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Assessing van Hiele's geometric thinking levels among elementary pre-service mathematics teachers

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ABSTRACT

Teachers' geometric thinking is crucial to teaching efficacy in geometry since teacher knowledge or thinking serves as a basis for the quality of instruction provided for students' learning. Teachers' thinking about geometry has attracted much attention among mathematics education researchers. This study therefore aimed at assessing elementary pre-service teachers' geometric thinking within the first three levels of van Hiele's model. The study was guided by three objectives. The objectives were to (1) assess the distribution of van Hiele's geometric thinking among the study participants, (2) determine if the participating pre-service teachers' geometric thinking is significant for teaching geometry, and (3) find out if any difference in geometric thinking of the pre-service teachers existed with regard to gender. The study used the descriptive survey design. The study participants were prospective mathematics teachers drawn from four Colleges of Education in the Bono Region and Ashanti Region of Ghana. The Colleges were randomly selected for the study. The study participants comprised 217 pre-service teachers. The van Hiele's test instrument was adapted and pilot tested to assess the internal consistency of the items in the various levels. The calculated reliability coefficient of the instrument ranged from 0.71 to 0.74. The instrument was administered to the study participants on the scheduled date. Data generated from the participants were analysed based on the study objectives. Findings from the analyses show that pre-service teachers have limited geometric thinking within the first three levels. However, their geometric thinking of the levels assessed was found to be significant which could have some impact on teaching geometry. Findings also reveal gender differences in pre-service mathematics geometric thinking. It is recommended that conscious effort must be made by mathematics teacher educators in the Colleges of Education to deepen the pre-service mathematics teachers' geometric thinking.

Keywords: Gender, geometric thinking, pre-service teachers, van Hiele's thinking levels.

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INTRODUCTION

Teachers' depth of knowledge is considered one of the most significant variables responsible for effective teaching. Teachers' understanding of the subject matter serves as an underlying factor for instructional quality, and also as a predictor of students' achievement (Hill et al., 2005; Robichaux-Davis and Guarino, 2016). Almost all classroom activities are centred on the knowledge base of the teacher. According to Krauss et al (2008), a high teacher knowledge base of the subject matter forms the pivotal knowledge for teacher competency. Although there are several knowledge-base a teacher needs to possess (Shulman, 1986), knowledge of subject matter is of critical concern to teacher education programmes (Aslan-Tutak and Adams, 2015). Aslan-Tutak and Adams (2015) emphasise that the subject matter knowledge of the teachers has a strong effect on their pedagogical practices. In effect, teachers must possess adequate knowledge of what they are supposed to teach before they can be in a position to determine how best to develop that knowledge among students.

The main reason for teaching mathematics is to enable students to develop high reasoning abilities in solving problems, and to enable them to utilise mathematics knowledge within and across all fields of study (Aslan-Tutak and Adams, 2015; Robichaux-Davis and Guarino, 2016). Mathematics curriculum frameworks, therefore, require that students are engaged in diverse thinking abilities to enable them to solve multi-step tasks (Robichaux-Davis and Guarino, 2016). Sibiya and Mudaly (2018) maintain that the role of mathematics education is to create and develop logical and critical thinking in learners. Research has shown that one of the topics that stands a high potential of developing learners' thinking skills is geometry (Sulistiowati et al., 2019). Learning geometry enables one to acquire thinking skills of being logical and analytical. Such skills are believed to be important tools for learning mathematics.

The utility of geometric knowledge in our daily lives cannot be underestimated. We live in a world of shapes and spaces and we can understand our world of living only when we have adequate and functional knowledge of geometry. Geometry which has its meaning as 'earth measurement' is aligned with the practicality of learning mathematics as we measure lengths, volumes and areas daily, using tools of the olden days which are still significant and are still in use. Understanding our world of the living or the living environment which is made up of shapes and artefacts and the relationships among their properties is heavily dependent on the study of geometry (Ngirishi and Bansilal, 2019).

