Beyond Skill: Students' Dispositions Towards Math

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ABSTRACT: Universities have been forced to confront the increasing placement of students into developmental courses and the subsequent lack of success students encounter. In an effort to better understand how students' attitudes and dispositions towards mathematics impact this trend, we explored the differences in nine characteristics between students in developmental and college-level mathematics courses from a sample of 676 freshmen at a large, public university. Findings showed that students in developmental math did have significantly less-favorable dispositions towards mathematics. However, a cluster analysis revealed that the students were not as homogeneous as may first have appeared, showing a mix of favorable and less-favorable attitudes and dispositions. Further, a regression analysis revealed three student characteristics significantly predicted students' placement into developmental courses: their perspective on the applicability of math to the world around them, low confidence, and a limited view of what constitutes valid mathematical solutions. Taken together, these results challenge traditional, skill-based assessments of postsecondary mathematics preparedness.

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Developmental mathematics courses comprise a significant share of all postsecondary mathematics courses taught in the United States of America. For example, in the 23-campus California State University system, the parent organization of the institution where we conducted this study, 17,406 first-time freshman students of a total 61,757, yielding 28.2%, were required to enroll in developmental education courses for the Fall 2016 academic semester ("Fall 2016 Final," 2017). This is slightly higher than national figures for the United States of America: Recent statistics (2011/2012) from the National Center for Education Statistics are that 20.4% of all first-year undergraduate students enrolled in a public four-year nondoctoral university took "remedial" (i.e., developmental) courses ("Percentage of First-year Undergraduates," 2017). Such developmental courses often bear no degree credit, resulting in increased time to degree completion. Additionally, students requiring developmental mathematics experience lower success in terms of persistence and completion than those who are not required to enroll in developmental mathematics courses (Bahr, 2013a; National Center for Public Policy and Higher Education, 2010).

Despite the large number of students enrolled in developmental mathematics, their vulnerability, and the large financial costs associated with developmental education, little research has been conducted on understanding characteristics of the students and how best to educate them (Rutschow & Schneider, 2011). Much of the published work on developmental mathematics education focuses on large scale trends in course taking and completion or success rate (Bahr, 2013a). However, how developmental mathematics education produces these trends remains a "black box" (Cox, 2015); it is clear that more effort needs to be expended to better understand the students taking these courses. Researchers have begun this work, focusing on such factors as students' extant mathematical skills (Givvin et al., 2011; Stigler et al., 2010) and the effects of their demographic and socioeconomic backgrounds (Bahr, 2010). However, far less is understood about students' affects (i.e., students' emotional associations and relationships with mathematics) and dispositions (nonemotional perspectives and relationships) towards mathematics.

Developmental practitioners appear to share a consensus regarding the affective and dispositional characteristics of developmental students. They acknowledge certain nonmathematical challenges faced by developmental students that often inhibit their abilities to fully engage with mathematics. Existing studies point to some affective and dispositional factors confronting developmental students (Bonham & Boylan, 2011). Among these are mathematics anxiety (Woodard, 2004), low confidence and persistence (Higbee & Thomas, 1999), low motivation (Moore, 2007), and other general factors (Code et al., 2016). It is well established that these traits can negatively impact learning and performance in mathematics (Aiken, 1972; 1976; Hembree, 1990; Tobias, 1978). In contrast, less research exists exploring how affects and dispositions differ between developmental and nondevelopmental students and how these potentially impact students' placement into developmental or college-level courses. One notable exception is the study by Hall and Ponton (2005) that reports calculus students have stronger mathematics self-efficacy beliefs than developmental mathematics students at the same institution. Such a result calls into question a belief

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that developmental and nondevelopmental students differ strictly in terms of mathematical skills and knowledge.

Our goal for this study has been to explore a range of affective, psychological, and dispositional characteristics of developmental mathematics students and, most importantly, to position these relative to nondevelopmental students. We acknowledge the research on the psychological affects held by developmental mathematics students has suffered from poor methodologies (Benken et al., 2015) and has been criticized for a lack of theoretical foundations (Chung, 2005; Chung & Higbee, 2005). Our intention here is to explore claims about developmental students' affects and dispositions towards mathematics, with the grander aim of informing more effective design of educational experiences for developmental students, for example, open-ended and student-centered authentic mathematics tasks (Maciejewski, Bragelman, & Bergthold, 2020; Maciejewski, Bragelman, Campisi, et al., 2020). Specifically, we sought to address the following:

1. Do students enrolled in a developmental mathematics course differ from those enrolled in nondevelopmental courses on a range of affective and dispositional factors?
2. Are there any affective and dispositional factors that are predictive of enrollment in developmental mathematics?

