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Mathematical Language of Students with Learning Disabilities in the Context of Length

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Mathematics is language dependent. Part of learning mathematics is learning the mathematical language. Learning disabilities are defined as disorders that are related to listening, thinking, speaking, writing, and reading that are seen in the processes of understanding or using a verbal or written language. In this context, the present study aimed to reveal the understanding and usage of the mathematical language of students with learning disabilities in the context of the concept of length. This study was conducted as a case study. The participants of the study were 4th grade and 5th grade students with learning disabilities. Data collection was conducted through semi-structured clinical interviews during a teaching experiment based on the concept of length and analysed using the content analysis method. According to the findings, the students with learning disabilities had a different and limited understanding, as well as usage of various mathematical terms including length, height, perimeter, half and centimetre.

Keywords: mathematical language, learning disabilities, length concept, mathematics education, special education

Introduction

Mathematics is known as a universal language and in practice, even though mathematics is believed to be independent of language, mathematics is very much language dependent (Kim, Ferrini-Mundy, & Sfard, 2012). Mathematics uses its own code and symbol systems that facilitate the transmission of ideas in a clear and precise way to formulate its own concepts and development (Sastre, D'Andrea, Villacampa, & Navarro, 2013). Learning mathematics means to learn the use of these systems, in other words, the language (Schleppegrell, 2007). Thus, the language affects learning mathematics (Kim, Ferrini-Mundy, & Sfard, 2012; Muzvehe & Capraro, 2012). As a matter of fact, Peng et al. (2020) found a moderate correlation between mathematics and language in their meta-analysis of 344 studies. Mathematical proficiency and language ability equally affect students' numerical abilities (Prince & Frith, 2020). As a matter of fact, even daily language skills affect mathematics learning. Abedi and Lord (2001) showed that students learning the language contained in mathematics tests were less successful than students fluent in that particular language. Similarly, a study by Seethaler, Fuchs, Star, and Bryant (2011) showed that students with stronger language skills benefited more from teaching and were more successful in problem-solving strategies than their peers with weaker language skills. Language processes have often been underestimated in teaching mathematics. However, if these processes

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are not organized carefully, various problems can arise (Morin & Franks, 2009). The predictive role of language acquisition in mathematics achievement in pre-school and primary school has been proven by recent studies (Purpura, Day, Napoli, & Hart, 2017; Vanluydt, Supply, Verschaffel, & Van Dooren, 2021). For example, the connections that students make between language and written symbols can differ from those made by adults (Muzvehe & Capraro, 2012). Vygotsky (1934/1986) stated that children used words in the same sense as adults could not be claimed (cited in Raiker, 2002). Shaftel, Belton-Kocher, Glasnapp, and Poggio (2006) supported this view, and according to their study conducted with 4th, 7th and 10th grade students they found that the 4th grade students were more affected by the complex language in verbal problems than the other students. They attributed this result to the less sophisticated verbal language skills of the 4th grade students. There is also powerful evidence that mathematical and language learning disorders are often (30-70%) seen together in individuals (Willcutt et al., 2013). Children with developmental language disorder underperformed consistently than their peers with typical development in arithmetic and story problem tasks. These children underperformed, especially in tasks with higher verbal demands (Cross, Joanisse, & Archibald, 2019). On the other side, students with learning disabilities have difficulty in distinguishing the sounds of spoken language and the three components of the language, namely form, content and usage (Schoenbrodt, Kumin, & Sloan 1997). One of the reasons students with learning disabilities might be having difficulty learning mathematics could be related to understanding and using mathematical language. Similarly, Butterworth, and Laurillard (2010) attributed the difficulties that students with learning disabilities experienced in learning arithmetic to their poor language skills. Additionally, Andersson (2010) found that reading and understanding language could affect improvement in arithmetic. The difficulties that students with learning disabilities experienced in problem solving were also attributed to the fact that problem solving requires language skills (Namkung & Peng, 2018, p. 38).

Miller and Mercer (1997) determined that language is a necessity in systematizing the use and recall of mathematical facts, rules and many steps in arithmetic and problem solving. Different learning domains of mathematics generally have their own vocabulary (Harmon, Hedrick, & Wood, 2005). For this reason, the literature indicates that the role of mathematical language in certain and different domains should be investigated (Purpura, Logan, Hassinger-Das, & Napoli, 2017). Besides, the studies conducted with students with learning disabilities in the literature recommend to concentrate on the learning measurement with students with learning disabilities to improve their language and communication competence (Cawley, Foley, & Hayes, 2009). The domain of measurement, that is the starting point of the development of geometry (Zacharos, 2006), forms the basis of quantitative reasoning, which includes relationships between ratio, proportion and variables, in addition to being significant for daily life skills. As the first concept encountered by students regarding measurement, the concept of length is very significant for students in terms of comprehending higher-level concepts such as area and volume and forming a basis for them. The measurement of length, which has a universal characteristic, is unique among spatial measurements (Van den Heuvel-Panhuizen & Elia, 2011). On the other hand, Smith and Heddens (1964) argue that mathematics is a special type of language with which spatial ideas are transmitted, mathematics is a visual language. In this case, the concept of length, which is at the center of spatial measurements (Smith et al., 2008), is one of the most suitable contexts for the examination of students' mathematical knowledge. Besides, when previous studies on learning disabilities are examined, this can be said that the existing studies focused on the cognitive functions and arithmetic skills of students with learning disabilities and that there are a limited number of studies on the other areas of mathematics. Particularly in the domain of measurement, there are few studies examining the developmental features of students with learning disabilities (Cawley, Foley, & Hayes, 2009). In the literature on learning disabilities, previous studies conducted in the domain of measurement were carried out only towards the concepts of perimeter and area. The aforementioned studies focused merely on calculation instead of the conceptual features of area and perimeter calculation. Therefore, this is thought that the present study, which examines the comprehension and use of the mathematical language by students with learning disabilities in the context of length, will provide a viewpoint regarding the students' perception of length. Accordingly, the present study aimed to reveal students with learning disabilities' understanding and usage of mathematical language in terms of the concept of length. The research problem of this study is how students with learning disabilities understand and use the mathematical terms of the concept of length.