The benefits of geometry, both in cognitive development and applicability to solving real-life problems, have given it critical attention in the mathematics education curriculum. As a result, there have been high expectations by curriculum frameworks in equipping learners with a good understanding of geometric knowledge across all levels of education. The realization of such expectations to a high extent depends on the level of understanding possessed by teachers, particularly, pre-service teachers who join the teaching service each year (Robichaux-Davis and Guarino, 2016). Pre-service teachers need to possess in their knowledge repertoire, deep understanding of geometric contents and be capable of representing the content in multiple perspectives to be able to competently facilitate or guide learning among learners.

The model of geometric thinking developed by van Hiele and his wife has gained strong acceptance in the mathematics research community of which several studies have been conducted to determine learners', pre-service and in-service teachers' geometric thinking (Armah and Kissi, 2019; Ngirishi and Bansilal, 2019; Robichaux-Davis and Guarino, 2016; Sulistiowati et al., 2019). The authors outlined five levels in their model for assessing geometric thinking in the geometry curriculum. These levels are visualisation, analysis, ordering/informal deduction, deduction or formal deductive and rigour (van Hiele, 1986). This study however focused on the first three levels which "identify thinking within the capacity of elementary school learners" (Spear, as cited in Robichaux-Davis and Guarino, 2016, p. 12). In addition, a study of the elementary school geometric curriculum necessary for understanding high school geometry falls within van Hiele's first three geometric thinking levels.

It is imperative therefore for the pre-service mathematics teachers to possess adequate and functional thinking of geometry at the minimum of van Hiele's first three levels. Pre-service teachers need this knowledge to facilitate elementary learners' geometric understanding in order to be successful in higher geometric courses in high school and beyond. This study then aims to determine the acquisition of the first three van Hiele's levels of geometric thinking by pre-service mathematics teachers. Since the teaching force is made up of both males and females, the study also focused on the difference in geometric thinking of gender, since it has been an important issue in mathematics education (Ma et al., 2015).

Statement of the problem

Mathematics, which has received much attention in education due to its importance in a nation's development and economic growth, has shown to be a subject that produces an unsatisfactory performance of learners in many parts of the world (Aslan-Tutak and Adams, 2015; Robichaux-Davis and Gaurino, 2016; Sibiya and Mudaly, 2018). Mathematics educators will agree that geometry is one of the topics that most learners experience difficulties in learning and understanding. However, geometry can be conjectured as one of the topics in the mathematics curriculum frameworks that develop critical reasoning abilities among learners. It develops spatial and visualisation abilities, learning tools necessary for learning mathematics in general. Research has shown that knowledge of geometry positively correlates with mathematics performance (Biber et al., 2013; Lutena, 2015; Robichaux-Davis and Gaurino, 2016). It will therefore be desirable to emphasise the teaching and learning of geometry as a means of enhancing performance in mathematics.

In the mathematics curriculum, geometry is considered either as a separate course (high school) or integrated with other topics (elementary school). For instance, learners who graduate from high school in many states in the United States are required to achieve some standards in geometry taken as a separate course (NCTM, as cited in Martinovic and Manizade, 2018). In Ghana, geometry is integrated into the mathematics curriculum at the basic and second cycle level (Lower Grade up to Grade 12 equivalence) and hence its score is embedded in general mathematics score. Whatever the case has been, concerns have been raised about learners' poor performance in geometry across the globe (Martinovic and Manizade, 2018; Ngirishi and Bansilal, 2019). Although the Ghanaian mathematics curriculum considers geometry as integrated with the other topics, learners' difficulties with solving problems in geometry have, in many years, been reported by the Chief Examiner of mathematics for the examination conducted by the West African Examinations Council ([WAEC], 2007, 2008, 2012, 2017) at the elementary school level.

