"UDL is the What, Design Thinking is the How:" Designing for Differentiation in Mathematics

Rachel Lambert
University of California Santa Barbara
Kara Imm
City University of New York
Rachel Schuck
University of California Santa Barbara
Sunghee Choi
University of California Santa Barbara
Avery McNiff
University of California Santa Barbara

Received: 6th February, 2021/ Accepted: 28th April, 2021

In order to transform mathematics classrooms for students with disabilities, teachers must design based on learners at the margins. Universal Design for Learning (UDL) is a framework built on these principles (Meyer et al., 2014), but lacks a robust process through which educators can reimagine mathematics. We theorize that when educators merge the principles of Universal Design for Learning in mathematics with the process of Design Thinking, more accessible designs for curriculum, routines, tools, and spaces would result. Using design research methodology, we analysed a 6-week summer course for mathematics educators on UDL and Design Thinking. Data sources included class video recordings and transcripts, class artefacts, as well as pre- and post-participant surveys. Findings indicate shifts in participants' knowledge of UDL from a passive to an active role, empathy as a central guiding principle, and the potential of using Design Thinking as a process to enact UDL.

Keywords · Differentiation · Disability · Universal Design for Learning · Mathematics · Design Thinking

Introduction

Differentiation for all learners necessitates a radical rethinking of mathematics teaching and learning. Gervasoni and Lindenskov (2011) have argued for the creation of a category of students with special rights in mathematics for traditionally underserved populations in mathematics, including students with disabilities and those from marginalized populations. Creating equity in mathematics requires radical shifts in the way we teach so that students with special rights in mathematics are given access to meaningful mathematics (Gervasoni & Lindenskov, 2011). This argument has been echoed by others in mathematics education, including an argument to rehumanise mathematics for marginalized students and those with disabilities (Tan et al., 2020; Lambert et al., 2018). Students with disabilities are offered a more procedural mathematics with less access to conceptual instruction (Jackson & Neel, 2006; Kurz et al., 2014). Classroom practices must shift from those that serve only some students to those that better serve the natural variety of knowing and doing mathematics across students. This requires shifts in teacher identity and classroom practice (Bobis et al, 2020).

Universal Design for Learning (UDL) is a framework built on the idea of planning based on learners at the margins (Rose & Meyer, 2005), rather than around "typical" students (Meyer et al., 2014). UDL allows educators to design mathematics classrooms that include all students in meaningful, inquiry-based mathematics (Lambert, 2020). However, to truly apply the principles of UDL to classrooms and schools, educators need a specific, flexible, and replicable process that will allow them to redesign instruction in a variety of ways for a variety of users. Conformity with a checklist is not enough, as students with disabilities face problems not just in mathematics curriculum, but also in the systems, routines, and practices that create the conditions for unequal access to meaningful mathematics. Even when curriculum is designed with a UDL lens, classroom teachers will need to engage in additional cycles of testing and redesign to make sure it is "working" in their contexts with their users.

This project studied a virtual course in the summer of 2020 for 45 mathematics educators in the United States. We designed a course to respond to the heightened needs felt by educators in the summer of 2020, as they paused from a spring of distance learning during a crisis and braced themselves for the uncertainty of the academic year ahead. Our course was created to help educators plan for their virtual work in the fall, both as classroom teachers and as professional development providers. We focused on the integration of Design Thinking (IDEO 2012) with UDL. Design Thinking provides a collaborative process that helps teachers address persistent challenges within their practice by beginning with empathy for users, then carefully defining the problem and generating a diverse set of innovative solutions. Our theory of action was that when educators merge the principles of Universal Design for Learning in mathematics with the process of Design Thinking, more accessible designs for mathematics curriculum, routines, tools, and spaces would result. We were guided by the following research questions.

Before and after participation in a virtual course on Designing Mathematics for Inclusion, how do participants conceptualise:

1. Universal Design for Learning,
2. Design Thinking,
3. The intersection between the two?

Conceptual Framework

In the sections that follow, we offer our conceptual framework which expands our theory of action, while integrating relevant, current literature in each of the domains.

Disability and Mathematics

We conceptualize disability through a social model of disability (Linton, 1998). Instead of focusing on individual deficits, we draw attention to how social contexts like classrooms construct both ability and disability. This dynamic exists in mathematics classrooms; in one study, we found that how students with disabilities were framed as able or disabled in mathematics shifted as the pedagogy of the classroom shifted (Lambert, 2015). At the same time, disability is also seen by self-advocates as a positive identity and a community. This positive reframing of physical, emotional, and cognitive differences is necessary in schools. A final important concept in disability is learner variability (Rose, 2015), which emphasizes the natural variability in human cognition, emotion, and bodies. In short, we attempt to honour disability as a political and social identity while also adapting environments to be more accessible.

Students with disabilities have less access to challenging and meaningful mathematics and are more likely to be given mathematics instruction that is below grade level and focused on memorization (Jackson & Neel, 2006; Kurz et al., 2014). In previous work, this problem has been located in ideological and methodological differences between research in mathematics education and special education (Lambert & Tan, 2020). Inclusion of students with disabilities in meaningful mathematics is impacted by deficit thinking by educators, particularly myths about whether inquiry mathematics is suitable for students with disabilities (Lambert, 2018). In order to include students with disabilities in challenging, engaging mathematics, teachers must grapple with deficit conceptions of students with disabilities.

Students with disabilities are also parts of complex systems such as special education which may contribute to this lack of access. Students with disabilities may also be taken out of classrooms during inquiry work to receive more procedural individualized instruction. These larger issues remind us that we need to consider this problem not only in the classroom, but also in teacher education, professional development, and larger systems such as special education.

Differentiation in Mathematics

Returning our focus to mathematics classrooms, the problem of how to include a heterogeneous group of students in collective learning in mathematics is a long-standing issue in education (Scherer & Krauthausen, 2010). Perhaps the most dominant strategy in the United States has been differentiation. Differentiation tends to refer to work in which the content, process, and product of instructional tasks are modified in such a way as to better address the range of learners (Tomlinson, 1999). Misconceptions tend to cloud teachers' understanding of differentiation. These include the belief that differentiation requires individualization for every student and, therefore, is too time-consuming (Dack, 2019).

A different conception of differentiation has emerged within mathematics education. Natural differentiation (Scherer & Krauthausen, 2010) refers to the inherent complexity within high-quality mathematical tasks, which create conditions for students to make choices about how they engage in the problems. Instead of providing multiple worksheets or creating ability groups, for example, all students receive the same learning offer. A teacher presents a task that offers the possibility of multiple questions to be explored and multiple solutions to be generated by students (Sullivan et al., 2020). Enabling and extending prompts can serve to promote participation in open-ended mathematics tasks. Enabling prompts support students experiencing challenges without the use of explicit direction, while extending prompts provide extension tasks for students who finish early (Sullivan et al., 2006b). As such, this "low floor, high ceiling" design can make mathematics more accessible to students with a wider range of prior knowledge. Sullivan and colleagues have also studied how to work individually with students doing such problems. When teaching with challenging tasks such as these, teachers also need to be prepared to engage in questioning that will support and extend student thinking (Sullivan et al., 2006). Teachers can determine these prompts while planning for differentiation and make these questions available to all students during the lesson. This teaching strategy gives students choice, which promotes a sense of autonomy and empowerment. It may also limit teachers' use of explicit guidance (Russo et al., 2020). This notion of a common mathematical offering as a form of differentiation stands in contrast to the typical use of the term: Instead of creating different learning offers for students at different levels, the learning offer is the same, and students enact the differentiation through their own choices.

However, teachers may believe that this type of challenging task is not appropriate for students with low prior achievement in mathematics (Russo et al., 2020). This general reluctance to provide challenging problems to students with disabilities can become more pronounced as teachers follow guidance, from special education research, that evidence-based practice for mathematics instruction should only be explicit or direct instruction. While there is a significant research base on direct instruction with students with disabilities, there is a much smaller set of research on including students with disabilities in inquiry mathematics (Lambert & Tan, 2020).

We have argued that this imbalance has affected access for students with disabilities to meaningful, challenging, standards-based mathematics, as some educators falsely assume that inquiry mathematics is somehow inappropriate for students with disabilities (Lambert, 2018).

Universal Design for Learning

Universal Design for Learning (UDL) is an approach to understanding classrooms and pedagogy grounded in the learning sciences and neuroscience (Meyer et al., 2014). UDL emerged from Universal Design, a movement in architecture and product design that sought to find elegant and effective ways to maximize the use of buildings and products. Rather than focus on the deficits of individual students, UDL refocuses on designing educational spaces and curriculum to include a wide range of learners from the outset. Instead of locating disability within individual students, disability is located in inaccessible classrooms, curriculum, and spaces. This is a radical reframing of disability from a medical to a social model. UDL is a useful and radical framework because of this shift in where disability is located—the potential of UDL rests in its power to redesign classrooms, curriculum, and systems to work better for students with disabilities.

