria is the architectural models that could be ancient, modern, based on mathematical concepts, freedom of choice, or even creating their own houses. The third module allows the teachers to define the implementation environment whether in a classroom, outdoor, online, or even in a museum. The fourth and final module is the technical specification which could be digital technology or physical technology, and how teachers can represent the same architectural model in many forms allowing the explorations of the 3D transformations. These options reflect our intentions in providing teachers with the flexibility and freedom to apply creative techniques that could reach the utmost goal of architecture modelling [20]. After the teacher specifies all the needed criteria for defining the STEAM practice, they then press on the button "Get the Link", as shown in Figure 3. This button generates links to redirect the teacher to a GeoGebra book.

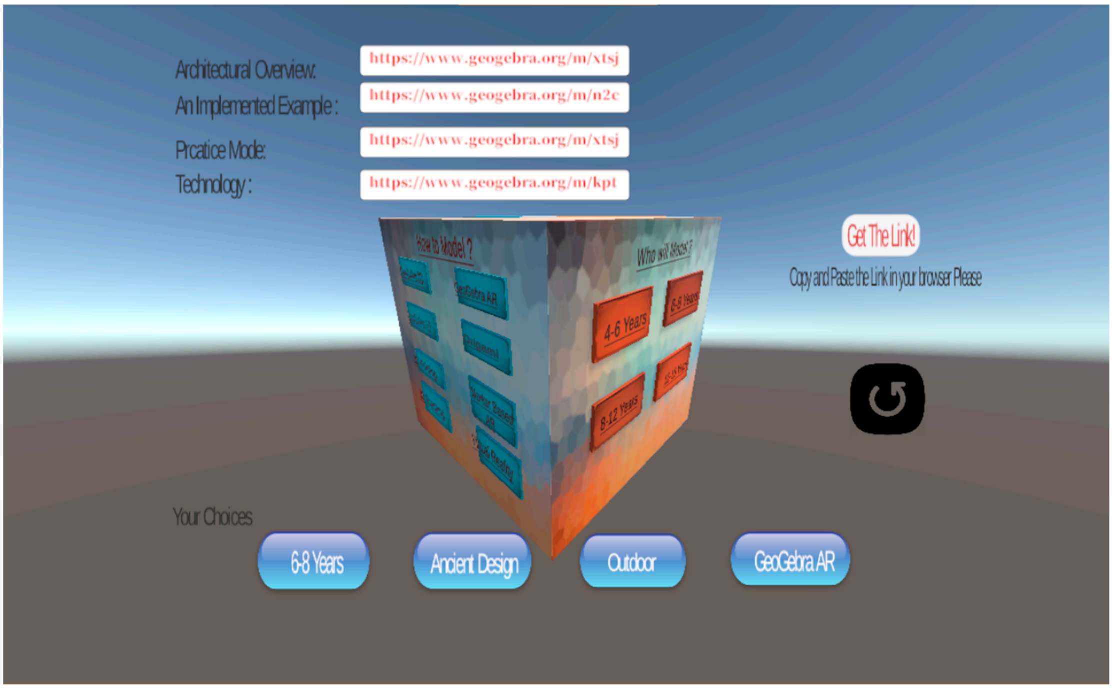


Figure 3. The DLP web interface with all the criteria provided to the teacher to navigate to.

The GeoGebra book contains specific implemented examples and instructional texts according to the teacher's chosen criteria. This book is to guide and provide the main foundational knowledge for teachers and practitioners of these practices. Therefore, we will now explore the reason behind adopting GeoGebra as the modelling tool in these practices.

2.2. GeoGebra A Problem-Solving Instrument

We are using GeoGebra in these STEAM practices because it may serve the architectural modelling based on mathematics knowledge. The meta-analysis work by D'Angelo et al. [21] compared the effects of using computer-based simulation versus no computer-based simulation in mathematics learning. Overall findings show that the computer-based simulations increased the students' improvements in the provided learnings more than the non-computer-based simulations.

Olsson and Granberg [22] describe the benefits of using dynamic software, such as GeoGebra, which will allow students to work with unguided tasks (tasks that do not have a defined outcome), which is the case in the architectural modelling practices. However, they propose that some challenges have to be met to ensure the success of such unguided tasks using dynamic software. First, the provided task needs to be designed to align with the student's prior knowledge. In our case, we try to meet this challenge in the DLP when the teacher provides the student's age as a first step when answering the question "Who will model?" in order to continue the lesson design while taking into consideration the student's age and accordingly their mathematical knowledge.

Another challenge that teachers are faced with, as presented by Olsson and Granberg [22], is guaranteeing their support as they solve the provided unguided tasks without giving too much guidance. Thus, the teachers can only guide students in highlighting valuable tools or techniques they learned in their math classrooms. Moreover, help them to come up with ideas and various approaches for solving the modelling tasks. GeoGebra will help teachers to allow students to engage in exploratory task-solving ways by asking them to clarify the approach they had in mind while solving a task. Olsson and Granberg believe that students would benefit from a dynamic software's potential if the tasks provided to them are unguided and with no defined solution method; it could hold various ways and approaches. According to their findings, the challenge is for teachers to design a convenient task that requires feedback to support the students without giving a full guided task. Additionally, capitalizing on their findings and suggestions, we consider the DLP to help and support teachers in creating unguided tasks to be practiced using dynamic software, such as GeoGebra. We refer to the tasks mentioned by Olsson and Granberg earlier, as in our case the architectural modelling tasks, which have a problem-solving nature because they could provide various solving or achieving approaches and thereby, we encourage teachers to ask students about their approaches in solving these modelling tasks and consider their various approaches in assessing these tasks.

When the participants are busy with the modelling process, most of the reflections can be captured as mathematical, cultural, and historical when the participants connect to the architectural model they are modelling. Olsson [23] noted that when students are engaged in solving the problem and using various features and tools from GeoGebra, it is advisable to understand how students react to that and not assume something. Furthermore, we want to capture the participants' reflections by designing questions during the modelling process to help capture the reflections and insights from the participant's answers inside the GeoGebra classroom. According to Olsson [23], other researchers proved that non-routine tasks provided to students who do not have the answer or ways to solve them are essential for learning and that students should discover their own methods and solutions to tasks as this will foster problem solving techniques for the students. Although other studies suggest that when students are given non-routine tasks, the teachers can provide them with initial hints for getting the task started, suggesting some ways to be used in the solving process, and offering some instructions for making suitable illustrations.

Therefore, in these proposed practices in this research, the participants are provided with implemented examples that explain step-by-step how to model architectural examples using the GeoGebra 2D/3D tools, for example, illustrating the polygon, extrusions, rotation, translation tools, and many others. Thus, this foundational step takes place in the form of a workshop for the participants. We present to the participants a guided architectural modelling task with a simple step-by-step approach. To aid the participants in designing their own architectural models, the unguided tasks were provided to them later. This provided knowledge can act as the foundation for the intervention and guide the participants towards applying them to any other architectural modelling task. According to Polya [15], this foundational step can help participants in "Understanding the Problem" and seeing many modellings and implemented examples using GeoGebra, this is the first step-in problem-solving strategies. The second step referred to by Polya is "Devising a Plan" and this step can differ from one participant to another in how they analyze the architectural models to start with the modelling process, so some may rely on guess and check and others may use symmetries or finding patterns, or using models or pictures from many views as site maps to be able to trace the architectures, other participants may choose to work backwards instead of starting at the base of the architecture to reach the top, they may start by modelling another part of the architecture. Therefore, we always advise our participants to start by importing images of the real architecture into GeoGebra. All these help in constructing a plan and a modelling approach that may ease the modelling process. The third step is the execution or "Carrying out the plan" as it is called by Polya, in this step, participants start by applying the tools and functions to follow their constructed plan using GeoGebra in modelling architectures. The fourth and final step is "Looking Back", and this happens in the comparison process when finalizing the modelling and trying to find ways to enhance it and trying to come as close as possible to replicating the real-life architectural example followed during the modelling process.

Lieban and Lavicza [14], who use Polya's problem solving strategies to use GeoGebra as an instrument, adopted these practices which are referred to later in this paper in more detail.