Literature Review

As mentioned previously, developmental mathematics education is a large, primarily American, enterprise, enrolling approximately one-third to one-half of all postsecondary students nationally per year and costing over a billion dollars (Bailey, Jeong, & Cho, 2010; King, McIntosh, & Bell-Ellwanger, 2017). Students are typically placed into these courses based on their performance on an entrance exam, though there is an effort in some jurisdictions to improve placement through considering a more comprehensive evaluation of the students' academic backgrounds and current abilities. For example, the California State University (CSU) system has recently stopped requiring entering students to write an entry-level mathematics exam, with their placement into mathematics courses determined by high school course credits and performance (California State University Executive Order 1100 [CSUEO], 2017). This ostensibly discontinues developmental mathematics placement practices and courses at all 23 CSU campuses. Students deemed to not have sufficient mathematics preparation are now entitled to enroll in a credit-bearing course, at college-level content and rigor, in their first semester. This policy increases the need for faculty to better understand and respond to the needs of students who would historically be classified as developmental.

Studies on the effectiveness of developmental courses have yielded mixed results (Scott-Clayton & Rodrigues, 2012). In a large-scale, longitudinal, quantitative study, Bahr (2008) found that students who successfully completed a developmental mathematics course sequence had outcomes in subsequent courses comparable to those who never took a developmental math sequence. Bahr (2008) concluded that developmental mathematics is effective at treating skill deficiencies. However, Attewell et al. (2006) reported that only 30% of developmental mathematics students pass their developmental mathematics course sequences, putting into question the effectiveness of such courses. If completion rates have remained as low as reported in Attewell et al. (2006), then developmental mathematics courses have likely been falling short of meeting their intended purpose. Further, placement exams are known to be highly inaccurate indicators of mathematical knowledge and poor predictors of subsequent college performance (Belfield & Crosta, 2012; Scott-Clayton, 2012). Taken together, unexplored, noncognitive factors may be influencing students' experiences during placement—via testing or other holistic measures—and in the resulting developmental courses.

> *Individual-level characteristics…explain more variance in student passing rates than institutional-level characteristics.*

Presently, not much is known about students who take developmental mathematics courses. This is especially true at the university level, as a substantial majority of the developmental mathematics research literature focuses on two-year college students (Boylan, 2002; Rutschow & Schneider, 2011). As such, for the purposes of this paper, we are content with drawing on the community college literature while recognizing the possible limitations of doing so. What is known is mostly related to demographic and socioeconomic characteristics.

For example, Fong et al. (2015) have found that individual-level characteristics, such as age, sex, race/ethnicity, enrollment status, and assessment scores and placement, explain more variance in student passing rates than institutional-level characteristics, such as institutional size and student composition. Therefore, the individual-level characteristics seemed to be vital in understanding students' progression through college mathematics. In a similar vein, Bahr (2010) identified substantial racial disparities in successful remediation in math: African American and Latinx students were less likely than others to complete developmental mathematics within 6 years of enrollment. Bahr (2010) concluded that developmental mathematics served to increase racial disparities in mathematics achievement. Other studies that examined such demographic and socioeconomic variables reported similar conclusions (Kao & Thompson, 2003). Although a well-developed understanding of the interaction between demographic and socioeconomic variables of students with academic achievement in developmental, and subsequent, mathematics has been reported, research on the effects of noncognitive factors is less developed, with few exceptions (Benken et al., 2015; Hall & Ponton, 2005; Higbee & Thomas, 1999).

Hall and Ponton (2005) performed a comparative analysis of 185 developmental and nondevelopmental students' mathematics self-efficacy, the belief in one's ability to perform a mathematical task. They found that nondevelopmental students exhibited significantly higher mathematics self-efficacy. Higbee and Thomas (1999) measured 20 affective factors of a group of 23 developmental math students, finding that math anxiety is negatively correlated with test performance. Further, they found the prior academic achievement metrics of high school GPA and SAT scores had no significant correlation to students' final grade in the course. The researchers argued this could be due to affective factors taking precedence over academic factors, deducing that "[m]any students enrolled in developmental education programs have ability but lack the motivation or confidence to achieve" (p. 14). In a similar vein, Benken et al., (2015) evaluated developmental students' self-perception of their mathematical skills, enjoyment of mathematics, confidence in mathematics, and comfort with mathematics. After a semester of developmental math, the participants rated their skills and enjoyment of math higher but their confidence and comfort lower. However, these studies (Benken et al., 2015; Higbee & Thomas, 1999) did not address the role of noncognitive factors for nondevelopmental, college-ready students.