Despite some literature suggesting that teachers face lots of difficulties in teaching geometry (Adolphus, 2011), which may be attributed to inadequate teacher preparation in such topics (Martinovic and Manizade, 2018), there seem to be few studies on assessing pre-service teachers' geometric thinking in Ghana. The aim of this study, therefore, is to investigate elementary pre-service teachers' geometric thinking.

Research question

1. What is the level of van Hiele's geometric thinking among pre-service mathematics teachers?
2. The participating pre-service teachers' geometric thinking is not significant for teaching geometry.
3. There is no gender difference in geometric thinking among the participating pre-service teachers?

LITERATURE REVIEW

Theoretical framework

This study was conducted through the lens of the levels of geometric thinking developed by two Dutch couple, van Peire van Hiele and Dina van Hiele-Geldof, in the late 1950s. The model on geometric thinking has been widely accepted by the mathematics research community and has been an informed model or framework for assessing learners and teachers' geometric understanding in diverse ways (Armah and Kissi, 2019; Ngirishi and Bansilal, 2019; Robichaux-Davis and Guarino, 2016; Sulistiowati et al., 2019). Several countries have used this model to develop a geometry curriculum for mathematics education (Ma et al., 2015). The model comprises five levels of geometric thinking that learners develop in understanding geometric concepts (van Hiele, 1986). The following sections provide brief explanations of the first three levels due to the focus of this study.

The first level, visualisation, is characterised by learners' ability to judge figures/shapes by appearance. Thinking at this level is based on what is seen, hence the term, visual skills. Learners' thinking about shapes is based on resemblance. A shape may be considered a rectangle because it looks like a door and not because it has four sides. Learners may have no idea about the properties of the shapes. They can only classify shapes in groups. At this level, learners will not be able to recognise a shape as a rectangle when tilted to stand on one of its corners.

Level 2 of van Heile geometric thinking, called descriptive or analytical, is characterised by learners' abilities to recognise or identify properties of geometric figures/shapes. Learners operating at this level are said to possess both drawing and verbal skills. Learners' thinking about shapes is influenced by their properties. They can list the properties of shapes but will not know the relationship between them. For instance, they may not conceptualise that a good definition of a rectangle is 'a figure with four sides and four right angles.' No implications are drawn among properties of shapes and a learner operating at this level may not understand why a square is also a rectangle.

At Level 3, ordering/informal deductive, learners can think of properties of geometric shapes and their relationships with and between the shapes. Learners understand and can formulate meaningful definitions of geometric shapes. They can follow an informal deductive argument. For instance, 'all rectangles are parallelograms, but not all parallelograms are rectangles', or all squares are rectangles, but not vice versa. Learners are able to hold informal justification of their geometric reasoning and cannot construct formal proofs in geometry.

Geometry content knowledge

Researchers agree that among the knowledge-base of the teacher (Shulman, 1986, 1987; Ball et al., 2008), subject matter knowledge also known as content knowledge is of much critical concern in teacher education (Aslan-Tutak and Adams, 2015). Aksu and Kul (2016) define content knowledge as the "organisation and amount of knowledge in a teacher's mind" (p. 35). The content knowledge of a teacher serves as a determinant of all classroom activities. According to Brown and Borko (as cited in Aslan-Tutak and Adams, 2015), pre-service teachers with limited content knowledge face lots of difficulties in pedagogical training. Several studies show that weak content knowledge affects teachers' use of pedagogical tools (Aslan-Tutak and Adams, 2015). It is therefore important that pre-service teachers possess a good understanding of the content or subject matter to ensure their competency development in training.

According to Aslan-Tutak and Adams (2015), in spite of the many applications of geometry in our daily lives, it seems to be a topic that is mostly neglected in the mathematics classroom. The authors add that a great deal of research on teacher knowledge of geometry concludes that “beginning teachers are not equipped with an adequate content knowledge of geometry” (p. 303). Content knowledge of geometry comprises concepts, facts, skills, theorems, theories and knowledge of associations among the topics in geometry (Sunzuma and Maharaj, 2019). Knowledge of associations deals with how specific content is associated with other concepts in geometry and other disciplines of study. Teachers need this knowledge in order to be effective in teaching.