The paper will present two case studies focusing on architectural modelling that took place in Upper Austria and in the Middle East and North Africa (MENA) region. Finally, the paper will present the outcomes and the analysis that took place in both interventions with an emphasis on the problem-solving strategies applied.

3. Theoretical Framework

The theoretical framework for this research track is to govern the main notions introduced and the interplay between them. Therefore, EL Bedewy et al. [5] highlights the theoretical framework of the proposed practices that combine the following theory of didactical situations (TDS) [24] to govern the DLP and the lesson planning part, in allowing the teacher to construct the milieu for their students to apply such practices. Moreover, capturing the reflections from the teacher milieu after the application of the proposed practices.

Technology, pedagogy and content knowledge (TPACK) [25] is implemented to regulate the technology integration in these practices and to develop the self-assessed TPACK for the teachers' training and development.

The adaptive, meaningful, organic, environmental-based architecture for online course design (AMOEBA) [26] theoretical framework is to regulate the cross-cultural interventions as has been applied in this paper the intervention took place in Upper Austria and in the MENA region.

How these frameworks take place in the practices and how they take a sequence of implementation and alternation is justified by the use of the integrational program development cycle (Demir [27]). Figure 4 shows the interplay of these practices with each other.

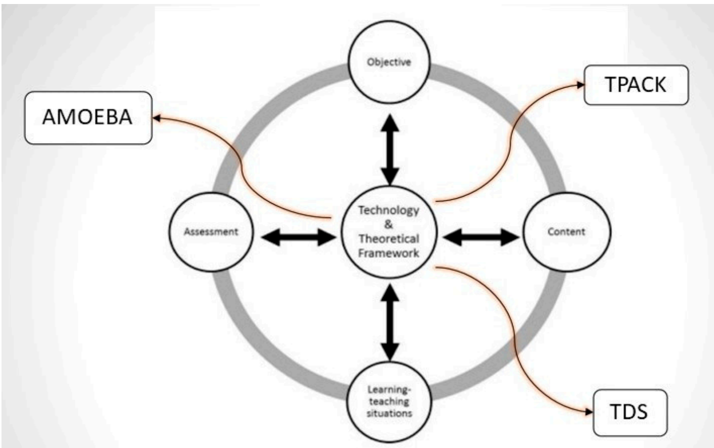


Figure 4. Integrational program development cycle inspired from (Demir [27]) with integrated proposed practices frameworks connection to the cyclic model.

The interplay between the theories that take place in different phases of the integrational program cycle which refer to our proposed practices are shown in Figure 4. The AMOEBA takes place through all the phases of the cycle to regulate cross-cultural variations. The teacher's TPACK takes place in the initiation and definition of the objectives and content of the program, then in the technology integration and finally in the assessment part. The TDS occurs when capturing the teacher's hypotheses from the objectives and defining the content and finally after the assessment takes place when we gather the teacher's reflections of their own milieu. In the next section we will highlight the methodology behind this research track.

4. Methodology and Methods

For the presented research approach in this paper, we follow the design-based research (DBR) methodology which has a dualism nature. The DBR is an iterative process which uses multiple components with diverse types that helps in tuning practice to meet what really works and to affect theory [28]. The DBR does not just contribute to the design process but rather provides a justification for the proposed designs. The DBR features for this research purpose are a good match because it allows for guidance through the design phase of the interventions from the beginning of the lesson planning till the end of the data collection from the participants; and what we believe is a strong feature to the DBR is the theory impact on the design phase and later on the theory contribution based on the research findings [29]. Moreover, DBR includes an exploratory nature in order to carry out the design phase and exploring the implementation possibilities as well as the practice results [29]. The research carried out is exploring integrating the proposed STEAM practices in mathematics education with a connection to culture and history. Because these practices are considered new, we will therefore try to explore its components, notions, and its applicability. Therefore, this would lead us to the measurable variables behind our research track which are architectural modelling based on mathematical knowledge encapsulation. We try to cover that by introducing GeoGebra modelling and explaining to the participants in the foundational stage how to use their mathematical knowledge while modelling. This can be measured from the participants' architectural modelled outputs. Another variable of focus is the cultural and historical integration, this can be measured from the participants' gathered knowledge about the selected architectures and from the survey and interview questions and finding the cultural relationship with the selected architectures. Teacher motivation towards these practices is usually measured during the intervention on three levels—before, in the middle, and after the intervention—to see if there are any motivational, beliefs, and behavioral changes that took place while introducing such practices. In this research we follow a mixed methods approach—mainly qualitatively for assessing the teachers interviews and quantitative for analyzing the survey questions from teachers and students. One of our core questions that we always asked participants is if they felt that they are doing any problem solving while they are modelling these practices and to provide an analysis to their modelling approaches. The answers to these questions were the highlights of the data represented in this paper and to develop the research question.

The research question proposed for the applications of these practices in this paper's contexts is: How does GeoGebra serve as an architectural modeling instrument for teachers to use in promoting mathematics learning in STEAM practices that fosters problem solving strategies?

We performed two different cycles with different participants, one in Upper Austria and the other one in the MENA region. We will provide a detailed description of the participants in the coming section.

Participants Sampling

From the proposed research question, we would like to clarify our main sample of participants, which is teachers who are interested in the proposed STEAM practices. The students are of minor focus in this research, therefore, we do not sample them, but the teachers do. Thus, in the intervention which will be presented in detail in the coming section which took place in Upper Austria, the teacher was teaching many classes in different grades. She chose this group of students because of her own criteria which she shared with us. The teacher's sampling criteria for choosing this group of students was their age, which she thought is suitable for these practices, and this group was newly introduced to modelling in the time of the intervention and some of them had prior GeoGebra knowledge.

For the MENA region intervention, the criteria in selecting the sample of participants was those who were taking the GeoGebra course. We were tracking the teacher's modelling attempts in the GeoGebra classroom during the course and therefore, the selected sample for interviewing and for data representation in this paper was not random, it was based on the teachers who did several attempts in the GeoGebra classroom and had the best modelled architectures in their final projects as will be displayed in later sections.

Part of the design process of the presented research is the data collection methods and instruments used which were interviews, questionnaires, and questions that are divided through the sections of the GeoGebra classroom to capture the participant's reflections before, in the middle, and after the interventions. These sources help us triangulate the data and come up with a sufficient analysis.

The upcoming section will describe the two interventions with a data analysis to summarize the authors' views and reflections.

5. Upper Austria Intervention Overview

In this section we will describe the intervention that took place in Upper Austria. The intervention applied the proposed practices in Upper Austria with a female teacher and her 35 students in HTL school, which is equivalent to the high school level in Austria. The intervention lasted for four months. During that time, we met the teacher four times. Each interview was recorded to capture reflections from the teacher on the technological, mathematical, cultural, and conceptual levels. The interview questions were piloted with other candidates and categorized based on the themes reflected later in the data analysis section. We introduced the research idea and the DLP to the teacher, which she used to choose the criteria she wanted to apply during the intervention with her students. The intervention criteria from the DLP were as follows: the students' aged 15–16 years old; the ancient architecture to be modelled is the Carnuntum in Upper Austria; and for the practice environment she chose the hybrid part to be online and another part to be in the classroom, this was due to the pandemic limitations during the intervention period. The last criteria the teacher chose was the technologies to be used. She chose the digital visualization to be used in the GeoGebra 3D modelling and the GeoGebra AR. The AR feature in GeoGebra was proven to help students in understanding the geometry lessons over conventional means as discussed by Series [6].

Due to the pandemic limitations, the teacher could not choose any technology that could physically represent the architectural models, such as 3D printing, origami, or 4D frames.

5.1. Upper Austria Intervention Implementation

This section will discuss how the practices' implementations took place in this intervention. We met the teacher twice to design and set the DLP criteria in order to plan the practices' design. We were projected in the classroom for half of the students who were physically in the classroom and joined by the other half who were at home on Microsoft Teams. The practice took one and a half hours, where we presented simple architectural constructions to the students to elaborate GeoGebra tools that can be encapsulated in the modelling process. We then took the Carnuntum from Upper Austria according to the teacher's criteria as the ancient architecture. Finally, we presented some historical and cultural facts about this ancient architecture to crystallize the bond between architecture, culture, and history. The teacher encouraged her students to find the culture and history connection with the architectural models that would be represented in their final projects.

