Productive Failures: From Class Requirement to Fostering a Support Group

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Abstract

Mistakes occur frequently in mathematics. Reframing mistakes into positive moments can be psychologically important in a student's educational journey. We investigated two tertiary math classes that explicitly valued mistakes through a pedagogical requirement called "productive failure". For a percentage of their grade, students demonstrated how they made mistakes in their problem solving and, most importantly, how they overcame those mistakes. Through interviews, video-stimulated recalls, and evaluations of the course all from students, we initially looked for affectual responses to the pedagogical allowance and student-led demonstration. Many of the responses, both benefits and drawbacks of the productive failure, were interpreted by the research group to resemble the psychology literature on peer-led support groups. Descriptions of both productive failure and support groups, as well as quotes from the students, aim to shed light on psychological benefits of valuing mistakes. Finally, we believe that productive failures benefitted many students because it made the human aspect of mathematics more explicit.

Keywords: Mathematics education, Productive failure, Affect, Inquiry-based

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Fallos Productivos: del Requisito de Clase a la Promoción de un Grupo de Apoyo

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Resumen

Los errores ocurren con frecuencia en las matemáticas. Replantear los errores en momentos positivos puede ser psicológicamente importante en el viaje educativo de un estudiante. Investigamos dos clases de matemáticas terciarias que valoraban explícitamente los errores a través de un requisito pedagógico llamado "fracaso productivo". Para un porcentaje de su calificación, los estudiantes demostraron cómo cometieron errores en su resolución de problemas y, lo más importante, cómo superaron esos errores. A través de entrevistas, retiros de video estimulados y evaluaciones del curso, todos de estudiantes, inicialmente buscamos respuestas afectivas a la asignación pedagógica y la demostración dirigida por estudiantes. El grupo de investigación interpretó que muchas de las respuestas, tanto los beneficios como los inconvenientes del fracaso productivo, se asemejan a la literatura de psicología sobre grupos de apoyo dirigidos por pares. Las descripciones de los grupos de fracaso productivo y de apoyo, así como las citas de los estudiantes, tienen como objetivo arrojar luz sobre los beneficios psicológicos de valorar los errores. Finalmente, creemos que los fracasos productivos beneficiaron a muchos estudiantes porque hizo que el aspecto humano de las matemáticas fuera más explícito.

Palabras clave: Educación matemática, fracaso productivo, afecto, basado en la investigación

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> **So, then I tried to do something else**, like I tried to do a different function, but it didn't work, I don't even know really what this part was. **Then I was so sure it couldn't happen**. I kept trying these two functions (Laughs) and this is not possible, it's not going to happen. – Third Author, minute 1, productive failure demonstration[4]

At one point in their life every student or mathematician will reach a mathematical impasse when attempting to solve a problem. Being "stuck" on a problem is inevitable. But some mathematicians have developed self-efficacy and strategies to overcome impasses (Savić, 2015a). How do mathematicians develop this confidence to overcome, and can it be developed in the tertiary levels?

Schoenfeld (1989) surveyed 215 secondary students, and the longest response to the question "'What is a reasonable amount of time to work on a problem before you know it is impossible?' was 20 minutes" (p. 345). These students might be entering universities with a belief that many problems are quickly done, and if problems are not solved the first time, then the problem-solving process stops. Again, this is different than mathematicians, which go through a cyclical problem-solving process (Carlson & Bloom, 2005). What students do after an impasse might define how they view mathematics and themselves as mathematicians (Omar et al., 2019).

If overcoming impasses are incredibly important in mathematics, what can instructors do to cultivate students' perseverance on problems? We studied a pedagogical action of requiring students to demonstrate their problem-solving impasses and reframe their struggle positively in class. This struggle was referred to in class as a "productive failure." We conjecture that the act of a productive failure and the requirement to demonstrate it in class is similar to peer-led support groups such as Alcoholics Anonymous. While the two do not equate on a societal or requirement level, the characteristics and both positive and negative effects seemed to align with the support-group literature. We attempt to strengthen the conjecture of the link between productive failure demonstrations and peer-led support groups by using qualitative data collected in two uniquely separate classes and methods. Finally, we end with

[4] These quotes are dialogue from a productive failure presentation in an Abstract Algebra course. The third author was presenting on her proof of a theorem on a take-home test. We also claim that the reader does not need to know the specifics of abstract algebra, but rather notice (in bold) the problem-solving aspects of the student.

a discussion of how to address some of the negative effects, perhaps opening doors to rehumanize mathematics (Gutiérrez, 2018).