Literature Review

Concept of Length

The distance between the starting and end points of a linear object, and the distance between the start and end points of a non-linear object when made linear is the length of that object (adapted from Argün, Arıkan, Bulut, & Halıcıoğlu, 2014, p. 543). Length is a comparable or quantifiable (measurable) feature that involves the volume of the object in the one-dimensional space between its start and end points (Szilagyi, Clements, & Sarama, 2013). Due to the fact that the concept of length is used in daily life with the same meaning, length is regarded to be more advantageous compared to the concepts of area and volume (Zacharos, 2006). Thus, there are studies in the literature that examine the representations and measurement units used by the public in daily life regarding the concept of length (e.g. Saraswathi, 1989).

Traditional measurement teaching aims to provide students with computational competence and teach them formulas for length, area and volume measurement (Nitabach & Lehrer, 1996). The latest curriculum (e.g., Van de Walle, Karp, & Bay-Williams, 2010, p. 369) suggests a grading system in which students compare lengths, make measurements with non-standard units, combine the use of manipulative standard units and make measurements with a ruler (Clements, 1999). Studies in the literature have started to emphasize the necessity of developing conceptual building blocks that channel meaningful estimation and measurement, and providing students with the conceptual insights underlying measurement instead of merely teaching them the use of rulers, standard units and the conversion of units (Stephan & Clements, 2003; Smith, van den Heuvel-Panhuizen, & Teppo, 2011). Instead of questions such as "How many sticks does the length of this pen correspond to?", indirect comparison activities such as "Is the door wide enough for the table to pass through?", which will develop students' mental features involving transitivity and accumulation of distances, should be implemented (Kamii, 2006). As argued in the literature, unit features should be verbally discussed in the classroom environment (Barrett & Clements, 2003). In this context, the ability of students to comprehend and use terms related to the concept of length comes into prominence. For example, in terms of length, students are expected to be familiarized with terms such as width, height, depth, perimeter, thickness, and distance as the various representations of the concept (Outhred, Mitchelmore, McPhail, & Gould, 2003). Students should be able to recognize these representations on objects and understand their use in length measurement. In a sense, this is possible by the correct comprehension and implementation of the terms related to length.

Mathematical Language

There are several difficulties in creating a detailed description and definition of mathematical language and its content (Morgan, 1998). For a certain period of time, mathematical language has been approached as a model that people have to adhere to and characteristic features of this language have been determined as syntax, semantics and vocabulary. Recently, not only the syntax, semantics and vocabulary of this language, but also important features of language related to its use have been taken into consideration by studies conducted on the language of mathematics (Morgan et al., 2005). The meanings, purposes and functions of the words, sentences and texts may change according to the situations and practices (Moschkovich, 2007; Rowland, 1995). Not only "words" but also the cultural practices that involve mathematics and the language used in communication are of importance (Sarama & Clements, 2009). Thus, mathematical language includes more than a customized vocabulary (Morgan, 2005) but also language systems including written and spoken (verbal) language, symbolic notations, visual representations and even gestures and mimics (Morgan et al., 2005).

Language is important in learning mathematics. For example, language is necessary for a systematic understanding of numbers (Wiese, 2003). Similarly, the development of spatial concepts is affected by language (Bowerman, 1996). In their study examining proportional reasoning skills, Dooren Vanluydt, Supply, Verschaffel, and Van Dooren (2021) demonstrated that the relevant mathematical vocabulary in the first year of primary school predicted the skills in the second year. Muzvehe and Capraro (2012) state that the language used by students, provides insight into their implicit perception of concepts. For example, spatial language is based on pre-developed spatial concepts (Bowerman, 1996; Regier & Carlson, 2002). Since there are many terms of which children in particular have limited understanding, paying attention to language is important. Language supports mathematical thinking and learning mathematics, however, language does not explain them on its own (Sarama & Clements, 2009). Nevertheless, the fact should not be forgotten that language can influence thought (Vygotsky, 1934/ 1986). Similarly, the early storytelling skills of pre-school children predict their mathematical skills after two years (O'Neill, Pearce, & Pick, 2004). Purpura, Day, Napoli, and Hart (2017) found that even in a preschool school year, poor performance in mathematical language assessment in autumn was an overwhelming predictor of low math skills in spring.