Mathematics education has long been focused on engaging in challenging problems, modelling real and realistic situations mathematically, and justifying and defending one's thinking to others (e.g., Schoenfeld, 1988). This focus emerges from research in the learning sciences, which have documented the importance of engagement in these practices (National Academies of Sciences, 2018). UDL is similarly influenced by research in the learning sciences, and thus focuses on developing students to be life-long expert strategic thinkers (Meyer et al., 2014). As a result of this goal, UDL has transformative power within mathematics education to shift the field away from the procedural instruction more commonly used with students with disabilities towards instruction that engages all learners in mathematics that lives beyond the classroom.

The developers of UDL also drew on neuroscience to identify three networks that factor significantly in learning: the affective, representation, and strategic networks of the brain (Meyer et al., 2014). Attending to the critical role that our affective networks, or socioemotional processes, play in learning helps teachers ensure that mathematics class is a purposeful, welcoming space (Bobis et al., 2011). Mathematics learning is about developing understanding of representations and the connections between them, making representation a central concern in mathematics (Lambert, 2020). Strategic networks relate to the role of student choice in how they solve mathematical problems and is also critical to developing their understanding (Scherer & Krauthausen, 2010). Students also need development in strategic planning, important in developing the executive functioning and self-regulation skills necessary for success in mathematics (Lambert, 2020). These three networks work together in the complex process of learning in content areas such as mathematics. Part of our project is the creation of UDL Math Design Elements, which connect the UDL Guidelines regarding the three networks (CAST, 2018) to research specific to mathematics (Figure 1) (Lambert, 2021).

Despite its radical promise, there are critical issues in UDL implementation. First, some practitioners mistakenly believe UDL is a framework only for students with disabilities and/or within special education, even though UDL applies to all learners. Perhaps as a response, work on UDL tends to reference "all learners" and may not even mention students with disabilities, thus sometimes erasing disability as a central concern (Dolmage, 2017). A second issue is that UDL is not tied to any particular content area and instead presents general guidelines that need to be applied across content areas. This can make it challenging, for example, for teachers to see how to apply the guidelines to mathematics. The final issue is the focus of this paper: the lack of a design process in UDL. Despite the inclusion of the word "design" in the title, UDL lacks a replicable but flexible approach. Smith and colleagues (2019), identify the tension between seeing

UDL as passively following the UDL guidelines versus seeing UDL as a design process: "UDL is not simply a *listing* (emphasis added) of various flexible options and strategies; rather, it is a *process* (emphasis added) of designing intentionally to reduce cultural, cognitive, behavioural, and physical barriers" (Smith et al., 2019, p. 177). Moore (2017) similarly notes that there are two general ways of approaching UDL: first seeing it as passive alignment with the guidelines and, second, as a design process. Smith and colleagues also noted a need to understand UDL beyond the design of curriculum, to incorporate/redesign/consider systems and policies that are barriers for students at the margins.

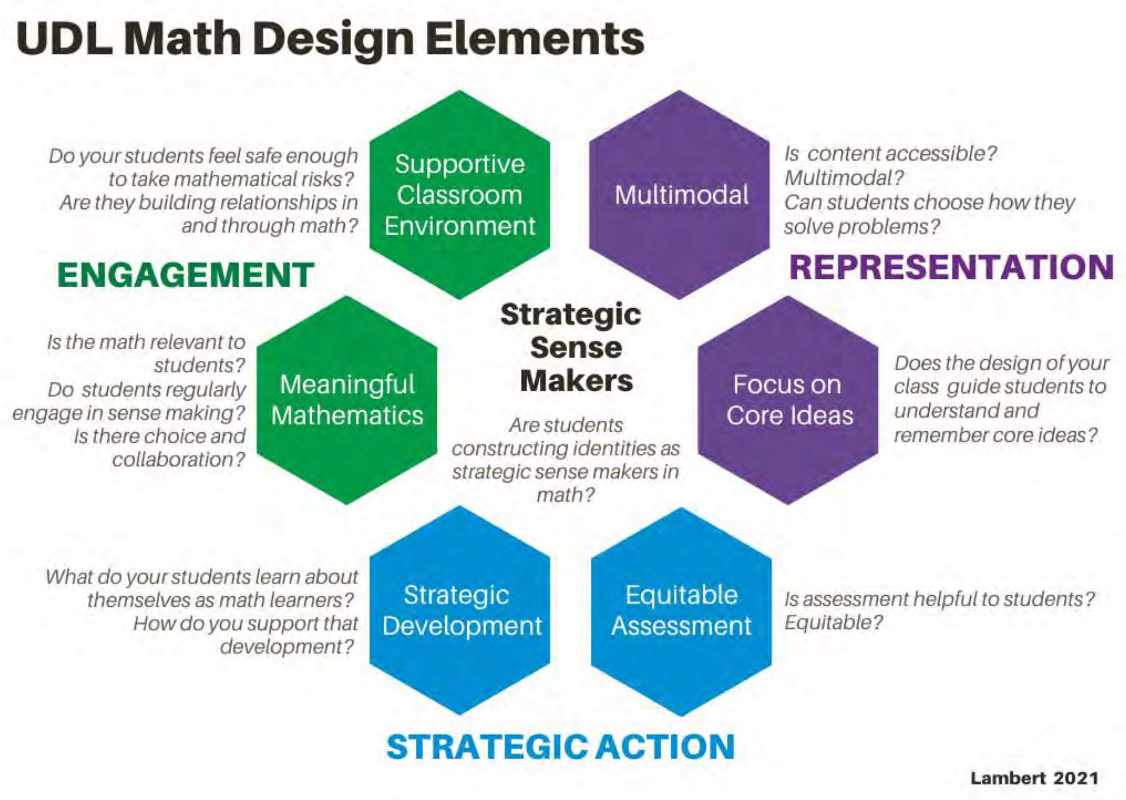

Figure 1. UDL math design elements.

While UDL emerged from Universal Design—a design process intended to create accessible architecture and products—it does not offer a specific process for teachers who want to use UDL to create curriculum, tools, spaces or systems. Some professional texts address this issue by proposing specific processes related to lesson planning (e.g., Ralabate, 2016) or curricular design (Basham et al., 2016). Edyburn (2010) notes that UDL must be recognized as a design process, but specifically associates the role of the designer with those who do instructional design, rather than the teacher. Across our work, we see teachers as designers and attempt to position and support them as such. However, this focus on a design process is not widely found in research on UDL and mathematics. In many cases, the only identifiable process is to look for alignment between lesson planning/instruction and the UDL guidelines (e.g., Courey et al., 2013; Owiny et al., 2019).

We believe that this process of alignment will not produce the radical redesigns possible with UDL.

One reason we do not see alignment with the guidelines as sufficient is learner variability. Disability is complex, multifaceted, and differs in context. What teachers might consider the "needs" of students with disabilities and students' actual "needs" (according to students themselves) are likely very different (Naraian, 2019). Experiences of those with disabilities are varied, and teachers cannot assume that what works for one student will work for all. Similarly, there are no set of UDL guidelines that allow teachers to understand the needs of students who are Black, Indigenous and People of Colour (Indar, 2018). These students are also students with special rights in mathematics (Gervasoni & Lindenskov, 2011), who have been systematically excluded from high quality instruction. Using UDL as a framework, teachers will need to develop empathy and understanding of the experiences of students at the margins of their classrooms through engagement in a design process.

This problem also connects to a problem of scale: how large or generalizable is the item being redesigned. For example, texts like Ralabate (2016) focus on the redesign of lesson plans. However, the disabling aspects of school go well beyond the unit of lesson plans. The UDL framework does as well. For example, the emphasis on engagement and motivation suggest the need to make larger changes in schools than in lesson plans. "Lesson plan" is too small a design space. It doesn't take into consideration the effect of sequences of experiences or systems or patterns that exist within schools, such as segregated classrooms for students in special education. These are "wicked problems" (Buchanan, 1992), which is where Design Thinking comes in.

Integrating Design Thinking

Design Thinking, as a concept, was first noted in 1987 in the work Design Thinking (Rowe, 1987) and originated in architecture, design, and art. Given its popularity and accessibility, it has since expanded well past its original scope to include the realm of education for both teachers and students alike. When used in educational settings Design Thinking mirrors the original process but with two subtle differences: the users tend to be students (not customers or clients) and the design space tends to include experiences, routines, tools and systems that support the larger goals of learning, participation and community (not products or services). As such, Design Thinking among teachers is seen as a collaborative, human-centred problem-solving approach, popularized by institutions such as the Stanford University Design School (or "d.School") and design firms such as IDEO. Design Thinking has positioned both teachers and students to see themselves as designers. While several variants exist, Waloszek (2012) suggests that the main stages of the process are the same: understanding the problem; observing users; interpreting the results; generating ideas (ideating); developing prototypes; and testing and improving the design. These stages are flexible and iterative — not a "predefined series of orderly steps" but rather a "system of spaces" (Brown, 2008, p. 88) where designers might re-enter the process to gather more information or generate more ideas.