> **I tried** five or six functions, and I was thinking about the isomorphism and how $ab\mathbb{Z}$ had to be the kernel, and so well that means I have to map the elements of $ab\mathbb{Z}$ to the identity... and then I was like no, there is no homomorphism**, this is not true**, it's not going to happen. **Then I stopped** doing the homomorphism, and I went directly to the isomorphism**, so I started listing out what these two things were. If I could map them.** – Third Author, minute 2, productive failure demonstration

Background Literature

What is a Productive Failure?

The notion of using mistakes, difficulties, and impasses as productive has been discussed in many capacities, often with success. However, both what constitutes "productive" and what kind of difficulty arises, varies in the literature. For example, Granberg (2016) defined productive failure as a "result in the restructuring of mental connections in more powerful, useful ways through which the problem at hand would make sense and new information, ideas and facts would become assimilated" (p. 34). Granberg, again, stated that errors play a large part: "It appears that making, discovering and correcting errors may generate effort that can engage students in productive struggle" (p. 34). However, productive to Granberg meant to obtain a correct solution. Other authors, such as Heibert and Grouws (2007), stated that struggle is necessary for learning math and is about the process of expending energy to make sense of mathematical concepts (as cited by Warshauer, 2015). Finally, the NCTM principles to action explicitly stated that productive struggle was "struggling at times with mathematics tasks but knowing that breakthroughs often emerge from confusion and struggle" (NCTM, 2014, p. 52).

The process of problem solving is often more important than product in the classroom for two reasons. First, answers are more readily available to students in the age of ubiquitous access to the internet (Linshi, 2016; Williams, 2020). Second, if problem solving is improved on in class, the products that result will improve due to students' checking and re-evaluating their own work – an action that is common in the problem-solving literature (Savić, 2015b). Liljedahl (2004) stated that checking is where the “solver re-engages with the problem,” and that solution can yield much more exploration and learning (Knuth, 2002). This requires active participation by both the instructor to value the problem-solving process as well as the student to be metacognitive. However, this metacognition must be productive, or else it might feel like a waste of time to the student.

What must occur for a student to be productive in their failure is a recognition of the failure or mistake (the “checking” phase of Carlson and Bloom’s (2005) problem-solving process), subsequent recovery or additional approach (the cycle back to “planning” and “executing” phase of Carlson and Bloom (2005), and the metacognitive awareness of modifying their approach for future problem solving. Research has suggested that during productive struggle, students activate prior knowledge and intuitive ideas (Kapur & Bielaczyc, 2012; Kapur, 2014). Furthermore, the more problem-solving methods that students construct during their struggles, the more prior knowledge is to be activated (Kapur, 2014).

Metacognition is one piece of the puzzle – how a student feels during the process can also indicate future problem-solving success. For example, say a student tries for more than 20 minutes on one problem, and has no breakthrough. Does that person feel like they can persevere and solve the problem, or feel like it is the “deepest darkness,” as one participant in the Furinghetti and Morselli (2009) study stated in a problem-solving session? Perseverance to overcome darkness is one aspect of a psychological construct known as affect.

Affect as Another Piece of Handling Failure

McLeod (1992) stated that the definition of affect “refers to a wide range of beliefs, feelings, and moods that are generally regarded as going beyond the domain of cognition” (p. 576). He goes on to state that there are three general categories to the affective domain: beliefs, attitudes, and emotions. While others have added categories to the domain (namely, values, motivation and engagement (Attard, 2014)), for the purposes of this article, the focus will be on these three categories, and on affect as a whole.

Attitudes are “develop[ed] from several similar and repeated emotive responses to an event or object” (Grootenboer & Marshman, 2016, p. 19). An example of an attitude developed about problem solving could be a dislike of using fractions. Emotions are more visceral and momentous. Positive emotions include AHA! moments (Liljedahl, 2013), while negative emotions involve frustration. Negative emotions can largely contribute to how students approach problem solving tasks: "Furner (2000) suggested that two-thirds of Americans either hate or loathe mathematics" (Grootenboer & Marshman, 2016, p. 21). Both attitudes and emotions towards mathematics can forge beliefs of mathematics.