Learning Disabilities

In Individuals with Disabilities Education Act of 2004[1] learning disability is defined as:

> "a disorder that can manifest itself in an imperfect ability to listen, think, speak, read, write, spell, or to do mathematical calculations or in one or more of the basic psychological processes involved in understanding or using a spoken or written language."

In some studies, cases such as experiencing difficulties in certain learning areas compared to others despite appearing to be capable, variability in performance and a contradiction between success levels in different areas were used to describe students with learning disabilities (Lewis, 2014). American Psychiatric Association (APA, 2013) defines learning disabilities as the performance of a student "persistently and consistently" failing to reach the expected level. In the literature, learning disabilities in general are defined as biological, behavioral conditions that are yet to have a formal universal definition (Mazzocco, 2007). Learning disabilities were also defined as the demonstration of unexpected, typical learning failure with uncertain reasons (Fusch, Mock, Morgan, & Young, 2003). As stated, learning disabilities can affect academic fields that include reading, writing, areas of mathematics and language (Kavale & Forness, 2012). While the reasons behind learning disabilities are unknown, multiple potential causes are emphasized rather than a single cause (Namkung & Peng, 2018). Andersson and Östergren (2012) state that there can be no single central cause of learning disabilities and that multiple deficiencies can lead to learning disabilities. For example, learning disabilities may be accompanied by sensory impairment, mental retardation and social and emotional disturbances or outside factors such as cultural differences and inadequate or improper education, however, learning disabilities is not a direct cause of these factors (NJCLD, 1988, p. 67). Individuals with learning disabilities form a heterogeneous group and they may experience difficulties in reading, mathematics, language or oral language (Namkung & Peng, 2018; Pierangelo & Giuliani, 2006). Based on the interactions and observations with students with learning disabilities, misspelling or incorrect copying of numbers, difficulty with

[1]Individuals with Disabilities Education Improvement Act, 2004, Pub. L. No. 108-446, 118 Stat. 37. https://sites.ed.gov/idea/.

mathematical step sequences, difficulty in naming terms or operations, misinterpretation and incorrect conversion of mathematical contexts into mathematical symbols, incorrect use of arithmetical signs or numerical symbols and incorrect calculations can be listed as the observable features of the said students (Sullivan, 2005). As understood from the definitions and literature, although the main difficulties in mathematics for students having learning disabilities are in the forefront of number knowledge, counting, arithmetic operations and fluency in calculation, the link between mathematics development and language should not be forgotten. As seen in the literature that students with both reading and mathematics difficulties have more difficulty in mathematics than those with only mathematics difficulties (Andersson, 2010; Jordan & Hanich, 2000). Thomas, Van Garderen, Scheuermann, and Lee (2015) reported that students with learning disabilities may have difficulty in grasping the meaning of words, mathematical meanings of words, semantic structure of mathematical language, mathematical terms, and in particular terminology with multiple meanings. In this context, students with learning disabilities' understanding and usage of mathematical terms of the concept of length, is worth to be researched.

Turkish Education System and Learning Disability

The basic organization of the centrally managed Turkish education system is the Ministry of National Education (MNE) in Turkey. MNE is responsible for planning, implementing, controlling education and training services (Binbasioglu, 1995). At the age of 5-6, children start formal education. In 1983, "Children in Need of Special Education Law" entered into force, and in 2000 the Ministry of Education Special Education Services Regulations issued and learning disabilities have been described. For diagnosis, learning disabilities are generally tried to be determined based on intelligence tests such as the Wechsler Intelligence Scale for Children-Revised (WISC-R). In Turkey, a student with learning disability needs to go through medical examination and educational diagnosis stages in order to be diagnosed and to benefit from special education services. After the medical diagnosis, the student should be subjected to educational evaluation and diagnosis in the local Counselling and Research Centres (CRC) (Görgün, 2018, p. 63). At this point, the student's teacher or family should initiate this process.

Schools and institutions made mainstreaming practices for the education of students who need special education in Turkey are located. These schools and institutions create an Individualized Education Program Development Unit and develop and implement an individualized education program for students who need special education (MNE, 2006, art. 72). Therefore, the teaching to which individuals in need of special education will be subjected is prepared by the student's teachers through a unit established by the school. In inclusive classes in special education, the majority of the student population consists of students with learning disabilities (MNE, 2010).

Methodology

Research Design

This qualitative study that was part of a doctoral dissertation that aimed to construct the learning trajectories based on the concept of length of students with learning disabilities was conducted as a case study (Stake, 1995; Yin, 2013). The mathematical language of students having learning disabilities was examined without any intervention in the study (Stake, 1995; Yin, 2013). The case of the study was two Turkish students with learning disabilities, one of which was a 4th grade student and the other of which was a 5th grade student. The students with learning disabilities' understanding and usage of the mathematical language, was the unit of analysis.

Participants

The purposeful sampling strategies-criterion sampling and convenience sampling methods were conducted for determining participants (Patton, 2005). The inclusion criteria of the study were as follows: voluntary participation, students who were recognized by the local CRC as having learning disabilities, who do not have any other accompanying disabilities and who are in the 4th and 5th (MNE, 2013; 2018). Within this context, two students, one female and one male one, were selected. The names were given pseudonyms, Mert for the male and Eda for the female. Mert was 9 years old and a student in the 4th grade, while Eda was 10 years old and a student in the 5th grade.