5.2. Upper Austria Intervention Problem Solving Approach

We started to model the Carnuntum in two ways, and this approach was intended to stimulate the participant's problem-solving essence. They were to visualize the idea and concept of having many options to model and several approaches to follow during their modelling process. To allow the students to question and inquire about the wide range of application, possibilities for the GeoGebra tools were presented during the intervention. The first modelling approach followed a more complex method: starting the architecture modelling in the 3D view, not in the 2D view. This approach is complex because they interact with 3D objects and adjust the points and their relation to the planes in the 3D view. In this approach, we used curves to connect two separate polygons constructed in the 3D view as visualized in Figure 5. The second approach, and a more logical one, is constructing the whole shape in the 2D view by adding polygons and points, then extruding and translating it into the 3D view as visualized in Figure 6. Although the two approaches took time, we believe they may have given the participants an idea of the endless possibilities each participant can follow. It is crucial to show participants that there is no single way or approach of modelling and that this would encourage them to do more problem-solving to solve the problem of architectural modelling.

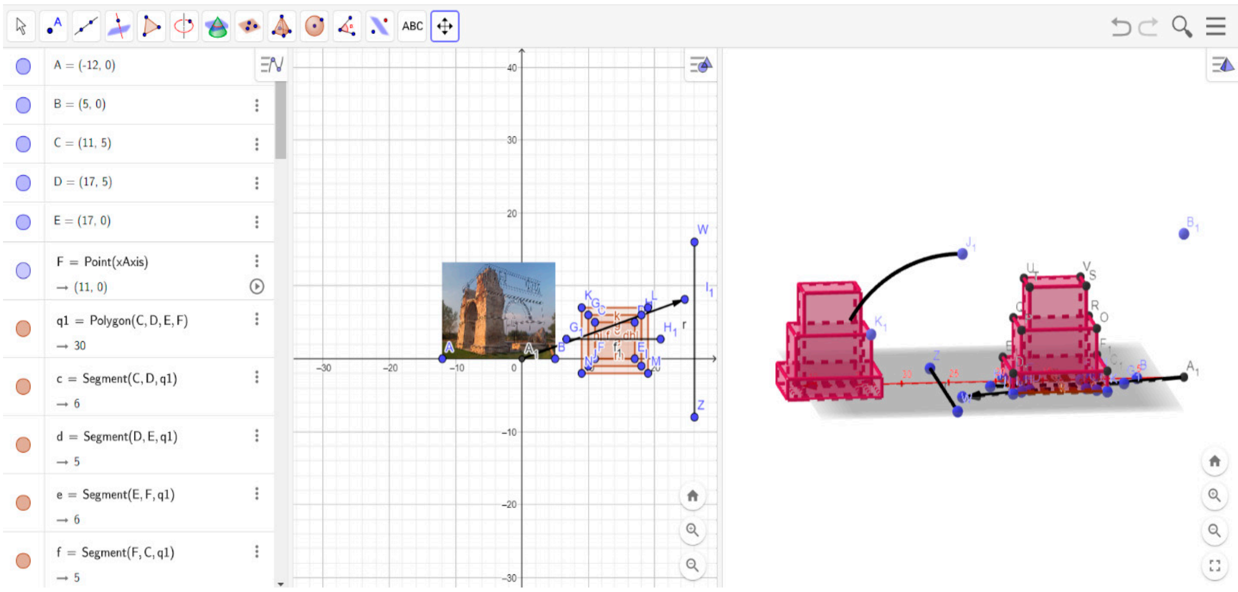


Figure 5. First modelling approach, in the 3D view using curves to connect polygons.

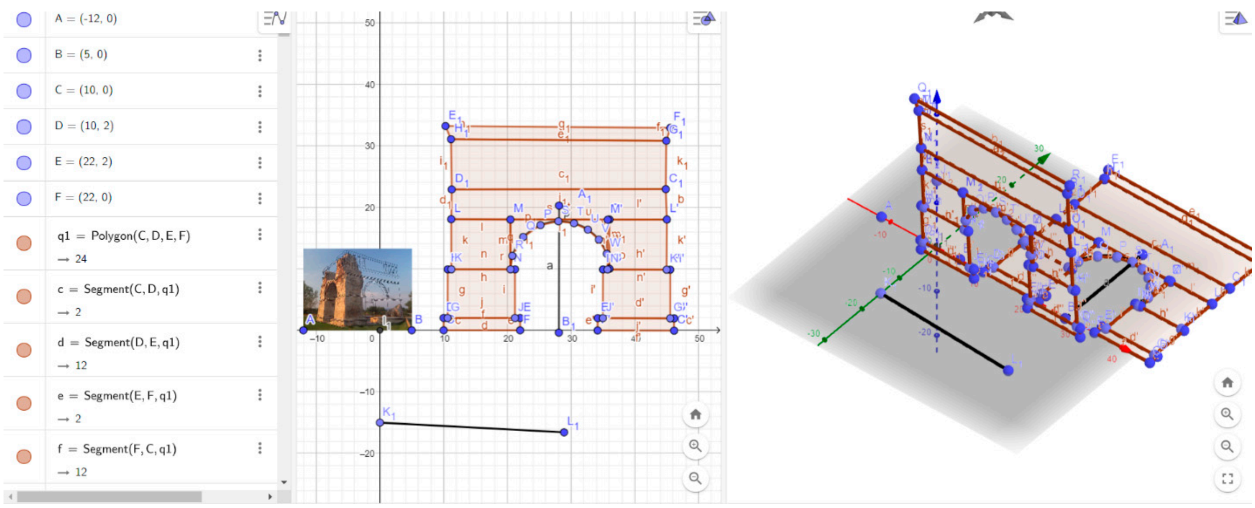
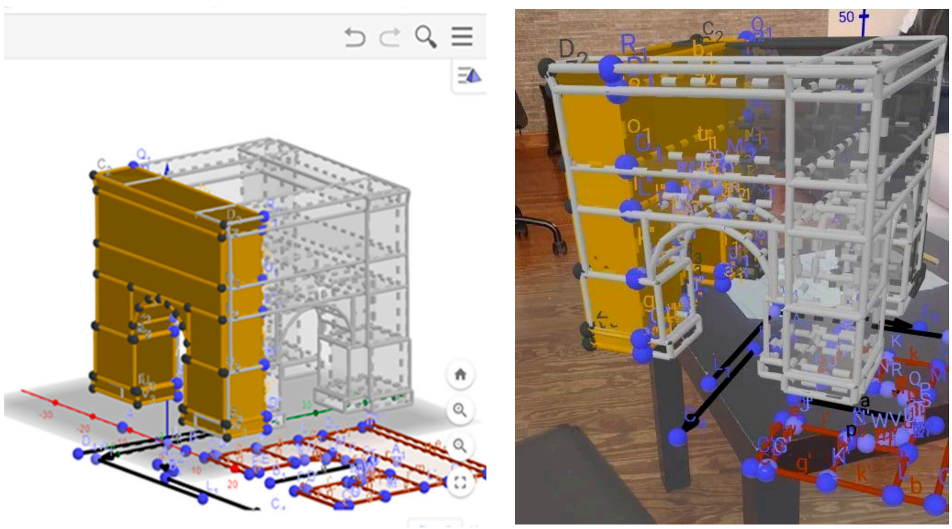


Figure 6. Second modelling approach, drawing the shape outline in the 2D view followed by rotation and extrusion in the 3D view.

After we presented the modelling approaches, we visualized how to change the colors and opacity of the polygons. It was intentional to give the Carnuntum a grey transparent look because this is how it was originally reconstructed. We aimed to connect the participants to real world symbols as architectures and by trying to represent these architectures by mathematical modelling. Moreover, in this step, we encourage the participants to simulate the actual architectural models as much as they can using GeoGebra tools. For digital visualizations, the teacher chose the GeoGebra AR. Therefore, we displayed the Carnuntum using the GeoGebra AR feature using the GeoGebra 3D calculator step-by-step, as shown in Figure 7.



