AN ALTERNATIVE MATERIAL FOR TEACHING PRIME NUMBERS: PRIME FACTORS CHART

Satı Ceylan Oral[1]

ABSTRACT

This study aimed to evaluate the effectiveness of the Prime Factors Chart (PFC) as an alternative teaching material, which was developed for teaching the concepts within the "factors and multiples" unit in the middle school mathematics curriculum. Twelve middle school mathematics teachers used the PFC to teach concepts, such as prime number, prime factor, the highest common factor, and the least common multiple, relatively prime numbers, factors, and multiples during face-to-face education in 4 lesson hours. The teachers and their students (n=90) evaluated the PFC based on the framework of the principles of material development. The participant teachers' and students' opinions indicated that the PFC is simple and understandable, suitable for the learning objectives and outcomes, suitable for the developmental characteristics of the students, and simple enough to be used by students as well as teachers.

Keywords: prime factors chart, prime numbers, mathematics, material development.

ASAL SAYILARIN ÖĞRETİMİNDE ALTERNATİF BİR MATERYAL: ASAL ÇARPAN KARTELÂSI

ÖZ

Bu çalışmanın amacı, ortaokul matematik dersi öğretim programında yer alan "çarpanlar ve katlar" ünitesindeki kavramlara yönelik alternatif bir öğretim materyali olarak geliştirilen Asal Çarpan Kartelâsının (AÇK) sınıf içerisinde kullanılabilirliğini değerlendirmektir. Yüz yüze eğitim sürecinde 4 ders saati boyunca 12 matematik öğretmeni ve 90 öğrencisi ile farklı zaman dilimlerinde asal sayı, asal çarpan, aralarında asal sayılar, en küçük ortak kat, en büyük ortak bölen, çarpanlar ve katlar gibi kavramları öğretmek amacıyla kullanılan AÇK, materyal geliştirme ilkeleri çerçevesinde değerlendirilmiştir. Öğretmen ve öğrenci yanıtlarına göre AÇK'nın basit, sade ve anlaşılabilir, dersin hedef ve davranışlarına uygun, öğrencinin gelişim ve öğrenim özelliklerine uygun, öğretmenler kadar öğrencilerin de kullanabileceği düzeyde basit olduğu söylenebilir. Öğrencilerin sürece ilişkin görüşleri incelendiğinde AÇK ile işlenen derslerin heyecan verici, merak uyandırıcı, öğretici, faydalı, eğlenceli olduğunu ifade ettikleri tespit edilmiştir. Öğrencilerden bir kısmı da boyama sürecinin gereksiz ve sıkıcı olduğunu belirtmişlerdir.

Anahtar kelimeler: asal çarpan kartelâsı, asal sayılar, matematik, materyal geliştirme.

Article Information:
Submitted: 04.20.2021
Accepted: 10.14.2021
Online Published: 10.29.2021

[1] Dr., Burhaniye Science and Art Center, ceylansati@hotmail.com, ORCID: https://orcid.org/0000-0002-3376-1709

INTRODUCTION

According to Altun (2001), the primary purposes of mathematics education are to provide students with the mathematical skills necessary in life, teach them to solve real-life problems, and enhance their ability to use a problem-solving approach when analyzing situations. Hence, it is of paramount importance today. However, many people have prejudices towards the abstract nature of mathematics because engaging in mathematics requires higher-order mental processes, such as problem-solving skills, and uses special symbols and signs to express concepts and ideas (Yıldırım, 1996). An increase or decrease in these prejudices can be directly related to how mathematics is taught (Dale, 1946; Hare, 1999). In many countries, traditional approaches that design mathematics instruction based on unconnected learning objectives have been replaced by constructivism-based approaches that emphasize conceptual and relational learning. Indeed, curricula developed based on traditional methods caused mathematics education to be trapped in a vicious circle (Boz, 2008). Developing teaching materials that students can use effectively in the classroom environment are listed among the factors that might save mathematics instruction from this vicious circle (Tuncer, 2008).

Many studies suggest that including materials in teaching environments comes with many benefits. According to the results of these studies, effective use of teaching materials in the learning environment facilitates learning and comprehension (İşman, 2005; Koşar et al., 2003), supports individual work, increases interest and motivation (Tuncer, 2008), provides learning experiences compatible with daily life (Gürbüz, 2007), improves critical thinking, problem-solving, and creativity skills (Körükçü, 2008), reinforces knowledge, and contributes to students' involvement in the learning process (Aslan & Doğdu, 1993). Teaching materials should be included in the instruction as objects that students can interact with to make sense of the mathematical concepts through their kinesthetic and visual senses (Sowell, 1989). Teaching materials can be used with different purposes including modeling the relationships between the sub-topics of a subject, involving students in the learning environment actively, and concretizing abstract concepts that are difficult to comprehend (Van de Walle, 2007; Yazlık, 2018).

Seferoğlu (2011) suggests that it is necessary to benefit from the prerequisites and principles of material development to design and produce a material that will contribute to effective teaching. According to the principles of material development; the material should be simple, plain, and understandable, suitable for the learning objectives and outcomes, have visual features highlighting the key points of the material, suitable for the developmental characteristics of students, provide students with the opportunity to practice concepts, simple enough to be used by students as well as teachers, and easily developed/revised as needed. During the material development process, the topics such as target analysis, identifying the learners' characteristics, content analysis and design, integrating the content and the tool, and transferring the material to the learning environment should be addressed.

Teaching the Concepts of Factors and Multiples

In the middle school mathematics curriculum, the topics of "Factors and Multiples" are prerequisites for some curriculum standards (e.g., writing a number using exponents), and therefore, they are listed before the algebra topics in the sixth and eighth grades (Ministry of National Education [MoNE], 2018). Factors and multiples are important topics of the number strand in mathematics and are often included in both national and international exams (Tatar et al., 2008). However, students have learning difficulties related to these two concepts.

Prior research studies examined students' understanding of factors and multiples using different teaching approaches and participants (Bilge, 2005; Bolte, 1999; Korkmaz & Korkmaz, 2017; Özdeş, 2013). Özdeş (2013) found that some students thought that the number 1 was a prime number and the number 2 was not a prime number because it is even. The study also reported that according to some students, all odd numbers should be considered prime because they are not divisible by 2 and the negatives of prime numbers are also prime. The students had misconceptions about relatively prime numbers, prime factorization, the highest common factor (HCF), and the lowest common multiple (LCM). Among the students' misconceptions were that relatively prime numbers must be prime numbers, they confused the concepts of division and divisibility, and they did not comprehend that number 1 is relatively prime with other numbers (Bolte, 1999; Yağmur, 2020).