Data Collection

The data was collected through semi-structured clinical interviews conducted twice a week over 32-36 individual teaching sessions during a teaching experiment based on the length concept performed over a period of four months. The data of the study were recorded in video and audio. Additionally, the field notes that are taken by the researcher and worksheets of students were utilized as the document data. The audio recordings were used to listen to the parts that could not be heard in the video recordings and to back up the data as a separate source.

Students' knowledge regarding the terms of mathematics, what they understood from these terms and how they used them were examined in the interviews. For this purpose, they were firstly asked to define the concept of length and the different terms or representations of length such as unit, meter, and centimetre. Then, activities that encouraged the students to use the words in context were carried out. These activities helped to reveal the students' understanding and usage of the words. As an example of the interview questions and activities, the students were asked "What do you understand from the word *width*?" Then, they were presented with various shapes and objects and asked to group the wide ones together and the narrow ones together. They were then asked to explain how they grouped the objects and on what they based their decisions on. The students were also made to watch a cartoon about width and narrowness and then discuss it. Then they were indirectly asked questions to make them explain the mathematical terms.

The fact that the students were active in the teaching experiment increased the communication based on the mathematics between the student and researcher and allowed many conversations in different contexts. The activities designed for teaching the length concept allowed the students' language use to be examined in depth and in terms of all the terms of the concept and its different contexts. However, in this study, while the students' understanding and usage of mathematical language was investigated without any intervention, their mathematical language improvement was not. In addition, the focus was on the students with learning disabilities' verbal language rather than written texts.

Data Analysis

Data analysis was conducted through the content analysis method. The video recordings were watched, transcribed verbatim and read several times. The documents of activities and the field notes were analysed simultaneously with the transcriptions for the support in identifying the patterns. The students' expressions were examined in depth, their understanding and usage of mathematical language in terms of length was coded and the patterns between the codes were investigated. Thus, the data triangulation provided the trustworthiness of the study (Patton, 2005). The codes were the meanings the students' attributed to certain terms. Thus, the contexts in which they used the mathematical terms and words were analysed, and in this way, their understanding and usage of the terms were uncovered. Data analysis was performed by microanalysis, in which each successive teaching session is analyzed separately for each student (Barrett et al., 2012). Firstly, student behaviors for each activity were described in detail. Patterns regarding the terms used by the students were revealed with a detailed analysis of these descriptions. The schemes including the students' mental relations network connecting concepts and processes were examined and their comprehension of the terms they used was revealed.

In the present study, an expert and a second coder were referred to for the reliability of the data analysis (Miles & Huberman, 1994). 10% of the data was presented to a research associate with a doctorate degree in mathematics education and coded. The second coder is a mathematics educator who specializes in students who need special education. The agreement between the researcher and the second coder was calculated with Kendall's coefficient of concordance. The coefficient of concordance was calculated as above 0.85 and a high agreement was found between the coders (Abdi, 2007).

Findings

The students' understanding and usage of the words and terms related to mathematical language were discussed within the context of the concept of length, different representations of length, direct and indirect comparisons, expressing measurement results and unit concept categories. The categories obtained and student language usage and understanding of each category are summarized in Table 1. In the ongoing headings, the findings in the table are detailed.

Table 1. The Findings Belonging to Students' Mathematical Language

Categories	**Students' Mathematical Language according to Categories**
Length	- Defining with *Length* (is one of the dimensions of a 3-dimensional object, while the others are width and height) (Both students) - Defining with being too long (Both students) - Aware of the situations in which the distance between certain points was the length of the object (Both students)
Different Representations of Length	- Using the concept of width instead of area or volume (Eda) - Defining width, perimeter and area in terms of each other (Mert) - Explaining thickness as "being rough, being puffy" (Mert)
Direct and Indirect Comparing	- Using the word flat to express linearity (Both students) - Describing comparing by using expressions such as "this is big" or "this is small" (Both students)
The Concept of Unit and Expressing Measurement Results	- Writing the term centimetre as "centi meter" (Eda) - Non-awareness about that centimetre is a name given to a certain length. (the length of 1 cm was "the length of a line" (Eda) - Thinking that centimetre is a name given to equal parts rather than the name of a standard unit rather than that centimetre is a name of a certain length (Eda) - Expressing a measurement result using only the number of units like "5" (Eda) - Expressing the measurement in the unit that was used to measure the object (for example, expressing "3 pencil" instead of "3 cm" (Mert)
Usage of Other Mathematical Terms	- Pronunciation the word "completely (tamamen in Turkish)" as "wholetely (tümamen in Turkish)" (Eda) - Referring to any part of an object as half- Non-awareness about the expressions of "half" and "one half" as two equal parts of an object (Eda)

Length Concept

When Eda and Mert were prompted to define the concept of length, their responses were simply "*Length*[2]". Therefore, the students defined the term length with the length attribute and focused only on the representation of length.

Researcher (R): What comes to mind when I say length?
Eda (E): *Length*
or
R: What is length?
M: *Length*.