Evaluations of developmental math curricular innovations often highlight the need to consider noncognitive, affective factors in developmental math course design and instructional practices (Bonham & Boylan, 2011; Deka & Lieberman, 2013; Gula et al., 2015; Maciejewski, 2012). However, claims of the importance of attending to the affective aspects of learning have not yet been fully substantiated in the literature. That is, there are few studies of developmental mathematics students' noncognitive, affective, and dispositional factors in the literature or comparative studies of developmental and nondevelopmental mathematics students.

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Methods

In our study, we have built on the work of the reviewed studies (Benken et al., 2015; Hall & Ponton, 2005; Higbee & Thomas, 1999) to compare noncognitive factors of developmental and nondevelopmental students. Further, we sought to identify the noncognitive factors unique to developmental students across a broad spectrum of affective traits and determine what, if any, of these factors may attribute to placement in developmental courses.

Participants

All students ($N_{total} = 2379$) enrolled in a "freshman" (i.e., first-year) mathematics course at San José State University in the 2016/2017 academic year were contacted via email and invited to complete a survey online at the start of the Fall 2016 semester. In this freshman cohort, 676 students were enrolled in developmental math (abbreviated DEV) and 1703 were enrolled in nondevelopmental math (Non-DEV), which includes college algebra, precalculus, calculus, and general education mathematics; 23 students were unable to be reached. Considering developmental mathematics students were of primary concern, these students were additionally incentivized by having their lowest homework score in their course dropped from the course grade calculation upon completion of the survey. In all, 686 students completed the survey—344 developmental mathematics students and 342 nondevelopmental mathematics students—for a response rate of 28.8%.

Instruments

The survey used in this study consisted of two sections. The first was composed of basic demographic questions and was not the focus of this report. The second was the combination of two established survey instruments: (a) the Mathematics Attitudes and Perceptions Survey (MAPS; Code et al., 2016) and (b) the Abbreviated Math Anxiety Rating Scale (AMAS; Hopko et al., 2003). Upon conclusion of the survey, responses were matched with institutional records on student demographics and socioeconomic data and linked to course grades. Overall, this created a rich dataset that served to address the research questions of the present study.

The MAPS instrument was initially designed as an adaptation, to the context of postsecondary mathematics, of the Colorado Learning Attitudes About Science Survey (CLASS; Adams et al., 2006), a widely implemented survey about students' attitudes and dispositions towards learning in scientific fields at the postsecondary level. The intended use of the CLASS was to evaluate students on a number of factors established in the research literature as indicators of success in science learning. Through multiple rounds of data collection, analysis, and survey revision, detailed in Code et al., 2016, the MAPS instrument has come to consist of 7 subscales: Growth Mindset, relationship between mathematics and the Real World, Confidence in mathematics, Interest in mathematics, Persistence in problem solving, Sense Making, and the Nature of Answers.

Items in the Growth Mindset category, such as "Math ability is something about a person that cannot be changed very much," determine students' beliefs about whether mathematical ability is fixed or can be developed, in accordance with the work of Dweck (2008) and Boaler (2015). Items within the Real World category, such as "Reasoning skills used to understand math can be helpful to me in everyday life," assess students' perspective on the relationship between mathematics and the real world. Items assessing Confidence in mathematics, such as "No matter how much I prepare, I am still not confident when taking math tests," determine students' perceptions about their ability to engage with mathematics tests successfully. Items within the Interest in mathematics category, such as "I only learn math when it is required," evaluate students' interest in engagement with mathematics. Items in the Persistence in problem solving category, such as "If I get stuck on a math problem, there is no chance that I will figure it out on my own," determine students' approach in solving nonroutine mathematical problems and their continued selection and use of solution strategies. Items in the Sense Making category, such as "In math, it is important for me to make sense out of formulas and procedures before I use them," assess students' perspectives on the nature of their mathematical knowledge and its structure. Finally, items in the Nature of Answers category, such as "I expect the answers to math problems to be numbers," categorize students' perceptions on the nature of solutions to mathematical problems. It is worth noting that the MAPS instrument positions responses against an abstract "expert." Whereas Code et al. (2016) explain the rationale for this in detail, we mention it here only to underscore its importance as a relative, fixed datum for comparison of groups.

> *The MAPS instrument positions responses against an abstract "expert."*

As mentioned previously, mathematics anxiety has been recognized in the research literature as a possible hindrance to mathematics learning and performance (Lyons & Beilock, 2012; Hembree, 1990; Tobias, 1993). It is therefore important to assess and monitor students' mathematics anxiety and how this might be shaped by their educational experiences. To evaluate the differences in anxiety between developmental and nondevelopmental students, we employed the AMAS (Hopko et al., 2003). It is comprised of two subscales, one measuring students' perceived anxiety when learning mathematics and the other measuring perceived anxiety when experiencing assessments related to mathematics, with respective examples such as "Watching a teacher work an algebraic equation on the blackboard" and "Being given a 'pop' quiz in math class."

