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Examination of 4th Grade Students' Definitions for Square, Rectangle and Triangle Geometric Shapes

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Abstract
The aim of this study is to obtain information about students' definitions, mistakes, misconceptions, and van Hiele geometry thinking levels by using the definitions of 4th grade students for geometric shapes of rectangle, square, isosceles triangle, equilateral triangle, and scalene triangle. The study was carried out with 156 primary school 4th grade students. In the study, the case design, one of the qualitative methods, was used. Students were asked to describe geometric shapes. It was observed that most of the 4th grade students participating in the study were in the visualization stage of van Hiele. There are very few definitions of the hierarchical structure in the study. Most correct definitions are in the partitional form. In the study, misconceptions were detected in some of the students.

Keywords: Error, Misconception, Van Hiele Geometry Thinking Level, Hierarchical Definition, Partitional Definition

1. İntroduction

Geometry is the ‘study of shapes, their relationships, and their properties’ (Bassarear, 2012, p. 463). Geometry is one of the important fields of mathematics. Knowledge of geometry remains a prerequisite for working in fields such as physics, astronomy, art, mechanical drawing, chemistry, biology, and geology (Luneta, 2015). “Geometry and spatial sense are fundamental components of mathematics learning. They offer ways to interpret and reflect on our physical environment and can serve as tools for the study of other topics in mathematics and science” (National Council of Teachers of Mathematics, 2000 (p. 41)). The hierarchy in van Hiele's geometry thinking model is an important determinant in the planning of the geometry curriculum. The stages in Van Hiele's theory are visualization, analysis, abstraction, deduction, and rigor, respectively. These levels are in hierarchical order (Van Hiele, 1986). When students learn geometric concepts, they progress through five hierarchical reasoning levels (Mbusi, 2015). The child cannot pass the abstraction stage before the visiulation stage occurs. These stages develop according to the experience of the child, not according to age. The development of a geometric concept takes place with the formation of other concepts related to this concept. Geometry subjects consist of abstract concepts and proceed in a sequential and stacked order. The learning of geometrical concepts requires high mental ability. Learners should acquire to be able to conceive, analyze them, as well as to recognize their relationships (Jacobson & Lehrer, 2000). Students may experience difficulties while learning concepts related to geometry. Not making the concepts meaningful enough may cause errors and misconceptions. A misconception could be the result of a misapplication of a rule, an over- or under-generalization, or an alternative conception of the situation (Drews, 2005). Although error and misconception are perceived as the same, they are different from each other. It is necessary to distinguish whether these problems that arise in students' learning are caused by errors or misconceptions. Error is the mistakes in the answers given by the students to the questions, and misconceptions are the internalized information that prevents the student from learning. While the error arises out of carelessness without the awareness of the student, the misconception emerges as a conscious action. Misconceptions are mistakes, but not all mistakes are misconceptions (Eryılmaz, 2002). Misconception is that the intuitive ideas that students have constructed for themselves as a result of experiences with their physical environment (Gilbert and Watts, 1983). Tall and Vinner (1981) formulated a distinction between formally defined mathematical concepts and the cognitive processes for which they are designed. The term concept image is used to describe the total cognitive organization associated with the concept, which includes all mental pictures and associated features and processes. This image is formed over time through all kinds of experiences that change as the individual encounters new stimuli and matures. The student's concept image is the form of the words he uses for his own explanation. Whether the definition was given to him or made by him, it may change from time to time. In this way, a personal concept definition may differ from a formal concept definition. For these reasons, the most important factor in the formation of misconceptions is the concept images that students have about the concept. Concept definition based on student's self-image does not accept that a square is a rectangle. A potential conflict factor occurs when the personal concept image conflicts with the formal concept definition itself. If a student is presented with an image that causes cognitive conflict with his own concept image, then he or she will experience difficulties (Kembitzky, 2009). Since students internalize their ideas about the concepts they have learned, it is very difficult to change their misconceptions. Once misunderstandings occur, they are highly resistant to change (Clement, 1982; Shaughnessy, 1981). This is because students are reluctant to change the wrongly constructed concepts (Kembitzky, 2009). Misconceptions prevent learning the truth of concepts and cause new misconceptions to occur. When the teacher is aware of possible concept images, he or she can rationalize conflict through confrontation and communication to bring false images to the surface (Tall & Vinner, 1981). With the student's reflection own self, be can balance the concept image that they will reconstruct with the mathematically accepted concept image. In the Luneta, (2015) study, most students in grade 12 did not understand many of the basic concepts in Euclidean transformation. Most of the errors are conceptual and suggested that the students did not understand the questions and did not know what to do as a result. It is also stated that when students lack conceptual knowledge, their results are so severe, and they hardly answer the questions in the exam. Luneta (2015) concluded that the majority of the students in the 12th grade were in second level of van Hiele geometry thinking model. Ningrum, Yulianti, Helingo and Budiarto (2018) indicated that 8th grade students had misconceptions in understanding the properties of rectangles. Students thought that rectangles were always regularly shaped. They assume that the rectangle was always in the same position and has the same shape as any other rectangle (Ningrum et al. 2018; Ryan & Williams, 2007). In the study of Ubuz (1999) in the 10th and 11th grades, the biggest reason for the mistakes of the students was that the visuality level was not fully formed in the Van Hiele geometry thinking model. Students evaluated shapes not by their features but by their appearance. Pickreign (2007) investigated 40 pre-service teachers' perceptions of the relationships between parallelograms and the properties of parallelograms; found that only 9 pre-service teachers made adequate definitions for rectangle and only one for rhombus. He stated that the definitions made were generally at the lowest level of Van Hiele geometric thinking. Keiser (2003) determined that students have difficulties in the concept of angle. He attributed the reason for this to the fact that the definitions related to the concept of angle are given in a single way in the textbooks. Fujita (2012), in his study with 19 new teachers, found that most of the participants knew the definitions correctly, but they evaluated the quadrilaterals by considering the prototype examples and shapes. Fonseca and Cunha (2011) examined the approaches of 79 pre-service teachers to geometric subjects. It was observed that most of the participants knew the definitions, made analogies, understood the operations and were able to apply geometric rules. However, he found that they had difficulties in visualizing, reasoning, explaining the thinking style he was processing, and making connections with various mathematical subjects. Ward (2004) found that students have problems even in basic polygon definitions.