The Sieve of Eratosthenes, a material that has been used for centuries in learning the concept of prime number, was designed as the elimination of multiples of prime numbers less than k to find prime numbers up to n where $k<\sqrt{n}$, and was introduced by the Greek mathematician Eratosthenes in around 250 BC (Lambert, 2004). The Sieve of Eratosthenes is limited to detecting prime numbers from 1 to n. There is a need for a material to teach the other concepts (factor, multiple, divisor, prime number, common divisor, common multiple, HCF, LCM, and relatively prime numbers) related to factors and multiples. In this context, this study aimed to evaluate the effectiveness of the Prime Factors Chart as a teaching material, which was developed as an alternative tool for teaching the concepts within the "factors and multiples" unit in the middle school mathematics curriculum.

THE DEVELOPMENT PROCESS OF THE PRIME FACTORS CHART

The material was developed based on the framework suggested by Seferoğlu (2011). Expert opinions were obtained at every stage of the process, which was carried out in accordance with the prerequisites of the material development process. These experts were two academicians and two mathematics teachers who were teaching the sixth and eighth-grade classes at the time of the study. The material, originally designed to teach only the concepts of prime number and prime factor, was improved based on expert opinions. Experts asserted that the material could not be "easily developed and revised as needed," and upon this, a meeting was organized with the experts to discuss what other concepts and procedures can be taught using the material. In the meeting, we agreed that the Chart could be used not only to teach prime numbers and prime factors but also to teach the concepts of the factors, multiples, divisibility rules, common multiples, common factors, relatively prime numbers, and HCF and LCM. Additionally, the material was finalized with its simple and plain form, which fits an A4 paper in light of the expert opinions. Due to the colorful images in its content, the designed material was named the "Prime Factors Chart (PFC)." Based on the prerequisites of the material development and expert opinions, the target audience of the material was determined, the characteristics of the learning environment and the learners were examined, and the material was transferred to the classroom environment after content analysis and design as explained below.

Target Analysis

This study aimed to develop a fun and effective material suitable for the developmental level of students to teach the concepts of factors and multiples. Therefore, we decided to design a material that would support students in cognitive, affective, and psychomotor learning domains.

Learner Characteristics

The topics of factors and multiples are mentioned for the first time in the curriculum in the sixth grade under the unit "*M.6.1.2. Factors and Multiples"* and then in the eighth grade under the unit *"M.8.1.1. Factors and Multiples*" (MoNE, 2018). Therefore, the target audience of the material developed in the current study is the sixth and eighth-grade students. The material was designed considering the students' attitudes, motivations, and readiness towards the mathematics course.

Content Analysis, Design, and Transferring the Material to the Learning Environment

The material evaluation principles (Seferoğlu, 2011) guided the design and evaluation stages of the material. The researcher and the experts agreed on its final form and how to transfer it to the classroom environment.

ACTIVITY IMPLEMENTATION

This study was conducted in the fall semester of the 2019-2020 academic year. The necessary legal permissions for the study were obtained from the relevant Directorate of National Education. In the study, a mixed-method research method was used. The quantitative data were obtained from the scores given by 12 teachers (Table 1) and 90 students (Table 2) within the framework of the material development principles, and the qualitative data were collected using open-ended questions administered to 15 eighth-grade students (Table 3) who participated in the sample implementation carried out by the researcher herself.

The current study started with a webinar in which the Prime Factors Chart (PFC) was introduced to 12 middle school mathematics teachers working in the Balıkesir province of Turkey. All of the teachers used the PFC in their lessons, and after the lessons, both they and their students evaluated the PFC using a questionnaire designed based on the Principles of Material Development (Seferoğlu, 2011). The material was used with the sixth and eighth-grade students. Each implementation lasted 4 lesson hours.

Table 1. Teacher Information in the Study Group

Group	**Teachers (n)**	**%**
Male	5	%41.6
Female	7	%58.4
Total	12	%100

Table 2. Student Information in the Study Group

Group		**Students (n)**	**%**
Male	51	30 sixth graders 21 eighth graders	%56.6
Female	39	19 sixth graders 20 eighth graders	%43.4
Total	90		%100

Table 3. Students Who Responded to Open-Ended Questions

Group		**Students (n)**	**%**
Male	8	eighth graders	%53.3
Female	7	eighth graders	%46.7
Total	15		%100

The Material Evaluation Form (Appendix 1), which was designed based on Seferoğlu's (2011) principles of material development, was used to obtain the teachers' and students' evaluations of the PFC. In order for the evaluation to be measurable, the material development principles were transformed into a Likert-type questionnaire. The participants' responses to questionnaire items were analyzed by calculating the mean scores.

Sample Activity Implementation

The researcher, the mathematics teacher of the participating students, carried out the sample activity implementation with 15 eighth-grade students attending a public school during 4 lesson hours. In order to help the students build new knowledge on their existing knowledge, the activity tasks focused on the concepts included within the Factors and Multiples Unit of the sixth and eighth-grade curriculum. These concepts are "*M.6.1.2. Factors and Multiples Unit / Terms or Concepts: Factor, multiple, divisor, prime number, common divisor, common multiple"* and *"M.8.1.1. Factors and Multiples Unit / Terms or Concepts: The highest common factor (HCF), the least common multiple (LCM), relatively prime numbers*" (MoNE, 2018).

In the preliminary stage of the activity, the teacher told the students that they needed 25 different colored crayons for the next day. She added that if they could not find that many colors, they could obtain a new color by mixing the already existing ones. On the day of the activity, the blank version of the PFC (Figure 1, Appendix 2) was distributed to the students. The students were given the opportunity to examine the Chart. After an examination that lasted for about 2 minutes, the teacher asked the students what they noticed in the Chart. The following dialogue took place (all names are pseudonyms):

Teacher: What do you see in the Chart in front of you?
Furkan: The numbers from 1 to 100 are written in a certain order and divided into rectangles of equal size. These rectangles are also divided into boxes of different numbers and sizes.
Teacher: Great, what else?
Eren: I see that some numbers are not separated into any boxes.
Teacher: Hmm, do these numbers have a common property?
Ege: Yes, these numbers are prime.
Teacher: Has everyone noticed this? Ok, write down the definition of a prime number in your notebook.

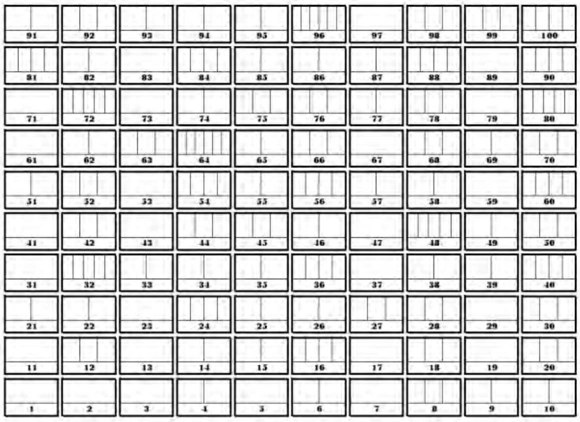


Figure 1. The In-classroom Version of the PFC

The PFC is indeed designed in a worksheet format, in which rectangles of the same size from 100 to 1 are divided into different sized segments. In this part, the teacher examined the prime number definitions that the students wrote in their notebooks and asked the whole class to write down all the prime numbers in the Chart in their notebooks as well. At this point, she tried to help the students realize that 25 different colors would represent 25 different prime numbers. The smaller boxes represent the prime factors of non-prime numbers. The teacher initiated a discussion in the class for students to discover this fact.