Educators often face problems of practice that are complex and varied, requiring flexible, interdisciplinary, and creative strategies. These kinds of "messy" problems — where no clear "right" or "wrong" solution exists — are well-suited to Design Thinking as a problem-solving tool (City et al., 2009). While the general premise of "teacher as designer" is well accepted, the applicability of Design Thinking in education is still developing. Cross (2001) argues that the Design Thinking approach may be difficult for teachers to make sense of on their own. Koehler and Mishra (2005) also note that teachers must experience designing as a learner in order for them to consider themselves designers. This conception of "teacher as designer" may challenge more traditionally held views of "teacher as doer" — that is, the design aspect of teachers' work is distinct from the implementation aspect. Designing requires the active construction of new ideas, while implementing often assumes that these ideas already exist (Carlgreen, 1999). In fact, argues Kirschener (2015), teachers are both designers and implementers.

Research on the impact of Design Thinking with in-service teachers is emerging. Henricksen, Gretter and Richardson's (2018) study followed 22 K-12 teachers in a semester-long course which documented how Design Thinking impacted both their thinking and their problem of practice. They found gains in three major ideas: valuing empathy, becoming open to uncertainty, and seeing teaching as design. The practice of empathy was found to be not the idea of caring for students, but rather the idea of understanding students more authentically, including seeing educational experiences from the students' perspectives. The teachers in the course began to shift their self-identities from "doers" to "designers."

Design Thinking became a central feature of our course for several reasons. We believed, based on research, theory, and experience, that it could help teachers:

- respond to the sudden and startling uncertainty that a global pandemic engendered
- identify the "solvable" problems within their purview and generate real solutions in the form of prototypes
- centre their teaching on students who were most excluded from high-level learning in mathematics

Our theory of action assumed that Design Thinking would offer participants a specific and replicable process to be used again and again within their local context. By experiencing Design Thinking for themselves—within collaborative teams, grounded in problems of practice of their own making, supported by our guidance—we believed that teachers might take up a "teacher as designer" identity. Finally, the historical moment of our course suggested the urgency for a problem-solving approach for teachers that was innovative and forward-looking. While other methods might have focused on "what is," we trusted that Design Thinking would support participants to envision "what might be" (Owen, 2005).

Methods

Design research studies have been described as iterative, process-focused, utility-oriented, and theory-driven (Cobb et al., 2003). The iterative aspect of design research manifests as a series of design-analysis-redesign cycles that the researcher typically undergoes. Design studies are considered process-focused in that they trace, for example, a teacher's understanding of a concept across time. They are utility-oriented in their attempt to engineer and improve outcomes, for example, the effectiveness of instruction. Finally, design research is theory-driven: hypothesizing, testing, and ideally advancing theory about instructional design. Given our desire to study the development of teachers' understanding of UDL and Design Thinking within the context of mathematics, a design research study provided the best method to investigate questions while generating some emergent theories as well.

Within mathematics education, teaching experiments within design research provide opportunities for researchers to experience students' mathematical learning and reasoning firsthand (Steffe & Thompson, 2000). Teaching experiments are poised to "develop theories, not merely to empirically tune 'what works'" (Cobb et al., 2003, p. 9). These theories are typically not "grand" in nature but do contribute to domain-specific ideas about learning, knowing and understanding. In this study we wanted to test our theory of action—that when teachers are taught the tools of Design Thinking, with the theoretical framing of UDL, they may be empowered to design innovative curriculum, experiences, spaces, tools and systems. Our summer course was therefore a teaching experiment in which we tested this theory of action, using cycles of design-analysis-redesign, which are a hallmark of design research. Our collective expertise and experience in mathematics education, disability studies, and Design Thinking was a strength of this design, allowing us to craft a theory of action from which to simultaneously build, modify, and study a professional development course for teachers.

Teaching Experiment

Our teaching experiment was a 6-week course for educators in the state of California in summer 2020. The class was designed and facilitated by the first author, a researcher and professional developer in UDL and mathematics, the second author, a researcher and professional developer in mathematics and Design Thinking, and an additional facilitator who was an expert in designing professional development in mathematics and UDL. Course outcomes, related to both UDL and Design Thinking, for participants were:

1. We see ourselves as designers.
2. We build a design community.
3. We take up design thinking as a problem-solving practice.
4. We listen to users and design for them.
5. We break stuff and have fun.
6. We rethink our ideas about disability.
7. We appreciate, celebrate and make use of learner variability.
8. We understand Universal Design for Learning as a design process.
9. We develop new understandings about how UDL applies to math teaching and learning.
10. We create equitable, inclusive mathematics experiences for children, families and teachers.

The course was designed to be highly interactive, with an emphasis on collaborative work (Table 1). Participants used Zoom, Google Docs, and Mural, a collaborative whiteboard space. Activities included short video lectures on UDL, access and empathy, and learner variability. We also included activities to introduce participants to the disability rights movement.

Throughout the course, participants collaborated in design work. In session 1, for example, participants engaged in a one-hour rapid design challenge, re-imagining and prototyping a "work from home" environment for their colleagues. Next, participants conducted empathy interviews, first with colleagues, then identifying real-world users to interview. By session 3, all participants were assigned to a Design Team based on who they hoped to design for (users) and what they hoped to design (curriculum, spaces, interactions, or systems). Having experienced the entire Design Thinking process (through simulation and other activities), teams used the remaining sessions to work through the process at their own pace. Each team created their own How Might We question to investigate that was connected to the problems of practice they were facing in terms of teaching mathematics through distance learning. Each team was supported by one of the course facilitators who provided feedback and guidance along the way: helping to develop a problem of practice, analysing empathy maps, or joining them for "wild brainstorming." In the final session, each Design Team presented their prototype for feedback from other teams. Prototypes included redesigns of classroom curriculum, engagement with families, and professional development for teachers.

Consistent with design research, our team of facilitators met frequently, before and after each class to debrief and plan. Since participants initially reported deeper knowledge of UDL compared to Design Thinking, we prioritized Design Thinking in our lesson plans.

Table 1: Description of class sessions

Class Session	Topics	Homework due this session
1	Introduction to UDL Rapid Design Challenge	
2	Rethinking Empathy and Accessibility in Mathematics Empathy Interviews	Reading: articles on UDL and Mathematics, disability and mathematics. Writing: Core beliefs about teaching and learning mathematics
3	Learner Variability in Mathematics *Defining the Problem*	Watching or Listening: Disability Rights podcast or documentary Writing: What is not working in your practice right now?
4	UDL Book Groups *Analyse Empathy Interviews* *Developing How Might We Questions*	Reading: Choice between four different books on UDL and/or disability rights in education *Conduct empathy interviews with your users*
5	Designing from the Edges in Mathematics *Developing Prototypes*	*Design Team meetings with facilitators* *Wild Brainstorming*
6	*Sharing Prototypes*	*Final Prototype*

Note. Items in italics were done with the Design Team.

Participants

Participants were recruited through a state-wide mathematics education organization. An email with the opportunity was sent to all members of the organization. As most members of this community were out-of-classroom educators working in county or district offices, we invited members of the collaborative to bring classroom teachers with them to the class. We also invited classroom teachers from another research project on UDL and mathematics to join. We did not exclude any participants.

We asked participants (N=45) to self-identify in terms of gender, race, and disability status. We use participants' own identifiers as much as possible in this demographic data. The majority identified as women (35), three identified as men, and seven did not state their gender. Participants indicated their race as White (n=17), Latinx (n=8), Asian-American (n=7), African-American (n=1), Native American (n=1), Middle Eastern (n=1), and one participant described themselves as "mixed." Nine did not state their race. Four participants identified as disabled.

As a whole, our participants were a highly experienced group of educators. Ninety-three percent of participants had more than five years of teaching experience in K-12 special education and/or mathematics classrooms, and 75% had more than ten years of teaching experience. Twenty-three were currently employed as administrators or in curriculum positions at the county or district level, six were currently working as mathematics coaches, and 16 were classroom teachers. Twelve of the 45 participants had a special education teaching credential.

Data Collection

Course development data

The course facilitators met frequently to design, teach, study, and refine the course. Data on our course development include field notes, transcripts of video-recorded meetings, and emails. Other sources of data on our course were video recordings of class sessions, including Design Team meetings, and class artefacts such as participants' reflective writing, in-class activities, and evolution of prototypes. We also asked participants for feedback after each session, which became part of the data.