Most of the studies examining the classification, errors, misconceptions, and van Hiele geometry thinking levels of students regarding geometric shapes were conducted with 6, 7 and 8 grades and high school students. In addition, many mistakes and misconceptions were studied with pre-service teachers. At the 4th grade level, studies examining students' knowledge of geometric shapes (classification, error, misconception, and van Hiele geometry thinking level) are almost non-existent. The aim of this study is to obtain information about how the students make the definitions, mistakes, misconceptions, and van Hiele geometry thinking levels by using the definitions of the 4th grade students for the geometric shapes of rectangle, square, isosceles triangle, equilateral triangle, and equilateral triangle.

2. Method

2.1. Research model

The data of the study were collected from the students in the form of documents. The analysis of the answers given by the students to the questions about geometric shapes was made with the case study pattern, which is one of the qualitative methods. There are multiple definitions of the case study. In qualitative research, qualitative data collection methods such as observation, interview and document analysis are used, and perceptions and events are presented in a realistic and holistic way in the ordinary environment (Yıldırım & Şimşek, 2008). The research that focuses on a determined aspect of a current event, phenomenon or situation is called a case study (Yin 2009). According to McMillan (2004), a case study is an in-depth study of the places and times of more than one event, setting, and individual.

2.2. Research sample

The study group of the research consisted of 156 students selected by systematic sampling method among 780 students in the 4th grade. In systematic sampling, all of the items needed for the sample are systematically selected from the sampling frame (Creswell & Clark, 2016). This is the path for the selection of 156 students in this study. The sampling interval is 780/156=5. Students are divided into groups of 5 people. One student was randomly selected from the first 5 people group. Then, the 5th student right after that student is selected and this continues until the 156th student is selected (Bernard, 2011). A possible bias is prevented by systematic sampling at regular intervals (Cochran, 2007).

2.3. Data Collection Tools

In the questions developed to collect data, it was tried to reveal the types of errors, misconceptions and definitions that students may have about geometric shapes (rectangle, square, equilateral triangle, isosceles triangle, and scalene triangle). The data of the study were collected with 18 questions about rectangular, square and triangular geometric shapes. The questions detailed in Table 1 include definition, properties, and drawing. In the questions, the reasons for the answers given by the students were asked. In addition, students were asked whether they were sure of their answers or not. During the preparation of the questions, the opinions of three field experts were considered.

Table 1: Contents of the questions

Task	Content of the questions	Number of questions
1	Definitions of square, rectangle, isosceles triangle, equilateral triangle, and scalene triangle	5
2	Properties and relationship between geometric shapes	8
3	Drawings of geometric shapes	5

2.4. Analysis of data

18 questions about geometric shapes were answered by 4th grade students in the form of written documents. In the study, the data obtained from the questions containing the definitions and properties of rectangle, square, equilateral triangle, isosceles triangle, and scalene triangle were evaluated by content analysis. Thus, the hierarchical and partitional definitions that students have about these concepts were tried to be revealed. In the partial definition, almost all the features of the relevant geometric shape are specified, while in the hierarchical definition, special cases of the geometric shape are mentioned. The fact that a square is also a rectangle, parallelogram, and trapezoid, quadrilateral is related to the hierarchical definition. Features such as the sides of the square being of equal length, the sides being perpendicular to each other, four sides and corners are included in the partitional definition. The correct hierarchical and partitional definitions given by the students for the geometric shape were analyzed and the appropriate van Hiele geometry thinking level was determined. In the study, misconceptions were detected by the simultaneous realization of the following 3 situations:
1. Incorrect definition or incorrect feature writing for a geometric figure,
2. Consistent repetition of incorrect or faulty features,
3. Be sure of the answer.

In the student's response, only the first two of the three situations may have occurred. However, if the student is not sure of his/her answer, there is no misconception. Or the student can be sure of the wrong answer given. However, if the wrong answer is not consistent with the answers in the other questions, it is not seen as a misconception. The data obtained in the study were evaluated and classified by two researchers. In this section, the percentage agreement formula is used to determine how compatible these two people are with each other. The percentage of agreement, in other words, the reliability of the study was calculated using the formula of Miles and Huberman (1994) (P=(Nax100) (Na+Nd) (P: percent agreement, Na: agreement amount, Nd: amount of disagreement)). The agreement percentages of two researchers who made the evaluation were found to be 89% at the beginning. With the collaboration of two researchers, agreement was achieved for different results.

3. Findings

In this section, the findings, and interpretations of the analysis of the data obtained for the rectangle, square and triangle are given, respectively. The answers given by the 4th grade students (156 students) to the question of what are rectangles, squares, equilateral triangles, isosceles triangles, and scalene triangles; were analyzed into the categories of correct, incomplete, incorrect and no answer. The results obtained here are presented in Table 2.

Table 2: Examination of the answers given by the students to the question of what are rectangle, square, equilateral triangle, isosceles triangle, and scalene triangle according to the categories of correct, incomplete, incorrect and no answer.