Analysis

We analyzed the data using three primary methods: (a) an overall comparison of MAPS and AMAS data for developmental and nondevelopmental students; (b) a cluster analysis of all MAPS and AMAS data; and (c) a regression analysis of MAPS and AMAS data. We completed he analyses to determine which subscales best predict developmental status. Each of these is described in the following.

Survey Response Comparison

Each of the seven MAPS subscales comprised the sum of the responses to a number of questions on the MAPS survey, scaled relative to a set of expert responses (Code et al., 2016). For example, Growth Mindset was a sum of scaled responses to questions 5, 6, 22, 31; Real World was the sum of scaled responses to questions 13, 15, 21. Therefore, a student's score on any one of these subscales was necessarily discrete, taking on the values X divided by the number of questions comprising the subscale, where X could take on the values 0, 1, 2, …, (# of questions comprising the subscale). Consequently, care had to be exercised when comparing populations on the MAPS subscales: the discrete nature of the subscales violated the continuity of data assumptions made in, for example, standard comparison of means tests. To this end, a Kruskal-Wallis test (1952) was performed on the difference between developmental and nondevelopmental responses for each subscale. We used a similar approach to compare developmental and nondevelopmental math students on the AMAS.

Cluster Analysis

Cluster analysis, a suite of methods used to describe data in terms of homogeneous groups according to some specified criteria, was also conducted to improve understanding of community college students. The goal of the cluster analysis in this study was to find homogeneous groups of students, based on MAPS and AMAS scores and independent of DEV/Non-DEV status.

The data were of mixed type, specifically, ordinal and continuous variables. Given the nature of the data, K-medoids, or partition around medoids (PAM; Kaufman & Rousseeuw, 1990), were the best fit. With mixed type data the mean could not be used as a center of the cluster; a *medoid* should be used instead. A medoid is a measure of centrality similar to a median but must also be a point in the dataset. K-medoid clustering partitioned the units minimizing the distance between the points in the cluster and the medoid. The result of this analysis was a partition of the units in homogeneous groups with respect to MAPS and AMAS. To choose the number of clusters we used the average silhouette value (Rousseeuw, 1987), a measure of the quality of the cluster partition. The range of *s(i)* was [-1,1] and was calculated as

$$s(i) = \frac{b(i) - a(i)}{\max\{a(i), b(i)\}}$$

by a silhouette value above s(i) = 0.5. The average silhouette value is a measure of the quality of the partition. We chose the number of clusters characterized by the highest average silhouette value.

Logistic Regression

To identify which variables potentially inform students' placement into developmental or nondevelopmental mathematics tracks, we used a logistic regression model (Cox, 1958) with a binary dependent variable and numerical independent variables. The dependent variable in our model was DEV/Non-DEV status, and the independent variables were the MAPS and AMAS scores. The latent variables MAPS and AMAS were obtained as a combination of several measured integer numerical variables and therefore could be treated as continuous in the logistic regression.

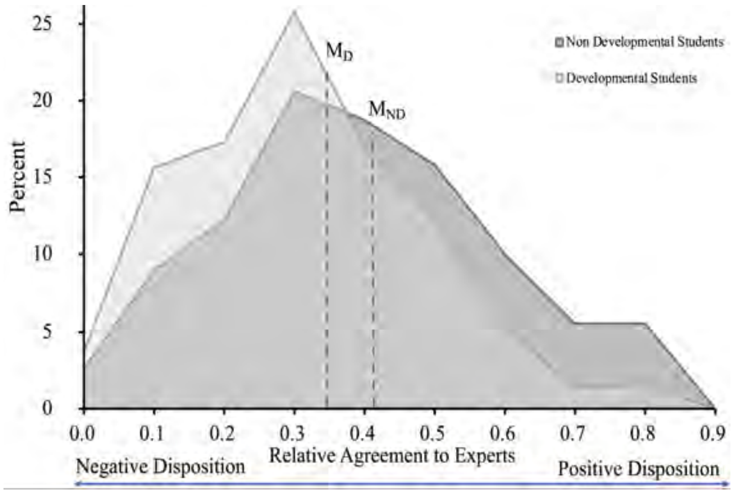

Figure 1. Overall "expert like" dispositions toward mathematics.