Course survey

Participants filled out surveys on Qualtrics before and after participating in the professional development course. Forty-four participants completed the pre-course survey, and 33 completed the post-course survey. At both timepoints, participants were asked to rate their familiarity with central course ideas on the following scale: none at all, slightly, somewhat, moderately, extremely. They were also asked to answer open-ended questions regarding their knowledge of key topics. In the post-course questionnaire, participants were also asked to rate how much they felt they had experienced growth (none, some, or definite) in the course outcomes. See Appendix A for a list of survey questions.

Data analysis

Our analysis of Design Teams took the shape of individual case studies of each Design Team. After collecting data on each team (e.g., researcher field notes, transcripts of Design Team meetings, artefacts of their planning process, and final prototypes), we compiled an analytic memo for each team focused on their process (Maxwell, 2013). This analysis of the design process is not a central concern of this paper, which is focused on participant conceptions of UDL and Design Thinking. However, we use the example case study of one representative Design Team to assist the reader in better understanding the course.

Participant response patterns to quantitative survey questions are displayed visually. Qualitative analysis of textual survey questions included both data-driven and theory-driven codes. We conducted open and axial coding of transcripts, with themes emerging from the data (Corbin & Strauss, 1990). We also created a priori, theory-driven codes related to the constructs of UDL and Design Thinking (DeCuir-Gunby et al., 2011). One member of the team read through all responses, extracting emergent themes and categories from each excerpt (defined as the response to a single question by one participant). We then used axial coding to combine codes into broader categories, re-coding all excerpts with the new categories. We discussed codes in multiple team meetings, coming to consensus on all analyses. We conducted focus groups approximately 6 months after the end of the course as a member check.

Findings

Overview of Findings

We begin by presenting an analysis of the process of one Design Team through the course. This analysis, in the form of a case study, provides a specific case of the trends that appear in both the qualitative and quantitative data across all study participants and provides additional context to better understand our course. Next, we address our research questions regarding shifts in how participants conceptualized UDL and Design Thinking before and after our course using pre- and post-survey data. Finally, we explore how participants understood the relationship between UDL and Design Thinking, with a specific discussion on the role of empathy across these processes.

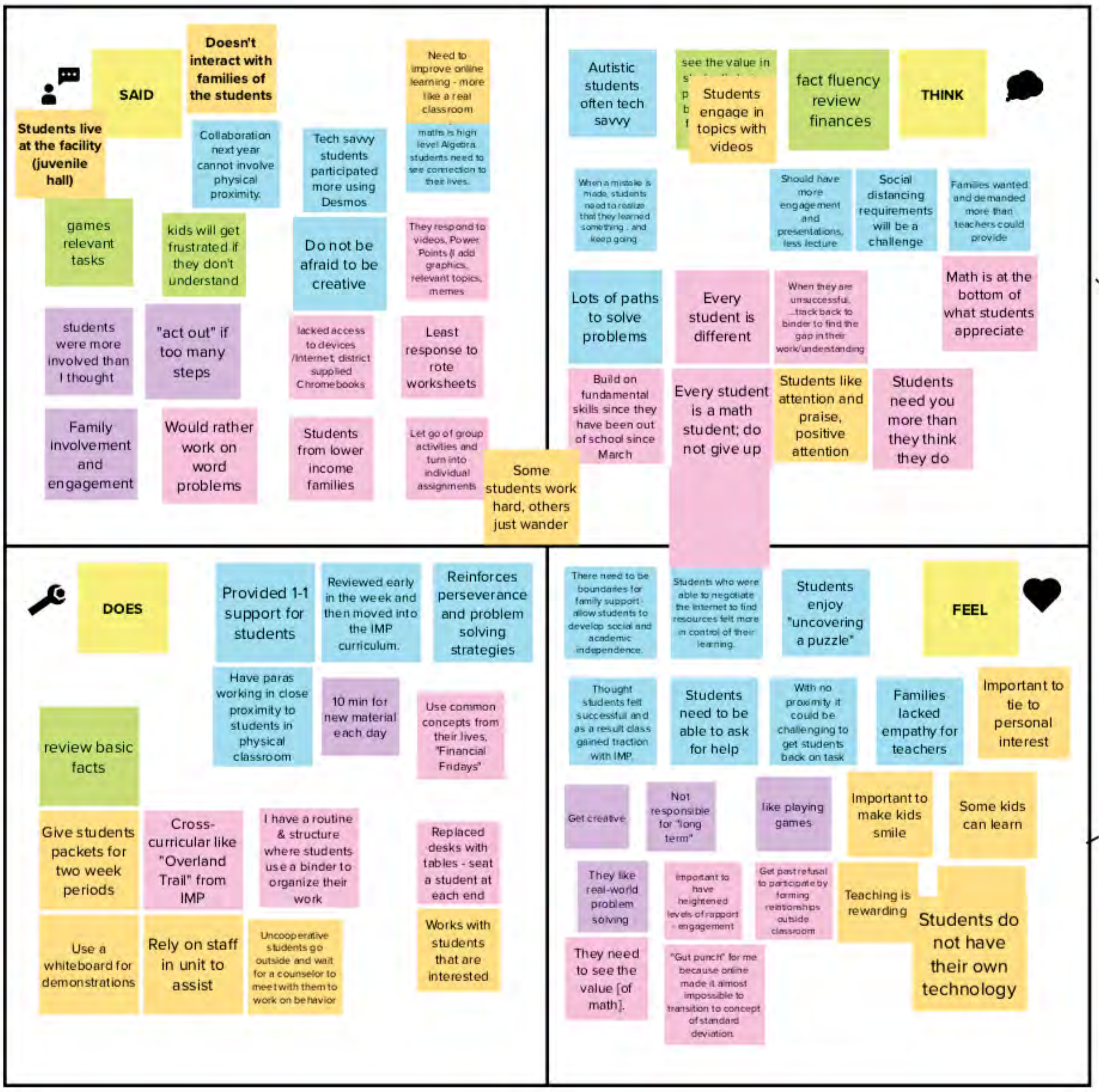

Figure 2. Design Team 1 Analysis of empathy interviews with special education mathematics teachers after spring 2020.

Case Study of a Design Team

Our case study focuses on a single design team, Design Team 1 (DT1). We chose this team because their process involved themes that were consistent across design teams, making them a representative case. The team consisted of four experienced educators: three professional developers and one high school classroom special education teacher who taught mathematics. Their shared goal was designing professional development for special education teachers to prepare them for teaching mathematics through distance learning in the fall, as all participants were engaged in district level professional development, including the classroom teacher. As part of their design process, members of DT1 conducted empathy interviews with five special education teachers to learn more about the experience of teaching mathematics online during spring of 2020.

The interview notes were codified into an empathy map — a Design Thinking tool that is used to capture salient thoughts and feelings of the interviewee, as well as analysis of body language during the interview (Figure 2). Each colour in the map indicated a different teacher that was interviewed. In their discussion of the interviews, members of DT1 focused on how the interviewed teachers expressed a need to get "back to basics." This was upsetting to these participants, whose work had long focused on helping teachers shift towards the use of challenging problems with low floors and high ceilings. From the perspective of the DT1, distance learning was moving these special education teachers to a more procedural mathematics — exactly the wrong direction. Having completed and analysed several empathy interviews, the team began to define the problem and wrote the following problem statements:

- Teachers feel like students are more engaged when they have videos, games, are problem solving and are solving real world, relevant problems.
- Teachers feel like they need to spend the majority of their time with fact fluency.
- Teachers want to change activities to individual problem-solving instead of group problem-solving because of distance learning.
- More difficult to re-engage students with distance learning (paraeducators not in close proximity).

Consistent with the Design Thinking process, they then used these problem statements to construct a How Might We question (their design question): How might we help teachers understand "what mathematics is"? Our initial observations of DT1 suggested that they shared the idea that "fixing" these teachers, or finding a way to prove to them that they were wrong to move away from inquiry-based mathematics, was the goal. Such a goal, however, led them away from the core principles of Design Thinking, in which the users' experiences and ideas supersede the designers'.

A course facilitator invited the team to explain the connections between the empathy interviews and their How Might We question. After explaining how their How Might We question came from users, through understanding their experiences with empathy, DT1 decided to change their question. They identified comments from the special education teachers that indicated their desire to give challenging problems, coupled with concerns about how to do so online. The team decided that the teachers did not know how to differentiate using challenging tasks in distance learning and needed to experience solving tasks online. As a result of this subtle intervention and subsequent discussion by the team, the How Might We question shifted from How might we help teachers understand "what mathematics is"? to How might we build opportunities for teachers to engage students in problem solving in the math classroom?

The team's next task was to ideate (or "wild brainstorm") possible solutions to their question. As their conversation developed, DT1 discussed how the special education teachers they interviewed had little or no experience doing challenging problems virtually, and so would, of course, have difficulty imagining this shift. This recognition of teachers' experiences was another important shift away from a deficit conception of these special educators towards one grounded in an empathetic listening to their experiences. As they selected ideas from their brainstorm to prototype, the team decided to create teams of special education teachers, paraprofessionals, and students who would collaboratively tackle challenging problems virtually through a virtual mathematics competition. Their design demonstrates how closely connected their prototype was to their empathy maps, as they included the empathy map in their final image (Figure 3).