Mert exemplified the term as "the *length* of a human being". The students' focus on only *length* representation of the term length may be related to the fact that they frequently encounter this usage of the term in their daily life and that the term *length* is often used instead of length in daily life (Güven & Argün, 2018). However, when students were asked to explain the concept once more, they

[2]*Length* is one of the dimensions of a 3-dimensional object, while the others are width and height.

focused on the root of words and associated nouns with adjectives in the context of length. For example, Eda thought that the term length (uzunluk in Turkish) described the situation of being long (uzun in Turkish). In Turkish, length is uzunluk, and long is uzun (See Table 2). And morphologically, uzun is the root of the word uzunluk:

R: So, what do you think length is?
E: Length is something that is too long.
R: Anything else?
E: That's it.
R: So, if an object or shape is too long, then we're talking "length"?
E: Yes.

Mert was asked by drawing a shorter line:

R: So, what do we call this?
M: That's shortness.

Therefore, the students described that length is as a term used to describe being long and shortness as a term to describe being short. As the students thought that length described the state of being long, they felt the need to call a shorter length as shortness. In this context, the students are thought to be associated the roots of the terms with their actual meanings. The students can be thought to overgeneralize this conception by associating width (genişlik in Turkish) as being wide (geniş in Turkish) and height (yükseklik in Turkish) as being high (yüksek in Turkish) because of the structure of the Turkish language. In Turkish, these terms have same roots and the spellings are the same. English and Turkish equivalents of the mathematical terms for the different representation of length are exemplified in Table 2. Table 2 can be expanded with other length attributes. In this context, the realizing that the concept of length is independent of being long or short, as rather than 'expressing' long or short, and that the length is a feature that can be quantified is important for students.

Table 2. English and Turkish Equivalents of the Mathematical Terms for the Different Representation of Length

Mathematical Terms for The Different Representation of Length	
In English	**In Turkish**
Length & long	Uzunluk & uzun
Height & high	Yükseklik & yüksek
Thickness & thick	Kalınlık & kalın
Depth & deep	Derinlik & derin
Width & wide	Genişlik & geniş

In terms of the concept of distance, Mert was aware of this concept. For example, when talking about a nonlinear object:

R: Yes. I'm not saying the length of this object is from here (starting point) to here (ending point). Because it's different.

M: It’s the distance.

Additionally, Mert was asked about the distance between two people on an activity sheet and correctly determined it. Eda said "length is actually distance" about the relationship between length and distance. Then, the following dialogue took place:

R: Can we say that the length of an object is the distance between the endpoints of it?
E: Yes.
R: So, what do you understand from the term distance?
E: The end point and the starting point. It’s something like, being far.

Eda tried to explain distance by emphasizing the state of two points being far away from each other.

R: […] For example, this pencil […] Between the endpoints?
E: Distance.
R: Can we call this distance as the length?
E: Yes.
R: But can we say that this is not linear? Can we say that the length of a nonlinear object is the distance between its endpoints?
E: No.
R: Why?
E: Because it’s non-linear. I mean we have not made the object linear yet.
R: You're right, we have not made the curve linear yet [a curve and a line aligned with the endpoints of the curve are drawn]. Look this (linear curve) started here and ended here. Can we say that this is the length [by showing the distance between them]?
E: No.
R: Why not?
E: Because we only did this from here, but the curve will be much longer when we make the object linear.

As can be seen, Eda was able to compare the length of the nonlinear object and the linear object in the light of her understanding that the length was a quantitative additive. She was aware of the situations in which the distance between certain points was the length of the object. She knows the difference between the distance between two points and the length of the object. Eda recognized when the distance between two points referred to length and when the distance did not. Therefore, as observed, her understanding and usage of the term distance was sufficient.

Different Representations of Length

The students’ understanding and usage of different representations of length such as height, width, thickness, thinness, narrowness and perimeter was investigated. Mert, the 4th grade student, was less aware about the meaning of these terms than Eda, the 5th grade student. Although Eda was aware that the other representations were related to length, she could not make sense of the term width. For example, Eda showed the width of the room by raising her hand and saying "This is its width", thus using the concept of width instead of area or volume. Mert, on the other hand, defined the concepts of width, perimeter and area in terms of each other.

R: What is width?
M: The area is wide and not narrow.
R: [A sheet is displayed] What is the width of this?
M: Perimeter [draws curves on paper]

When asked "Why the area?", Mert replied "Because the surrounding, area is the surrounding, environment" and when asked "What is perimeter?" he replied "Perimeter is the environment". Therefore, Mert defined the concepts of width, perimeter and area in terms of each other. As seen, Mert associated area and perimeter by using the concept of environment. However, Mert directly stated that width was an area saying "The width of the room is its area". He made definitions including "width is area", "area is environment" and "environment is perimeter". However, he did not use these terms in a mathematical sense. He explained perimeter as an area, region or "The things we see in nature". This might be because of the science lessons he receives at school or daily language. As a matter of fact, when Mert was asked "What is area?", he replied "The surrounding" or the next lesson:

R: What do you mean by region?
M: The sum of the distances we walk around.
R: Region is something else. You should not call region for the distances. What you mean is something different. [A planar region is drawn and smeared] This is a region. Is that what you mean by region?
M: No, the surrounding. Not the region, the surrounding.
R: For example, what do you mean by perimeter? Please show me on this. Which lengths do we add up?
M: The places we visited (Figure 1a).

Figure 1. Examining the Concept of Region with Mert

(a) (b) (c)

Source: Güven, 2018.