Results

In the following section, we present our results by research question. Within the first research question we present an aggregate comparison of developmental and nondevelopmental students' scores on the MAPS instrument and the AMAS instrument. We separate results for each instrument due to their intended design and use. Next, we present differences between groups of the subscales within each instrument. Following, we present a cluster analysis of the survey data to more deeply explore dispositional differences. Finally, under the second research question, we present the logistic regression model and the significant factors that predict placement.

Though the results are presented separately, they should be taken as a totality: Considering only, for example, the overall DEV/Non-DEV comparison could be misleading and ought to be considered in the light of the complementary results from the cluster analysis. That is, there is no singular comparison that accurately distinguishes between developmental and nondevelopmental students.

Research Question 1 (RQ 1)

Do students enrolled in a developmental mathematics course differ from those enrolled in nondevelopmental courses on a range of affective and dispositional factors? The results from all three analyses related to RQ 1 are presented in the following subsections.

RQ1: MAPS Results

For differences in aggregate responses to the MAPS instrument, the median for developmental students was 0.34 whereas the median for nondevelopmental students was 0.41. A Kruskal-Wallis H test showed the difference between groups was significant ($\chi^2(1) = 36.073$, $p < 0.001$), suggesting incoming students who placed into developmental coursework exhibited overall lower dispositions towards mathematics than students who placed into nondevelopmental coursework (see Figure 1) The relative percentage on the x-axis describes the percent of responses by the participant who agreed with an expert in the field of mathematics.

RQ1: AMAS Results

Similarly, for cumulative mathematics anxiety, the median for students who placed into developmental courses was a 29, whereas the median for students who placed into nondevelopmental courses was a 28. A Kruskal-Wallis test showed the difference, although numerically small, to be significant ($\chi^2(1) = 8.36$, $p < 0.01$), suggesting students who placed into developmental courses experienced higher levels of mathematics anxiety compared to students who did not place into developmental courses (see Figure 2). Of note, the x-axis displays the overall anxiety score a participant attributed to mathematics, with a 9 representing low-anxiety across every item and a 45 representing maximum anxiety for every item.

RQ1: Instrumental Subscale Results

Turning to the subscales within each instrument, we found significant differences between developmental and nondevelopmental students across all seven subscales of disposition (see Table 1, p. 22). In other words, developmental students' dispositions were comparatively lower and less-similar to expert responses for the MAPS items than nondevelopmental students. This suggested that developmental students in this study not only exhibited a lower overall disposition towards mathematics than nondevelopmental students, they also exhibited individually low scores on all subscales including Growth Mindset, relationship between mathematics and the Real World, Confidence in mathematics, Interest in mathematics, Persistence in problem solving, Sense Making, and the Nature of Answers. For the individual subscales in the anxiety (AMAS) instrument, we found no significant difference between developmental and nondevelopmental students on the Evaluation Anxiety subscale ($H = 2.638$, $p = 0.104$). However, the median on the Learning Anxiety subscale, in comparison, is significantly higher for developmental students ($\chi^2(1) = 8.503$, $p < 0.01$), suggesting that developmental students felt significantly higher anxiety when exposed to learning environments for mathematics than nondevelopmental students.

These results suggest that developmental students, although significantly different than nondevelopmental students in terms of their disposition towards mathematics, do not seem distinctly different in any

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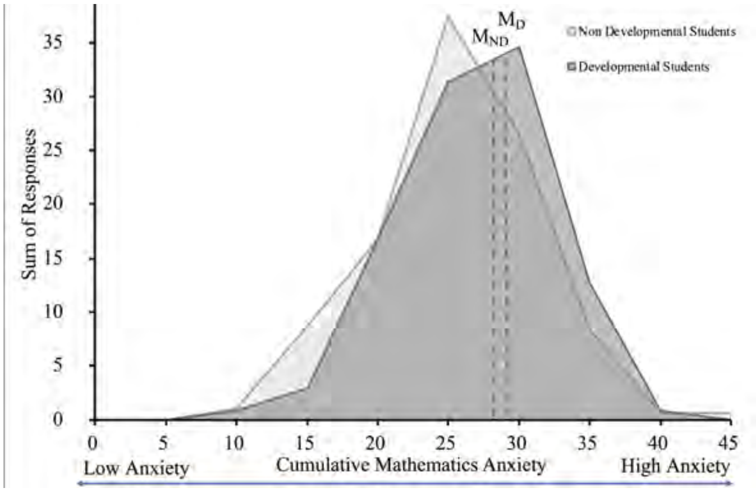

Figure 2. Anxiety (AMAS) data: developmental and nondevelopmental students.