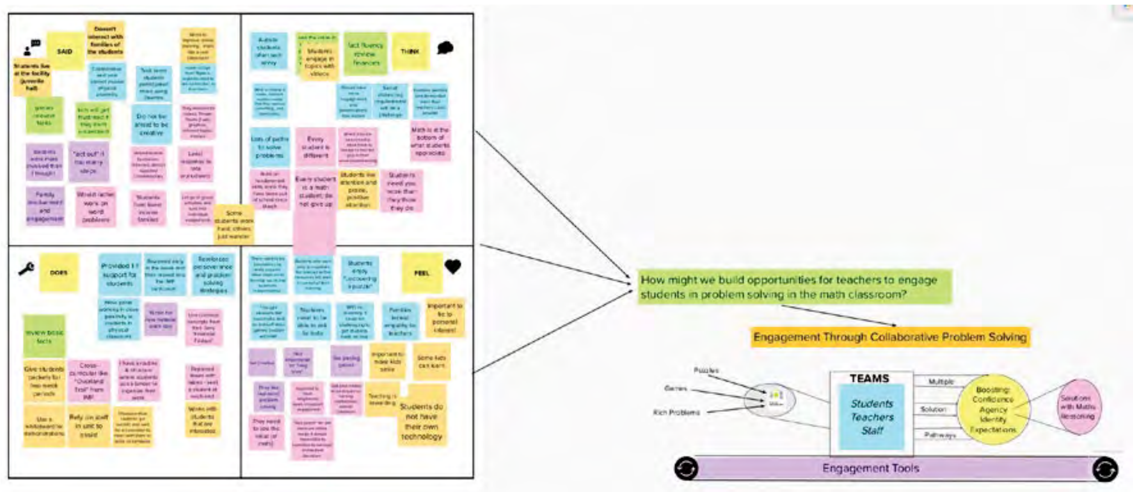

Figure 3. Design Team 1 prototype

We saw a shift in this team from seeing teachers as broken, so much so that they felt they needed to teach the teachers what mathematics actually was, to a perspective on teachers that acknowledged their complexity as mathematics teachers. The difficulty doing challenging problems online was reconceptualised from a lack of understanding to a lack of experience. Their solution was designed around the area of engagement in UDL, noting that collaborative mathematical problem-solving would uniquely create the conditions for more challenging problem solving by creating confidence, increasing mathematical agency (Boaler & Sengupta-Irving 2016).

The innovation that the teachers, paraprofessionals, and students would tackle problems together is an important shift, which could create new conditions for mathematical teaching and learning for both students and educators.

Overall Findings on Course Outcomes

Many of the trends that appeared in the analysis of DT1 were found in analysis of the survey data across all participants. Participants reported growth on all ten course outcomes (Table 2), with the majority of participants indicating "definite growth" on most outcomes. Less than 50% of participants indicated "definite growth" in just three course outcomes (outcomes #5, 8, and 9). Course outcome survey results are discussed throughout the remainder of the findings section where relevant to qualitative findings.

Table 2: Number of participants indicating growth in course outcomes

Course Outcome	No Growth	Some Growth	Definite Growth
1. We see ourselves as designers.	1	8	24
2. We build a design community.	0	11	22
3. We take up design thinking as a problem-solving practice.	0	8	25
4. We listen to users and design for them.	0	7	26
5. We break stuff and have fun.	3	20	10
6. We rethink our ideas about disability.	0	6	27
7. We appreciate, celebrate, and make use of learner variability.	0	8	25
8. We understand Universal Design for Learning as a Design Process.	0	19	14
9. We develop new understandings about how UDL applies to math teaching and learning.	1	19	13
10. We create equitable, inclusive experiences for children, families, and teachers.	1	16	16

Shifts in Conceptions of UDL

Few participants came into the course with no knowledge of UDL (Figure 4), with most indicating they were somewhat (29.5%) or moderately (34.1%) familiar with UDL. Post-course responses demonstrated a shift towards familiarity.

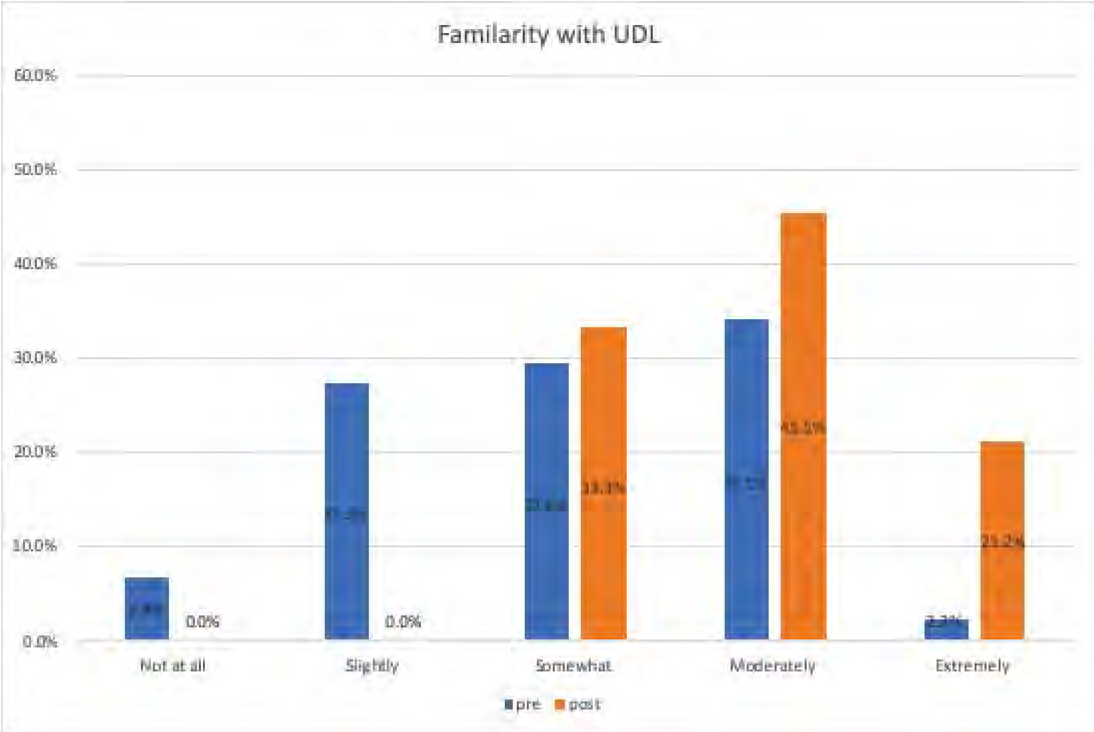

Figure 4. Familiarity with UDL at pre- and post-surveys

We also analysed textual responses pre- and post-course for the question: What is Universal Design for Learning? for the 33 participants who completed both timepoints. Prior to participating in the course, many respondents defined UDL as a framework or approach to lesson planning and teaching. Descriptions of UDL related to the idea of framework included: approach, principles, guidelines, and structure. One participant provided the following definition: "UDL is a framework for planning & delivering instruction that intends to teach to ALL students." Defining UDL as a framework alone suggested that participants may have interpreted the practice of UDL as a passive practice or system to follow precisely, rather than as a process in which mathematics educators actively engage. In the post-survey, participants used the term "framework" less frequently. In the pre-survey, 20/33 responses were coded as passive and 9/33 as active (4 did not provide enough data to code in this area). Following the course, 9/33 were coded as passive and 24/33 as active. In passive responses, participants seemed to perceive the UDL framework itself (not the teacher) to be the driving force behind creating an equitable and accessible classroom environment. In the active definitions, educators applied UDL and often used verbs to describe the term. For example, a participant defined UDL as "Designing for the jagged profile of each learner; designing learning opportunities that remove potential barriers for learners." The following pre- and post-responses to the question What is UDL? further illustrate this shift in conceptualization:

- Pre-survey: Universal Design in Learning is an approach to teaching where lessons are presented in a way to teach all learners.
- Post-survey: Designing classrooms and curriculum to reach all students by designing around student's needs.

Some participants (7/33) framed UDL passively in both pre and post-surveys. The majority of these participants described UDL as a framework, approach, or set of guidelines.

Shifts in Conceptions of Design Thinking

Compared to UDL, participants came into the course with limited knowledge of Design Thinking (Figure 5), with the majority indicating that they were not familiar (36.4%) or only slightly familiar (32.8%) with it. After the course, over 70% of participants indicated moderate or higher familiarity. Most of the participants who completed the post-survey indicated significant growth in their understanding of themselves as a designer (Table 2).