Beliefs are "psychologically held understandings, premises, or propositions about the world that are thought to be true" (Philipp, 2007, p. 259). For example, an instrumentalist belief of mathematics states that mathematics is all about rules and procedures. Schoenfeld (1989) found that "[secondary] students believe that solving mathematics problems depends on knowing the 'rules'" (p. 345). These beliefs are entrenched, and it takes significant effort to change (Schoenfeld, 2011). However, Grootenboer and Marshman (2016), citing Pajares (1992), stated that "because central beliefs have been developed through experience, new activities giving rise to positive experiences and reflection upon those experiences is critical to belief change" (p. 17). While not impossible, beliefs can be changed, and one of the main catalysts for change in students is by the instructor. It is in courses where much of the beliefs of mathematics solidify, because it is where the most engagement of mathematics occurs (Brown, 2019).

The Value of Valuing Mistakes in Class

As a pedagogical tool, there is an indication in previous literature that an environment structured for utilizing failures or mistakes can be successful in refining students' problem-solving skills. For example, an explicit incentive to correct their mistakes can be an effective formative assessment tool (Black & Wiliam, 2009). This incentive could be points or other credit in the course: "Offering grade incentives to diagnose and correct mistakes can go a long way to close the performance gap between struggling and high-performing students" (Brown, Singh, Mason, 2015, p. 4).

A by-product of pedagogical actions focused on productive failure is that it can create "failure tolerance" (e.g., Clifford, 1984; 1988), turning potentially negative occurrences into positive outcomes. This can blend into students' affect; positive outcomes may be associated with positive emotions, which in turn can lead to positive attitudes toward problem solving.

Specifically, presentations in class on struggle may be risk-averse if cultivated non-judgmentally. Shultz and Herbst (2020) stated that "having students give presentations might align with an instructor's desire to have students experience struggle, but not enough to risk that students could feel anxious about being judged" (p. 535). However, Tulis (2013) stated that research into pedagogical actions on failure and mistakes is scarce: "little is known about adaptive classroom practices for dealing with errors and the reciprocal effects of students' and teachers' attitudes towards learning from mistakes" (p. 56). We hope to make a connection between the actions of an adaptive classroom and fostering a support group, demonstrating that the attitudes towards learning from mistakes are the same effects that support groups have on their participants.

> **So I did this, and then I tried something new.** And it wasn't (Laughs). There's another one, so I think we're up to five or six here. So then I was like, no this [theorem] is false. I know this is false. **I know he [the instructor] told us to prove it, but it's false. (Laughs) … I'm going to show him that this is false. (Laughs)** – Third Author, minutes 2-3, productive failure demonstration

What is a Support Group?

Support groups are characterized in the psychology literature as "caring and support for its members… whereby they can learn and share with each other… [offer] pragmatic approaches to certain life challenges and needs… [and] emerge from members' struggles to create some degree of order in their social lives" (Mohr, 2003, p. 676). By the theory of Schopler and Galinsky (2014), support groups have certain characteristics that include:

- "organizational sponsorship or be the creation of an innovative practitioner" (p. 4)
- being member-centric, with members providing experiences, information, advice, and occasionally leadership within the group.
- leaders sharing authority with the members, having their legitimacy often being based on training
- providing a supportive environment and a means for developing coping abilities

Support groups can be in the community such as Alcoholics/ Narcotics Anonymous or a Caregiver Support Group (Ebenstein, 2006), or can be used in higher education at the counseling level (Ribeiro, 2018). The long-term benefits of a support group include empowerment, gains in empathy, and validating those difficult experiences (Ribeiro, 2018).

In this study, we used Schopler and Galinsky's (2014) theoretical study of support groups. They proffered that there are positive effects to support groups that include "greater social resources, increased knowledge about the focal concern, a sense of relief and reassurance, and enhanced skills for coping" (pp. 6-7), which also agree with other support group literature (e.g., Docherty, 2004; Adamsen, 2002). There are also negative effects; participants felt "pressure to conform, stress related to group obligations, feeling overwhelmed and less adequate, learning ineffective and inappropriate responses, embarrassment, and overconfidence" (Schopler & Galinsky, 2014, p. 7). This paper takes into consideration the individualistic perspective – indeed there could have been other approaches to researching the demonstrations of productive failure, including a social identity from shared stories (Rappaport, 1993).