Geometric shapes	Correct definition number of students		Number of incomplete definition students	%	Incorrect or very insufficient definition number of students	%	No answer Number of students	%	Total number of students
Rectangle	11	%7	80	%51,2	55	%35,2	10	%6,4	156
Square	24	%15,3	83	%53,2	42	%26,9	7	%4,5	156
isosceles triangle	56	%35,8	50	%32	35	%22,4	15	%9,6	156
Equilateral triangle	61	%39,1	39	%25	31	%20,6	25	%16	156
Scalene Triangle	55	%35,2	40	%25,6	35	%22,4	26	%16,6	156

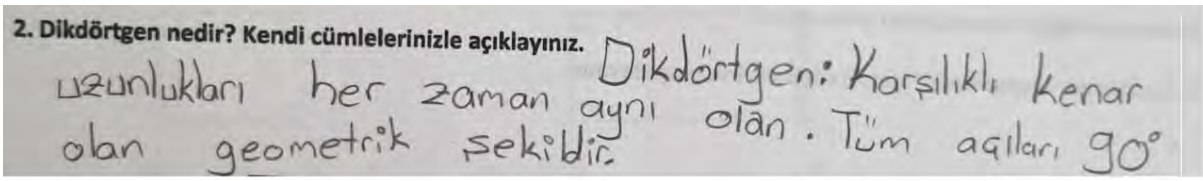
Definitions related to triangles have higher accuracy rates than definitions of rectangle and square. The reason for this may be that the definitions of rectangle and square at the 4th grade require more detail than the definitions of equilateral, isosceles and equilateral triangles. For example, in the definition of a square, having equal sides, having four sides, and having 90 degrees of angles includes more features than the definition of isosceles triangle. The most correct definitions were made for equilateral triangles (39.1%) and the least correct definitions were made for rectangles (7%). Similarly, wrong or very inadequate definitions were made for the rectangle (35.2%) the most and for the equilateral (20.6%) the least.

3.1. Findings for the rectangle

Table 3: Examination of the definition types of students who answered correctly to the definition of rectangle

Definition type	Expressions	Number of students	van-Hiele level	Number of students
Hierarchical definition	It is a special quadrilateral.	1	Abstraction	2
Hierarchical and partitional definition	Opposite sides are equal and parallel, sides are perpendicular to each other.	1		
Partitional definition	A geometric shape whose opposite sides are of equal length and all angles are 90 degrees.	6	Analysis	9
	A shape whose opposite side lengths are equal and parallel, all angles are 90 degrees.	2		
	A geometric shape with four sides, four corners, four right angles, two opposite sides, and two diagonals.	1		

11 students gave the correct definition to the question of what is a rectangle. In Table 3, the numbers of students who gave correct definitions are given according to their definition types. A student gave a hierarchical definition by stating that a rectangle is a quadrilateral. 9 students gave partitional definitions. In the definition of 1 student, both hierarchical and partitional definitions appear. A student's partitional definition is given in Figure 1.



2. What is a rectangle? Exspress it in your own words.

Answer: Rectangle: Opposing side lengths are always the same. It is a geometric shape with all angles of 90.°

Figure 1: Partial definition of student for rectangle

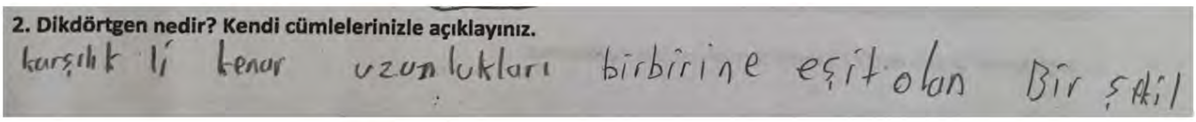
80 students gave incomplete partitional definitions for the rectangular geometric shape. Incomplete partitional definitions with similar structures are classified in Table 4. There are 34 (21,7%) students who said that the most

striking situation in the incomplete partitional definitions is that the rectangle has 2 long and 2 short sides. This situation is seen as an obstacle for students to reach the level of analysis, which is the 3rd level of van Hiele. Grade 4 students are expected to see the difference between an object and a geometric figure. According to incomplete definitions, the most problematic concept is angle. Only 3 students mentioned that the angles of the rectangle are 90 degrees. However, there are overall expectations related to the angle in the mathematics teaching program in the 3rd and 4th grades. 11 students out of 156 students who participated in the study gave the correct definition. It can be said that the students who gave the correct definition mentioned the concept of angle. Accordingly, only 14 students out of 156 mentioned the angle feature of the rectangle.

Table 4: Expressions stated in the definitions of students who gave incomplete partitional definitions for rectangle.

Expressions for incomplete partitional definitions for rectangle	Number
A quadrilateral with opposite sides equal in length and opposite sides parallel.	3
It is a quadrilateral with opposite sides equal to each other.	9
It is a shape with 2 short sides and 2 long sides.	31
It is a four-sided geometric shape with two equal long and 2 short sides.	3
A four-sided, four-sided shape whose opposite sides are equal.	3
A shape with four corners, four sides, and angles of 90 degrees.	2
All angles are 90 degrees.	1
It has 4 vertices and 4 edges.	5
A figure with 4 sides, 4 vertices, 4 angles, and opposite sides parallel.	1
A shape with four equal sides and opposite sides.	2
It is a shape with opposite sides equal.	10
Opposite sides are equal and the sum of the interior angles is 360 degrees.	4
It is a geometric shape with four corners and four sides.	2
A shape whose interior angles add up to 360 degrees.	1
Its sides are parallel to each other.	2
It is a polygon with four sides.	1
Sum	80

In Figure 2, there is an answer of a student who gave an incomplete partitional definition. This student sees the rectangle as a shape with only opposite sides equal. Other features are not mentioned. This student did not specify an angle in the rectangle drawing.



2. What is a rectangle? Explain in your own words.

Answer: A shape with opposite side lengths equal to each other.