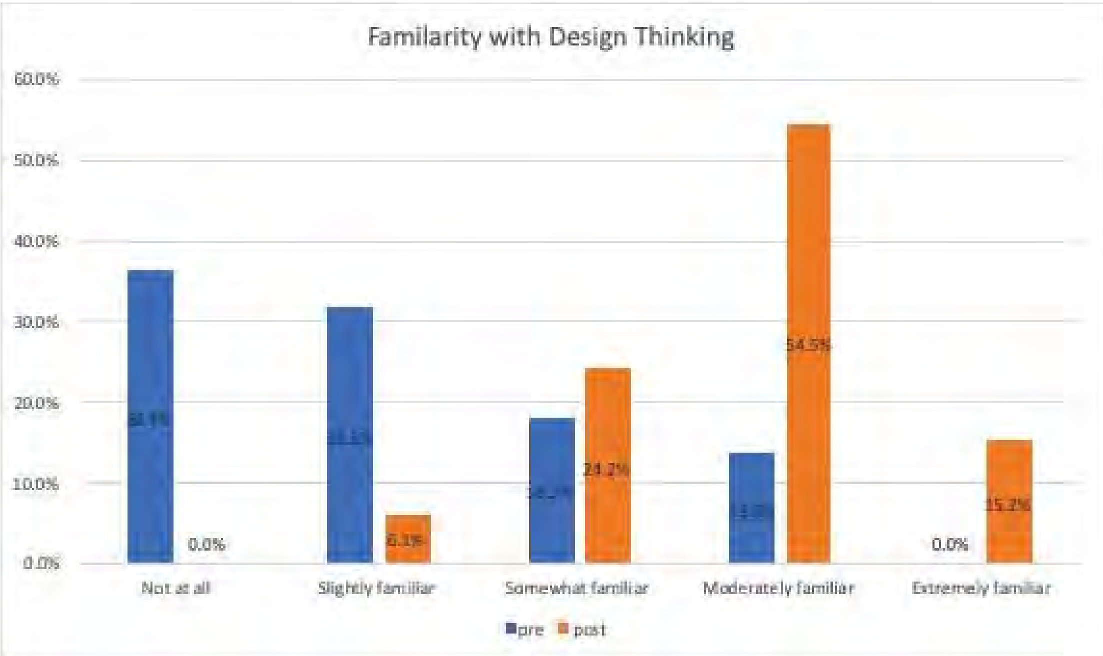

Figure 5. Familiarity with Design Thinking at pre- and post-survey

We also analysed responses to the question, What is Design Thinking? What is its purpose? in pre- and post-course surveys. Initially, teachers' conceptions of Design Thinking varied. Those who reported no or limited knowledge of Design Thinking tended to describe it in general terms such as "looking at problems from many different ways" or "designing many things with the end user in mind." These teachers tended to note some kind of creative process that was focused on a specified user, sometimes an "individual" and other times an "audience." For those who reported more prior knowledge of Design Thinking, they referred specifically to some part of the process, including but not limited to developing empathy, designing prototypes or gathering feedback.

Data from the post-survey suggests some conceptual shifts. Most teachers used the phrase "user" or "user-centred" or even "human-centred" and proceeded to explain how designs were focused on such users' needs. Several named all five stages of the process: "empathize, define, ideate, prototype, test" with empathy being the aspect that appeared most frequently. Teachers also noted that the design process was not "a one-time act" but rather "iterative," "cyclical," "continual," or that it "repeats."

Absent from the initial data was any reference to Design Thinking as a collaborative endeavor. In the post-survey, however, there was some mention of this important facet, which emerged in the following ways:

- change in pronouns from singular to collective (from "as a designer, I…" to "we are designing for…")
- explicit mention of design teams

Defining the Relationship between UDL and Design Thinking.

While growth in understanding of UDL and Design Thinking are positive outcomes, our ultimate theory of action was to connect the two disciplines so that UDL might be used as a design process. In this section, we discuss shifts in participant conceptualizations of the intersections between UDL and Design Thinking. Our analysis revealed that, in fact, many (though not all) participants saw the connections and had clear ideas on how to use them in their future work.

In response to the post-survey question, "What connections do you see between UDL and Design Thinking?" almost half of participants explicitly mentioned empathy and putting users at the centre. A representative response included, "The fact that both are centred on the unique needs of the user; the ways in which both draw on understanding the user first and last." Responses mentioning student-centred or user-centred methods often overlapped with characteristics of an empathetic approach. As one participant explained, "I now see over and over again people designing learning experiences from their perspective without being able to see how the user might experience it. I can't unlearn starting with the user and what they need."

When asked, "How might this course impact your work as an educator?", the most pronounced set of impacts were connected to empathy (17/33). Participants noted how focusing on the user, beginning with empathy interviews, could impact their practice: "I think I will be a better listener-and have more curiosity about my student's ways of knowing." Participants also linked empathy to Design Thinking in the school context:

> It has helped in giving me the lens of starting with empathy. Of course all educators empathize with their students, but I loved the idea of really sitting with students and problem solving based on their individual perspective instead of just my own and patterns in data. It is going to give my work more heart.

Participants' recognition of the importance of empathy within Design Thinking and UDL also emerged in responses to the question regarding the course outcome "We listen to users and design" (Table 2).

In addition to an increased recognition of the centrality of empathy in both Design Thinking and UDL, participants began to theorize the relationship between Design Thinking and UDL. Specifically, 7/33 responses described Design Thinking as the how behind the what of UDL. To illustrate, one participant wrote, "UDL is both the Why and some How, where DT is a How for me and for systems." Others made connections between the Design Thinking process and the application of UDL, such as: "UDL is designing from the margins. Design thinking is a process that we can use to design from the margins if we are intentional about the process."

This finding was further bolstered by responses to other questions, such as What is Design Thinking? In response, a few teachers explicitly articulated the relationship between Design Thinking and UDL by suggesting that Design Thinking "allows UDL to concretely be done." That is, Design Thinking is how we achieve the goals of the UDL framework. Said differently by another participant, it is a "process that we can apply to instructional or other design work we do to put the user at the center, think broadly and flexibly, then collaborate about possible solutions to design challenges we identify."

Some participants stated that they would use UDL and Design Thinking in tandem. They also seemed to express a greater appreciation for the connections between UDL and Design Thinking. Learning about Design Thinking may have enhanced or revitalized participants' understandings of UDL and its application. One participant stated,

> Without a doubt this course has both refreshed where I was in my UDL learning and deepened both my understanding about the why and how - with Design Thinking - to make UDL a practice that is the system-wide expectation in mathematics and other classes.

Other participants focused more on how the design process would impact their future work: "I am excited to use the design process to examine my classroom (virtual and IRL [in real life]) and create new systems that are more inclusive, intentional, and beautiful for students in my school."

Although less than 50% of participants indicated definite growth in course outcome "We understand Universal Design for Learning as a Design Process," we did see evidence in participants' open-ended responses that interweaving UDL and Design Thinking gave educators a process that they could use to challenge broken educational systems and curriculum. When asked about the relationship between UDL and Design Thinking, one participant noted, "They go hand in hand, you cannot have one without the other. To design for universal learning is to think outside the box and challenge traditional education methods." Participants noted that in order to solve intractable educational problems, you needed to radically rethink them. Design Thinking provided a means for doing so. One participant stated,

> Design thinking allows for people to look at the underlying causes of a problem and then brainstorm ways to overcome the issues. In the classroom, the issues are generally around increasing learner outcomes. By looking deeply at the problem, teachers can begin to design multiple paths for all students to accomplish the learning goals, so that each student can make the journey and feel accomplished.

Participants also shared that they would be more reflective in their planning and teaching. A participant wrote, "It's making me more thoughtful, I'm questioning practices, not that they are wrong, just reflecting, seeing things with new eyes."

We also saw some evidence that the combination of practices was particularly useful in "uncharted times", as one participant reflected on how UDL and Design Thinking could be used in tandem to redesign lessons based on empathy interviews:

> While planning a lesson, simply asking students to reflect on their experience in the classroom (empathy interview) and then thinking outside the box on how to resolve some sticky points would help us design lessons to better meet students' needs, especially in this uncharted time where we don't really know what does/doesn't work for students.

Again, we saw that participants began to conceptualize the relationship between Design Thinking and UDL as sharing a focus on empathy for the users.

Discussion

Differentiation can offer meaningful inquiry-based mathematics instruction for students at the margins. In order to implement natural differentiation within mathematics education, teachers need the knowledge of and a process for designing accessible and engaging mathematical curriculum, routines, tools and spaces. Our theory of action was that by bringing together the principles of Universal Design for Learning in mathematics and the process of Design Thinking, participants would gain the conceptual understanding and experience with UDL and Design

Thinking that would in turn allow them to design high-quality mathematics instruction for students with disabilities.

Across our data, we saw evidence that participants deepened their understanding of both UDL and Design Thinking and began to see how they might interact to enact change. While these findings mattered particularly in the immediate context of a global pandemic, we argue our findings have power post-pandemic as well. The exclusion of students with disabilities within mathematics education existed prior to the pandemic, and any return to "normal" will likely only exacerbate this problem. Students with disabilities and those with special rights in mathematics have long been shortchanged in their access to challenging mathematics (Gervasoni & Lindenskov, 2011; Lambert, 2018; Jackson & Neel, 2006). In order to shift traditional practices, we need to empower educators to challenge unjust systems and to redesign them to better serve students at the margins. That Design Thinking gives UDL a process not only for redesigning curriculum and spaces but also for systems is perhaps its greatest potential.

In this paper, we detailed growth in participant knowledge of both UDL and Design Thinking. Perhaps more importantly, we noted a shift in how participants understood the relationship between them. As noted earlier, participants' perceptions of UDL shifted from a passive (something received) to an active (a way to make change or solve problems) process. Perceptions of Design Thinking became more collaborative, with a more developed understanding of the process of Design Thinking. Recall also that one of our course goals was to enable mathematics educators to see themselves as designers, as has been documented in other research on Design Thinking (Henricksen et al., 2018). We posit that these shifts in discourse do suggest a shift towards seeing oneself as a designer, as suggested by the 24 participants who noted "Definite Growth" in the related course outcome (We see ourselves as designers).

Developing understanding of empathy as central to Design Thinking in education has been successful in previous research on Design Thinking (Henricksen et al., 2018). In our study, participants not only came to understand empathy as an important part of the design process, but as integral to UDL as well. While empathy has previously not been a core feature of UDL, we emphasized it, drawing from UDL's origins in Universal Design, which -- like Design Thinking -- is grounded in an empathetic stance. Our study suggests, however, that empathy is a natural and powerful aspect of the intersection between UDL and Design Thinking. We theorize that these strong feelings about empathy come from participants' empathy interviews with users. As we saw in the case study of Design Team 1, it is easy to shift back to one's own perspective. Being accountable to the actual expressed needs of the users is a powerful way to make mathematics educators accountable to the children they serve. Designing mathematics classrooms with and for students at the margins could be a transformative way to exact meaningful change. We hope to continue to study how the idea of an empathetic stance might transform mathematics teaching and learning.

We also saw evidence that a significant number of participants constructed an understanding of Design Thinking as the how for UDL. There has been critique of how UDL is presented to teachers as passive alignment with a set of guidelines (Moore, 2017) -- a passive way of engaging the teacher. Others have argued for the centring of a design process in UDL (e.g. Smith et al., 2019); however, such processes thus far have been focused on instructional design. Our vision was slightly different: that is, in order to design from the margins, we need to design more than curriculum. Further, all design needs to begin in users' experiences, not in standards. The course we designed, and the corresponding design experiment, was a manifestation of this vision—a chance to test these ideas with educators to advance this emerging theory.

In the case study of Design Team 1, we see the importance of both empathy and the design process. The deficit framings of special education teachers as needing to be fixed were shifted within the design team process. We saw that when a facilitator held participants accountable to explain the connections between their empathy interviews and their design question, the participants revised their design question to respond to users' actual words and experiences.

Overall, the results of our study suggest that this professional development can provide teachers with a process to: enact UDL in the classroom, design from the margins, and see themselves as designers. These ideas felt particularly timely and useful during the uncertain times of the COVID-19 pandemic.

Limitations

We acknowledge two notable limitations of the current study. First, while we demonstrated shifts in teacher knowledge related to UDL, Design Thinking, and the relationship between them, we were not able to study how these shifts impacted practice as K-12 classroom teachers and district leaders. Understanding the way participants made use of these new understandings is of critical interest, but remained outside the possibilities for this current study. Second, for the intersection of Design Thinking and UDL to have real power, we would need to demonstrate the effects of the prototypes designed in our course. Our intense six-session summer course did not allow participants enough time to test and refine their prototypes for their users. Understanding the effects of these designs on users is vital, but was not part of the current iteration of our research. These prototypes have the potential to create greater access for students with disabilities, their teachers, and their families, and we look forward to studying the impact of such prototypes in future studies.

Implications

We end with a series of questions to guide future work. If educators took up the notion of teacher as designer, and saw themselves as designers of things beyond curriculum, how might they rethink and re-imagine new spaces, experiences, structures, relationships, and systems for students with disabilities? If teachers began to see the power of empathy, not simply as caring for students but also as deeply understanding their experiences, how might they dismantle pervasive deficit models in lasting ways? Finally, if educators were reminded of the joyful and creative power they have, as many in our course expressed, how might they unleash this power to differentiate for a wider range of students?

References

Basham, J. D., Smith, S. J., & Satter, A. L. (2016). Universal design for learning: Scanning for alignment in K-12 blended and fully online learning materials. *Journal of Special Education Technology, 31*(3), 147–155. https://doi.org/10.1177/0162643416660836

Boaler, J, & Sengupta-Irving, T. (2016). The many colors of algebra: The impact of equity focused teaching upon student learning and engagement. *The Journal of Mathematical Behavior, 41*, 179–90. https://doi.org/10.1016/j.jmathb.2015.10.007.

Bobis, J., Anderson, J., Martin, A. J., & Way, J. (2011). A model for mathematics instruction to enhance student motivation and engagement. In D. J. Brahier (Ed.), *Motivation and disposition: Pathways to learning mathematics* (pp. 31-42).National Council of Teachers of Mathematics.

Bobis, J., Khosronejad, M., Way, J., & Anderson, J. (2020). "Sage on the stage" or "meddler in the middle": Shifting mathematics teachers' identities to support student engagement. *Journal of Mathematics Teacher Education, 23*(6), 615–632.

Brown, T. (2008). Design thinking. *Harvard Business Review, 86*, 84-92.

CAST (2018). *Universal design for learning guidelines version 2.2* [graphic organizer]. Author.

City, E. A., Elmore, R. F., Fiarman, S. E., & Teitel, L. (2009). *Instructional rounds in education: A network approach to improving teaching and learning*. Harvard Education Press.

Cobb, P., Confrey, J., DiSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher, 32*(1), 9–13.

Corbin, J. M., & Strauss, A. (1990). Grounded theory research: Procedures, canons, and evaluative criteria. *Qualitative Sociology, 13*(1), 3–21. https://doi.org/10.1007/BF00988593

Courey, S. J., Tappe, P., Siker, J., & LePage, P. (2013). Improved lesson planning with Universal Design for Learning (UDL). *Teacher Education and Special Education, 36*(1), 7–27. https://doi.org/10.1177/0888406412446178

Cross, N. (2006). *Designerly ways of knowing*. Springer-Verlag London.

Dack, H. (2019). Understanding teacher candidate misconceptions and concerns about differentiated instruction. *The Teacher Educator, 54*(1), 22–45. https://doi.org/10.1080/08878730.2018.1485802

DeCuir-Gunby, J. T., Marshall, P. L., & McCulloch, A. W. (2011). Developing and using a codebook for the analysis of interview data: An example from a professional development research project. *Field Methods, 23*(2), 136–155.

Dolmage, J. T. (2017). *Academic ableism: Disability and higher education*. University of Michigan Press.

Edyburn, D. L. (2010). Would you recognize universal design for learning if you saw it? Ten propositions for new directions for the second decade of UDL. *Learning Disability Quarterly, 33*(1), 33–41. https://doi.org/10.1177/073194871003300103

Gervasoni, A., & Lindenskov, L. (2011). Students with 'special rights' for mathematics education. In Atweh B., Graven M., Secada W., Valero P. (Eds.), *Mapping Equity and Quality in Mathematics Education* (pp. 307–323). Springer. https://doi.org/10.1007/978-90-481-9803-0_22

Henriksen, D., Gretter, S., & Richardson, C. (2020). Design thinking and the practicing teacher: Addressing problems of practice in teacher education. *Teaching Education, 31*(2), 209–229. https://doi.org/10.1080/10476210.2018.1531841

IDEO. (2012). *Design thinking for educators.* http://designthinkingforeducators.com/.

Indar, G. K. (2018). An equity-based evolution of Universal design for learning: Participatory design for intentional inclusivity. *Proceedings of UDL-IRM International Summit 2018.* ttps://udl-irn.org/wp-content/uploads/2018/04/Done_INDAR.EDIT_.DH_.JEG-copy.pdf

Jackson, H. G., & Neel, R. S. (2006). Observing mathematics: Do students with EBD have access to standards-based mathematics instruction? *Education and Treatment of Children, 29*(4), 1–22.

Kirschner, P. A. (2015). Do we need teachers as designers of technology enhanced learning? *Instructional Science, 43*(2), 309–322.

Koehler, M. J., & Mishra, P. (2005). What happens when teachers design educational technology? The development of technological pedagogical content knowledge. *Journal of Educational Computing Research, 32*(2), 131–152.

Kurz, A., Elliott, S. N., Lemons, C. J., Zigmond, N., Kloo, A., & Kettler, R. J. (2014). Assessing opportunity-to-learn for students with disabilities in general and special education classes. *Assessment for Effective Intervention, 40*(1), 24–39. https://doi.org/10.1177/1534508414522685

Lambert, R. (2015). Constructing and resisting disability in mathematics classrooms: A case study exploring the impact of different pedagogies. *Educational Studies in Mathematics, 89*(1), 1–18. https://doi.org/10.1007/s10649-014-9587-6

Lambert, R. (2018). "Indefensible, illogical, and unsupported"; Countering deficit mythologies about the potential of students with learning disabilities. *Education Sciences, 8*(2).

Lambert, R. (2020). *Increasing access to Universally Designed mathematics classrooms.* PACE. https://edpolicyinca.org/publications/increasing-access-universally-designed-mathematics-classrooms

Lambert, R. (2021). The magic is in the margins; Universal Design for Learning Math (UDL Math). *Mathematics Teacher: Learning and Teaching Pre-K–12, 114*(9).

Lambert, R., Tan, P., Hunt, J., & Candela, A. (2018). Rehumanizing the mathematics education of students with disabilities; Critical perspectives on research and practice. *Investigations in Mathematics Learning, 10*(3), 1-4. https://doi.org/10.1080/19477503.2018.1463006

Lambert R. & Tan, P. (2020). Does disability matter in mathematics educational research? A critical comparison of research on students with and without disabilities. *Mathematics Education Research Journal, 32*, 5-35. https://doi.org/10.1007/s13394-019-00299-6

Linton, S. (1998). *Claiming disability: Knowledge and identity.* NYU Press.

Meyer, A., & Rose, D. H. (2005). The future is in the margins: The role of technology and disability in educational reform. In DH Rose, A Meyer, C Hitchcock (Eds), *The universally designed classroom: Accessible curriculum and digital technologies* (pp. 13–35). Harvard University Press.

Meyer, A., Rose, D. H., & Gordon, D. T. (2014). *Universal design for learning: Theory and practice*. CAST Professional Publishing. http://udltheorypractice.cast.org/

Moore, E.J. (2017). UDL in 3D! *Proceedings of UDL-IRN Summit, 2018*. https://udl-irn.org/2018-summit-proceedings/

Naraian, S. (2019). Precarious, debilitated and ordinary: Rethinking (in)capacity for inclusion. *Curriculum Inquiry, 49*(4), 464–484. https://doi.org/10.1080/03626784.2019.1659100

National Academies of Sciences, Engineering, and Medicine (2018). *How people learn II: Learners, contexts, and cultures.* The National Academies Press

Owen, Ch. L. (2005, Oct. 1) Design thinking. What it is. Why it is different. Where it has new value. [Conference keynote speech]. I*nternational Conference on Design Research and Education for Future*, Gwangju, Republic of Korea.

Ralabate, P. K. (2016). *Your UDL lesson planner: The step-by-step guide for teaching all learners*. Brookes Publishing.

Rose, T. (2015). *The end of average: How we succeed in a world that values sameness.* HarperOne.

Rowe, P.G. (1987). *Design thinking.* MIT Press.

Russo, J. A., Bobis, J., Downton, A., Hughes, S., Livy, S., McCormick, M., & Sullivan, P. (2020). Students who surprise teachers when learning mathematics through problem solving in the early primary years. *International Journal of Innovation in Science and Mathematics Education, 28*(3). http://dx.doi.org/10.30722/IJISME.28.03.002

Russo, J., Minas, M., Hewish, T., & McCosh, J. (2020). Using prompts to empower learners: Exploring primary students' attitudes towards enabling prompts when learning mathematics through problem solving. *Mathematics Teacher Education and Development, 22*(1), 48-67. https://mted.merga.net.au/index.php/mted/article/view/592

Scherer, P. & Krauthausen, G. 2010. Natural differentiation in mathematics: The NaDiMa project. *Panama-Post, 29*(3),14-26.

Schoenfeld, A. H. (1988). *Mathematical problem-solving*. Elsevier.

Smith, S. J., Rao, K., Lowrey, K. A., Gardner, J. E., Moore, E., Coy, K., Marino, M., & Wojcik, B. (2019). Recommendations for a national research agenda in UDL: Outcomes from the UDL-IRN Pre Conference on research. *Journal of Disability Policy Studies, 30*(3), 174–185. https://doi.org/10.1177/1044207319826219

Steffe, L. P., & Thompson, P. W. (2000). Teaching experiment methodology: Underlying principles and essential elements. In R. Lesh & A. E. Kelly (Eds.), *Research design in mathematics and science education* (pp. 267- 307). Erlbaum.

Sullivan, P., Bobis, J., Downton, A., Feng, M., Livy, S., Hughes, S., McCormick, M., & Russo, J. (2020). Characteristics of learning environments in which students are open to risk taking and mistake making. *Australian Primary Mathematics Classroom, 25*(2), 3–8.

Sullivan, P., Mousley, J., & Zevenbergen, R. (2006a). Developing guidelines for teachers helping students experiencing difficulty in learning mathematics. In P. Grootenboer, R. Zevenbergen & M. Chinnappan (Eds.), *Identities, cultures and learning space: Proceedings of the 29th annual conference of the Mathematics Education Research Group of Australasia* (pp. 496–503). Sydney: MERGA.

Sullivan, P., Mousley, J., & Zevenbergen, R. (2006b). Teacher actions to maximize mathematics learning opportunities in heterogeneous classrooms. *International Journal of Science and Mathematics Education, 4*(1), 117-143. https://doi.org/10.1007/s10763-005-9002-y

Tan, P., Padilla, A., Mason, E., & Sheldon, J. (2019). *Humanizing Disability in Mathematics Education: Forging New Paths.* National Council of Teachers of Mathematics.

Tomlinson, C. A. (1999). *The differentiated classroom: Responding to the needs of all learners*. Association for Supervision & Curriculum Development.

Waloszek, G. (2012, September 12). *Introduction to design thinking*. Retrieved from http://www.sapdesignguild.org/community/design/design_thinking.asp
Zydney, J. M., & Hasselbring, T. S. (2014). Mini anchors: A Universal Design for Learning approach. *TechTrends, 58*(6), 21–28. https://doi.org/10.1007/s11528-014-0799-5

Authors

Rachel Lambert
University of California Santa Barbara
Santa Barbara, CA 93106, United States
email: rlambert@ucsb.edu

Kara Imm
City University of New York
425 E 25th St, New York, NY 10010, United States

Rachel Schuck
University of California Santa Barbara
Santa Barbara, CA 93106, United States

Sunghee Choi
University of California Santa Barbara
Santa Barbara, CA 93106, United States

Avery McNiff
University of California Santa Barbara
Santa Barbara, CA 93106, United States

Appendix A

Survey Questions

1. What is Universal Design in Learning?*
2. What is Design Thinking? What is its purpose?*
3. What do you design in your current work?*
4. What do students with disabilities need in math?*
5. Rate your familiarity with the following topics. - Design Thinking*
6. Rate your familiarity with the following topics. - Universal Design for Learning*
7. Rate your familiarity with the following topics. - Research base of UDL*
8. Rate your familiarity with the following topics. - UDL Principle of Engagement*
9. Rate your familiarity with the following topics. - UDL Principle of Representation*
10. Rate your familiarity with the following topics. - UDL Principle of Action and Expression*
11. Rate your familiarity with the following topics. - Using UDL in Math Classrooms*
12. Rate your familiarity with the following topics. - Learner variability*
13. Rate your familiarity with the following topics. - Designing from the edges*
14. Rate your familiarity with the following topics. - Neurodiversity*
15. Rate your familiarity with the following topics. - Social Model of Disability*
16. Rate your familiarity with the following topics. - Connections between UDL and Culturally Responsive Pedagogy*
17. What did you learn about disability in this course? Does that matter in your work?
18. How might this course impact your work as an educator?
19. If this course was 10 sessions rather than 6 sessions, what content areas would you like expanded? Why?
20. What connections do you see between UDL and Design Thinking?
21. What did you learn about UDL in this course? Was the content or presentation different than in previous learning about UDL? How would you improve the work on UDL in the course?
22. What did you learn about Design Thinking for Educators in this course? Was the content or presentation different than in previous learning? How would you improve the work on Design Thinking in the course?
23. This version of the course included district and county administrators, curriculum specialists, coaches, and teachers. If the course was focused on teachers, what (if anything) do you think we would need to change?
24. Anything else you would like us to know?

Note. *Question appeared on both the pre- and the post-survey. The remaining questions were only on the post-survey.