In conclusion, the process of a person demonstrating a productive failure requires meta-cognition in understand what they did, affect in understanding the beliefs, attitudes, and emotions of their failure and success, and a mathematical struggle. There seem to be benefits in the math education literature about valuing productive failures, but there needs to be more research on students' effects from teachers explicitly incorporating productive failures in classrooms. Finally, we hope to explain the effects of demonstrating productive failures by framing those effects with peer-led support group theory. The next three sections address why the productive failure demonstration was a support group, including how it lined up with the characteristics, positive, and negative effects outlined by Schopler and Galinsky (2014).

Methods

The Pedagogical Action - Students Demonstrating Productive Failures

We focus on two mathematics classes that had the same requirement of a productive failure: undergraduate/graduate abstract algebra (Fall 2015) and calculus II[5] (Spring 2016). These classes occurred in a United States, medium-sized, research-intensive, public university. Demonstrating a productive failure in front of the class accounted for 5% of their final grade, with 2% extra credit in the calculus course if the student demonstrated in front of the large classroom (137 students) instead of the discussion sections (roughly 25-30 students); the algebra course had 32 students and did not have the 2% extra credit opportunity. Students only had to demonstrate once in order to receive credit, although some could volunteer multiple times. Courses were 50 minutes in length and occurred on Mondays, Wednesdays, and Fridays of the week, and there were 15 weeks of classes. All participants were enrolled in one of these two courses and participated in an Institutional Review Board-certified investigation with written consent. Since all were over 18 years of age, consents are required of them but not any family members.

Productive failures generally occurred in the same manner. The instructor asked if any students had a productive failure, and if one did, the instructor would ask them to come to a document camera located in front of the class. While demonstrating the productive failure, students would describe their mistakes and were encouraged to reflect on them. The instructor would usually sit in a desk and watch the presentation with the rest of the class. These demonstrations lasted for an average of five minutes. Unless already explicitly mentioned, they were typically asked why the failure was productive for them. After questions from other students and the instructor, the presenting student would walk back to their seat while their peers applauded. Often the problem or theorem in question was an entry point to discuss the topic for that day.

Data Collection and Techniques

The first author was the instructor for both courses and taught using inquiry-based mathematics education (formerly Inquiry-Based Learning; Cook, Murphy, & Fukawa-Connelly, 2016; Laursen & Rasmussen, 2019). The second author researched the calculus course, taking observation notes of daily classes, interviewing four students (including the fourth author), conducting an online survey (different than the end-of-course evaluations), and interviewing two students (including the fourth author) months after the

[5] The abstract algebra course used the Inquiry-Oriented Abstract Algebra materials (Larsen, Johnson, & Weber, 2013) and focused on beginning group theory including the isomorphism theorems and quotient groups; the calculus course used Stewart's (2012) textbook covering from the definition of definite integral to integration techniques.

course ended. The third author was an undergraduate student in the Abstract Algebra course. Her productive-failure presentation has been interwoven into the paper to demonstrate an example of how one was presented to the class[6]. She presented this productive failure in week 10 because “it ha[d] to be good enough failure” according to her (video-stimulated recall). The fourth author presented her productive failure to the large calculus II class before the first test. We wanted both authors to be participant-researchers (Probst, 2016) in order to collect, enhance, and check the meta-cognitive reflections of their presentations, along with experiences in researching in mathematics education.

Both the third and fourth authors were asked to participate in a video-stimulated recall (Theobald, 2012) session with the first author about seven months after their demonstrations, where they discussed the demonstration of the failure, the reactions that they had during the time, and future effects. We report on one of those reflection sessions between the first and third authors. They watched the video of the third author’s productive-failure presentation (collected for another project) twice and discussed occurrences. The third author then transcribed that discussion. The reader can refer to Table 1 for a summary of the data collection techniques, and Table 2 for a summary of the five participants’ demographic data.

Table 1
Data Collection by Course

Class	**Data**	**Collected/Transcribed (Authors)**
Abstract Algebra	Video of Productive Failure	First and Third
	Video-stimulated recall	First and Third
Calculus II	Interviews with Four Students	Second (Fourth participated)
	Video-stimulated recall	First and Fourth
	Online surveys	Second
	Follow-up Interviews	Second (Fourth participated)

[6] These “failures” were in proving a theorem in Abstract Algebra: If a and b are relatively prime, then $\mathbb{Z}_{ab} \cong \mathbb{Z}_a \times \mathbb{Z}_b$.