(**a**) (**b**)

Figure 7. (**a**) The Carnuntum in GeoGebra 3D, (**b**) The Carnuntum in GeoGebra AR.

After the intervention, the teacher gave her students an unguided task to model any architecture, represent their motivations, and collect some historical facts about these architectures. The students handed in their final projects in documents and GeoGebra files.

5.3. Upper Austria Teacher's Mathematical Modelling Reflections

This theme is generic, tackling all the possibilities the participants may face during their modelling that is based on mathematical concepts. First, we will display some general thoughts from the teacher's reflections about the modelling and then we will focus on the GeoGebra 3D modelling in particular. The teacher lacked GeoGebra knowledge, but her motivation increased after the workshop. She started to do some modelling and to guide the students when they had questions. The teacher compared the 3D modelling in GeoGebra to the 3D modelling in Autodesk Maya (https://www.autodesk.com/products/maya/overview accessed on 30 November 2021). She said that the main difference is that in GeoGebra, everything has to have a mathematical basis and foundations, unlike Autodesk Maya. This is because GeoGebra is a mathematical platform, it gives higher priority to showing algebraic equations and mathematics behind the modelling process; unlike other modelling software giving more focus to simplifying the modelling process and handling all the logical equations and operations in the background without displaying them to the end-user. The teacher sensed that GeoGebra's modelling behavior is based on a different logic than standard software such as Autodesk Maya and Adobe Illustrator (https://www.adobe.com/mena_en/ accessed on 30 November 2021). GeoGebra 3D modelling would support mathematics more and visualize the equations, functions, and formulas allowing users to link to their mathematical knowledge while modelling [8].

The teacher told us that she believed that the learning outcome could be excelled by using these practices and she said that GeoGebra 3D modelling is quite beneficial as a first start to introducing 3D modelling concepts and software; and to introduce to the students the connection to mathematics and to see what is going on in the backgrounds of other modelling software by applying these practices and using GeoGebra 3D modelling. We will now explore the problem-solving strategies expressed in these practices.

5.4. Upper Austria Teacher's Problem-Solving Behavior

We will now highlight the teacher's common behavior and attitudes towards problem solving tasks. During the interviews with the teacher, we referred to questions that had to do with teaching methods and the type of tasks the teacher usually practices with her students. Additionally, the teacher's answers from the first interview highlighted that she in her regular teaching method tends to be flexible when defining the tasks; she allows her students to talk and express what they know about the given topic or problem before she explains it. She believes the student's answers would even guide her in which direction she would go, and which aspects of the problem need more focus than others. Moreover, she tries to map how the brain usually works and how it can retain information longer. Thus, she encourages her students to describe the problems and to try to approach them on their own, by reading books, watching videos, taking notes, creating mind maps that can help in solving problems, and collecting all the related aspects to a certain problem and finally presenting it.

Therefore, when we presented to the teacher the practices that we are introducing, she was ok with the idea of providing the students a problem, a foundation, and allowing them to come up with ideas and solutions on their own. Rott [30] defines the teacher's behavior that applies to problem solving strategies: they afford their students to participate with the problems and try to find a solution to the problem on their own. The teacher was also very flexible in her DLP choices, she wanted the students to have a freedom of choice when it came to the architectural models, the technology, and even the environments of practice, but unfortunately the pandemic had a limiting effect on the technologies and environments choices.

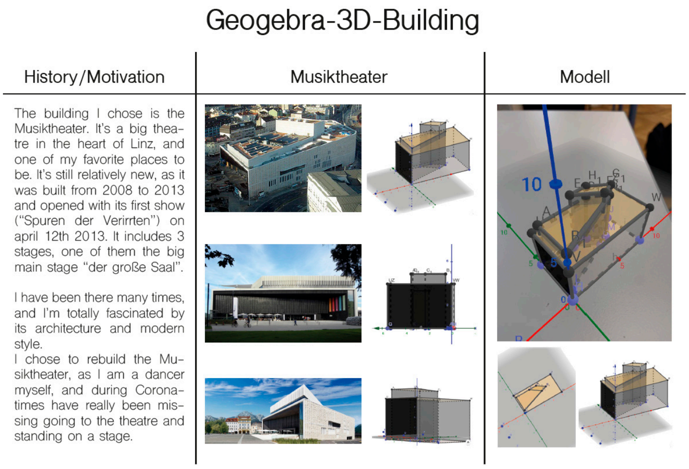
Rott [30] describes the types of behaviors of teachers and the last behavior he stated is what we can also adopt for the teacher's behavior during this intervention: "*There are teachers that try to give as little feedback as possible in order to not influence their students' individual ideas*" [30].

5.5. Upper Austria Final Projects

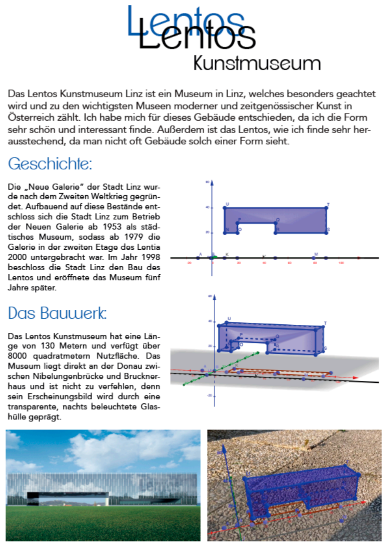
This section displays some of the student's projects after the intervention to solve the assigned unguided tasks. Figure 8 shows the submitted documents of the architecture modelling, GeoGebra AR visualization, motivation, and some historical facts.

In the following section we will explore the MENA intervention and focus on one of the participants' work and reflections.

Figure 8. *Cont.*



(**e**)



(**f**)

Figure 8. Some samples of the Upper Austria students' final projects. (**a**) Ancient Egyptian Pyramid, Egypt (**b**) Donau Tower, Austria (**c**) Big Ben Clock Tower, England (**d**) Berlin TV Tower, Germany (**e**) Music Theatre Linz, Austria (**f**) Art Museum Linz, Austria.

6. MENA Region Intervention Overview

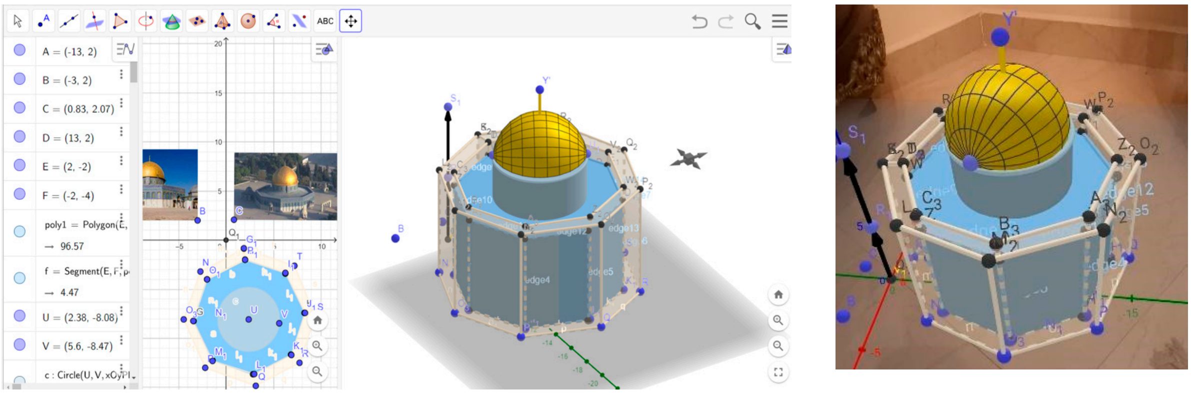
We will now present another intervention that took place with 9 in-service mathematics teachers from various countries from the MENA region. They were middle aged teachers all working as mathematics teachers in their own countries and were teaching different age groups. This intervention was part of an ongoing GeoGebra course that lasted for two months. Our contribution was three sessions—each one lasted for two and half hours. The sessions took the form of a workshop allowing the teachers to work simultaneously in the GeoGebra classroom that was designed and provided to them earlier.