Figure 2: The response of a student who gave an incomplete partitional definition

In Table 5, the definitions and numbers of students who gave wrong or insufficient definitions for the rectangular geometric shape are given.

Table 5: Incorrect or very insufficient definitions for rectangular geometric shape

Incorrect or very insufficient definitions for rectangular geometric shape	Number
It is a rectangular geometric object.	10
It is a geometric solid with opposite sides equal.	2
It is a geometric object consisting of 4 sides and 4 vertices.	3
It is a geometric object with opposite sides parallel to each other.	1
It is a geometric object with a rectangular, four-cornered, and four-sided surface.	1
A shape with a long side and a short side.	2
It is a rectangular geometric shape.	7
A rectangle is a shape with 4 corners, 4 sides, and 4 edges.	6
It has two sides, two corners.	1
It is a slightly longer shape than a square.	21
Opposite surfaces are an equal shape.	1
Sum	55

There are 7 students who see the rectangle as a geometric object. 21 students gave a shape definition slightly longer than a square. Students who gave wrong or very inadequate definitions could not make the concept of rectangle meaningful in the learning process. It can be said that 55 (%35,2) students who defined rectangular geometric shapes in this table did not reach the visualization stage of van Hiele. 10 students did not write an answer to the question of what is a rectangle.

3.2. Findings for the square

When Table 2 is examined, 24 (15.3%) students out of 156 students gave the correct answer to the question of what is a square. The answers given are hierarchical; partition; hierarchical and partitional definitions are considered. In Table 6, 24 students who gave the correct answer to the question of what is a square showed that hierarchical (9); partition (10); hierarchical and partitional (5) definitions are specified.

Table 6: Examination of the definition types of students who answered the square question correctly

<table>
<tr><th>Definition type</th><th>Expressions</th><th>Number</th><th>Van Hiele level</th><th>Number</th></tr>
<tr><td rowspan="4">hierarchical definition</td><td>The square is the special case of the rectangle.</td><td>3</td><td rowspan="4">Abstraction</td><td rowspan="4">14</td></tr>
<tr><td>A square is a special quadrilateral.</td><td>4</td></tr>
<tr><td>A square is a parallelogram.</td><td>1</td></tr>
<tr><td>A square is a rhombus.</td><td>1</td></tr>
</table>

Hierarchical and partitional definition	A square is a quadrilateral with four corners, four sides of equal length, and sides perpendicular to each other.	2		
	A square is a rectangle with four corners, four sides of equal length, and sides perpendicular to each other.	3		
Partial definition	It has four corners. It has four edges. The edges are perpendicular to each other. The sides are equal.	10	Analaysis	10

As a hierarchical definition in Table 6, 3 students saw that the square is a special form of the rectangle. 4 students gave the definition that a square is a rectangle. One student gave the definition of a square parallelogram and another student gave a definition of a square rhombus. 5 students gave hierarchical and partitional definitions together. hierarchical for square; 14 students giving hierarchical and partitional definitions are at van Hiele's informal deduction (abstraction) level.

For the correct partitional definition of the square, at least four corners, four equal sides, and the features that the sides are perpendicular to each other are accepted. In Table 6, 10 students out of 156 students who participated in the study gave the correct partitional definition. Definitions are at van Heile's level of analysis (description).
Four students gave a hierarchical and partitional definition for the square. Two of these students have a square with four corners, four equal-length sides, and a quadrilateral whose sides are perpendicular to each other. 2 students gave the definition that a square is a rhombus with four corners, four equal-length sides, and sides perpendicular to each other. In Figure 3, there is the answer of a student who gave the correct hierarchical definition for the square.

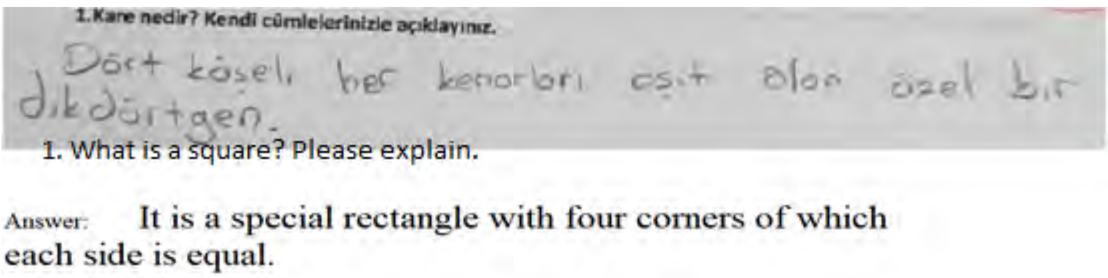


Figure 3: The answer of a student who gave the correct hierarchical definition for the square

83 students gave incomplete partitional definitions. In Table 7, the content of the definitions of the students who gave incomplete partitional definitions for the square is indicated.

Table 7: Statements in the definitions of students who gave incomplete partitional definitions for the square

Expressions for missing partitional definitions for square	Number
Side lengths are equal, opposite sides are parallel and shape.	2
A shape with 4 vertices and sides, opposite sides parallel to each other.	4
Its sides are equal to each other.	9
It is a geometric shape with equal diagonals.	4
A shape whose diagonals are equal and bisect each other.	1
It has 4 vertices and 4 edges.	12
It is a shape with opposite sides equal.	5

It is a shape with four corners and four equal sides.	9
The sides are perpendicular to each other.	2
It is a shape with four sides.	5
A shape whose four sides and interior angles add up to 360 degrees.	3
It is a geometric shape with four corners.	3
A shape whose four corners and interior angles add up to 360 degrees.	2
All sides are equal and the sum of the interior angles is 360 degrees.	4
All angles are 90 degrees.	2
A shape with four corners, four sides, and angles of 90 degrees.	4
It is a shape with four corners and all angles of 90 degrees.	5
It has four sides and 2 diagonals.	4
A geometric figure with four equal sides and diagonals bisecting each other.	1
A shape with four sides and equal interior angles.	2
Total	83

The students who gave the missing partitional definition in Table 7 could not see all the features for the square together. For example, 12 students stated in their definitions that a square is only four-cornered and four-sided. These 12 students did not mention that the sides are equal to each other and that each interior angle is 90 degrees. The number of students who gave incomplete partitional definitions was 83 (53.2%).