Table 2
Demographic Data of Participants

Person	Gender	Race/Ethnicity	Course
Emily (Third Author)	Woman	White	Abstract Algebra
Ariana (Fourth Author)	Woman	Latina	Calculus II
Student 2	Woman	Native American	Calculus II
Student 3	Man	Black	Calculus II
Student 4	Man	White	Calculus II

The data was then coded for instances of affect. For example, in the video-stimulated recall session, the third author was describing what was going through her mind when standing up in front of the class after presenting: "I really do think it impacted me. I don't know if it impacted other people, but I think that specific instance has changed how I perceive problems when I see them. I had a lot more success in Abstract Algebra 2 I think because of it." This was coded as an affective utterance, specifically as a belief since she believed that the course had such a lasting impact. Then, the affect quotes were coded using the positive and negative effects of support groups that Schopler and Galinsky (2014) stated. For example, the previous quote by the third author was coded as "increased knowledge of the focal concern," because, in this case, the focal concern was about making failures productive, and she changed her approach to future problem solving. All of the data was coded by the first two authors separately, and then the two met to discuss and agree upon codes for both iterations until full agreement. This approach "rel[ied] on intensive group discussion and simple group 'consensus' as an agreement goal" (Harry, Sturges, & Klingner, 2005, p. 6; as cited by Saldaña, 2009, p. 28) instead of Cohen's Kappa for interrater reliability.

> But really, **when I tried to prove that it was false was when I found out it was actually true**. So then I tried this [finding the orders of the elements]. The hard thing was that two of them had order 3… Finally, I figured it out, the isomorphism. But this was for n=2 and m=3. So, then I had to find the general function, and **this is where I figured it out here. (Points to OH! on paper)** (Laughs) – Third Author, minute 3, productive failure demonstration

Results

Why was Demonstrating Productive Failures a Support Group?

In the section above, Schopler and Galinsky (2014) stated that there are certain organizational characteristics of support groups. In this section, we attempt to satisfy those requirements. The first author implemented the productive failure requirement in his courses beginning Spring 2016 but was influenced by the Inquiry-Based Learning (IBL) community (e.g., Yoshinobu, 2014), Burger and Starbird (2012) in learning to fail, and previous literature about impasses (e.g., Savic, 2015a). This wasn't necessarily the "creation of an innovative practitioner," but a practitioner that created the productive failure requirement influenced by an innovative community.

All productive failures were done by the students, and frequently ended with a round of applause from the majority of the students, hence were "member-centric." In the video-stimulated recall, the third author stated, "We have to clap! This person did such a good job! I was so excited for anyone to get up there and do it that even if it was horrible." Also, in the quotes of the productive failure demonstrated by the third author throughout this article, there is laughing being done not by the presenter but the audience as well.

The instructor frequently sat with the students when a productive failure was presented and had experience in teaching inquiry-based mathematics education. In the productive failure demonstration that flows throughout this article, the third author stated that her goal was to prove the instructor wrong. Therefore, there was a sharing of authority as well as legitimacy of the leader (the instructor). As for "providing a supporting environment and a means for developing coping abilities," both may be apparent when discussing the positive and negative effects of the productive failure.

Positive Effects

When the first two authors investigated affect, multiple quotes from the students aligned with each or multiple benefits of a support group. For example, a benefit of a support group is greater social resources, meaning that there are others who can provide support and help around you. The fourth author stated that "when you make a mistake [in other classes], people just look at you like 'wow, she's so dumb' and not in this class. Like, they [the students] actually value… when you make a mistake" (interview). The third author, whose presentation of the productive failure is demonstrated in the frequent quotes, had students laughing, smiling, and clapping along with her demonstration. She can then feel a sense that there are shared difficulties that many of the students were going through. Finally, Student 4 stated that "the entire class was answering questions, building on common mistakes people had and… doing a process of productive failures" (interview).