The intervention took the same approach as the one presented earlier in Upper Austria where we showed the participants some significant architecture modelling in a step-by-step approach from different countries in the Middle East during the first session as shown in Figure 9, to explain and visualize the idea of architectural mathematical modelling. This step was the foundational step for preparing the participants for the unguided tasks.

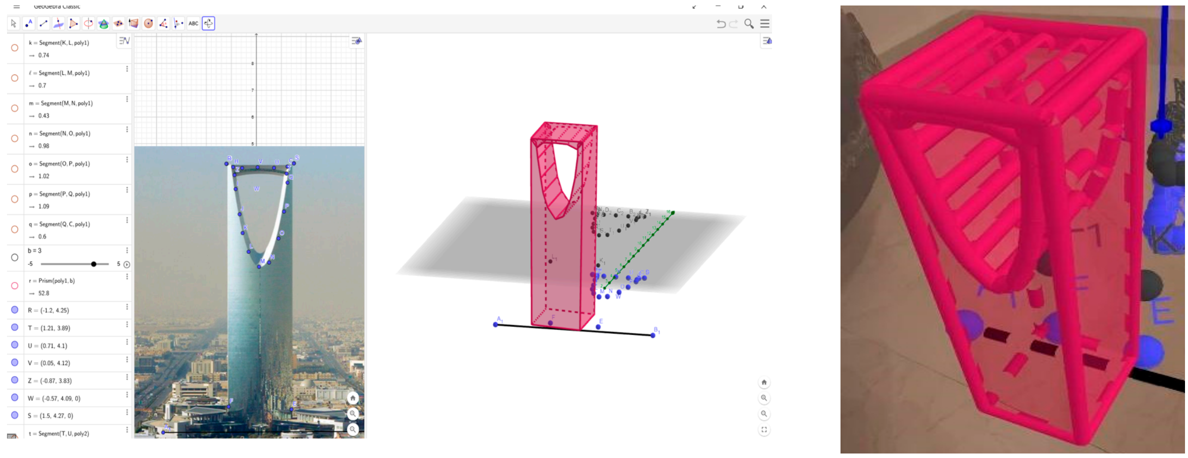
For the second and third sessions we got some input and feedback from the participants from the GeoGebra classroom which was left open for the participants to try to implement their own architectures. In the following sessions we took some examples from the participants' trials and implemented them.

6.1. GeoGebra Classroom as a Problem-Solving Mirror

The participants showed their problem-solving strategies unintentionally from their work inside the GeoGebra classroom during the intervention. Because we could monitor their progress, their attempts, and their failed and successful trials. The real time monitoring feature the GeoGebra classroom provided us with a good tool to see the participants' trials. Additionally, building on this experience we can reflect on Lieban and Lavicza [14] by adding that the GeoGebra classroom feature acts as a real time reflective mirror as a metaphor for problem solving strategies that teachers can consider for fostering reasoning and assessing unguided tasks.



(**a**)



(**b**)

Figure 9. The architectures practiced by the MENA region teachers during the workshop. (**a**) Dome of the Rock Mosque in Palestine, (**b**) Kingdom Centre Tower in Riyadh Saudi Arabia.

6.2. MENA Intervention Problem Solving Approach

In this paper we will shed light on one of the MENA participants' trials. She is a mathematics teacher from Libya. She had been learning GeoGebra for six months before joining this course. She was 42 years old and teaching sixth-grade mathematics in Libya. We chose this teacher because she had successive attempts that showed problem solving strategies in solving the modelling of the architectural model in Figure 10 in the GeoGebra classroom. The Libyan teacher chose to model the architecture in Figure 10, which is the center of a Libyan Mosque located in her city.

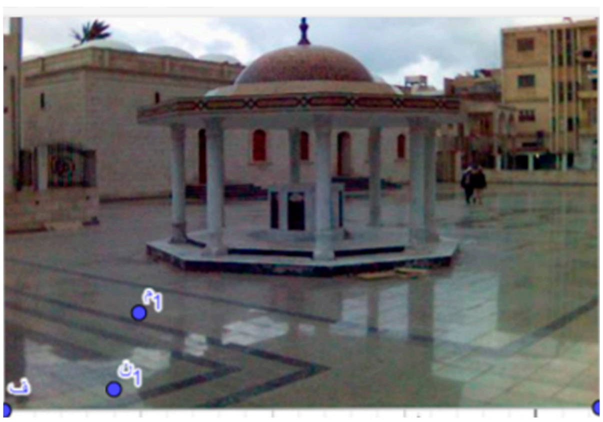


Figure 10. The Libyan mosque center.

Figure 11 shows her first attempt in modelling this architecture in the GeoGebra classroom, constructing the base of the architecture as an octagon polygon with its internal layers.

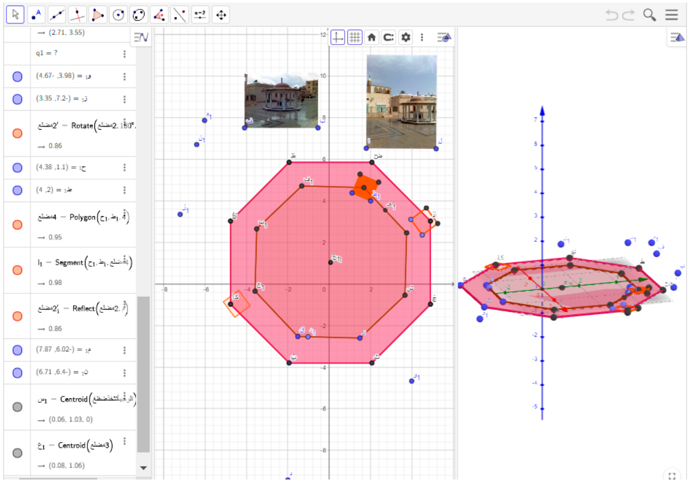


Figure 11. The Libyan teacher's first attempt at modelling the Libyan mosque center.

Figure 12 shows the teacher's second attempt by constructing the base of the columns and trying to construct the columns of the architecture.

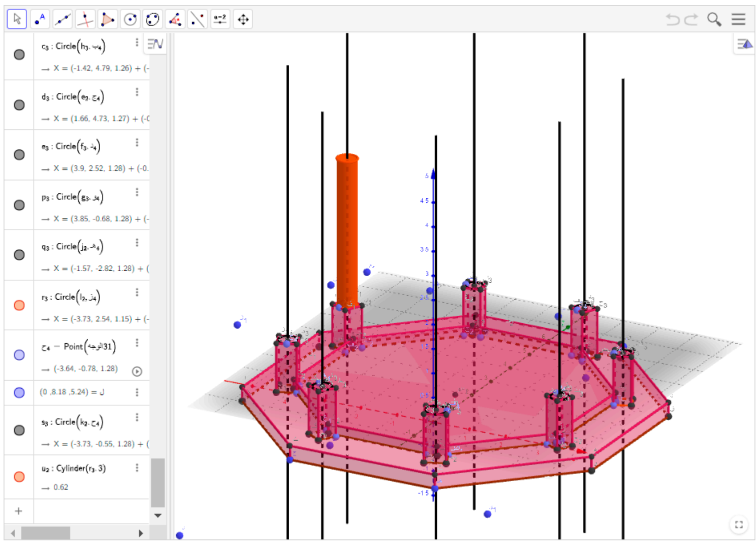


Figure 12. The Libyan teacher's second attempt in modelling the Libyan mosque center.

During our second session we took this architectural example to assist the Libyan teacher in her trial and motivate her to understand the main significant concepts behind modelling this architecture. We guided the participants in a step-by-step fashion to construct this architectural shape. Figure 13 shows the final architectural modelling after guiding the teacher and the participants in a step-by-step approach towards achieving this modelling. We then showed them the digital representation of this architecture using GeoGebra AR.