R: For example, I am an ant, I'm walking around (inside of the region). What do you understand when you say perimeter?
M: I understand the outside, I understand the point [points to the edges (Figure 1b)]

As seen in the dialogue given above, the student used the terms area and region as he constructed them in his own mind without being aware of their mathematical meanings. Although he expressed area as "the surrounding", he actually meant the length. Therefore, understanding that the same language is spoken with the student is important. According to Mert the concept of thickness was not a representation of length as he explained thickness as "being rough, being puffy". Mert was of the opinion that thickness was used for the thicker one, this is consistent with him associating length (uzunluk) with being long (uzun). Mert was influenced by daily language and the structure of words in understanding and using different representations of the length such as height, depth, and thickness.

Direct and Indirect Comparing

In order to examine the meaning and usage of the words, the students were asked to compare the lengths of various objects both directly and indirectly. The students should have used the word linear in these comparisons, however they used the word flat to express linearity. The term linear is more appropriate when describing length, whereas the term flat is used to characterize a surface. This usage of the students was thought to be due to the effect of daily usage and that the term flat is an alternative to the term linear in the Turkish language. As a matter of fact, when Mert and Eda used the term flat, they meant one dimension, not a surface. However, Eda described the term linear as being flat and often used the term with the phrase "going straight". For example, after the first sentence of the activity was read the following dialogue took place:

R: The table legs are linear. What do you think this means?
E: … means they're straight.

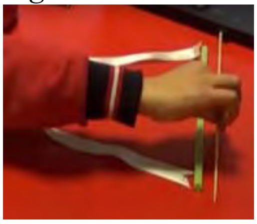
Therefore, Eda intuitively conceived the concept of linearity. Eda correctly selected the linear object in the images given in the activity. When asked to describe non-linear objects, she said "Curling like this" and showed the examples in Figure 2.

Figure 2. Examples of Non-Linear Objects by Eda

Source: Güven, 2018.

Mert used the term flat with a line: "Because as flat as a line shows this". When the students talked about the length of objects, they used expressions such as "this is big" or "this is small". For example, as seen in Figure 3, Mert took a stick and compared the length of the pipette and the ribbon with stick.

Figure 3. Mert's Comparison of the Lengths of the Ribbon and Pipette



Source: Güven, 2018.

> M: That's how we place thick and measure ribbon. That's how we measure how big this is.

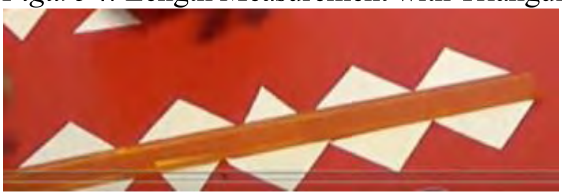
Similarly, Eda said "A ruler, then there was something else used to measure big things?" referring to the meter as a measuring instrument. Accordingly, students compared the objects in terms of their length by using the words big and small.

The Act of Measuring

The students' understanding and usage of measurement action terms was examined under the categories of measurement result, unit concept and other terms used.

The Concept of Unit and Expressing Measurement Results. For the concept of unit, Eda wrote the term centimetre as "centimeter". In this case, the probability of the inability to distinguish meters and centimetres may increase. According to Eda, the length of 1 cm was "the length of a line". The length of her forefinger was 5 cm and the length of her arm was 10 cm, thus, Eda could not construct a mental image regarding how long one centimetre was. This may be an example of how language affects understanding (Buss & Spencer, 2014; Sarama & Clements, 2009). However, when expressing the measurement results, Eda did not refer to the unit used (expressing a measurement result using only the number of units like "5"), while Mert expressed the measurement in the unit that was used to measure the object (for example, expressing "3 pencil" instead of "3 cm"). Another example is the dialogue that took place regarding Figure 4.

Figure 4. Length Measurement with Triangular Blocks Carried out by Mert



Source: Güven, 2018.

> R: What is the length of this orange bar?
> M: Each of them makes something ... one, two, three, four, five.
> R: Five what?
> M: Five centimetres

For example, Mert said "2 cm" for a length of 2 millimeters. Immediately after, he was asked to measure a length of 2 cm and again said 2 cm for its length. Unlike Mert, Eda did not use the term centimetre even in her measurements with a ruler were in centimetres. When Eda was asked to estimate a length of 1 or 2 cm, she said "1 line". In an equal partitioning activity, the name Eda assigned to each equal part was "1 cm". In the clinical interviews, Eda tended to express the measurement results only by the number of units instead of using the unit name. For example, she said "14" as a result of a measurement. When asked "What is 14?" She said "Well, 14… might mean… length" or "14 measurements". However, similar to Mert, Eda's centimetre schema did not contain the knowledge that centimetre was a name of a certain length and she did not have a mental image of how long a centimetre was. Eda had the knowledge that centimetre was a name given to equal parts rather than the name of a standard unit. This situation was thought to result from the students' understanding of a unit, lack of sufficient knowledge about the function of a unit and the centimetres in the measurement action. The students were not aware that centimetre was a name given to a certain length. Therefore, this can be said that students' understanding of the unit affected how they expressed measurement results and thus their usage of mathematical language.