METHODOLOGY

The study followed the quantitative approach employing a descriptive survey design to investigate pre-service teachers’ geometric thinking. The study participants were prospective mathematics teachers randomly drawn from four Colleges of Education in the Bono Region and Ashanti Region of Ghana. The study participants were 217 who were undergoing training to be equipped in mathematics content knowledge and pedagogical practices to be competent and effective in teaching. The participants comprised 149 males and 68 females with participants’ average age of 21 years. For this study, it was ensured that the participants had studied geometry as part of the courses in their programme of study in the College to deepen their content knowledge and to have an increased ability in geometry experience beyond their knowledge acquired at the secondary school level.

Data were collected using an instrument developed by the researchers. The development of the instrument was partly guided by van Hiele’s model of geometric thinking instrument and the geometry content and expectations of the Ghanaian mathematics syllabus for Junior High school. Thus, modifications of the items in the van Hiele instrument were done to make it suitable and purposeful for the context in which the study was conducted. The instrument measured pre-service teachers’ visual, descriptive and informal deductive ability as the first three van Hiele geometric thinking levels and their ability to apply geometric properties to solve related problems in geometry. 40 multiple-choice items were constructed and followed with four options from which participants were to select the best one. The test instrument was in two sections. The first section requested the participants' bio-data which were gender and age. The second section of the instrument contained the 40 multiple-choice test items. Five (5) items for level 1, 6 items for level 2, 13 items for level 3, and 16 items required the participants to apply knowledge of shapes, properties, and their relations to solve related tasks in geometry.

To ensure that the instrument measured what it was supposed to measure, it was given to two mathematics teachers in a College of Education and one mathematics teacher who teaches at an elementary school to assess the quality of the items. Responses received were incorporated to enhance the quality of the data collection instrument. A further attempt was made to determine the internal consistencies of the items within the various levels by pilot testing the instrument. The calculated reliability coefficients of the items in levels one, two and three were 0.72, 0.74 and 0.71 respectively, which were found to be good according to a criterion determined by Wells and Wollack (2003).

The instrument was administered to the participants on the dates scheduled in the various Colleges for the data collection. The dates were determined after permission to conduct such a study had been sought. Participation in the study was voluntary and participants were assured of utmost confidentiality as the data were meant for academic purposes. Participants were given one hour to answer the questions after which the instruments were collected.

The scripts were screened to be sure that items that were not answered were not more than five. This step was taken due to the purpose of the study and the desire to obtain information about the true state of the pre-service teachers’ geometric thinking on the first three levels to enable accurate inference to be drawn from the study. This effort resulted in eliminating three scripts, yielding a total of 217 for analysis. The scored scripts were coded into SPSS version 22 and analysis was conducted in accordance with the research questions.

RESULTS

This section presents the results of the study that sought to assess pre-service mathematics teachers’ geometric thinking. The results are presented according to the research questions.

Research Question 1: What is the level of van Hiele’s geometric thinking among pre-service mathematics teachers?

Table 1 shows the mean performance of the pre-service teachers on geometry tasks. For the fact that the participants were being prepared to teach geometry at the elementary school level, the geometry test items covered the first three levels of van Hiele’s geometric thinking. The informed decision on this was that elementary students are supposed to acquire the first three levels of van Hiele’s geometric thinking as foundational knowledge to be successful in the study of higher geometry content in secondary school and beyond. Hence, the pre-service mathematics teachers were expected to demonstrate adequate knowledge of these levels for effective teaching of geometry. The results are presented in Table 1.

The results in Table 1 show that the participants showed adequate knowledge at level one of van Hiele’s

Table 1. Pre-service performance on geometric items.