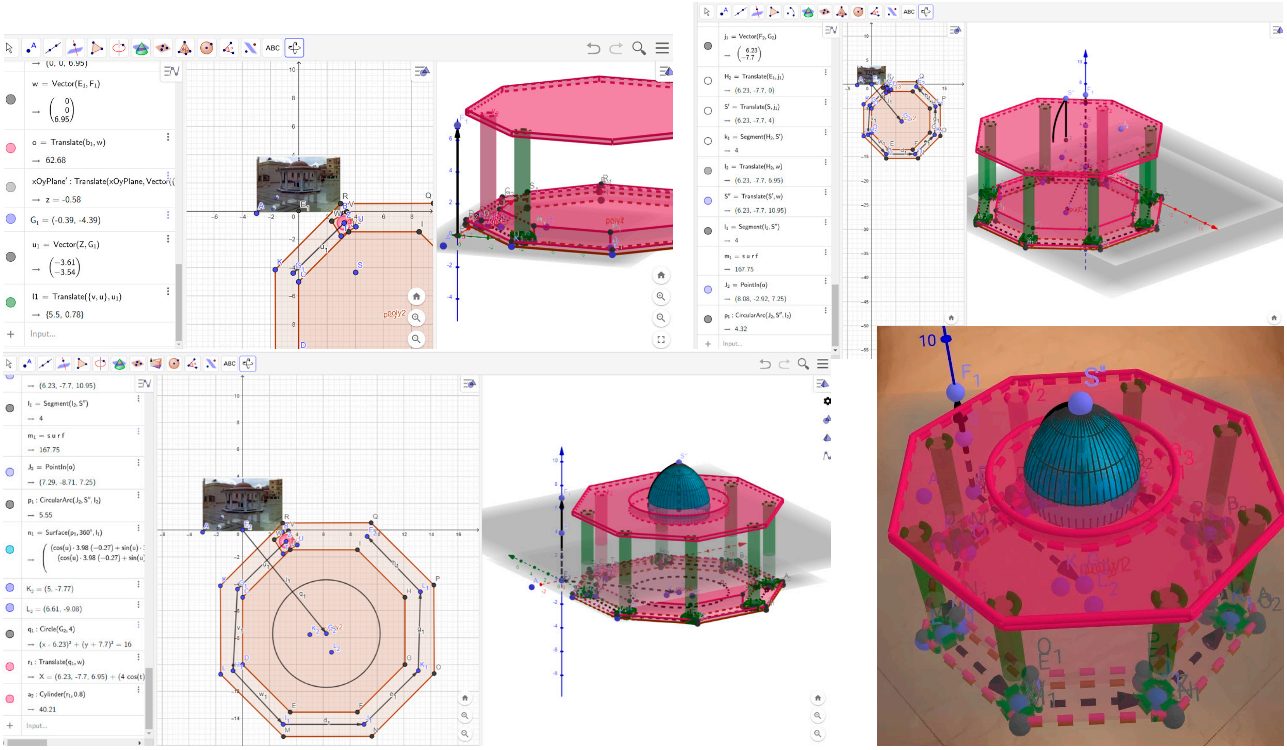


Figure 13. The steps to modelling the Libyan mosque center architecture and the AR representation.

6.3. The Libyan Teacher's Problem-Solving Behavior

We interviewed the Libyan teacher before the end of the course and after the final session to gather some reflections from her on the proposed practices. When we investigated the teacher's own teaching methods, we saw that she allows her students space for explorations, especially in the beginning of the lesson. She starts by asking the students about the lesson title and allows them to deduce the content and challenge them to see who will answer first, and then she always asks some IQ questions. The teacher reported that the students like this teaching style and wait for that part of her lessons. The teacher also uses the IBL techniques in allowing the students to prepare the lesson, gathering information about the topics before she starts explaining them. For the GeoGebra motivation, the Libyan teacher showed a huge interest in using GeoGebra; she also believed that it gave her another method of explaining mathematics in an interactive manner.

The teacher believes that GeoGebra solved many problems, for example, her son did not understand the pi factor so she could not explain it in normal ways, but once she learned GeoGebra she could do it easily and she said that now he had seen the visualization, he could understand it better.

The Libyan teacher also added that as a teacher, when she prepares her GeoGebra material she could alter simple equations and numbers that would give her new contents to practice with her students. The Libyan teacher showed interest in the proposed practices and gave a real-life example from a school in Turkey that she knows that practices these ideas of bringing real-life examples into the classroom and trying to model them. The teacher added that what interests her is: "*she said what is interesting is how the students do it and how they approach the models, it could be someone starts with a circle, another one with a square, but they all approach the model in a different way*".

The teacher thinks that such practices can start by drawing shapes and monitoring student's creativity some will have the capability of understanding such concepts and they have this talent while others do not, but she believes that such introduced practices could be of great help and that it fosters problem solving as well as expanding the student's reasoning.

We were keen on observing the teacher's normal teaching methods because it reflects on the problem-solving strategies that she uses in her own teaching, according to Rott [30], and on the teacher's own behaviors.

6.4. Libyan Teacher's Architectural Modelling

After the intervention, the participants submitted their final projects as architectural models and collected historical facts about these architectures. The teachers were motivated to submit the whole project as a GeoGebra file including the text, audio, and video along with their GeoGebra 2D and 3D representation. We will present the final work of the Libyan teacher to elaborate for the readers what these projects looked like.

Figure 14 shows the teacher's architectural choice, called "Ghadames City", located in Libya, one of the oldest cities in the world. The teacher collected cultural and historical facts about this city along with some pictures in a video using GeoGebra.