> Teacher: So, let’s think about what the little boxes mean. Does anyone have an idea?
> Onur: Factors, I guess.
> Teacher: What do you think about Onur’s idea?
> Ege: Let’s start with the smaller numbers one by one, ma’am. For example, the factors of 4 are 1, 2, and 4, so there are three numbers, but there are two boxes here. It can’t be factors.
> Zeynep: Oh, prime factors! But number 4 has one prime factor, which is 2. We have two boxes here.
> Onur: Couldn’t it be 4=2x2, then? For example, there are three boxes for number 8, and I think that 8=2x2x2.
> Students: Yes, product of the prime factors.
> Teacher: You guys are great!

The whole class discussion supported the students to discover the working principle of PFC. Indeed, the PFC is designed as boxes containing the product of prime factors of non-prime numbers. Figure 2 shows an example.

Figure 2. Examples of Product of Prime Factors

Figure 2 shows that the rectangle representing the number 60 is divided into four equal parts, and the rectangle representing the number 68 is divided into three equal parts. In short, each number is divided into segments, each representing a prime factor, and painted in the color of the prime number it represents. Photograph 1 presents an example student work.

Photograph 1. A Student’s Work on the PFC

The initial exploration of the PFC took about 20 minutes. In this phase, the teacher stated that the color of the box is their main concern, not the size so that the students do not develop misconceptions.

> Teacher: Now, we will fill in a part of the Chart together. Previously, we discussed why 1 is not a prime number. Let’s start with 2.
> Banu: 2 is a prime number. It consists of one box. Should we assign a color to it?
> Alp: Yes, shall we color number 2 in yellow?
> Teacher: Yes, guys, yellow can represent the number 2 for us.
> Banu: Let’s color the number 3 in blue, then.
> Teacher: If you have colored numbers 2 and 3 in yellow and blue, let’s move on to 4. What do you think of 4?
> Zeynep: If we write the number 4 as the multiplication of its prime factors, since 4=2x2, we write 2 in both boxes and color it in yellow.

Derin: There is something I don't understand. Why don't we think of it as 4=4x1?
Zeynep: Because 4 and 1 are not prime numbers. We will always write the prime factors in these little boxes so we know what color we can paint them.
Teacher: Yes, Derin, Zeynep is right. Come on; you tell us about the number 6 now.
Derin: So, since 6=2x3, we will write 2 and 3 in the boxes and paint them in colors that represent 2 and 3.
Teacher: Well done, kids!

After making sure that the students firmly understood the process and the common colors were determined for each prime number, they were given approximately 1 lesson hour to fill in and paint the entire Chart (Figure 3, Appendix 3). Some students first wrote down the prime factors and then proceeded to color them (Figure 4, Appendix 4). The teacher reminded the students about the divisibility rules:

Teacher: Guys, it will be easier to fill out the Chart if you know the divisibility rules.
Tarık: Yes, teacher, non-prime numbers from 1 to 100 are multiples of 2, 3, 5, or 7. That's why I always look to see if the numbers are divisible by these numbers.
Teacher: Great job, Tarık! Can you please tell your friends about the shortcuts for whether a number is divisible by 2, 3, or 5?

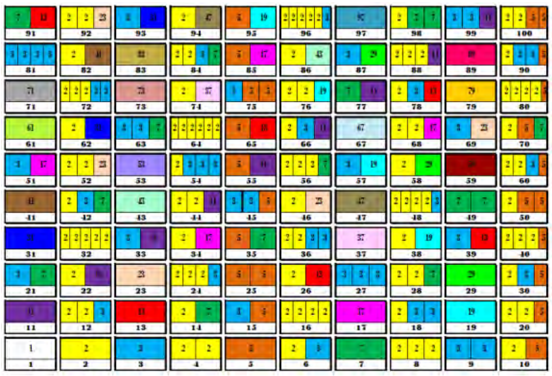


Figure 3. A Completed PFC

It is crucial that the teacher guides the students throughout the process and creates an environment where students can express themselves freely. A dialogue illustrating this environment is as follows:

Eylem: Teacher, I'm done.
Teacher: How did you finish it so fast?
Eylem: I found a shortcut, ma'am. For example, when factoring the number 78 into prime factors, I knew it was divisible by 2 because it was even. 78:2=39. I had already painted the Chart for the number 39. Look, it is 39=3x13. Then, 78=2x3x13.
Teacher: Of course, you can think of it that way. Nice one.

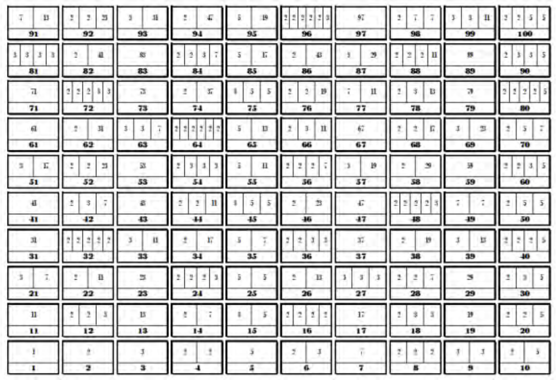


Figure 4. Prime Factorized PFC

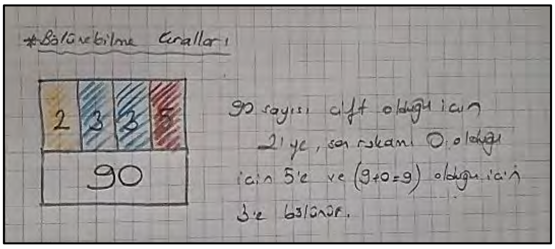
The first 2 hours of the activity were spent on discovering the working principle of PFC and writing and coloring the prime factors. In the next 2 hours, the students discussed which mathematical concepts they can explore using the PFC in addition to the concepts of prime numbers and prime factorization.

Teacher: Everyone should have completed their prime factor chart and brought it with them today. Now, let's discuss what else we can learn by using this Chart and briefly write down everything we find in our notebook. [Students are given some time to think about the question.] Yes, anyone wants to speak?
Elif: Ma'am, the PFC teaches us the concepts of prime numbers and prime factorization (Photograph 2).
Teacher: Now, everyone, draw and color a prime and non-prime number that you choose from your Chart.

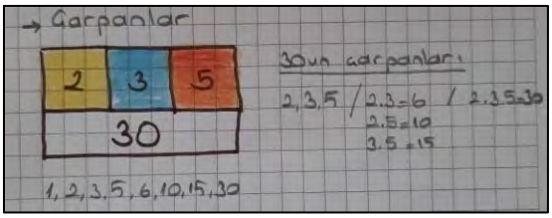
Photograph 2. Sample Student Work 1

Other concepts discovered with the students were noted in the student notebooks, along with their examples.