Usage of Other Mathematical Terms

For the unit concept, Eda wrote the term centimetre as "centi meter". In this case the students' understanding and usage of various mathematical terms emerged while carrying out measurements and expressing the measurement results. For example, Eda's pronunciation of various words was different. For example, she pronounced the word "completely (tamamen in Turkish)" as "wholetely (tümamen in Turkish)". Additionally, the words half and one half are often used in the iteration of units, in expressing measurement results and equal partitioning activities. When Eda was asked to show half of a length, she showed a completely different length (Figure 5). This was first thought to be the result of Eda's conception of equal partitioning or division action. In other words, Eda was thought to have difficulty in partitioning a whole into two equal parts, or divide the number of units into two.

Figure 5. Eda Shows Half of the Bar

Source: Güven, 2018.

R: Where exactly is half of this bar?
E: A place here (Figure 5).

Soon after, Eda was asked to divide the same length into two, which she did so correctly. Therefore, after sufficient examination, this was decided that she did not perceive the expressions of "half" and "one half" as two equal parts of an object. Eda referred to any part of an object as half (because Eda's representations were not consistent in itself).

Results and Discussion

The concept of length is important as the terms of this concept are used in the same sense in daily life as they are in mathematics. Szilagyi, Clements, and Sarama (2013) drew attention to the use of language when learning about length. For example, they stated that the term long is used for linear objects, which limits the understanding of the term. Similarly, Güven and Argün (2018) emphasized that the language used in daily life is important for primary school students having learning disabilities in understanding the length. Feza-Piyose (2012) discussed students' mother tongue as a psychological tool that enriches the mathematics learning and is especially important in conceptualizing length. Similarly, D'Amore and Fandiño Pinilla (2006) state that the difficulties, experienced by students regarding the relationship between perimeter and area, are also based on the linguistic development of the subject.

In the present study, students having learning disabilities' understanding and usage of mathematical language in the context of the concept of length, which is an important concept for the daily life of the students, was investigated. As stated in the literature that students with learning disabilities have difficulties in learning and using mathematics (Andersson, 2010; Jordan & Hanich, 2000; Thomas, Van Garderen, Scheuermann, & Lee, 2015). Considering the difficulties students with learning disabilities also have in learning verbal and written language, one of the difficulties they may experience in learning and using mathematics could be due related to the mathematical language. This language is important for understanding the world, arranging thoughts and expressing oneself. The present study contributed to the literature by providing an understanding into the mathematical language of students with learning disabilities. Similarly, Sarama et al. (2011) stated that word development was important for students to construct a higher-level understanding of length. As seen in the present study, the students having learning disabilities may have difficulties in understanding and using mathematical language and that they either use their own terms like Eda, or use mathematical terms without awareness like Mert. Mert's understanding and usage of the terms region, area, wideness, Eda's usage of the expressions half and one half and the usage of centimetres by both students were different from what is known and these results were new for the literature. Additionally, the findings were consistent with the view of Vygotsky (1934/1986) who stated that children use words in the same sense as adults cannot be claimed (cited in Raiker, 2002).

Eda and Mert, both of them defined length using the *length* representation of the word. This is the same misconception regarding students having learning disabilities reported by the study of Güven and Argün (2018). This misconception may be caused by daily life and the usage of daily language. The fact that the concept of width is defined by the Turkish Language Association (TLA) as "the antonym of *length*, length" is the display of the interchangeable use of *length* and length. Considering the interchangeable use of the words "linear" and "flat" in Turkish, this can be said that these uses in daily life also support the idea of Sarama and Clements (2009) that some languages make mathematics easier while some make mathematics harder. As a matter of fact, misconceptions can arise from both the classroom and the physical and social world in which the learner interacts (Smith III, DiSessa, & Roschelle, 1994). According to Clements and Sarama (2014), some learning domains are more affected by cultural and social experiences. Therefore, one of these learning domains is thought to be length. When pre-school students aged 5 and 6 were asked to exemplify measuring with real-life photographs, the fact that the characteristic most commonly observed by the children was comparison was found (MacDonald, 2012). The language used in these actions will undoubtedly contain the terms "length, height, higher, wider, long, short". Considering that students with learning disabilities experience difficulties in verbal-linguistic perception, this is foreseeable that they can adopt various uses of these terms due to the fact that they interpret their experiences in school differently, and that these different uses can affect their daily lives negatively.