According to Table 2, 42 (26.9%) students gave wrong or very inadequate definitions. The expressions mentioned by these students in their definitions are given in Table 8.

Table 8: Incorrect or very insufficient definitions for the square geometric shape

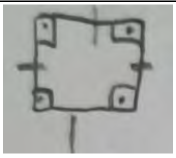
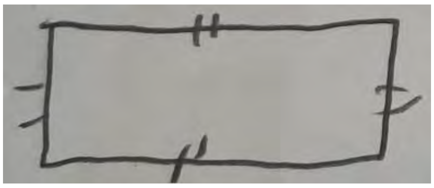
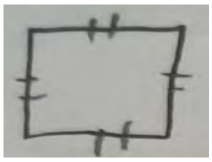
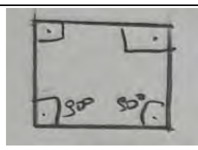
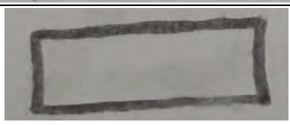
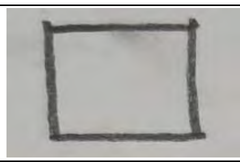
Incorrect or very insufficient definitions for the square geometric shape	Number
A square is a geometric object.	8
A Square is a cube.	3
A square is a 4-sided geometric object.	6
A square is a geometric object consisting of 4 edges and 4 corners.	1
A square is a geometric object with 4 corners.	3
A Square is the geometry.	2
A Square is geometric shape.	5
A Square is a shape.	5
A square is a 2-sided shape.	2
A square is a shape with 2 corners.	1
A square is a shape with 2 sides and 2 corners.	1
A square is a shape with two surface.	1
A shape with 4 corners the same length.	5
Total	42

21 students agreed that the square is a geometric object. It can be said that 42 students who defined the square in this table did not reach the visualization stage of van Hiele.
In Table 2, 7 students did not answer the question of what is a square.

3.3. Student's rectangle and square drawings

Rectangle and square lawns of the students are classified in Table 9. When the drawings were examined, 4 different drawings were determined for rectangle and square.

Table 9: Rectangle and square drawings

Rectangle	Number	Square	Number
	2		2
	2		2
	11		11
	141		141

All the students participating in the study made drawings. In the study in which 156 students participated, the number of students who drew correctly for square and rectangle was 2 (%1,2). These students paid attention to the side lengths and angles while drawing. For square and rectangle, 2 students only paid attention to the side lengths and did not show the verticality of the sides in their drawings. 11 students did not specify anything about the side lengths by only showing the angle in their square and rectangle drawings. However, the number of students stating that the angles are 90 degrees in the correct or incomplete partitional definitions is 14 for the rectangle. The number of students showing the 90 degree angle to their drawings is 13. 31 students mentioned that their angles are 90 degrees in their correct or incomplete partitional definitions. In the drawings, only 13 students showed that the edges are perpendicular. 141 students drew only one quadrilateral for rectangle and square.

3.4. Misconceptions for square and rectangle

Table 10: Misconceptions for square and rectangle

Incorrect results	Number of students with errors	Number of students with misconceptions
Not accepting a square as a rectangle	120	7
Considering rectangle and square as objects	33	5

Table 10 shows the errors and misconceptions that students have for rectangles and squares. Accordingly, it is a mistake not to accept the square as a rectangle at most. 120 students do not see the square as a rectangle. 7 of these 120 students have misconceptions. These 7 students did not accept the square as a rectangle, because they wrote stable answers such as square or all sides are equal. They did not think that the sides of the rectangle would be equal. For these students, the rectangle has two short and two long sides. They stated that they were sure of their answers. In the Figure 4 below, a student with this misconception answered the question of what is a rectangle, it is a slightly longer shape than a square. The photograph of this student's answer to the question "Which of them are rectangles, circle them?" is given in figure 4 below.