Teacher: What other concepts does the prime factors chart teach us?
Fuat: We repeatedly use the divisibility rules (Photograph 3). This reinforces our learning.
Derin: We can see all the factors of a number. For example, when we consider the number 30, we can say that 30 is divisible by 2, 3, 5, and their product that are 6, 15, 10, and 30. Each number is already divisible by 1 (Photograph 4).

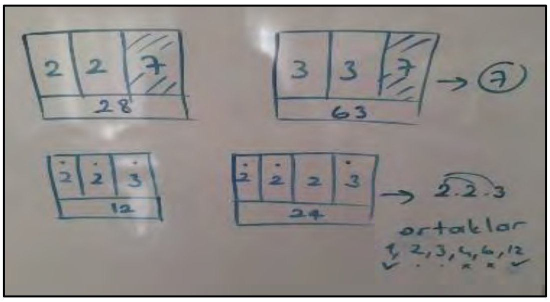


Photograph 3. Sample Student Work 2



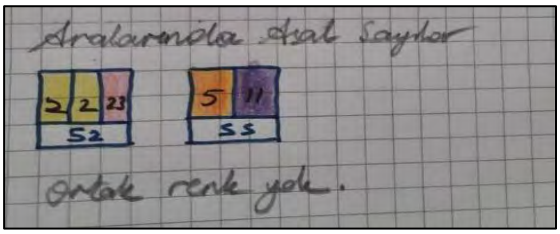
Photograph 4. Sample Student Work 3

The students explained their ideas to their classmates, and when necessary, they showed their work by writing them on the board.
Akın: We can also find the common factors of numbers. For example, when I look at the numbers 28 and 63, I can see that 7 is a common color.
Mustafa: That is easy. What about the numbers 12 and 24? They have a lot of common factors. It is not enough to say a common color and a common factor. Very complicated.
Akın: Teacher, can I show this on the board? (Photograph 5) 12= 2.2.3 and 24=2.2.2.3. What colors do they both have in common? (He asks her to look at the Chart) 2, 2, and 3. Just like we find all the factors, we will write down all the factors that we can form from these common colors. 1, 2, 3, 4, 6, and 12. These are common factors of the numbers 12 and 24.
Mustafa: We have also seen the highest of the common factors. Then, the highest number of common colors can also be the HCF. It is fine.



Photograph 5. Akın's Work on the Whiteboard

During the activity, the students were encouraged to discuss and learn from each other. In the current implementation, the students had difficulty seeing that the Chart could be used for relatively prime numbers and the LCM concepts. The teacher gave hints for how these concepts can be examined with the help of the PFC.
Teacher: Guys, can we see the relatively prime numbers when we examine the Chart?
Zeynep: Do the relatively prime numbers have to be prime, teacher?
Elif: No, I guess, they are not. There should be no common factors other than 1 for them to be relatively prime numbers.
Teacher: Is this clear to everyone? Eylem, do you understand the definition Elif explained? How can the Chart help us in this regard?
Eylem: Actually, I don't know. I need to think. Akın just said that the common factor is the common color. Shouldn't there be a common color then? [Looking at the Chart.] For example, 8 and 15 do not have any common colors. Can we call them relatively prime?
Elif: Of course!
Teacher: Well done, guys! Let's draw an example (Photograph 6).



Photograph 6. Sample Student Work 4

At the end of the lesson, to increase the students' motivation, the teacher mentioned the benefits of mathematics as a discipline that teaches interrelated concepts and develops the human brain. Photograph 7 shows samples of student work with the PFC.

Photograph 7. Students' Work on PFC

The PFC can be brought back to the classroom environment and used with students for exploring many other concepts and course objectives. *Examples of these concepts and objectives are given in Appendix 5.*

EVALUATION OF THE ACTIVITY

In order to analyze whether the PFC can be used in the classroom as an alternative teaching material, the evaluations of three different groups were taken into consideration. Table 5 presents information on these groups and related data collection tools.

Table 5. Groups Evaluating the Material

Group and No	Data Collection Tool
Teachers (n=12)	Material evaluation form (Quantitative data)
Students (n=90)	Material evaluation form (Quantitative data)
Students (n=15)	Open-ended questions (Qualitative data)

Twelve middle school mathematics teachers and their students (n=90) indicated in Table 5 used the PFC and evaluated it according to the material evaluation form. In this evaluation, each item was scored over 5 points (100%), and the mean scores were calculated (Table 6). Besides, 15 eighth-grade students who participated in the sample implementation lessons taught by the researcher evaluated the PFC by responding to open-ended questions.

Table 6. Teachers' and Students' Responses to the Material Evaluation Form

	Teachers		Students	
	Mean	**%**	**Mean**	**%**
* Is it simple and understandable?	3.33	83.33	3.67	91.67
* Is it appropriate for the learning objectives and outcomes?	3.58	89.58	3.83	95.83
* Do the visual features highlight key points of the material?	3.17	79.17	3.90	97.50
* Is it appropriate for students' developmental characteristics?	3.25	81.25	4.00	100.00
* Does it provide the student with the opportunity to practice and exercise?	3.15	78.75	3.93	98.33
* Is it simple enough to be used by students as well as teachers?	3.50	87.50	3.83	95.83
* Can it be easily improved and revised as needed?	2.42	60.42	3.50	87.50
Mean	3.20	% 80.00	3.81	% 95.24

Table 6 shows that both the teachers and the students evaluated the PFC with high ratings ranging from 2.42 to 4. Based on the teachers' and students' evaluations, we can infer that the PFC is suitable for the learning objectives and outcomes, is simple and understandable, and can be used by students as well as teachers. Additionally, participant teachers and students indicated that the PFC is suitable for the developmental characteristics of the students, the visual features highlight the key points of the material, and it provides the students with the opportunity to practice and exercise. The lowest rating was given to the last item on improving and revising the material by the participating teachers.

The eighth-grade students who participated in the sample activity implementation responded to open-ended questions about the skills they gained and the problems they experienced during the process. Table 8 shows the open-ended questions. The students answered the questions in writing.

Table 8. Open-Ended Questions

✓ What are your opinions about the lessons taught with the Prime Factors Chart?
✓ What skills did you learn as a result of participating in the Prime Factors Chart lessons?
✓ What problems did you experience in this process?

The students expressed that the lessons taught with the PFC were exciting, engaging, instructive, useful, and entertaining. Some of the students added that the process required patience. Examples of the student statements are as follows:

> I was very happy to attend this lesson. Gradually, the initially complex paper with numbers up to 100 that you gave us became meaningful. So, the lesson was interesting and fun. (Furkan).
>
> I found this activity very useful. First of all, it was very nice that our teacher encouraged us to question and listen to everyone's opinion. (Elif).
>
> I think this process was very entertaining and instructive. I realized that I understood some concepts that I had trouble with understanding before. (Eren).