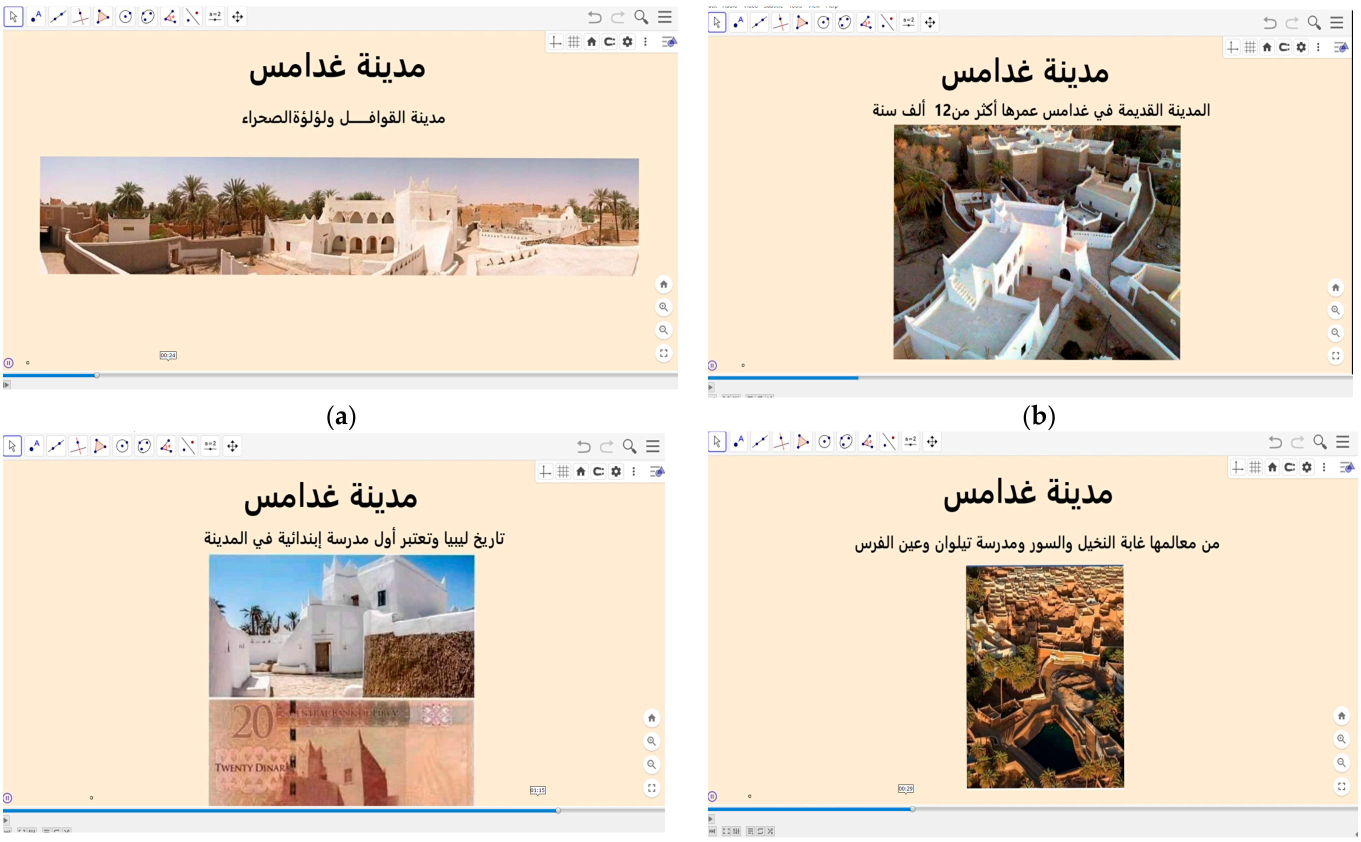


Figure 14. (**a**) Ghadames was called the city of tribes and the desert pearl, (**b**) the old city in Ghadames aged 12 thousand years, (**c**) the first primary school in Ghadames city was put on the Libyan currency, (**d**) the landmarks are the palm forest, the fence, Tyloan school, and "El Fares" eye.

The teacher wanted to represent this city in GeoGebra, thus she drew a concept map and collected some architectures from the Ghadames city and combined them in a small GeoGebra concept which features the houses' architectural styles with the shapes on the ceilings and house edges, stairs, colors, the water eye in the Ghadames city, and the sand pyramid. The city had many palm trees, thus the teacher tried to reflect that in her project to simulate the reality of Ghadames city in Libya as shown in Figure 15.

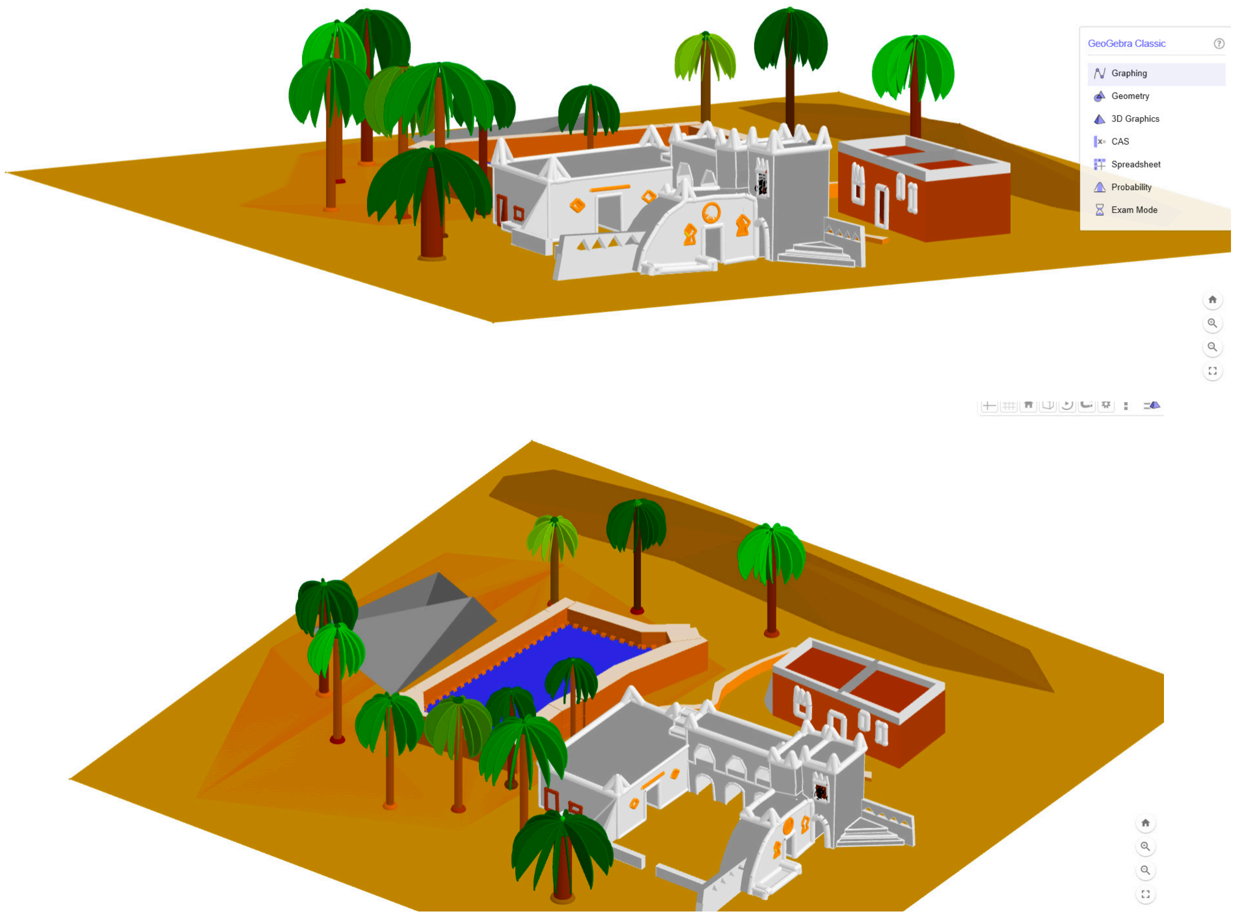


Figure 15. Libyan teacher's architectural modelling of the Ghadames city in Libya.

The teacher showed problem solving strategies in implementing her final project and in the idea of forming and modelling a concept map of the whole city with its main landmarks that reflects the Ghadames city identity. She approached this challenge, which is problem-based, by solving in the manner of combining and designing the architectural components in a single GeoGebra project.

7. Interventions Data Sources and Results Overview

In this section we describe the outcomes and results that we obtained from both interventions in Upper Austria and MENA and the type of the data retrieved.

The two interventions had slight differences in their research design implementation and that was due to the different circumstances. Thus, the first intervention that took place in Upper Austria was with a teacher and once she was convinced by the research idea after the first two meetings, she decided to introduce the research practices to her students in the form of a workshop. Thus, this intervention gave us access to the teacher's and students' data; while the second intervention that took place in the MENA region was part of an ongoing course for in-service mathematics teachers. Thus, in this intervention we collected data from teachers only.

The results of these practices can be captured from both interventions from the final projects outputs that were referred to earlier in this paper after each intervention. The results were their final projects which were further analyzed to extract the useful measurable variables of interest to us and to help us answer the proposed research question. In both interventions and both design approaches we collected data from multiple sources which helped us in triangulating the results. On one hand, the participant's final projects from Upper Austria, which were submitted in the form of documents and GeoGebra files, followed the document analysis techniques. As described by Baglama et al. [31], document analysis is a qualitative research method because the researcher gets to analyze the document content that could be written text, images or diagrams, and tables. Thus, these documents will be qualitatively analyzed. Another qualitative analysis took place on the interview data that was video recorded with the teacher.

On the other hand, the final projects from the MENA region were submitted in the form of GeoGebra projects that included text, video, audio, and the modelled architectures. Additionally, the Libyan teacher's interview was qualitatively analyzed. There is survey data from both interventions which will undergo quantitative data analysis.

The data was divided into two groups according to their method of collection, either interview data, documents, or GeoGebra projects. These were the data sources that helped us in retrieving the participants' reflections and their final projects in both interventions.

8. Discussion

We are reflecting on the two intervention outputs from a generic point of view and looking at the main aims these practices were trying to settle and achieve. As discussed earlier, these practices are eager to integrate architecture, culture, and history into mathematics education in a STEAM fashion.

Therefore, we will now display the significance of these notions and how they were translated into the participant's results.