Level	No. of Items	Short description	Mean	SD
1	5	Shapes are identified by appearance	5.00	0.000
2	6	Shapes are identified by properties	4.41	1.66
3	13	Able to think of properties of geometric shapes and their relations	6.78	2.36
Standard tasks	16	Ability to apply geometric properties to solve tasks	10.03	3.28
	40		25.39	5.81

geometric thinking. The average mean performance of 5.00 shows that the participants were able to identify shapes in their various orientations. This was evident by the standard deviation value which indicates a high state of similarity in performance of the pre-service teachers in this level.

The next set of items required the participants to show thinking of geometric shapes by their properties which were aspects of level two of van Hiele's geometric thinking. The results in Table 1 show that the participants obtained an average score of 4.41 out of six items, each attracting 1 mark. The mean score of 4.41 shows that the participants performed above the median mark of 3. This shows that most of the study participants scored more than half of the items at this level. The standard deviation of 1.66 compared with that of level 1 showed some emerging diversity in the responses among the pre-service teachers.

Thirteen items were used to assess the participants' geometric thinking at level three. At this level, participants were supposed to demonstrate a good knowledge of understanding geometric properties and their relationships. The results in Table 1 show a mean performance of 6.78 which indicates a moderate performance of the pre-service teachers. It could be observed from the increased value of the standard deviation that the participants thinking about geometry items at this level was getting more diverse.

The next set of questions tested the pre-service teachers' geometry thinking on standard questions that required some application of their knowledge in solving problems in geometry. The results in Table 1 show the participants' mean score of 10.03 out of 16 items, each attracting 1 mark. The mean score of 10.03 shows a moderate performance of the pre-service teachers on the geometry tasks.

A further attempt was made to determine the distribution of the van Hiele level among the pre-service teachers. As suggested by van Hiele, the criterion used for this distribution was 3 of 5 correct. Van Hiele also had a stricter criterion of 4 of 5 correct to be qualified as operating at a particular level. However, the criterion of 3 of 5 was used in this study. To fit into a particular level, say n, one must meet a criterion of n and n–1. In other words, placing a participant at level n required him/her to meet the criterion at n level and all lower levels of n. In this study, where the highest level considered was 3, one could be placed at level 3 provided that the person meets the requirement of level 3 and that of levels 2 and 1.

In this study, five items were used at level 1, visual level, for which the criterion of level 1 placement was 3 of 5 correct. Six items were used at level two, analytical level, which also required a performance criterion of 4 of 6 correct. The participants were placed at this level provided they scored 4 or more items correctly and met the level requirement. Level Three, which was measured using 13 items, required a criterion of 8 of 13 correct in addition to meeting the requirements of levels 2 and 1. In a situation where one meets levels 1 and 3, but not level 2, that participant is said to be operating on level 1. The results are shown in Table 2.

Table 2 shows that all the pre-service teachers progressed beyond level 1 of the van Hiele geometric thinking. A little above half of the research participants, 128 representing 59%, operated at level 2, analytical level. This description of participants at this level shows their ability to demonstrate a good understanding of geometric properties. Thus, they were able to classify geometric shapes based on their properties. 41% of the pre-service teachers demonstrated the ability to be operating at the informal deductive level.

Research question 2: The participating pre-service teachers' geometric thinking is not significant for teaching geometry.

Since the impact of teachers' knowledge on learners' performance has been increasingly researched, the researchers desired to determine if the participating pre-service mathematics teachers' geometric thinking could make any impact in teaching geometry contents in the mathematics curriculum. This was determined by using a one-tailed one-sample t-test with a predetermined value of 20. The significance of the pre-service teacher's thinking was measured by setting a minimum score of half of the total score in the test. The result is presented in Table 3.

Table 3 shows the mean score of pre-service mathematics teachers' geometric thinking (M = 25.37, SD = 5.81) with $t(216) = 13.62$ with p-value less than 0.05. Since $p < 0.05$, we reject the null hypothesis that the participating pre-service teachers' geometric thinking is not significant for teaching geometry in favour of their geometric thinking being significant for teaching

Table 2. Van Hiele level among pre-service teachers.

Thinking Level	Visualisation	Analysis	Informed Deductive
Number of Participants	0	128	89
Frequency of Participants	0	59	41

Table 3. Extract of one-sample t-test.

N	Mean	SD	d.f	*T*	*p*
217	25.37	5.81	216	13.62	0.000

Test Value = 20.

geometry. This indicates that the pre-service teachers' geometric thinking can make a good impact on teaching. Good impact to teaching shows that learners who would be taught by any of these pre-service teachers will be able to receive some good guidance as far as the teaching of geometry is concerned. This implies that the knowledge possessed by pre-service teachers will be able to make a positive impact on teaching geometry contents in the mathematics curriculum.

Research question 3: There is no gender difference in geometric thinking among the participating pre-service teachers?

Independent sample *t*-test was conducted to find out if there was any gender difference in geometric knowledge possessed by the pre-service teachers. Since the teaching profession is no gender bias, it is expected that both males and females will be able to show comparable competence in their geometric thinking for teaching.

Table 4 shows significant difference in gender and performance among the pre-service teachers with male (M = 26.19, SD = 5.7) and female (M = 23.59, SD = 5.69); $t(215) = 3.117$, $p = 0.002$. The p-value of $0.002 < 0.05$ gives the evidence to reject the null hypothesis which states there is no gender difference in geometric thinking among the pre-service teachers. Gender differences exist among the participating pre-service teachers in favour of the males. Thus, the males in the study outperformed their female counterparts. Since both males and females were undergoing training purposely to join the teaching profession, the researchers were curious to determine if their difference was statistically significant by further estimating the eta squared. The estimated eta squared value of 0.04 was interpreted as a small effect, following the guidelines for interpreting such values proposed by Cohen (as cited in Pallant, 2004). Thus, although there were gender differences in performance among the pre-service teachers, only 4% of the variance in performance could be explained by gender, which could be considered to be statistically insignificant.

Table 4. Extract of t-test.

Gender	N	Mean	S.D	d.f	*t*	*p*
Male	149	26.19	5.7	215	3.117	0.002
Female	68	23.59	5.69			

DISCUSSION

The purpose of the study was to investigate the level of van Hiele geometric thinking achieved by pre-service mathematics teachers. The study was guided by three objectives, which were to determine the distribution of van Hiele geometric thinking levels among pre-service teachers, the impact of the participated pre-service teachers' thinking would have on their students' learning, and possible existing gender differences in geometric thinking among the study participants. Results from Table 1 show that pre-service mathematics teachers' performances at levels 2 and 3, apart from level 1 were moderate. As evident from the Table, the pre-service teachers' geometric thinking could not be considered so high. A look at the mean performance shows a little above half of the awarded marks. Determining the van Hiele geometric thinking among the pre-service teachers, results show that less than half of the participants performed at the expected level concerning what elementary students are expected to achieve before entering high school. This shows that about half of the pre-service mathematics teachers who participated in the study were functioning at the first two levels of geometric thinking, namely, 'visualisation' and 'analysis'. Thus, the participated pre-service teachers can demonstrate geometric thinking related to shapes and their properties. A little below half was able to demonstrate thinking about deducing properties from others. The findings of this study support the findings of Robiachaux-Davis and Guarino (2016), whose study showed pre-service teachers to be operating at the first two levels of geometric thinking. Teachers' knowledge continues to receive significant attention in educational research due to its potential impact on students' learning (Hourigan and Leavy, 2017). Notable among teachers' knowledge is their thinking about the content. Content knowledge informs, basically, everything the teacher does in the classroom. Activities such as those required to enable set objectives to be achieved, what pedagogical consideration to settle on, materials to select and use to facilitate concept development, and the like depend on the teachers' explicit knowledge about the content. With regard to geometry in the mathematics curriculum, van Hiele identifies five hierarchical levels that students are supposed to possess as they experience geometric instruction. Acquiring knowledge on higher levels of geometry depends on earlier or preceding levels and results from learning experiences made available for students' learning. Teachers' ability to impact positively on learners depends on their ability to also demonstrate deep and flexible knowledge about geometry (Hourigan and Leavy, 2017).

Being much concerned about the significance of the pre-service teachers' geometric thinking to teaching, resulted in further analysis in this regard. The analysis shows that the state of the participants' geometric thinking would, to some extent, make a positive impact on their teaching abilities of geometry. However, one may question the quality of such an impact. This could be due to the participants' moderate performance in the tasks given and the fact that a little above half of the participants operated below level 3 of geometric thinking, the required level that elementary students are supposed to attain before entering high school. Why would one raise a question about the quality of impact these participated pre-service teachers' geometric thinking would make in their teaching? As indicated earlier, pre-service mathematics teachers' proficient geometric thinking makes a significant difference in the kind of instructional practices they make available for students' learning. It is worthy to note that the geometric concepts taught in class, how the concepts are presented, the necessary materials to facilitate the teaching and learning of geometry, and the quality of questions used to assess students' learning and understanding of the geometry concepts largely depend on teachers' competence in geometry. Mudaly (2015), who considered teacher thinking as a critical component of their knowledge, asserted that teacher thinking affects their preparatory activities towards lesson, classroom delivery, the teacher's actions, inactions and all kinds of learning experiences the teacher may provide. It is needful then for pre-service teachers who join the teaching service to possess adequate and competent knowledge to enhance teaching effectiveness.

Analysis of the study results on gender reveals differences in pre-service teachers' geometric thinking in favour of the males. The implication of this finding is that male pre-service teachers are likely to provide quality teaching and learning experiences for students' learning than their female counterparts. Noticing gender differences in pre-service teachers' thinking in geometry can be quite alarming since both are being trained to provide relatively equal teaching and learning opportunities for students learning geometry and mathematics in general. This finding partly mirrors that of Anas (2018) who conducted a study to investigate gender differences in reasoning among pre-service teachers. Although the study by Anas did not find any significant gender difference, there was some evidence of the males performing better than their female counterparts in the first three levels of van Hiele geometric thinking. The findings also support that of a study conducted by Usman et al. (2020) with pre-service mathematics teachers whose analysis of gender differences among those taught through conventional traditional approach showed that the males had a higher mean achievement score than their female counterparts.

Conclusions

This section concludes the studies based on the discussion of the study results. The study concludes that about half of the pre-service teachers in this study operate at van Hiele's second level of geometric thinking. This was evident through their answers provided to tasks measuring their geometric thinking on the first three levels which are the required levels to be achieved by elementary learners before entering high school. This shows that the participated pre-service mathematics teachers are not adequately prepared for teaching geometry at the elementary level given their limited geometric thinking.

Another conclusion drawn is that, despite the inadequacy of the pre-service teachers' geometric thinking, the analysis showed that their geometric thinking can make some positive impact on learners' learning experiences. Although, the question one may ask is the quality of such instruction.

A further conclusion drawn is that there exists gender difference in pre-service teachers' geometric thinking. Such existing difference in gender implies that there is the likelihood that their geometric teaching abilities may as well not be equal.

RECOMMENDATIONS

This study recommends that mathematics teacher educators at the participated Colleges of Education need to do more on content knowledge development among pre-service teachers, particularly, geometric thinking for effective teacher capacity building for the teaching profession. In so doing, attention may also be drawn to chief examiners' reports on learners' difficulty in solving problems in geometry to be incorporated into teachers' geometry content knowledge development.

Mathematics teacher educators at the Colleges need to focus teaching attention on trainees' understanding of the geometry content areas that would be required for teaching after completion. In this case, the pre-service teachers would be able to possess good knowledge of geometry and approach its teaching with a high level of confidence.

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