The students had some prior knowledge of factors and multiples before participating in the activity. They wrote that they made more sense of the concepts after engaging in the lessons taught with the PFC. They answered the question about what skills the lessons taught them by writing down all the concepts related to the Chart. The students explained that they learned the concepts such as prime number, prime factors (8 students), factors (7 students), divisibility rules (9 students), relatively prime numbers (8 students), and HCF-LCM (7 students). Some of the student statements on the interview form are as follows:

> First of all, I must state that we are expected to know the divisibility rules, prime numbers, and prime factors, which we had learned in the sixth grade. However, we don't really know these topics that much. Thanks to the PFC, I concretized the concepts I thought I knew, such as the common factors and relatively prime numbers… Everything was so clear… (Derin).
>
> The concept of the prime factor was interesting to me. Think about it; we can write all the numbers except 1 to infinity as the product of prime numbers. So weird and fascinating! (Mustafa).
>
> One of the most fundamental topics of the eighth grade math is factors and multiples. I understood a topic that I previously had difficulty with learning during a stressful time for us, as we are preparing for LGS [Nationwide High School Entrance Exam]. I learned relatively prime numbers, for example. If there is no common color, there is no common factor. These are relatively prime numbers. Very easy! (Elif).

The last question asked to the students in the interview form was the problems they experienced during the lesson taught with the PFC. While most of the students expressed that they did not encounter any problems, some stated that it was difficult to find 25 different colors, and they were bored with coloring all the numbers. Additionally, some students expressed that they had difficulty understanding some parts and did not like the questioning process created in the classroom. Some of the student statements on the interview form are as follows:

> Was it necessary to color all the numbers in the Chart? We could only write the prime factors and discuss them. I think that our teacher had it painted for fun, but I thought it was boring and unnecessary. I couldn't find 25 colors anyway. (Akın).
>
> Our teacher asked us everything. We tried to discover everything ourselves. I was not very active during that time, but after my friends explained the concepts, I understood them. (Eren).
>
> I had a hard time understanding some parts. That's why I can say I was bored. (Eylem).

CONCLUSION and SUGGESTIONS

This study used classroom implementations to evaluate the effectiveness of the PFC, developed for teaching the concepts within the factors and multiples unit in the sixth and eighth grades mathematics curriculum in Turkey. The PFC was designed based on the prerequisites of the material development process by following the phases of target analysis, determination of learner's characteristics, content analysis and design, integration of content and the tool, and transferring the material to the learning environment. Hence, aligned with the related literature (Seferoğlu, 2011; Yanpar, 2015), the material was designed to be simple, plain, and understandable, suitable for the learning objectives and outcomes, providing students with the opportunity to practice and exercise, and economical and ergonomic in a way that all students and teachers can easily use.

Twelve mathematics teachers used the PFC for teaching the concepts of prime numbers and prime factors. The teachers' and the students' evaluations of the PFC within the framework of the principles of material development revealed that the PFC is suitable for the learning objectives and outcomes, simple and understandable, simple enough to be used by students as well as teachers, suitable for the developmental characteristics of students, and has visual features that highlight the key points of the material. The participants' evaluations indicate that the PFC can be used to teach the concepts within the factors and multiples unit and will positively contribute to the teaching process. The findings of the current study are consistent with the findings obtained by Bilge's (2005) study conducted on teaching the concepts of prime numbers and prime factors through an active learning method. Kamii et al. (2001) found that most of the mathematics teachers in their study underlined that the use of materials contributed positively to students' mathematical thinking. A similar finding was obtained in this study based on the participating teachers' evaluations of the PFC.

The findings regarding the students' opinions on the activity showed that they found the lessons taught with the PFC exciting, engaging, instructive, useful, and entertaining. Some of the students thought that the process required patience. The positive effect of the mathematics lessons conducted with the PFC on students' motivation levels supports similar studies that reported a positive relationship between students' motivation and using materials in lessons (Keller, 2010; Yorgancı & Terzioğlu, 2013).

The related literature reported that students have misconceptions related to prime numbers such as the number 1 is a prime number, the number 2 is not prime because it is even, and all odd numbers are prime (Özdeş, 2013). It was found that according to some students, the negatives of prime numbers are also prime, and relatively prime numbers must be prime (Bolte, 1999). In light of the existing literature on students' misconceptions, the researcher emphasized the key points of the concepts during the lessons taught with the PFC. In the Chart, the number 1 is colored in white, and it is clear that it is not prime as no other number includes a white section. Additionally, the PFC consists of only positive numbers, and this gives an implicit message that there are no negative prime numbers. The fact that there is no requirement for relatively prime numbers to be prime is presented to the students with many examples according to the "common color means common factor" principle. From this point of view, the PFC has a structure that can eliminate students' misconceptions and can even prevent such misconceptions to occur.

The results of the current and previous research studies support that using materials in the learning environments facilitates perception and learning, arouses interest, and brings vitality to the lesson. Additionally, materials save time in the teaching process, help to consolidate the knowledge, and promote long-lasting learning. Hence, materials that can make learning permanent, effective, and enjoyable should be used in learning environments. Educators should receive training to design materials and gain knowledge about the importance and positive effects of using materials in their courses. The current research is limited to the "Factors and Multiples" concepts in mathematics. New course contents can be developed using materials for teaching other mathematics concepts or for other subject fields.

REFERENCES

Altun, M. (2001). *Matematik öğretimi [Teaching mathematics]*. Alfa Press.

Aslan, Z., & Doğdu, S. (1993). *Eğitim teknolojisi uygulamaları ve eğitim araç gereçleri [Educational technology applications and educational tools]*. Tekışık Publishing.

Bilge, O. (2005). *İlköğretim 6. sınıf matematik dersi asal sayılar ve çarpanlara ayırma ünitesinin hedef ve davranışlarını kazandırmada aktif öğrenme yaklaşımının etkisi [The effect of active learning approach in reaching the objectives and behaviors of the unit prime numbers and factorization in the sixth grade in primary school]* [Unpublished master's thesis]. Gazi University.

Bolte, L. (1999). Enhancing and assessing preservice teachers' integration and expression of mathematical knowledge. *Journal of Mathematics Teacher Education, 2*(2), 167-185.

Boz, N. (2008). Matematik neden zor? [Why is mathematics difficult?]. *Necatibey Faculty of Education Electronic Journal of Science and Mathematics, 2*(2), 52-65.

Dale, E. (1946) *Audio-visual methods in teaching.* The Dryden Press.

Gürbüz, R. (2007). Olasılık konusunda geliştirilen materyallere dayalı öğretime ilişkin öğretmen ve öğrenci görüşleri [Students' and their teachers' opinions about the instruction based on the materials on probability subject]. *Kastamonu Education Journal, 15*(1), 259– 270.

Hare, A. Y. M. (1999). *Revealing what urban early childhood teachers think about mathematics and how they teach it: Implications for practice* [Unpublished doctoral dissertation]. University of North Texas.