Mert was thought to be influenced by daily language and the structure of words in understanding and using different representations of length such as depth, height and thickness. The students may think that distance means to be far away, height means to be high and length means to be long. These ideas may arise from the fact that the students did not have sufficient knowledge of the concepts, but also from the fact that the mathematical language to which these terms belong leads them to think in this way. For the Turkish version of these terms, the students could have been affected by the roots of the words (e.g., yükseklik and yüksek). Similarly, as reported in other studies that students could be affected by daily language and mathematical language itself (Feza-Piyose, 2012; Sarama et al., 2011). However, there are no other comprehension examples in the literature where for example height was associated with being high were found except for students with learning disabilities (e.g., Güven & Argün, 2018). At the same time, as observed that Eda's pronunciation and her verbal comprehension was different. The students' understanding and usage of centimetres were also remarkable in terms of their differences. Therefore, whether these differences were due to the students' learning disabilities or not should be examined. Keijzer and Terwel (2004) observed that one student called Shirley, who had a low achievement level in math, called each part "quarter" or "piece" when naming fraction parts. This is similar to Eda's usage of the terms half and one half for all the parts in the activity of partitioning. Shirley was found to continuously use non-formal fraction names for a long time and this took a long time for her to use formal language (Keijzer & Terwel, 2004). Nonetheless, in the present study, Mert insisted throughout the teaching experiment to include the expression "centimetre" next to each measurement result. Hence, to change the schemes that the students constructed in their minds was difficult. If Shirley had no learning disabilities, when the findings of this study and the present study are examined, this can be said that students having learning disabilities and low level achievers could have similar characteristics in terms of language comprehension. In addition, although Mert and Eda had a different understanding and usage of centimetres, they were both different from the appropriate use, and the reason for this could be that both students were unaware that a centimetre is a quantitatively certain length. In this context, the students' understanding regarding the unit concept affected their language use. Consequently, as claimed in the literature, understanding affects language use as much as language use affects understanding (Buss & Spencer, 2014). For example, plural structures such as "pens" or quantifiers such as "some" and "all" establish a framework for the development of quantitative thoughts (Carey, 2004). Bowerman (1996) also states that the semantic organization of language influences the development of spatial concepts in students. On the other hand, students' uses regarding centimeter measurement may have multiple causes. For example, the usage can be interpreted as a vocabulary-related problem experienced by students in expressing measurement results. In other respects, the usage can be explained by the students returning to their previous schemes. Additionally, the reason may be that students have overgeneralized cm due to the fact that cm is the most emphasized unit in teaching the concept of length or the measurements made with different units or lengths are briefly mentioned and the measurements made with cm are more common. At the same time, after the action of equal-partitioning was emphasized, Eda called each equal part a cm measurement. Eda performed equal-partitioning using the iteration of units and length measurement. Therefore, she may have called these parts "1 cm" using the same scheme. Additionally, to consider cm as a name that should only be added to the measurement result may be wrong. That is because cm may be more than a name for students; cm can also be the name of each unit used. The fact that Eda called each equal part 1 cm in the equal-partitioning may be an indicator of this.

As a limitation of the study, identified students with learning disabilities' understanding and usage of mathematical language is specific for Turkish language. At the same time, the findings are limited to the activities chosen or designed by the researchers and the communication established. Considering that length is a concept that can be affected by cultural experiences, it can be thought that the results obtained in the study are also limited by the individual, environmental and socio-cultural factors of the participant students. In this context, the importance of these factors should be considered in utilizing the findings of the present study.

Conclusions

Effective communication is achieved through the consistent transmission of the message between the sender and the recipient (Schoenbrodt et al., 1997). As determined that the students were affected by the language used, being sure that the terms and the language have the same meaning for the students is important. In other words, in addition to making sure that students understand the language used, to understand the language used by them is necessary. In the study, the students with learning disabilities used words without being aware of the mathematical meanings and the differences in these usages were important as they indicated that the language used by the students should be understood and that the terms should not be left only in verbal-linguistic form. To accept that their world is different and to listen to them in different ways is necessary; for learning and understanding the thoughts of students (Greig, Taylor, & MacKay, 2007). Even though students learned the terminology, they were distant from the conceptual understanding of length. Students' knowledge of the terminology should not remain in verbal-linguistic form, but be internalized and moved to the upper form. In this case, to reveal what students understand conceptually from this terminology is important. The fact that the term of length has the same using in daily life may be an advantage of length according to area and volume characteristics (Zacharos, 2006). However, this should be remembered that there is a possibility that the term length can be misused in daily life or can be misunderstood by students with learning disabilities who have difficulty in verbal-linguistic processes. Additionally, as observed in the study, the students stated "This is big, this is small" when talking about the length of objects. Using only attribute-specific words, such as long, short, longer, shorter, in the case of length is important, because the words large and small can also be used for other attributes of objects. While this usage may be a sign that the student recognizes the quality; the usage by the teacher will help the student to recognize the quality and distinguish the quality from other qualities. Based on the mutual interaction between understanding and language, to better learn the concept of length, this is important for the students to realize that centimetre is a name given to a certain length. To realize that different length terms such as height, length and width are the measurable features of objects rather than expressing being long, shortness or another state is also important for the students. Short objects should be included as a length example during the first introduction of the concept. In addition, to make sure that even a simple word such as "half" or "completely" is understood correctly by the students with learning disabilities during the interactions is worth to remember. Additionally, as observed in the definition of length by TLA (2011) as the longest edge of the object, students may associate the concept with being long due to its usage in daily life and word stem (TLA, 2011). Similarly, as stated by Szilagyi, Clements, & Sarama (2013) that the perception of the word "long" is limited as the word is used to describe linear objects. Therefore, to give examples of length that also include short objects in the initial introduction of the concept of length may be helpful. According to Barrett and Clements (2003), measurement takes its meaning from the real objects comparison. Students' schematics of measuring length become more sophisticated when comparisons are based on real life situations (Barrett & Clements, 2003). In this context, to be combined with real-life situations and discussed clearly in order to enable students with learning disabilities to structure them correctly may be beneficial for mathematical terms. Only after understanding the concept of length can students justify its representations as units of length, and correctly structure and utilize the terms related to length.

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