The focal concern of the productive failure sessions was to gain strategies of overcoming mistakes – turning failures into something more productive. The fourth author expressed that they had "increased knowledge of the focal concern", stating that

> Once you fail you're gonna be like, "I'm so dumb! Like, I made this mistake." And once you see something similar somewhere else, you're not going to make the same mistake, because you're going to remember how you failed doing the other thing. So that's why I think it's important. (interview)

She then, in her follow-up interview months later, stated that productive failures carried with her in her other studies. When discussing doing homework in other classes, the fourth author stated that, "I'm not afraid of it, like I told you, anymore. Like, if I fail, well I fail. I just restart again. I put it on the side and go clear my head and then come back to it and try something different." Student 4 also stated that he "didn't feel all that bad about having a failure and a lot of people, [he felt], benefited from [him] going up, because a lot of people were making the same mistake as [he] was" (interview).

Schopler and Galinsky (2014) talk about a "sense of relief and reassurance," and both the third and fourth authors were adamant about their relief of presenting their mistakes. The third author had "OH!" in her writing and pointed to it in her demonstration. She then stated that "I had been battling it for like a day and a half. I was ready to, even for my own satisfaction, just to figure out what's going on. I was so excited… I know why it's true now" (video-stimulated recall). The fourth author exclaimed that it was odd that she "went to the board and presented my productive failure and didn't feel bad." She also stated that the first author "made sure that we understood that he was not going to punish us for our mistakes." Speaking about difficulty openly can feel like a weight off your shoulders, and each of the two authors demonstrated that relief.

Finally, there is a benefit of a support group of developing enhanced skills for coping. In this situation, coping can mean that when encountering a failure in the future, the student can handle the difficulty and more forward. The fourth author already stated that she now approaches problems with the sense of "If I fail, well I fail. I just restart again." The third author had a similar coping mechanism:

> I think before this course I would not have had the, I'm not even talking about mathematical skills, but the skills necessary to have gone through this process and taken all of this time an gone through the ups and downs without… you know I would have either given up or just not been able to do it. Even emotionally. But I think because in this course I knew that making mistakes was gonna make it… I was gonna get there. I felt the whole time that I was gonna get to the right answer whether it be that the theorem was false or not… I thought that I was gonna get there and I don't think that I would've thought that in another class. (video-stimulated recall)

Negative Effects

While the two authors and another student spoke about the benefits of productive failures, other students expressed negative effects of the productive failures similar to the effects Schopler and Galinsky (2014) stated in their definition of a support group (see Section above). For example, Student 2 stated that productive failures are "terrifying," and preferred a large class because she could "hide with all those people" (interview). Both are affectual responses that can be categorized as pressure to conform and feeling overwhelmed. Student 4, in his follow-up interview, stated that he enjoyed productive failures but did not find that it would transfer to his major in medicine, where he hoped to specialize in cardiovascular surgery. He stated that, "Best case - productive failure is they come back and you try a new medicine during residency, but there's a lot [of] worse [cases] in residency" (interview). A student evaluation of the course stated that "I don't feel like the productive failures are effective cause it's a hit or miss whether they'll explain it well," which can be categorized as learning ineffective and inappropriate responses. Ineffective responses was coded the most in terms of negative effects, and were summed up by another evaluation: "I do appreciate the productive learning but things get very hard to understand if a student shows the class how they did something wrong and then that is the only time we ever touch on a topic."

Discussion

We believe that, through this qualitative study, the pedagogical action of a productive failure demonstration seemed to create a support-group environment. This connection to the psychology literature has never been done before. However, Warshauer (2015), in her research, created a Productive Struggle Framework in which this demonstration might be consider a subset. The framework had teacher responses that went from Telling (a teacher directing or quickly answering the struggle) all the way to Affordance (building on student thinking, detailed justification from student, and affording for student thinking) (p. 387). Our demonstration falls into the affordance category, perhaps even going more due to the teacher not intervening as much. Students creating their own support group, with the requirement and fostering by the instructor, can add to the literature that is growing on supporting students' productive struggle, including other teaching actions such as providing time in class for struggle, discussion, questioning, and eliciting student thinking (Bobis & Tregoning, 2019, p. 138).

The affective statements by the two authors and other students show that many of the positive and negative effects occurred in the classrooms. While the pedagogical action was merely to assign, grade (for completion), and give time for productive failures in the classroom, the way the students handled the experience was transformative to them. This pedagogical action of a productive failure requirement may partly answer the question posed by Shultz and Herbst (2020): "What if we could give instructors data about how to best implement inquiry-oriented practices in ways that would build students' confidence instead of potentially embarrass them?" (p. 535).