İşman, A. (2005). *Öğretim teknolojileri ve materyal geliştirme [Instructional technologies and material development]*. PegemA Publishing.

Kamii, C., Lewis, B. A., & Kirkland, L. (2001). Manipulatives: When are they useful? *The Journal of Mathematical Behavior, 20*(1), 21-31.

Keller, J. M. (2010). *Motivational design for learning and performance: The ARCS model approach.* Springer Science & Business Media.

Korkmaz, E., & Korkmaz, C. (2017). Ebob-Ekok konusunun gerçekçi matematik eğitimi etkinlikleriyle öğretiminin başarı ve tutuma etkisi [EBOB –EKOK subject effect to success and attitude with teaching realistic mathematics education activities]. *Mustafa Kemal University Journal of Social Sciences Institute, 14*(39), 504-523.

Koşar, E., Yüksel, S., Özkılıç, R., Avcı, U., Alyaz, Y., & Çiğdem, H. (2003). *Öğretim teknolojileri ve materyal geliştirme. [Instructional technologies and material development]*. PegemA Publishing.

Körükçü, E. (2008). *Tam sayılar konusunun görsel materyal ile öğreniminin 6. sınıf öğrencilerinin matematik başarılarına etkisi [The effect of learning integers using visual materials on 6th grade students' success in mathematics]* [Unpublished master's thesis]. Marmara University.

Lambert, M. (2004). Calculating the Sieve of Eratosthenes. *Journal of Functional Programming, 14*(1), 759-763.

Ministry of National Education. (2018). *Matematik dersi öğretim programı (İlkokul ve ortaokul 1, 2, 3, 4, 5, 6, 7 ve 8. sınıflar) [Mathematics curriculum (Primary and middle school 1, 2, 3, 4, 5, 6, 7, and 8th grades)].* https://mufredat.meb.gov.tr

Özdeş, H. (2013). *9. sınıf öğrencilerinin doğal sayılar konusundaki kavram yanılgıları [Misconceptions of 9th class students regarding to natural numbers]* [Unpublished master's thesis]. Adnan Menderes University.

Seferoğlu, S. S. (2011). *Öğretim teknolojileri ve materyal geliştirme [Instructional technologies and material development].* PegemA Publishing.

Sowell, E. J. (1989). Effects of manipulative materials in mathematics instruction. *Journal for Research in Mathematics Education, 20*(5), 498-505.

Tatar, E., Okur, M., & Tuna, A. (2008). Ortaöğretim matematiğinde öğrenme güçlüklerinin saptanmasına yönelik bir çalışma [A study to determine learning difficulties in secondary mathematics education]. *Kastamonu Educational Journal, 16*(2), 507–516.

Tuncer, D. (2008). *Materyal destekli matematik öğretiminin ilköğretim 8. sınıf öğrencilerinin akademik başarısına ve başarının kalıcılık düzeyine etkisi [The effect of teaching material aided instruction on 8^{th} grade students' academic success and level of permanency]* [Unpublished master's thesis]. Gazi University.

Van de Walle, J. A. (2007). *Elementary and middle school mathematics: Teaching developmentally* (6th ed.). Pearson Education, Inc.

Yağmur, B. E. (2020). A game-based activity related to prime numbers. *Journal of Inquiry Based Activities, 10*(1), 18–30. https://ated.info.tr/ojs-3.2.1-3/index.php/ated/article/view/9

Yanpar, T. (2015). *Öğretim teknolojileri ve materyal tasarımı [Instructional technologies and material design].* Anı Publishing.

Yazlık, D. Ö. (2018). Öğretmenlerin matematik öğretiminde somut öğretim materyali kullanımına yönelik görüşleri [The views of teachers about use of concrete teaching materials in mathematics teaching]. *OPUS International Journal of Society Researches, 8*(15), 775-805.

Yorgancı, S., & Terzioğlu, Ö. (2013). Matematik öğretiminde akıllı tahta kullanımının başarıya ve matematiğe karşı tutuma etkisi [The effect of using interactive whiteboard in mathematics instruction on achievement and attitudes toward mathematics]. *Kastamonu Educational Journal, 22*(3), 919-930.

Yıldırım, A. (1996). Disiplinlerarası öğretim kavramı ve programlar açısından doğurduğu sonuçlar [The concept of interdisciplinary teaching and its consequences in terms of programs]. *Hacettepe University Journal of Education, 12*, 89-94.

Citation Information

Ceylan Oral, S. (2021). An alternative material for teaching prime numbers: Prime factors chart. *Journal of Inquiry Based Activities, 11*(2), 92-110. https://www.ated.info.tr/ojs-3.2.1-3/index.php/ated/issue/view/22