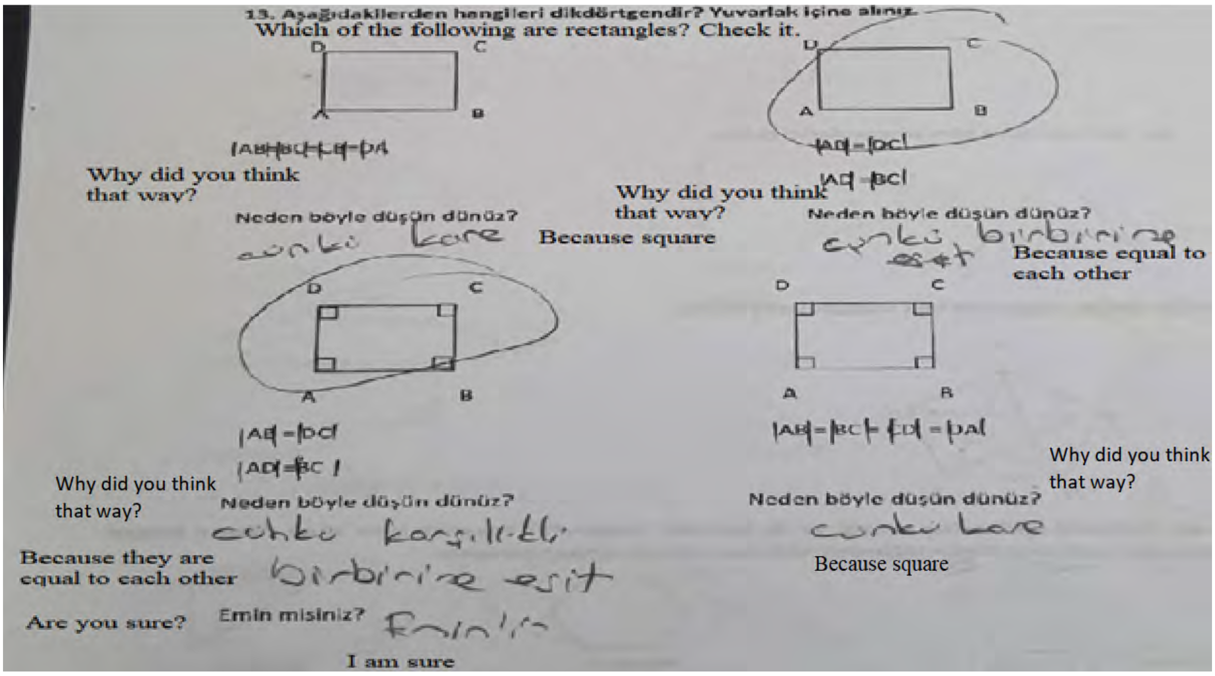


Figure 4: A student's answer to the question "Which of the following are rectangles"?

In figure 4, the student did not mark the square shape as he did not see it as a rectangle. He said that the sides are equal and square to each other as a reason. He also stated that he was sure of his answer. In addition, he considered the quadrilateral without angle information as a rectangle. He stated that the opposite sides are equal to each other for the reason of the shape he marked as a rectangle. This student is not aware that his correct generalization is valid in the square.

33 students saw these geometric shapes as objects in their definitions of rectangles or squares. 5 of these 33 students have the misconception that geometric shape is an object. These students used the word object in their definitions for both rectangle and square. These students stated that they were sure of the wrong answer they gave by using words such as cube and object frequently.

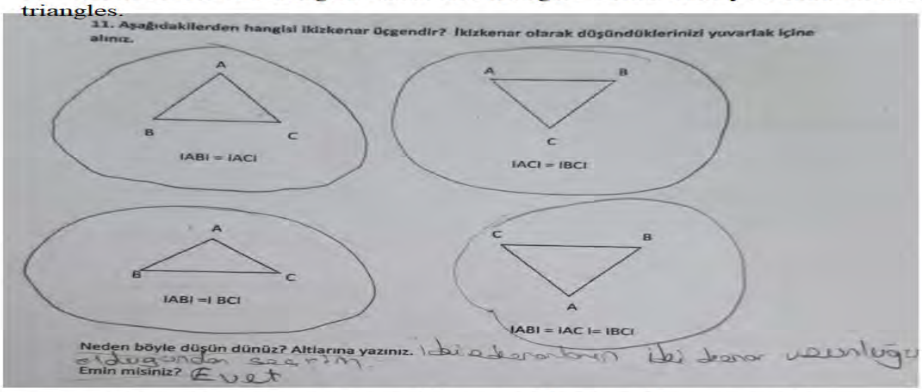
3.5. Findings about isosceles, equilateral, and scalene triangles

In Table 11, 61 students gave correct definitions for equilateral triangles, 56 students for isosceles triangles, and 55 students for scalene triangles. The correct definitions given are at van Hiele's level of analysis. An abstraction level definition is not given in the definitions. However, for the question of which of the following triangles is isosceles, only 5 students out of 156 stated that equilateral triangle is isosceles triangle. The answer of one of these students to this question is in Figure 5 below.

Table 11: Correct definitions for isosceles, equilateral, and scalene triangles

Equilateral triangle	Number of students	Isosceles Triangle	Number of students	Scalene triangle	Number of students
A triangle with all sides and angles equal.	18	Three-sided polygon with two sides of equal length	1	Triangle with three sides and different sides	5
A triangle with three equal sides, 60 degrees, and 180 degrees for the sum of the interior angles	8	Three-sided polygon with two sides and two same angles	10	A triangle with all sides and angles different.	17
Triangle with three equal sides	23	Triangle with two equal sides	39	Triangle with different sides	25
Triangle with each angle of 60 degrees	10	Triangle with two equal sides and two equal angles	5	Triangle with different angles	5
A three-sided geometric shape with equal sides and angles	2	It is a 3 sided shape with two equal sides and 3 corners.	1	Three sided geometric figure with unequal sides and angles	3
Total	61 %39,1		56 %35,8		55 %35,2

11. Which of the following are isosceles triangle? Circle what you think of as an isosceles triangles.



Answer: In an isosceles, I choose it because the two sides are equal in length.

Are you sure?

Answer: Yes

Figure 5: A student's answer to the question "which of the following triangles is isosceles"?

3.6. Expressions specified in the definitions of students who gave incomplete partitional definitions

Table 12: Incomplete partitional definitions given by students

Equilateral triangle	Number of students	Isosceles triangle	Number of students	Scalene triangle	Number of students
All sides and angles are equal	14	Triangle with the same side lengths	5	A shape with different sides and angles	11
A shape with all sides equal	25	Triangle with two equal sides and a third side of different measure	10	Shape with unequal sides	29
		A shape with two equal sides and two equal angles	3		
		shape with two equal sides	29		
		A geometric shape with two equal sides and an apex angle	3		
Total	39		50		40

In the incomplete definitions given for the triangle, the number of definitions such as a shape with equal sides or a different shape is 114. These students did not use expressions such as triangle or three sides in their definitions. Only 15 students who defined isosceles triangles used triangle expressions. 39 students for equilateral triangle, 50 students for isosceles triangle and 40 students for scalene triangle gave incomplete partitional definitions. It is observed that the analysis level of these students, which is Van Hiele's 2 levels, did not occur. 4th grade students are expected to reach this level comfortably.