The relationship of mathematics to architecture is obvious as geometry plays a crucial role towards students learning about architecture as they use it to help in expanding 3D knowledge and it helps them to construct their designs. [32,33]

Art and architecture can be a connection point to teach geometry in a more practical way referring to actual constructions, shapes, forms, and patterns according to Mcgowen [34]. They created several lesson plans, including the objective of the mathematical knowledge content and procedures for how to apply these lesson plans for mathematical educational purposes by using architectural and artistic examples.

When we focused on the architectural notion, and its relation to mathematics, this was significant in the nature of the mathematical modelling the participants tended to practice during these interventions. Thus, the participants did architectural modelling using GeoGebra as was displayed in the final projects the participants handed in. The architectural models were diverse due to the participants' freedom of choices. Due to the fact that architecture is connected to culture and history [35], the participants had the chance to represent their motivations behind choosing these architectures which for us reflected a cultural significance of these practices. For the historical notion, the participants collected historical facts and displayed historical knowledge about the architectures they represented. The amount of knowledge varies from one participant to another, but it proves the historical significance in these practices [36].

These practices' aim is to foster mathematical knowledge, therefore GeoGebra was chosen over any other modelling software [37]. The modelling strategy was based on mathematical notions and problem-solving practices as was displayed in the two interventions in Upper Austria and in the MENA region. The mathematical significance was obvious in the functions and tools used during the participants' modelling that are represented in their final architectural models.

The last main aim of these proposed practices is to allow participants to have sufficient modelling knowledge using GeoGebra to be able to model architectural constructions. This last aim is different from one participant to another because it depends on the mathematical and reasoning knowledge they possess and also the differentiation between a participant that could be a teacher or a student, as was represented in the diverse outputs that were featured from the two interventions represented in this paper. We saw architectural outputs from teachers in the MENA region and from students in Upper Austria.

We tended to measure the participant's modelling competencies, motivations, behaviors, and attitudes, which will be discussed in future papers while taking into consideration GeoGebra knowhow.

Problem Solving and Reasoning in the Proposed Practices

Lieban and Lavicza [14], in their work, considered instrumental genesis using GeoGebra and they connected that with the heuristic strategies of Polya's [15] work on problem-solving which needed more empirical research experimenting for its classroom application. Therefore, they suggested transforming these heuristic strategies into interventions to be practiced by teachers. They defined the problem as the 3D transformation of the digital and physical representation of the same models. Their geometric modelling strategies included various approaches which were an attempt to follow Polya [15] strategies that are related to backward thinking, generalization, specialization, and decomposing. They demonstrated that decomposing the problem is dividing it into smaller steps to find a solution to that particular problem while using the instrument, which in that case was GeoGebra. At the end they proved that Polya strategies served their goal in using GeoGebra as an instrument. Based on Lieban and Lavicza's [14] work in using Polya's problem solving strategies and using GeoGebra as an instrument we want to use this instrumental genesis justification in understanding the results generated from these proposed practices of applying architectural modelling using GeoGebra. From these findings we capitalize on their work and consider GeoGebra as an instrument to aid in architectural modelling, while connecting to the problem-solving heuristic strategies of Polya's [15], but from another perspective. For us involved in this research, the proposed problem is to model architectural buildings and attempt to simulate its constructional elements using GeoGebra. According to Polya, we can reflect on the decomposition step that took place for the participants while they were modelling architecture from the algebraic view in GeoGebra, which shows the modelling steps that took place. From GeoGebra's algebraic view we can explore how participants approach an architectural model; they usually start with some reasoning on the priority of the element's order as a first step. What should the base of the building be and what should then appear on top of the building is displayed in the hierarchy of the elements used. After prioritizing the architectural elements, in each stage of the modelling process the participants should wait and reason which tools and functions they should use from the wide options in GeoGebra to simulate the most similar constructional result as the architectural building's real shape. The participants may learn to specialize tools for their needs which may allow us to see the application of *"specialization"* in these practices. Thus, the whole architectural modelling process may need a continuous "*backward thinking*" strategy referred to by Polya, that participants have to think about in order to achieve the required decomposed step; which tools the participants should continuously revert to, explore, and make sure would satisfy or specify their final architectural goal.

The problem-solving strategies also apply in the first steps of these practices when the participants choose the architectural model. At this stage, when participants are familiar with GeoGebra's modelling capabilities, this information and foundational step that is acquired during the intervention would interfere in their final project's architectural choices because they may sense which architectural models can be modelled using GeoGebra and which ones would require another instrument to achieve success. Therefore, we always encourage participants in our interventions to look at the architectural models in a mindful way, to think and analyze the building components before approaching the modelling stage.

If we reflect on Polya's [15] strategies for the results that we gathered from both interventions, it helps in resolving the proposed research questions. We can see the steps followed from each participant in each intervention from the algebraic view of the modelled architectures Thus, some participants used various mathematical functions and operations that vary, and this could be due to the architectural nature that, for example, does not require reflection about a line but would require translation. Therefore, this analysis step and breaking the architectural modelling approaches down into smaller steps would require us and practitioners who are evaluating the participants models to follow Polya's strategies in doing a "*backward thinking*" strategy to always refer to the real-life architectural image the participants chose. We tend to use "*specialization*" strategies in evaluating the GeoGebra tools the participants adopted and which tools they specialized in, how they adopted them, and why. This type of analysis may allow teachers and practitioners of such practices to open the door to discussion of the various modelling approaches and possibilities each participant can follow. This discussion of various modelling approaches can be also useful when comparing two implemented models of the same architecture and exploring the various attempts and strategies by different participants. All these mentioned possibilities are a golden chance for application of problem-solving strategies by teachers using the GeoGebra instrument in the implementation, analysis, and discussion phases of the architectural modelling.

9. Conclusions

At the end of this paper, the introduced practices are attempting to promote problem solving strategies in order to solve the problem, which is architectural modelling. We discussed both interventions in Upper Austria and in the MENA region with the possibility of integrating practices that encapsulate the mathematical knowledge that also requires reasoning and problem-solving strategies to model architectural buildings. We explored that the architectural usage added another connecting point to these practices, which are cultural and historical advantages that were displayed in the participant's final projects in the motivations and historical facts in connection with the chosen architecture. We received diverse architectural representations from both interventions. Therefore, practices can allow a wide variety of technology exposure. The architectural modelling proposed could be modeled in various ways using a wide spectrum of technologies, allowing the application of 3D transformational representations from digital to physical.

The limitations of the proposed practices are that teachers mistakenly adopt them by leaving out the main notions. For example, by only focusing on the technical part of the GeoGebra modelling and, for example, not adopting architectural examples which may affect the link to culture and history or any of these notions, can be left out, which may affect the interdisciplinary approach behind these practices. Moreover, teachers can neglect the application of the problem-solving strategies by showing a single modelling approach to their students or by not promoting the problem-solving strategies while modelling and this may affect the educational benefits behind these practices.

The teachers who adopt these practices can be guided towards using a recommended rubric for these practices, if they wish to develop their own rubrics in the evaluation phase then it should cover the main notions of these practices which are the mathematical modelling, modelling approaches (problem solving), cultural and historical significance, and finally, the technology representation assessment, whether digital or physical. All these evaluation preferences can be decided by the teacher, and they may vary according to the culture, the teaching methods, and curriculums followed. These notions are the cornerstones of such practices and teachers should understand and learn how to introduce them to their students, implement them, and finally assess them. Our approach in overcoming these limitations is by providing teacher training in the foundational steps of these practices where we highlight the main notions and how they can be adopted moreover by providing directional URLs in the DLP to instructional texts and implemented examples in a preprepared GeoGebra book.

Our future work in this research track is to explore various cultures in the adoption of such practices. This would provide us with more diverse data to evaluate the applicability of such practices across cultures. This approach would make us connect more theoretically to the proposed theoretical frameworks in understanding the teacher's milieu in the TDS, in developing the teacher's TPACK, and finally in regulating the cultural variations using the AMOEBA framework. These theoretical connections can be presented in future publications and may help in crystallizing our thoughts.

From this point we believe that these practices may be of good use for teachers' adoption in their teaching that could connect to real-life examples as architectures while practicing technologies. That architectural modelling could be an out of the box practice that could foster problem solving, inquiry, and encourage creative solutions using GeoGebra.

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