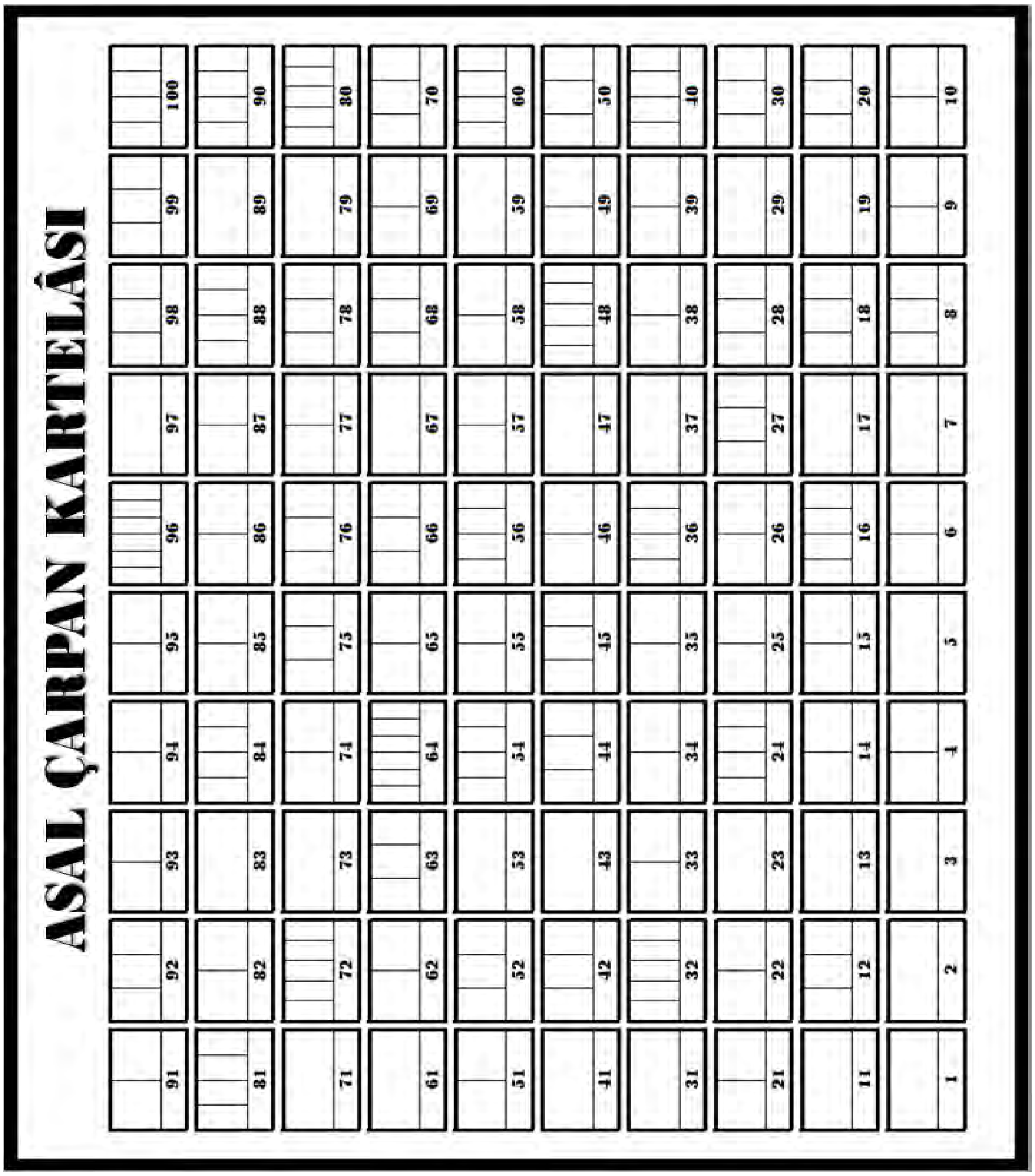
Appendix 1

Material Evaluation Form

	Definitely No	No	Partly	Yes	Definitely Yes
* Is it simple and understandable?					
* Is it appropriate for the learning objectives and outcomes?					
* Do the visual features highlight key points of the material?					
* Is it appropriate for students' developmental characteristics?					
* Does it provide the student with the opportunity to practice and exercise?					
* Is it simple enough to be used by students as well as teachers?					
* Can it be easily improved and revised as needed?					

Appendix 2

The In-classroom Version of the Prime Factors Chart



Appendix 3

Completed Prime Factors Chart

7 13 **91**	2 2 23 **92**	3 31 **93**	2 47 **94**	5 19 **95**	2 2 2 2 2 3 **96**	97 **97**	2 7 7 **98**	3 3 11 **99**	2 2 5 5 **100**
3 3 3 3 **81**	2 41 **82**	83 **83**	2 2 3 7 **84**	5 17 **85**	2 43 **86**	3 29 **87**	2 2 2 11 **88**	89 **89**	2 3 3 5 **90**
71 **71**	2 2 2 3 3 **72**	73 **73**	2 37 **74**	3 5 5 **75**	2 2 19 **76**	7 11 **77**	2 3 13 **78**	79 **79**	2 2 2 2 5 **80**
61 **61**	2 31 **62**	3 3 7 **63**	2 2 2 2 2 2 **64**	5 13 **65**	2 3 11 **66**	67 **67**	2 2 17 **68**	3 23 **69**	2 5 7 **70**
3 17 **51**	2 2 23 **52**	53 **53**	2 3 3 3 **54**	5 11 **55**	2 2 2 7 **56**	3 19 **57**	2 29 **58**	59 **59**	2 2 3 5 **60**
41 **41**	2 3 7 **42**	43 **43**	2 2 11 **44**	3 5 5 **45**	2 23 **46**	47 **47**	2 2 2 2 3 **48**	7 7 **49**	2 5 5 **50**
31 **31**	2 2 2 2 2 **32**	3 11 **33**	2 17 **34**	5 7 **35**	2 2 3 3 **36**	37 **37**	2 19 **38**	3 13 **39**	2 2 2 5 **40**
3 7 **21**	2 11 **22**	23 **23**	2 2 2 3 **24**	5 5 **25**	2 13 **26**	3 3 3 **27**	2 2 7 **28**	29 **29**	2 3 5 **30**
11 **11**	2 2 3 **12**	13 **13**	2 7 **14**	3 5 **15**	2 2 2 2 **16**	17 **17**	2 3 3 **18**	19 **19**	2 2 5 **20**
1 **1**	2 **2**	3 **3**	2 2 **4**	5 **5**	2 3 **6**	7 **7**	2 2 2 **8**	3 3 **9**	2 5 **10**

Appendix 4

PFC with Prime Factorized Numbers

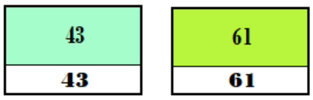
7 13 **91**	2 2 23 **92**	3 31 **93**	2 47 **94**	5 19 **95**	2 2 2 2 2 3 **96**	97 **97**	2 7 7 **98**	3 3 11 **99**	2 2 5 5 **100**
3 3 3 3 **81**	2 41 **82**	83 **83**	2 2 3 7 **84**	5 17 **85**	2 43 **86**	3 29 **87**	2 2 2 11 **88**	89 **89**	2 3 3 5 **90**
71 **71**	2 2 2 3 3 **72**	73 **73**	2 37 **74**	3 5 5 **75**	2 2 19 **76**	7 11 **77**	2 3 13 **78**	79 **79**	2 2 2 2 5 **80**
61 **61**	2 31 **62**	3 3 7 **63**	2 2 2 2 2 2 **64**	5 13 **65**	2 3 11 **66**	67 **67**	2 2 17 **68**	3 23 **69**	2 5 7 **70**
3 17 **51**	2 2 23 **52**	53 **53**	2 3 3 3 **54**	5 11 **55**	2 2 2 7 **56**	3 19 **57**	2 29 **58**	59 **59**	2 2 3 5 **60**
41 **41**	2 3 7 **42**	43 **43**	2 2 11 **44**	3 5 5 **45**	2 23 **46**	47 **47**	2 2 2 2 3 **48**	7 7 **49**	2 5 5 **50**
31 **31**	2 2 2 2 2 **32**	3 11 **33**	2 17 **34**	5 7 **35**	2 2 3 3 **36**	37 **37**	2 19 **38**	3 13 **39**	2 2 2 5 **40**
3 7 **21**	2 11 **22**	23 **23**	2 2 2 3 **24**	5 5 **25**	2 13 **26**	3 3 3 **27**	2 2 7 **28**	29 **29**	2 3 5 **30**
11 **11**	2 2 3 **12**	13 **13**	2 7 **14**	3 5 **15**	2 2 2 2 **16**	17 **17**	2 3 3 **18**	19 **19**	2 2 5 **20**
1 **1**	2 **2**	3 **3**	2 2 **4**	5 **5**	2 3 **6**	7 **7**	2 2 2 **8**	3 3 **9**	2 5 **10**

Appendix 5

Concepts, Curriculum Standards, and Examples Related to PFC

Concept to Teach	Curriculum Standard
Prime Number: Numbers that have only one section in the PFC are prime and are shown in a different color from other primes.	**M.6.1.2.3.** Students will be able to determine prime numbers with their properties. They also find prime numbers up to 100 with the help of the Sieve of Eratosthenes.