There can be changes in implementing the pedagogical practice of requiring a productive failure in class. For example, instead of presenting productive failures, one could incorporate a question of reflecting on mistakes, similar to one of Su's (2020) seven questions for a final exam. Options for both may be more beneficial due to the support group benefit received by presenting but may also allow students to complete the requirement in their own process. This may alleviate some of the negative effects, including the student who wanted to "disappear" and may gauge the cost-benefit of losing

5% of their grade in order to not present. An instructor could also pre-screen the productive failures in order to address the ineffective and inappropriate responses in class; it would require more time on the instructor's behalf.

Both the third and fourth authors stated that through their productive failures they gained better coping skills and have found a sense of power to handle failure. They also stated that it was their peers, and the environment that was created from productive failures, that contributed to their own cognitive and affective growth. Brown, Tang, and Hollman (2014), citing Brown (2009), stated that "part of [support groups'] strength lies in their empowering nature, where participants help each other as equals rather than taking on dependency roles where they rely on the advice of professionals" (p. 84). Therefore, in addition to inquiry-based mathematics education, demonstrations of productive failures may help shift power to create a more equitable classroom (Tang et al., 2017).

In a purely cognitive sense, the socio-mathematical norm (Yackel & Cobb, 1996) of learning from mistakes has effects on students' approaches to future problems. For example, the fourth author stated in her interview that she is "not afraid of failing," thus her self-efficacy may have increased for subsequent courses. Thus, there may be many metacognitive gains for students when demonstrating a productive failure. Seeing students fail may cause students to believe that they are not alone in their difficulties, and especially in an individual-centric classroom that mathematics breeds, this belief that others are having problems can cause relief. This relief and reassurance can counter the beliefs of math anxiety. Math anxiety is defined as "feeling of tension, apprehension, or fear that interferes with math performance" (Ashcraft, 2002, p. 181). The ways that math anxiety has been approached in the literature can be counter to long-lasting support: "[Previous literature] focus[es] on trying to avoid, reduce, redirect, or suppress the power of negative thoughts, which may not be as effective as developing patterns of positive thoughts that preempt negative thinking" (Regier, 2020, p. 2). Productive failures can be a step of developing those "positive thoughts" that can pre-empt the anxiety that progresses.

We acknowledge that there is at least one lingering difference between this course and other support groups in the community or in counseling. Part of the students' grades (5%) was associated with demonstrating a productive failure, which means that while the students volunteered when they would present, they were *required* to present. The third author stated that she had productive failures prior to the one discussed in this article, but the failure was not "good enough" to demonstrate. While the benefits and drawbacks seemed to align with support groups, the premise itself may not fully align. However, this requirement of presenting a productive failure may be an example of taking an extrinsic motivator (percent of grade) into more of an intrinsic motivator (understanding mathematics and being a better mathematician) (Ryan & Deci, 2000). The requirement may have been a catalyst for creating the support group.

Productive failure demonstrations allow mistakes to be open and psychologically constructive instead of damaging, give a platform and power that otherwise may not be available, and may influence both the presenter and their peers affectively. It seems to rehumanize mathematics – it "recogniz[es] mathematics as a living practice" and shows that mathematics has a "human element" (Gutiérrez, 2018, p. 6). Even the choice of words of "productive failure" is up for debate. For example, Granberg (2016) called it a productive struggle. However, we believe in keeping the words "productive failure" since it makes the deficit-laced term "failure" and cognitively reconfigures it into something that is helpful. This is directly in line with Adiredja's (2020) "anti-deficit" theoretical perspective and Burger and Starbird's (2012) failing to succeed.

The intention is to investigate and collect further data, especially for gains in problem solving. A conjecture is that as failures tend to be recast, more students will persist in their problem solving. Time and effort may improve their mathematical skills and allow them to grow to be more content with their abilities. Encouraging productive failures in a classroom can give students the affectual support to grow as practicing mathematicians, thus overcoming difficulties for now and for the future. As the third author ended her productive failure demonstration, she stated:

> **I don't know why I didn't do that from the beginning, but I guess it was more fun this way.** So then I figured out how it worked for the isomorphism, and use the fact of the [first isomorphism theorem] and try to use the same concepts as the [first isomorphism theorem], and it worked. And that was the end. **Onto was hard, but I proved it and homomorphism. And that's the end.** (CLASS CLAPS) – Third author, minute 5, productive failure demonstration

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