Table 1

Resulting Medians and Significant Tests for Individual Subscales across the MAPS and AMAS Instruments

Category	Dev median	Non-Dev median	Kruskal-Wallis (chi square) value	Kruskal-Wallis *p*-value
MAPS-Aggregate Expertise				
Growth Mindset	0.50	0.62	9.638	< 0.01
Real World	0.33	0.33	13.905	< 0.001
Confidence	0.25	0.50	26.829	< 0.001
Interest	0	0.33	20.592	< 0.001
Persistence	0.25	0.25	16.488	< 0.001
Sense Making	0.40	0.40	8.788	< 0.01
Nature of Answers	0.33	0.33	21.174	< 0.001
AMAS-Aggregate Anxiety				
Learning Anxiety	13	12	8.503	< 0.01
Evaluation Anxiety	16	16	2.638	0.104

Table 2

Results of Cluster Analysis across the MAPS and AMAS Instruments

Cluster	MAPS	AMAS	*N*	%DEV
1	Low	High	242	61.20%
2	Mid	Mid	257	51.80%
3	High	Low	187	33.70%

specific category. In other words, they did not show a unique difference in any single subscale. The distributions in Figures 1 and 2 were not bimodal, suggesting there were not two unique populations. In other words, the data shows there were developmental students with low anxiety and a positive disposition towards mathematics just as there seemed to be nondevelopmental students with high anxiety and a low disposition towards mathematics.

RQ1: Exploring Group Differences

To further explore this line of reasoning, we employed cluster analysis across the MAPS and AMAS data for both developmental and nondevelopmental students. A description of each cluster in terms of its relative position on each AMAS and MAPS category is presented in Table 2. Essentially, Cluster 1 is composed of students with a negative disposition towards mathematics and with high levels of math anxiety. Cluster 3 is composed of students with a positive disposition towards mathematics and with low levels of math anxiety. Finally, students in Cluster 2 fell in the middle. Figures 3 and 4 display box plots of the resulting clusters, the former for mathematics disposition (data from the MAPS) and the latter for mathematics anxiety (data from the AMAS). Figure 5 displays the distribution of each cluster by type of student. In short, developmental students were a majority in Cluster 1, roughly half of Cluster 2 and a one-third minority in Cluster 3.

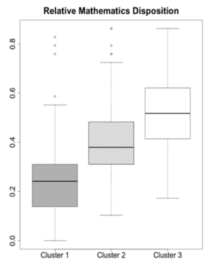

Figure 3. Relative Disposition

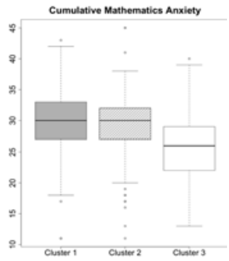

Figure 4. Cumulative Anxiety

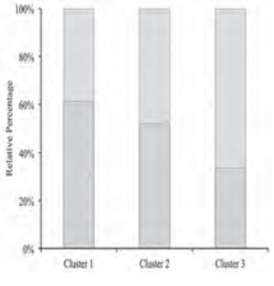

Figure 5. Percentage in each cluster.

RQ1: Discussion

In totality, our findings for RQ1 present a nuanced picture of developmental students' affect towards mathematics. Our initial tests show students in our sample who place into developmental courses have significantly lower disposition towards mathematics on average and in every subscale. They also experience significantly higher anxiety towards mathematics, primarily due to anxiety associated with the learning of mathematics. However, the cluster analysis suggests there is not a unique dichotomy between the two groups; each cluster—which can be thought of as archetypal groups of students—is split between developmental and nondevelopmental students. In short, there is a noteworthy percentage of students in developmental courses with a neutral to positive disposition towards mathematics and low-to-medium levels of anxiety towards mathematics. In addition, there is a noteworthy percentage of students in nondevelopmental courses with a negative disposition and high levels of anxiety. This suggests there is not a "type" of student found in developmental mathematics or in nondevelopmental mathematics, at least in regards to students' disposition and anxiety towards mathematics.

Research Question 2 (RQ 2)

Are there any affective and dispositional factors that are predictive of enrollment in developmental mathematics? We analyzed RQ 2 employing a logistic regression analysis.

A logistic regression analysis using placement into a developmental or nondevelopmental track as the dependent variable and the dispositional factors assessed by the MAPS and AMAS instruments as the independent variables found the model was significant ($p < 0.001$). Within the model, three subscales from the MAPS instrument significantly predicted students' placement into developmental courses: students' understanding of the relationship between mathematics and the Real World, students' Confidence in mathematics, and students' perception of the Nature of Answers to mathematical problems. The coefficients for the predictive MAPS subscales, their *p*-values, and their log-odds ratios are presented in Table 3 (p. 23). The odds ratio quantifies how much the probability of not being in developmental math increases for a unitary increase of the given variable. For example, in Table 3, for every unit decrease in students' responses to items in the Nature of Answers subscale, they were 3.23 times more likely to place into the developmental track. Similarly, a unit decrease in Confidence in mathematics is associated with 2.88 times greater chance of placing into developmental courses, whereas a unit decrease in the relationship between mathematics and the Real World category is associated with a 1.82 times greater chance of placement into developmental mathematics. Put another way, the lower the average score in any of the Real World, Confidence, and Nature of Answers MAPS subscales, the more likely the student was enrolled in a developmental mathematics course. These findings suggested that students' perception of mathematics and the Real World, their Confidence in mathematics, and their perception of the Nature of Answers to mathematics problems were significantly predictive of their placement in either developmental or nondevelopmental courses.

Summary of Results

Taken as entire groups, the developmental and nondevelopmental students differed on all subscales of the MAPS and AMAS instruments save one, anxiety associated with mathematics assessments. In comparison to nondevelopmental students, developmental students exhibited more negative feelings and higher anxiety towards mathematics. Our cluster analysis of the MAPS and AMAS subscales revealed three clusters: one with a negative disposition and high anxiety, one with a positive disposition and low anxiety, and one in the middle. Perhaps surprisingly, given the overall comparison result, the individual clusters did not fit on either side of the developmental/nondevelopmental divide. Rather, developmental and nondevelopmental students roughly split each cluster. This suggested a counterpoint to an initial interpretation of the comparison results: Students in developmental courses should not be considered as possessing a negative disposition or high anxiety towards mathematics.

Finally, of the nine subscales of disposition assessed, regression analysis revealed three from the MAPS subscale that were predictive of developmental status. Listed in decreasing power of predicting developmental status, these were: relationship between mathematics and the Real World, Confidence in mathematics, and the Nature of Answers.

Discussion

This study further develops previous research on psychological affects and dispositions of developmental students (Benken et al., 2015; Higbee & Thomas, 1999), how these differ in comparison to nondevelopmental students (Hall & Ponton, 2005), and how they impact students' placement into developmental courses due to their negative influence on performance in mathematics (Aiken, 1972, 1976; Hembree, 1990; Tobias, 1978). The results of this study confirm prior findings of comparative studies (Hall & Ponton, 2005): Developmental students have less favorable dispositions towards mathematics than their nondevelopmental counterparts. However, our cluster analysis suggests that students who place into developmental courses cannot be simply characterized as having a negative view of mathematics or as being highly anxious. Indeed, the mixed results of student success in developmental education (Scott-Clayton & Rodrigues, 2012) may be attributed to homogenizing the population of students who place into its programs. Curricula and interventions that approach students from this aggregate perspective do a disservice to a large percentage of students who place into developmental courses. The results from logistic regression analysis confirm this critique: students' perception of the relationship between math and the Real World, their Confidence in math, and their perception of the Nature of Answers to mathematical problems are all predictive of developmental mathematics enrollment.

Limitations

Naturally, there are limitations inherent in our work. First, our study was conducted at a single institution with a single year's cohort of students. We have no reason to suspect that our sample is biased to the point of invalidating our results, but we also do not have sufficient evidence to affirm that our sample is not biased. Second, the instruments we chose consider only certain affective and dispositional factors; others, discussed in the implications section, are certainly relevant.

Implications for Practice and Future Research

Complementing the research on the effects of disposition (Pajares & Miller, 1994) and anxiety (Higbee & Thomas, 1999) on students' performance in mathematics, our findings suggest students place into developmental courses due to dispositional characteristics and not solely to a "deficit" understanding of mathematics. Rather, , interventions targeting students' perceptions of mathematics and their confidence (e.g., the student-centerd teaching described in (Maciejewski, Bragelman, & Bergthold) and (Maciejewski, Bragelman, Campisi, et al., 2020)) would be most effective at scaffolding their transition to college-level coursework.. Curricular and pedagogical reforms that target these factors could be effective at shifting students' experiences with, and subsequently their performance in, mathematics. Practitioners who, in the aftermath of reform, find former "developmental" students newly included in their courses may draw from research on culturally responsive teaching (Gay, 2002). Gay advocates for situating mathematics teaching and learning "within the lived experiences and frames of reference of students" (p. 106). For example, practitioners may draw on students' mathematical funds of knowledge (cf., Gonzales et al. 2001) to target their perceptions of math and the Real World. Practitioners considering this approach would find value in Sleeter's (2012) examination of interventions and implementations.

Table 3

Results of Logistic Regression Analysis across the MAPS and AMAS Instruments

Category	Coefficient	*p*-value	Odds ratio
Interest	-1.05	< 0.001	0.35
Real World	0.6	0.01	1.82
Confidence	1.06	< 0.001	2.88
Nature of Answers	1.17	< 0.010	3.23

Alternately, practitioners may draw upon various well-established interventions and literature that addresses students' affects--for example, *mindset* interventions (Boaler, 2015), *grit* interventions (Duckworth, 2016), or *universal design for learning* interventions (Rose & Meyer, 2002)--to develop students' confidence in math. They may also draw upon notions of flexibility (Rittle-Johnson et al., 2015) to target students' perception of the Nature of Answers to mathematical problems. Developmental mathematics educators are encouraged to widen their focus away from a strict focus on mathematics skills teaching, and include a holistic consideration of their students' past and developing relationships with mathematics (Maciejewski, Bragelman, & Bergthold, 2020; Maciejewski Bragelman, Campisi et al., 2020). Improving students' relationships with mathematics should not be viewed as auxiliary to teaching mathematical content but rather as a central emphasis of developmental education. In addition, study findings related to affective characteristics and dispositions common to students placing into developmental mathematics may be helpful to other administrative and student support services personnel. For example, the findings might be used to inform and shape future institutional placement processes.

A direction for future investigation stems from a glaring omission in the current work: hat mathematics do developmental students know and how do they know it? In particular, how might mathematical knowledge(s) and performances be linked to the noncognitive factors identified in this study? The authors of two studies focused on measuring students' knowledge of mathematics (Givvin et al., 2011; Stigler et al., 2009) attribute the students' difficulties with mathematics to an over-reliance on procedures and limited conceptual knowledge. Though that may be the case, what is left unexplored is how the students' dispositions and affects towards mathematics might interact with their performance in mathematics. We have begun here a rigorous exploration of these dispositional and

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affective variables. Future work should investigate how dispositions and affects interact with knowledge and performance in mathematics.

Additional dispositional and affective factors should also be considered in future research, for example, motivation (Moore, 2007) and various other attitudes and beliefs (Hannula et al., 2016). Future work might be devoted to creating a more complete map of such factors and consider possible interactions and intersections. Gathering data through student interviews or free-form writing about their experiences with mathematics (Maciejewski, 2018) may generate further factors not considered here.

Conclusion

In the wake of recent critique of developmental education (Scott-Clayton, 2012), systems and states have moved to either reconceptualize it, abolish it, or make it optional. In our case, Executive Order 1110 (California State University Executive Order 1100, 2017) was produced from the California State University Chancellor's Office. This EO discontinues use of the Entry Level Mathematics (ELM) exam, calls for all courses offered at CSU campuses to be credit-bearing and stipulates that all entering students ought to be able to complete general education mathematics and English by the end of their second semester. In short, CSU's administration has eliminated developmental education at 23 campuses. Admission standards will remain unchanged, but no students will be identified as developmental through the use of the ELM; that is, the students who would have been classified as developmental before EO1110 will be present in credit-bearing university mathematics classes. Although we are left questioning how to support these students and to facilitate their success in the midst of their new-found integration, our findings suggest a possible answer.

A move towards inclusion of all students necessitates a close examination of our general mathematics education practices. Instructors and institutional administrators ought to consider the heterogeneity inherent in their general education student body and design curricula and other educational experiences that specifically recognize and leverage this heterogeneity. Until this is enacted, the general education offered in postsecondary institutions will never move beyond skill.

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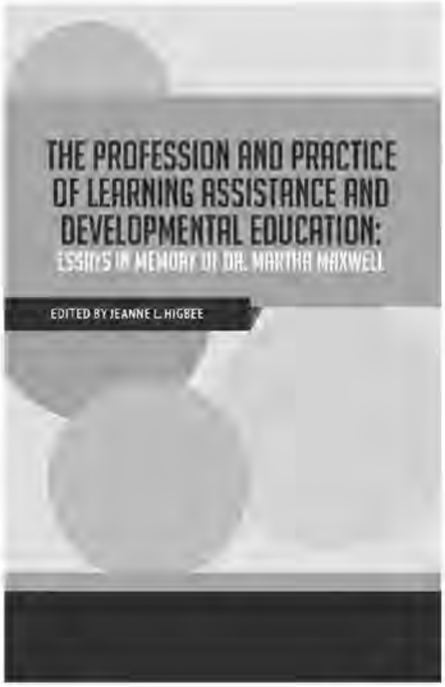

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