In Table 13, 31 students gave wrong or very insufficient definitions for equilateral triangle, 35 students for isosceles triangle and 35 students for equilateral triangle. It can be said that visualization, which is the first stage of van Hiele, did not occur in students who gave wrong or insufficient answers.

Table 13: Wrong or very insufficient definitions

Equilateral triangle	Number	Isosceles triangle	Number	Scalene triangle	Number
Opposite sides are equal to each other.	6	Opposite sides are equal to each other.	10	Shape with three corners	6
Shape with 3 corners	5	Triangle with equal opposite angles	2	a three-sided shape	2
Triangle and triangular shape	5	A shape with three corners	8	Object with different sides	10
An object with three equal sides	10	Ship shape with three sides and three corners	3	3 corners and 3sided shape	11
Shape with sides and corners	5	Geometric solid with sides and corners	3	Shape with sides and corners	4

A shape with 4 corners and four equal sides	1	Three-sided geometric solid with equal base angles	3	a four-sided triangle	1
		A shape with 2 sides, 2 corners	3	A different shape with 4 corners and four sides	1
		Shape with 4 sides and opposite sides equal	1		
Total	31		35		35

Table 14: Students' drawings of triangles

Drawing type	Equilateral triangle (number of students)	Isosceles triangle (number of students)	Scalene triangle (number of students)
Correct drawing	25	23	20
Wrong Drawing	131	133	136

In Table 14, the most correct drawings were for equilateral triangles (25 students). However, the number of students making the correct definition for the equilateral triangle is 61 in Table 2. Some of the students did not show that the edges are not equal on their drawings. A similar situation has been experienced for isosceles and scalene triangles. 23 students drew an isosceles triangle correctly. However, 56 students gave the correct definition for isosceles triangle. 20 students drew a scalene triangle correctly. However, 55 students gave the correct definition for the scalene triangle. Although the students made a correct definition, they could not reflect this situation in their drawings.

3.7. Misconceptions for triangles

Table 15: Misconceptions identified for triangles

Incorrect results	Number of students with errors	Number of students with misconceptions
Drawing a triangle as a quadrilateral	2	1
Seeing the triangle geometric shape as a geometric object	11	5
Not seeing equilateral triangles as isosceles	151	12

Misconceptions about equilateral, isosceles and scalene triangles were detected in the students participating in the study. 151 of the 156 students who participated in the study do not see the equilateral triangle as an isosceles triangle. 12 of these 151 students have misconceptions. The answer of one of the students with this misconception is given in Figure 6.

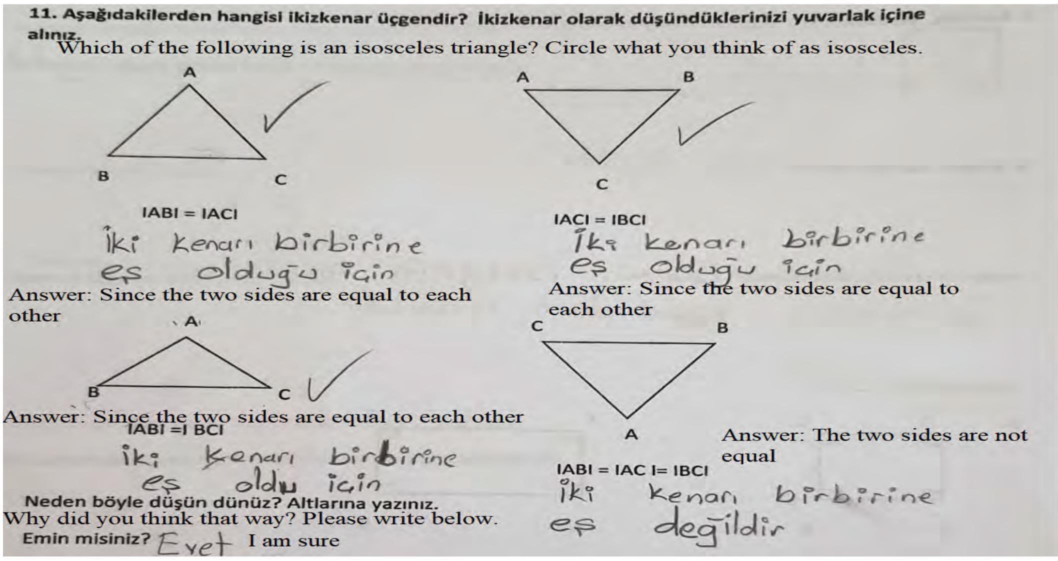


Figure 6: The mistake of not seeing the equilateral triangle as an isosceles triangle

This student does not see the geometric figure with three equal sides as isosceles in figure 6. He said that "the two sides are not equal to each other" as a reason. In other options, only two equal sides are given as isosceles. The student said that triangles with only two equal sides are isosceles because they have two equal sides. At the same time, the student has such a misconception because he cannot make the necessary analysis on the equilateral triangle. It can be said that this student, who stated that he was sure of his answer, had the misconception of not seeing the equilateral triangle as an isosceles.

11 students used the word "object" for the triangle geometric shape in their student definitions. 5 of these students saw these geometric shapes as objects in all definitions of rectangle, square and triangle. These 5 students have the misconception of seeing the geometric shape of the triangle as an object.

In the students' drawings of triangles, 2 students drew some of the equilateral, isosceles and scalene triangle types as quadrilaterals. One of these 2 students used expressions such as quadrilateral in all triangle drawings and quadrilateral and quadrilateral in their definitions. It can be said that this student has a misconception on this subject.

Conclusion

In the study, by using the definitions of the 4th grade students for the geometric shapes of rectangle, square, isosceles triangle, equilateral triangle and equilateral triangle, it was tried to obtain information about how the students made the definitions, mistakes, misconceptions and van Hiele geometry thinking levels. In the study, 11 (7%) students out of 156 students made the correct definition of the rectangle. Two of these students gave definitions at the 3rd level (abstraction) of Van Hiele geometry thinking level and 9 students at the 2nd level (analysis). Fujita and Jones (2007), Fujita (2012) stated that students have difficulties in hierarchical classifications. In the study of Öksüz and Başışık (2019), 14.5% of 5th grade students concluded that they defined the rectangle correctly. 14 students mentioned in their definitions that the sides of the rectangle are perpendicular to each other. Clements and Battista (1992) stated that students do not pay attention to orthogonality. 51.2% of the students who participated in this study gave an incomplete definition for the rectangular geometric shape. In the missing definitions, they could not give all the necessary and sufficient conditions for the rectangle together. It can be said that the 4th grade students who gave incomplete definitions could not reach the analysis level, which is the second level of Van Hiele. These students could not see all the features for defining rectangle. The most common definition in incomplete definitions is that it has two short and two long sides. 48 students gave definitions in this way. Berkün (2011) stated in his study that the majority of students had such an incomplete definition. In Berkün (2011) study, 41.6% of these students could not give an incorrect definition for the rectangle.

In the students' rectangular and square drawings, only 2 students drew correctly for both. 141 students drew only a quadrilateral. In these drawings, the rectangle is drawn longer than the square. It was observed that these students did not consider the properties for rectangle and square. In this case, it can be said that these 141 students are at the visual level.

In the study, 24 students (15.3%) gave the correct definition for the square. 9 of these students gave hierarchical, 5 hierarchical and partitional definitions, and 10 students gave partitional definitions. 83 (53.2%) students gave an incomplete definition for the square. Since these students could not adequately analyze the concept of square geometric shape, they could not see the features required for square together. It is seen that these students did not reach the level of analysis, which is Van Hiele's 2 levels. In the study of Öksüz and Başışık (2019), 15% of the students (5th grade) correctly stated the characteristics that the square should have. The most striking situation in the incomplete definitions of the students is that they did not see that the angles were 90 degrees. Of the 83 students who gave an incomplete definition for the square, only 15 students said that the angles were 90 degrees. The rate of students who mentioned the concept of angle together with the students who gave the correct definition is 25%. Berkün (2011); Öksüz and Başışık (2019) reached similar conclusions in their studies.

In this study, two misconceptions were identified for rectangle and square. The first of these misconceptions is not seeing the square as a rectangle. 120 (76.9%) students do not see the square as a rectangle. 7 (4.4%) of these students who have this error have misconceptions. Okazaki and Fujita (2007); Aktaş and Aktaş (2012) stated that most of the students had difficulties in perceiving the square as a rectangle. 33 (21.1%) students see squares andrectanglese as objects. They emphasized that they were objects in their definitions. It was determined that 5 of these students had misconceptions.

In the study, 39.1% of the students gave correct definitions for equilateral triangles, 35.8% for isosceles and 35.2% for scalene triangles. Of the 4th grade students participating in the study, 25 (16%) drew equilateral triangles, 23 (14.7%) an isosceles triangle, and 20 (12.8%) scaled triangles. These students said the features that should be in the definitions. However, they did not reflect these features sufficiently in their drawings.

Definitions related to triangles are more accurate than definitions of squares and rectangles. This is because there are less adequate descriptive properties for triangle varieties than for square rectangles. For example, the definition of at least two equal sides is sufficient for an isosceles triangle. In definitions of isosceles triangles, definitions such as triangle with two equal sides are generally used. 151 (96.7%) of the students who participated in the study did not see the equilateral triangle as an isosceles triangle. In addition, 12 of these students (7.6%) have misconceptions. Two of the students participating in the study drew quadrilaterals in all triangle drawings. 1 of these students has a misconception. This student stated that he was sure by mentioning 4 edges and four corners in the definitions. 11 students used the concept of geometric object in their definitions for triangle. 5 of these students used the concept of object in square and rectangle similarly and mentioned that they were sure. It can be mentioned that there are misconceptions in these students.

In the study, the knowledge of 4th grade students about the concepts of square, rectangle, equilateral triangle, isosceles triangle and scalene triangle was examined. According to the results obtained, it can be said that the students have serious problems with these concepts. On the other hand, 4th grade students can classify the properties of geometric shapes (MEB, 2018). Students' difficulties in learning geometry are related to their inability to understand and correctly interpret geometric concepts and their weaknesses in deductive reasoning (Gal & Linchevski, 2010; Miyazaki, Kimiho, Katoh, Arai, Ogihara, Oguchi, Morozumi, Kon, & Komatsu, 2012). Starting from the preschool period, mathematical concepts should be embodied. The cause-effect relationship is fundamental to induction and deduction. The role of teachers in reaching abstract mathematical concepts is very high. Toluk et al. (2002) attributed the inability of students to form such relationships on their own to the absence of classification activities in mathematics education in Turkey, and therefore not drawing students' attention to shapes and relationships between shapes. Therefore, students state that they see geometric shapes only separately and independently of each other. This situation creates an obstacle for making inference, which is van Hiele's 3 levels. Failure to provide adequate teaching environments for students to construct concepts meaningfully and teaching without associating them with their prior knowledge (memorization) may cause students to have serious problems with geometry. The fact that the stages that reveal metacognitive level behaviors in Bloom's taxonomy are not included in the curriculum or are few in number creates problems in the learning of students.

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