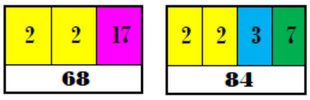
Example:



The numbers 43 and 61 are placed in one box in the material because they have no prime factors other than themselves.

Prime Factors: The Prime Factor Chart allows non-prime numbers to be written in terms of prime factors.	**M.8.1.1.1.** Students will be able to find the prime factors of a positive integer.

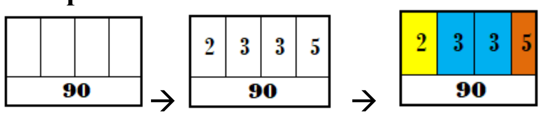
Example:



The prime factors of the numbers 68 and 84 appear clearly in the material.

Divisibility Rules: A student working with the Chart continuously repeats divisibility rules to see if numbers are divisible by 2, 3, 5, or 7.	**M.6.1.2.2.** Students will be able to explain and use the divisibility rules by 2, 3, 4, 5, 6, 9, and 10 without a remainder.

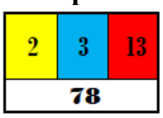
Example:



A student working on finding the prime factors of 90 and expressing it with appropriate colors will continuously repeat the divisibility rules to see if the number is divisible by the prime numbers 2, 3, or 5.

Factors: PFC helps to clearly see the factors of the numbers.	**M.8.1.1.1.** Students will be able to find positive integer factors of given positive integers, write the integer as the product of its factors using exponential expressions.

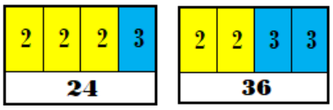
Example:



The number 78 is divisible by 2, 3, and 13, as well as 6 (2x3), 26 (2x13), 39 (3x13), and 78 (2x3x13).

The Highest Common Factor (HCF): HCF of any selected two numbers can be found.	**M.8.1.1.2.** Students will be able to calculate the highest common factor (HCF) and the least common multiple (LCM) of two whole numbers.

Example:

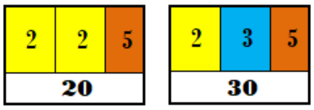


The greatest of the common divisors of numbers 24 and 36, that is, HCF, is 12 (2x2x3). In other words, it is the product of the numbers representing the common colors in both numbers.

Least Common Multiple (LCM): LCM of numbers from 1 to 100 can be found using PFC.

M.8.1.1.2. Students will be able to calculate the highest common factor (HCF) and least common multiple (LCM) of two whole numbers.

Example:



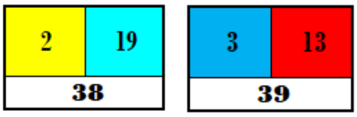
A student who wants to find the highest common factor of numbers 20 and 30 first finds the common colors in both boxes. These colors are yellow and orange for the numbers in question (2 and 5. 2x5=10 is the HCF of these numbers). Then, yellows and oranges are discarded in any box, and all remaining numbers are multiplied. Therefore, LCM of (20, 30) = 2.2.3.5 = 60.

Relatively Prime Numbers: Relatively prime numbers have no common divisor other than 1. If the common divisor is considered a common color, numbers that do not have a common color in the PFC are relatively prime.

M.8.1.1.3. Students will be able to determine whether two given whole numbers are prime or not.

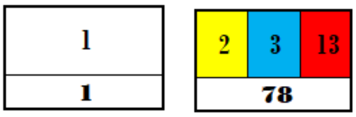
Examples:

- **Consecutive numbers are relatively prime.**



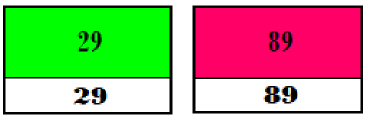
The numbers 38 and 39 do not have any common colors. No common color means no common factor, and these numbers are relatively prime.

- **Number 1 is relatively prime with all numbers.**



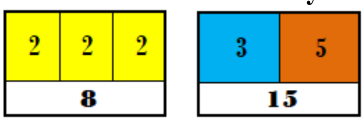
The white color belonging to the number 1 does not exist in any other number in the Chart. So, number 1 is relatively prime with all numbers.

- **Prime numbers are relatively prime.**



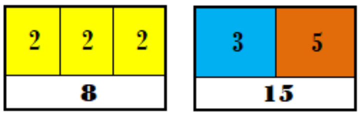
Since each prime number is represented by a different color, they cannot be expected to have common colors. This shows that all prime numbers are relatively prime.

- **The HCF of Relatively Prime Numbers is 1.**



The common colors of 8 and 15 expressed their HCFs. Numbers with no common color are relatively prime, so it is clear that these numbers have no common divisor other than 1.

- **The LCMs of Relatively Prime Numbers are found by multiplying these numbers with each other.**



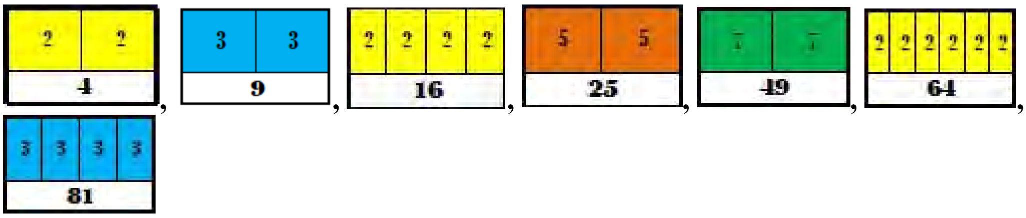
Since the numbers 8 and 15 have no common color, there will not be a common color to be discarded to find the least common multiples, and thus all the factors of the numbers for LCM will be multiplied with each other. Then, LCM is (8.15)= 120

Square Numbers: The Chart allows students to directly see square numbers from 1 to 100.

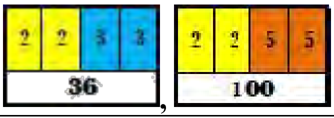
M.8.1.3.1. Students will be able to determine the relationship between square numbers and their square roots.

Example:

The same colors with even numbers



The different colors with even numbers

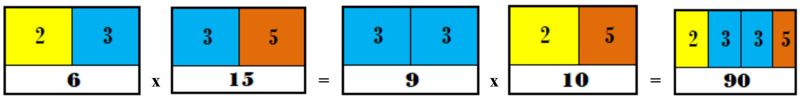


Conceptual Multiplication: While doing multiplication, students can change places between the multipliers to make the operation easier.

M.5.1.2.7. Students will be able to determine and use the appropriate strategy in mental multiplication and division with whole numbers.

Example:

For the 6x15 process, a student can follow a path as follows.



Simplification: It can be used to see common factors from the PFC while simplifying fractions.

M.5.1.3.4. Students will be able to understand that simplification and expansion will not change the value of the fraction and create fractions that are equivalent to a fraction.

Example: