Education Quarterly Reviews

Gök, Mustafa, and Demir, Nihal. (2021), The Opinions of Mathematics Teachers about Using Mathematical Modeling in the Solution of Daily Life Problems and an Application of the Chinese Remainder Theorem. In: *Education Quarterly Reviews*, Vol.4, No.3, 100-116.

ISSN 2621-5799

DOI: 10.31014/aior.1993.04.03.322

The online version of this article can be found at:
https://www.asianinstituteofresearch.org/

Published by:
The Asian Institute of Research

The Asian Institute of Research
Education Quarterly Reviews
Vol.4, No.3, 2021: 100-116
ISSN 2621-5799

DOI: 10.31014/aior.1993.04.03.322

The Opinions of Mathematics Teachers about Using Mathematical Modeling in the Solution of Daily Life Problems and an Application of the Chinese Remainder Theorem

Mustafa Gök[1], Nihal Demir[2]

[1] Faculty of Education, Van Yuzuncu Yil University, Van, Turkey
[2] Hurriyet Middle School, Van, Turkey

Correspondence: Mustafa Gök, Faculty of Education, Van Yuzuncu Yil University, 65080, Campus, Van, Turkey Tel:+905457264621. E-mail: mustafagok@yyu.edu.tr

Abstract
The opinions of mathematics teachers about using mathematical modeling (MM) in daily life problems and their use of MM in solving a daily life problem were examined within the scope of the graduate-level MM course in this study. The research was designed as a case study. Participants are five mathematics teachers selected by the purposive sampling method. The data were obtained through a structured form containing questions about a daily life problem and using MM in a daily life problem. Teachers were given one week to answer the questions on this form. Descriptive analysis was performed on the data obtained from the teachers' problem solving, and content analysis was carried out on the data containing the teachers' opinions. The findings determined that although the modeling processes were generally used appropriately by the teachers in solving a daily life problem, the process of understanding the problem was quickly mentioned, the teachers had difficulties in giving the most effective solution during the evaluation stage, and the communication stage was mostly ignored. It was also found out that MM could be beneficial in many ways in solving daily life problems. On the other hand, it was indicated that there may be difficulties in terms of student-teacher-environment and that teachers give very limited space to MM in classroom practices. It can be stated that the tendency of teachers to use the approaches they are accustomed to instead of MM in the solution of daily life problems is effective in the emergence of this situation.

Keywords: Mathematical Modeling, Daily Life Problems, Problem-Solving, Teacher Training

1. Introduction

Today, studies to solve daily life problems gain importance in mathematics education. Factors such as the encouragement of the use of different representations (verbal, symbolic, graphic, and table) of daily life problems, the possibility of students to experience these problems, and the fact that they can be presented in many different contexts (The National Council of Teachers of Mathematics [NCTM], 2000) have been effective in the emergence of this situation. In support of this, the indicator of mathematical competence in the mathematics curriculum is associated with solving daily life problems (Ministry of National Education [MoNE], 2018).

In many studies in the literature, it is emphasized that daily life problems should be used in mathematics teaching (e.g., Karataş, & Güven, 2010; Verschaffel et al., 2000). On the other hand, it is stated that students experience many difficulties in solving daily life problems (Berenger, 2018; İncikabı, Ayanoğlu, & Uysal, 2020; Vijayan, & Joshith, 2018).

An attempt to overcome the difficulties experienced in solving daily life problems in mathematics education constitutes the motivation of this study. One way to overcome this problem is suggested to be mathematical modeling (MM).

It is a known fact that MM has a close relationship with daily life problems (Ang, 2009). It was justified in many studies that MM offers a way to solve daily life problems (e.g., Berry, & Nyman, 1998; Haines, & Crouch, 2007; Schroeder, & Lester, 1989). On the other hand, the prerequisite for the effective use of MM in real-life problems in mathematics education lies in teachers' experiences and perceptions of MM. It is important to note that MM education is included as a compulsory course at the undergraduate level in teacher education programs renewed in 2018 in Turkey to raise this awareness (Demir, Ertem Akbaş, & Gök, 2021). Therefore, in the following chapters of the study, the relationship of MM with daily life problems and the use of MM in daily life problems in the context of teacher education are analyzed.

1.1 Relationship of Mathematical Modeling with Daily Life Problems

Mathematical modeling (MM) can be briefly defined as the art of applying mathematics to real-life problems to better understand a particular problem (Ang, 2009). In this context, it offers an effective approach in revealing the relationship between mathematics and real-life (Heymann, 2003).

The use of the MM approach in teaching is designed by MM activities. These activities are generally studied as problems of daily life. In other words, modeling activities should include situations that can be encountered and interpreted in real life (Bukova-Güzel, Tekin-Dede, Hıdıroğlu, Kula-Ünver, & Özaltun-Çelik, 2016). Thus, MM activities offer students a way to realize and understand different aspects of mathematics in real life, in addition to learning mathematics (Lingefjärd & Holmquist, 2005).

When MM processes are examined, it is seen that mathematical modeling is an important approach for problem-solving (PS). It also provides a new perspective for PS (Lesh, & Zawojewski, 2007). In this regard, school mathematics emphasizes that students should use mathematical relations and use mathematical models in PS processes (NCTM, 2000). In support of this, Han and Kim (2020) accepted MM proficiency as a subcomponent of mathematical PS proficiency. To this end, it can be concluded that modeling is part of the PS process (Hartono, 2020).

Distinguished mathematician George Polya (1973) conceptualized the PS process as four steps: understand the problem, devise a plan, carry out the plan, and look back. As a PS approach, the MM process was cascaded in different ways by different researchers (e.g., Ang, 2001; Berry, & Houston, 1995; Blum, & Leiß, 2007; Doerr, & Pratt, 2008; Hıdıroğlu, 2012; Mason, 1988). In line with the approach suggested by De Corte, Verschaffel, and Greer (2000), the steps of the modeling process are shown in Figure 1 within the scope of this study, which are understanding, modeling, mathematical analysis, interpretation, evaluation, and communication.

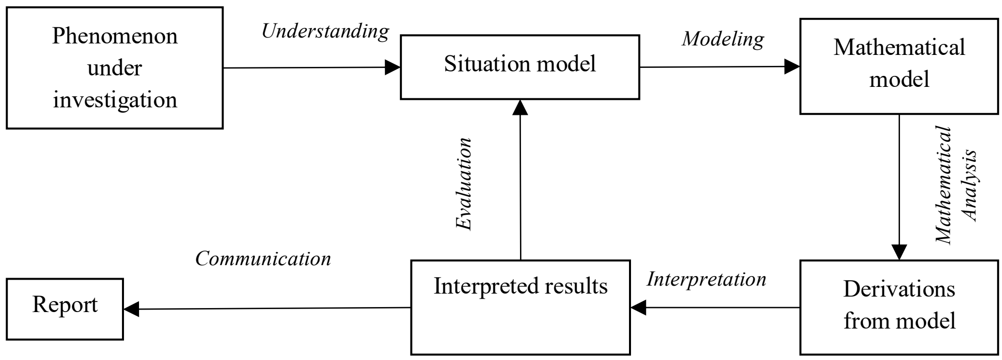

Figure 1: Schematic diagram of the modeling process (De Corte, Verschafeller, & Greer, 2000)

According to this approach, students first decide which elements specified in the problem to understand the phenomenon under investigation are necessary or less important. In this direction, they develop a situation model. In the next stage, the situation model is modeled by expressing it with mathematical equations according to the relevant elements, relations, and conditions, and thus a mathematical model is created. Then, mathematical analyzes are conducted on the mathematical model, and the derivations from the model are revealed. The results of the analysis are interpreted according to the problem situation, and interpreted results are obtained. It is evaluated by checking whether the interpreted mathematical results are appropriate and reasonable for the problem situation. Finally, the solution to the problem is reported at the communication stage.

1.2 Use of Mathematical Modeling in Daily Life Problems in The Context of Teacher Education

When the literature is reviewed, although MM offers a PS-oriented approach in general regarding teachers' use of MM in solving daily life problems (Hartono, 2020; Heymann, 2003; Lesh, & Zawojewski, 2007), there are difficulties experienced in the MM process (e.g., Hıdıroğlu et al., 2017; Işık, & Mercan, 2015; Kertil, Erbaş, & Çetinkaya, 2017). In this direction, in the research conducted by Aztekin and Taşpınar-Şeker (2015), it was determined that most of the studies on MM in Turkey were carried out with prospective teachers.

For example, Hıdıroğlu et al. (2017) examined the modeling process in seven stages (understanding the problem, choosing variables and making assumptions, mathematizing, constructing mathematical models and correlating them, working mathematically, interpreting solutions, validating the model) and found that prospective teachers had difficulties in many stages of MM in PS. Similarly, Kertil et al. (2017) concluded that prospective teachers had difficulties in interpreting the mathematical relationship in the MM process. In another study, it was found that secondary school mathematics teachers had general knowledge about models and modeling; however, it was observed that there were deficiencies in their knowledge about which examples can be qualified as models (Işık & Mercan, 2015).

In a study that reports how middle school mathematics teachers explain models, it is shown that teachers can have multiple model understandings that are active at different times and reflect various perspectives (Wilkerson et al., 2018). Furthermore, Anhalt and Cortez (2016) asserted that prospective teachers could develop an understanding of correct modeling with the given education.

When the literature is reviewed, it is understood that it is important to experience the MM approach by the teachers and reveal the solution processes and their opinions on the process. From this point of view, this study aims to examine the situations of mathematics teachers in solving a daily life problem within the scope of MM education they have received at the graduate level and investigate their opinions on the use of the MM approach in solving daily life problems.

The research questions to be answered in line with the purpose of the research are as follows:
1) What is the process of solving a daily life problem of mathematics teachers with MM?
2) What are the opinions of mathematics teachers about examining daily life problems in the context of MM?

2. Method

A case study, one of the qualitative research methods, was used in the study in which a solution to a daily life problem was sought through MM after MM training, and the opinions of teachers on the use of MM in the solution of daily life problems were expected to be revealed. A case study is defined as an in-depth description and examination of a limited system (Merriam, 2013). Thus, the unit of analysis in the study is the perceptions of teachers about using MM in daily life problems.

2.1 Study Group

The study participants consisted of teachers who took the MM course, and the participants were selected by purposive sampling (Christensen, Johnson & Turner, 2015). Mayring (2011) noted that a small number of participants in such studies allows researchers to quickly understand the event that needs to be described and analyze it faster. Therefore, in this study, data on the perceptions of five mathematics teachers were collected in line with the analysis unit. Table 1 contains participant demographics.

Table 1: Participant demographics

Participant Code	Gender	Graduated High School	Graduation Area	Type of Employed School	Graduated Faculty	Professional Experience
P1	Female	Anatolian High School	Science	Middle School	Faculty of Education	Two years
P2	Female	Anatolian Teacher Training High School	Science	Middle School	Faculty of Education	Five years
P3	Female	Anatolian Teacher Training High School	Science	Middle School	Faculty of Education	Two and half years
P4	Female	Anatolian High School	Science	Middle School	Faculty of Education	Three years
P5	Male	Anatolian Teacher Training High School	Science	Middle School	Faculty of Education	Three years

When the participants are examined, it can be assumed that they reflect a homogeneous group. Although this does not show that the inferences about the investigated phenomenon can be generalized, it can be argued that it can strengthen the inferences that can be obtained.

2.2 Application Process

In the application process of the study, the participants stated that they wanted to take part in the research voluntarily in the context of the MM course. Therefore, with regards to this course, what MM is, MM competencies, its process and components, and the models presented by researchers about MM from past to present are examined. Accordingly, sample applications involving daily life problems are included. In this process, a book about MM and the activities in it were used (Bukova-Güzel et al., 2016).

2.3 Data Collection Tools and Application

A structured form was developed by one of the researchers to collect the teachers' opinions about the use of MM in the solution of daily life problems. A pilot study was conducted by applying this form to one teacher. This pilot study was examined by two expert researchers, and arrangements were made for a clearer understanding of the questions in the form. This form consists of demographics, questions about the use of MM in daily life problems, and a daily life problem. Participant teachers were given one week to answer the structured interview form. The research data were obtained using this form. In case of studies, it is frequently used to test the validity of the data obtained for the investigated phenomenon. In this context, teachers were asked to solve a daily life problem in a structured form in the context of MM, and the validity of their opinions on this topic was examined.

The research problem used in the study was adapted from Gök (2020). In this study, four different ways to solve this problem are given. This shows that teachers have multiple ways to solve the problem. Besides, by giving a long period of time, such as one week, to solve the problem, teachers are provided with the opportunity to solve the problem in multiple ways. The data obtained in the study were collected through

- A daily life problem and
- An interview form about MM in solving daily life problems in the structured form.

The problem given to the teachers in the structured form is presented below.

How many people will be at the meal?

Students from secondary schools A, B, and C came to a scout camp. Although it is not known how many students from each school attended the camp, the estimated numbers from the form filled in by the students at their school before the camp (attending, not sure, or not attending) were determined as follows:

- Minimum 30 and maximum 45 from school A,
- Minimum 50 and maximum 90 from school B,
- Minimum 25 and maximum 75 from school C,

A nature trip was organized for all the students participating in the camp to explore the environment. In this nature trip, students crossed a suspension bridge and got on a boat and a cable car. A maximum of 3 students can cross side by side on the suspension bridge. A maximum of 5 students can fit on the boat and 7 students on the cable car. Since this nature tour is aimed to be completed as soon as possible, students crossed the bridge in threes, got on the boat in fives, and boarded the cable car in sevens. However, this rule was not followed for the students who stayed in the last row of this trip. In this process, it is known that

- 1 student remained at the end of the suspension bridge,
- 2 students on the last trip of the boat, and
- 4 students on the last trip of the cable car.

At the end of the trip, the cook must know the exact number of students to prepare a meal. However, the cook of the camp is undecided about what to do about how many people he will cook with this information. In light of this information, could you help the cook of the camp regarding the number of students participating in the trip?

First, the daily life problem reflects an application of the Chinese Remainder Theorem mathematically. Secondly, the interview questions of teachers regarding the use of MM in solving daily life problems include "What are the benefits of MM in solving daily life problems?" "What are the difficulties that may arise if MM is used in solving daily life problems?" and "Do you use MM to solve daily life problems in your classroom? Explain with examples."

2.4 Data Analysis

In the study, a daily life problem was analyzed with descriptive analysis in the analysis of the data obtained from the solution approaches of teachers through MM, and the data containing the teachers' opinions on the examination of daily life problems with MM were analyzed through content analysis.

Descriptive analyzes were carried out based on De Corte, Verschafeller, and Greer's (2000) model. The processes in this model (such as understanding, modeling, and mathematical analysis) are classified as complete, incomplete, and no explanation. For example, if the participant teachers perform the process completely, it is coded as complete, making it in a single direction or incompletely was coded as incomplete, and not providing any explanation or stating it incorrectly was coded as no explanation.

For the data analysis of the teachers' opinions, the data was first transferred to the computer. Then, content analysis of the data was carried out by two experts. In this context, codes were created, and themes were obtained from these codes through reduction and combination (Saldaña, 2011). In the findings, the results were supported using the themes and direct quotations from the teachers' opinions on these themes.

2.5 Validity and Reliability

The research validity was ensured using more than one data collection tool, presenting the findings with evidence, and analyzing the data according to a specific approach. In this direction, the use of MM in daily life problems was not only limited to a sample application but also reflected whether the teachers actually used the modeling processes or not. The consensus among coders regarding the analyzes performed was calculated (Miles, & Huberman, 1994). The intercoder agreement was determined as 94%. This result indicates that the analysis performed is reliable. Additionally, in terms of providing the credibility criteria put forward by Lincoln and Guba (1985) for case studies, the data were studied for a long time, the emerging issues were constantly observed, the data were checked with the participants, different methods were used in the data analysis, the findings were systematically tested, the findings were reviewed by the other expert, the findings were presented in detail, and the results were kept open to audit.

3. Results

This research aims to investigate the case of mathematics teachers in solving a daily life problem within the scope of MM education they have received at the graduate level and reveal their perceptions regarding the use of MM in such problems. In this context, the findings are presented under two sub-headings: "the processes of solving a daily life problem with MM" and "the opinions of mathematics teachers about examining daily life problems in the context of MM."

3.1 The Mathematics Teachers' Processes of Solving a Daily Life Problem with MM

Within the scope of this study, the solution of a daily life problem by mathematics teachers with MM was analyzed with the themes of understanding, modeling, mathematical analysis, interpretation, evaluation, and communication in line with the approach suggested by De Corte, Verschaffel, and Greer (2000). Findings related to solution processes are presented in Table 2.

Table 2: Modeling processes that teachers use in solving a daily life problem

Themes	Criteria	P1	P2	P3	P4	P5
Understanding	Complete		✓	✓		
	Incomplete	✓			✓	
	No explanation					✓
Modeling	Complete	✓	✓	✓	✓	✓

	Incomplete					
	No explanation					
Mathematical Analysis	Complete	✓	✓	✓	✓	✓
	Incomplete					
	No explanation					
Interpretation	Complete	✓	✓	✓	✓	✓
	Incomplete					
	No explanation					
Evaluation	Complete		✓			
	Incomplete	✓		✓	✓	✓
	No explanation					
Communication	Complete				✓	
	Incomplete					
	No explanation	✓	✓	✓		✓

It is observed that teachers generally include MM processes in solving the processes of a daily life problem. Besides, it was determined that especially modeling, mathematical analysis, and interpretation processes were fully implemented, but some teachers provided incomplete explanations about other processes. The understanding process is usually found to be implicit because it is not possible for other processes to emerge fully without understanding. It is an indication of this that teachers show a correct approach until the evaluation in the following processes. However, the deficiencies in the evaluation and communication stages are remarkable. During the evaluation process, it is observed that the teachers only make attempts to check the solution they offer. Nevertheless, it was observed that they could not propose different solutions and had difficulty in developing an argument about why their own solution was the ideal way. Similarly, the explanation regarding the communication process was given by only one teacher. Based on this data, it can be deduced that in the solution of daily life problems, teachers hastily start solving the problem without paying the necessary attention to the stage of understanding the solution of the problem. Besides, it is understood that the evaluations about why the solutions offered for the daily life problem are the most effective are lacking. These inferences can be observed more clearly in the citations of teachers in the modeling processes.

> P1: Since I was familiar with the problem situation, I had no trouble understanding it. First, I formulated the problem after understanding the problem. (*understanding stage-incomplete explanation*)
>
> P4: While solving the problem, I first determined what was given and what was requested. (*understanding stage-incomplete explanation*)
>
> P3: I fully understood the problem when I read it for the second time. First of all, I found out the maximum and the minimum number of students who participated in the trip. Then I realized how many trips the suspension bridge, boat, and cable car made and realized that the number of students passing through all of them would be equal. (*understanding stage - complete explanation*)

Based on these quotations, it can be stated that although P1 and P4 teachers expressed that they understood the problem, they did not clearly reveal the conditions in the problem and what was requested. On the contrary, the P3 teacher offered explanations that fully express the conditions given in the problem and what is asked for.

It was determined that the teachers presented similar explanations with slight differences in the modeling, mathematical analysis, and interpretation stages. Explanations on this subject are given in the excerpts below.

> P4: The mathematical model I used is the equations I wrote to find the common term of three different number patterns in a particular range [105, 210]. (*modeling stage-complete explanation*)
>
> P1: To solve the inequality I created, I brought the algebraic expressions to the multiples by adding 38 to each algebraic expression and expanded the LCMs of the coefficients by 2 to fall within the inequality range. (*mathematical analysis-complete explanation*)

P5: Considering both conditions, the number of students could be 67, 172, 277, 382, … (3 minus 2, 5, 8, and 11 multiples of 35). Based on the estimation of the students' numbers, the number of students should be 172. (*interpretation stage-complete explanation*)

It is seen that teachers produce similar arguments in the modeling, mathematical analysis, and interpretation stages of the modeling process, albeit in different ways. This solution bears the traces of an approach commonly used in the curriculum. In this context, teachers determined the range of the estimated number of the group. Additionally, they set up three different equations from the given ones and focused on their common solutions.

In the evaluation and communication stages, it is observed that only one teacher could give a complete explanation, and the others gave incomplete or no explanation. Therefore, the teachers' opinions on this subject are given below.

P2: 172 is between 105 and 210, 1 plus of 3 multiples, 2 plus of 5 multiples, and 4 plus of 7 multiples. The suitability of the result found for the given cases can be verified… However, I added 38 to be an exact multiple of 3, 5, and 7. There may be a mathematical process to doing this, and the mod technique can be used. This is what I can recall now. The mod technique could be used twice. Maybe it could be solved according to the remainder. T= {...1, 4, 7, 10, 13.....}. I don't think there is any need for this solution. It could be used for larger numbers. (*evaluation stage-complete explanation*)

P1: I had difficulty in the verification stage of the mathematical modeling in solving this problem because I could not find a way to verify the result I found, other than re-do the mathematical analysis. If the result was verifiable from other sources, the verification step would be easier for me. (*evaluation stage-incomplete explanation*)

P4: Report: The cook needs to cook for 172 students. If you want to be sure of the result I have reached, the control part in the solution I have sent will help you. (*Communication- complete explanation*)

P2 indicates that there can be an arithmetic solution. In this context, an attempt to support her solution with a different solution is highlighted. Teacher P4 tried to establish a relationship between the result and the situation as an indication of the idea of helping a client by associating the result she found with the given problem situation. This situation reflects the need to use modeling processes related to the given problem.

It can be stated that teachers can generally use modeling processes appropriately in solving daily life problems. On the other hand, it was determined that the process of understanding the problem was skipped quickly, the teachers had difficulties in giving the most effective solution during the evaluation stage, and the communication stage was mainly ignored.

3.2 The Opinions of Mathematics Teachers about Examining Daily Life Problems in the Context of MM

The opinions of the mathematics teachers about the examination of daily life problems in the context of MM were collected in line with the knowledge and experience they gained within the scope of MM education they received at the graduate level. In this context, first of all, the teacher's opinions about the benefits of using MM in solving daily life problems are given in Figure 2.

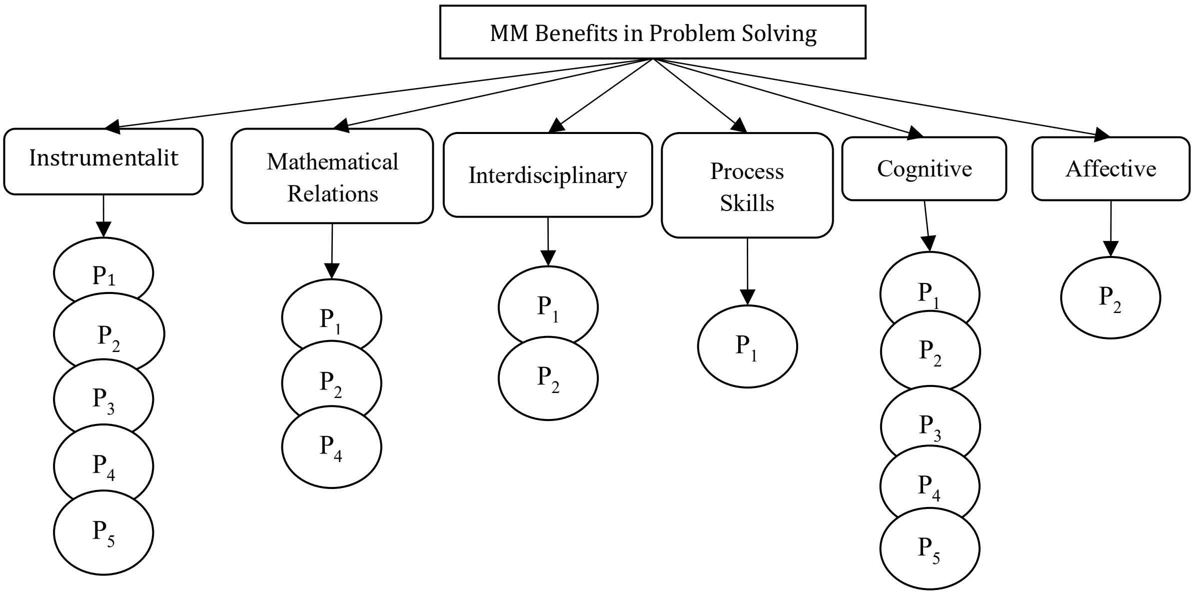

Figure 2: Themes regarding the benefits of teachers' use of MM in PS

Figure 2 indicates that teachers activate especially cognitive skills in MM in the solution of daily life problems, and it can be used as a tool in solving the problem. On the other hand, it was pointed out that they have the potential to notice mathematical relationships in daily life problems, establish connections between different disciplines, activate mathematical process skills, and affect affective skills positively. Relationships to these themes are clearly seen in the teachers' opinions given below.

> P3: We benefit greatly from modeling when transferring abstract concepts to younger students. (*Instrumentality*)
> P1: The mathematical modeling process allows the teacher to understand, simplify, interpret, and verify the problem he encounters in daily life more than a teacher who does not know the process. (*Mathematical Relations*)
> P2: As individuals who can model, we realize the importance of the relationship between mathematics and other disciplines, and we can cooperate with other teachers. (*Interdisciplinary*)
> P5: It makes learning permanent. (*Cognitive*)
> P1: Using mathematical modeling in the classroom as a tool for problem-solving will push individuals to think actively. (*Cognitive*)
> P1: Using mathematical modeling in problem-solving will bring mathematics to daily life; thus, it will make the problems interesting and make individuals want to study. (*Affective*)

MM is a way of teaching abstract concepts in the solution of teachers' daily life problems, and it can be used to perceive and analyze mathematical relations in these problems. This approach also activates the thinking processes of the students and allows creating a connection of mathematics with other disciplines. While emphasizing that permanent learning emerges at the end of this process, it is referred to the potential to positively affect student interest, primarily associating problems with daily life.

The teachers' opinions about the problems they may encounter in the process of using MM in the solution of daily life problems were collected. The analysis of the data obtained in this context is presented in Figure 3.

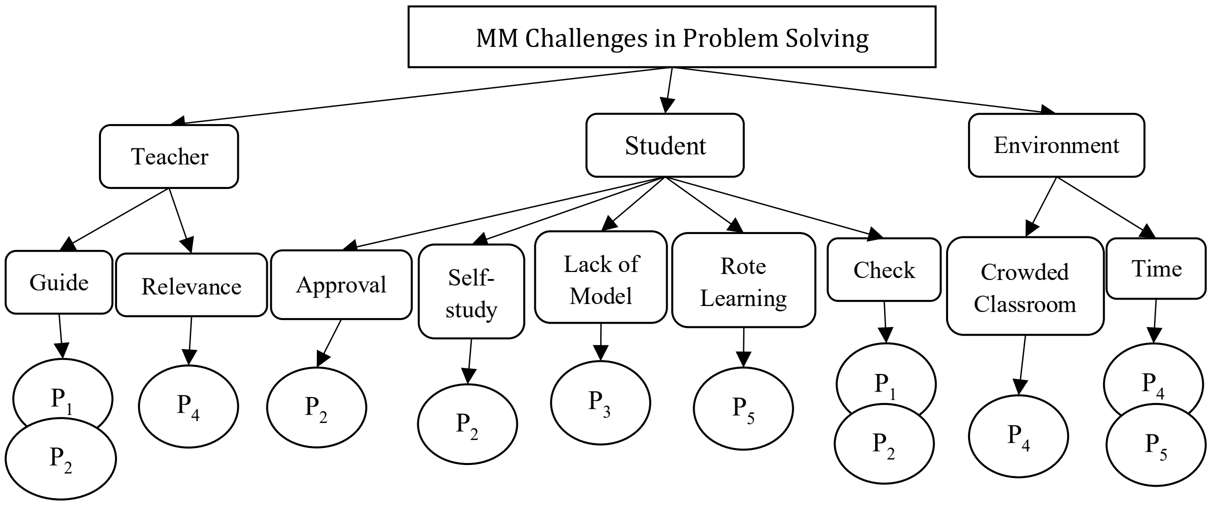

Figure 3: Challenges faced by teachers regarding MM in PS

The challenges that teachers may encounter regarding the use of MM in daily life problems are categorized under the perspective of teacher, student, and environment. It was suggested that most of these difficulties might arise related to the student stakeholder. Regarding the student's use of MM in PS, the desire to get approval for the solution performed, seeking an individual solution and self-study by not participating in the group, not being able to find a suitable model for the desired situation, inability to adapt to the use of MM due to the traditional teaching method being used to rote learning, and failure to control what is achieved by following a results-oriented approach rather than a process are shown as the prominent challenges. In terms of the teacher, it was expressed that the organization may be inadequate in problem-solving, and there may be problems with the problem being appropriate for the grade level. Finally, it is explained that sufficient time in crowded classrooms may pose a problem in solving such problems. These explanations can be seen in more detail in the following teacher quotations.

> P2: Students may try to get approval from the teacher in this process. They may also want to get information about the correctness of the solution. (*Getting approval*)
> P2: Students may prefer self-study; thus, there may be problems in group work. (*Self-study*)
> P3: Sometimes, mathematical models can be more challenging to understand than the problem. This may be due to the difficulty in finding a suitable mathematical model for the problem. (*Lack of model*)
> P5: It can be challenging and time-consuming for students who previously used memorizing and rote learning rules. (*Rote learning*)
> P2: Students proceed towards the result and do not usually check the correctness of the solution. (*Check*)
> P4: I think that the only difficulty that can be caused by using mathematical modeling in problem-solving may be the balance of time-class size. (*Crowded classroom-Time*)

Based on these explanations, teachers implied that students might want to receive feedback from the teacher regarding the correctness of their actions regarding the use of MM in solving daily life problems, have difficulty in finding a suitable model, it is not easy to establish MM as a method, and this step can be ignored in MM because the solution is not checked in the solution approach that students are normally used to. Besides, it was emphasized that the teacher should consider the students' level in the selection of the problem, present it by making the necessary adaptations in the problem, and give appropriate feedback where necessary during the solution process. Finally, it was noted that the implementation of such applications in crowded classrooms might cause problems in terms of time.

Teachers were asked about their use of MM in solving daily life problems and how they used it. The analysis of the data obtained in this context is presented in Figure 4.

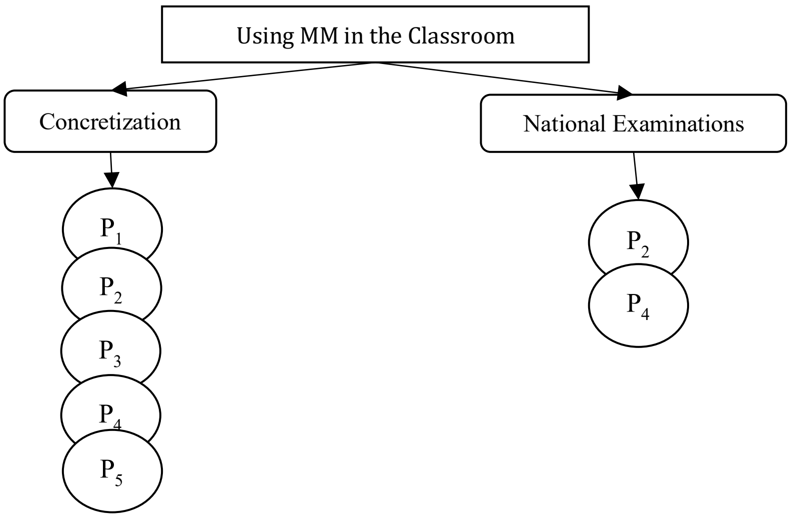

Figure 4: Teachers' use of MM in the classroom

It is understood that teachers use MM in two contexts in their classrooms. These are expressed as examining and concretizing the problems in the national exams called the high school entrance exam (LGS). The following excerpts provide clues about how teachers use MM in the classroom in these contexts.

> P2: Obviously, the LGS exam, in which "new generation" questions are asked, requires proficiency in solving complex problems such as reading comprehension, interpretation, and solution development in a practical way. In this process, we include questions that we can use mathematical modeling in the classrooms. (*National Examinations*)
>
> P4: For example, we follow a modeling process with paper activities to discover identities in the textbook. (*Concretization*)
>
> P5: I use it not only with concrete materials or models with figure content but also with verbal expressions. (*Concretization*)

It is understood that teachers use MM in their classrooms to prepare for national examinations with complex questions and with the idea of concretizing abstract mathematical objects. Hence, teachers use MM for very limited purposes in their classroom practices. For example, a practical method for solving complex questions, empirical demonstration of the correctness of identity, and verbal explanations can be considered useful in this context.

4. Discussion

The results obtained in the study are presented under two sections: "The results of the mathematics teachers' processes of solving a daily life problem with MM" and "the results of the opinions of mathematics teachers about examining daily life problems in the context of MM."

4.1 The Results of the Mathematics Teachers' Processes of Solving a Daily Life Problem with MM

When the solutions of mathematics teachers for a daily life problem were examined in the context of the MM process, it was seen that they generally performed the modeling, mathematical analysis, and interpretation processes adequately. However, they had problems in other processes (understanding, evaluation, and communication) due to deficiencies in content knowledge or not paying attention to MM processes.

First of all, the rapid skip of the comprehension process suggests that this process may have taken place implicitly. The proof of this situation can be explained as the correct application of the next three processes by the teachers. It is underlined that a problem in the transition from the phenomenon under investigation, which reflects the first process of the modeling, to the situation model negatively affects other stages. In support of this, it is stated in different studies that problems in the first steps of modeling have negative effects on the next steps (e.g., Deniz,

& Akgün, 2018; Hıdıroğlu et al., 2014). It is important to note that since a context is created for teachers to help a client, although teachers should provide explanations showing that they are aware of this process, only P2 and P3 teachers fully reflect this process. It was determined that other teachers quickly moved to other steps with no explanation or insufficient explanations regarding whether they understood the problem. To this end, the research results are partially similar to those of studies (Hıdıroğlu et al., 2017; Peter-Koop, 2004) that indicate that there is no difficulty in understanding the problem in studies that seek solutions to daily life problems with a modeling approach. On the other hand, in different studies, it was seen that the participants (prospective teachers and teachers) were deficient in understanding modeling activities and tended to take the easy way out (e.g., Duran et al., 2016; Ural, 2014; Urhan, & Dost, 2016). In this study, in addition to these results, it was determined that some teachers moved on to other processes without providing any explanation that they understood the problem they were familiar with. This may be due to the fact that the participants are used to solving problems directly without thinking too much about them (Blum, & Borromeo-Ferri, 2009; Eraslan, & Kant, 2015). In this study, teachers' rapid transition to the solution can be associated with their familiar approaches to PS or their carelessness about understanding from modeling processes.

In the evaluation process, only one teacher expressed that there could be different solution methods by giving examples and made a complete evaluation about the correctness of her solution. It was observed that other participants did not include explanations that there might be different solutions during the evaluation stage and were limited to checking the mathematical operations in their solutions. Therefore, it was out of the question for teachers to take the initiatives to reveal the most effective solution among different solutions. Although there are multiple solutions to the problem presented to the teachers, the fact that almost all of them similarly solved the problem can be explained by the fact that this solution is dominant in the curriculum. However, the absence of other solutions can be interpreted as the teachers' content knowledge and their inability to use the connections between mathematical concepts effectively. This result shows parallelism with the results of previous studies that there may be deficiencies in interpreting mathematical results based on the actual situation in the evaluation process (e.g., Berry, & Houston, 1995; Çakmak-Gürel, & Işık; 2018; Duran et al., 2016; Hıdıroğlu et al., 2014; Hıdıroğlu et al., 2017; Kapur, 1982; Peter-Koop, 2004; Sekerak, 2010; Tekin-Dede, & Yılmaz, 2013). Different models or solutions were developed by the participants in different studies; that is, there is no single correct model or solution in MM (Hıdıroğlu et al., 2017; Yanagimoto, 2005). In this respect, it is noteworthy that for a problem with multiple answers (Gök, 2020), creative solutions are not found at the end of the process, and even a different solution is not even proposed. This deficiency in the evaluation process may be an effect of the traditional rote-learning approach, or it may be due to the lack of content knowledge or the insufficient internalization of the modeling process. Hence, Tanju (2020) concluded that prospective teachers had memorized information about the mathematical concepts in the given problem, and they failed in model creation activity.

The fact that teachers solve the problem in a single way may also be due to their lack of group work. In this study, the teachers individually sought a solution to the daily life problem with the MM approach. In other words, group work was not allowed. Group work has the potential to create a rich environment for problems by enabling participants to express their ideas more quickly, close each other's gaps, and develop different perspectives (Ärlebäck, 2009; Karaci-Yasa, & Karataş, 2018; Korkmaz, 2010; Peter-Koop, 2004). In this direction, the fact that different solutions are not presented may also be due to individual work rather than group work.

In the communication stage, only one participant wrote a report that responded to the client by associating the result with the given problem situation. This step was ignored by the other participants. This may be another indicator of their inability to internalize the modeling processes. This lack of internalization can be seen as a result of not giving enough place to MM in school mathematics. Additionally, in parallel with the results obtained in different studies on the MM process, it was observed that prospective teachers were unsuccessful in using verbal explanations (Tekin-Dede, & Yılmaz, 2013; Kertil, 2008).

Consequently, teachers can generally use modeling processes appropriately in solving a daily life problem; however, some teachers have deficiencies in the stages of understanding, evaluation, and communication.

4.2 The Results of the Opinions of Mathematics Teachers about Examining Daily Life Problems in the Context of MM

It was found out that all teachers agree on using MM in solving daily life problems, activating cognitive skills, and using it as a tool in PS. Besides, teachers noted that the use of MM in daily life problems could be beneficial in terms of recognizing mathematical relationships, establishing interdisciplinary links, activating mathematical process skills, and developing affective skills. In past studies in literature, it was concluded that the association of MM with daily life led to positive development in terms of the activation of cognitive and affective skills (Doruk, & Umay, 2010; Işık, & Mercan, 2015; İncikabı, & Biber, 2020; Muşlu, & Çiltaş, 2016; Özer, & Bukova-Güzel, 2016; Şahin, & Eraslan, 2019; Şahin et al., 2019; Türker et al., 2010; Urhan, & Dost, 2016; Zbiek, & Conner, 2006), being a tool for solving daily life problems (Hartono, 2020; Heymann, 2003; Kertil, 2008; Lesh, & Zawojewski, 2007; Tekin-Dede, & Bukova-Güzel, 2013) and establishing interdisciplinary cooperation and communication by developing interdisciplinary skills (Deniz, & Akgün, 2017; Kertil, 2008; Suh et al., 2021; Şahin, & Eraslan, 2019; Takaoğlu, 2015; Tekin-Dede, & Bukova-Güzel, 2013). In this context, the research results show parallelism with the studies in the literature.

As another result of this study, the challenges of MM in solving daily life problems emerged as the desire for approval, seeking individual solutions by not participating in the group, not being able to find a suitable model, not adapting due to the traditional teaching's use of rote learning, and not checking the solution due to being result-oriented. In parallel with the research results, different studies also suggested group work (Deniz, & Akgün, 2017; Şahin, & Eraslan, 2019), inability to find a suitable model (Zulkarnaen, 2018), and the continuation of the traditional-rote learning approach (Kertil, 2008; Urhan, & Dost, 2016) as the difficulties of MM. To this end, the desire for approval, difficulties in working with the group, and the lack of check by acting result-oriented can reflect the traditional approach. In this direction, the source of the difficulties that may be experienced in terms of students is the tendency to use the traditional method widely in the teaching environment.

The challenges of MM for teachers in solving daily life problems are expressed as the inadequacy of the organization and the inability to develop activities suitable for the grade level. In the literature, it was highlighted that it is vital for teachers to organize the modeling process (such as planning and guidance) (Borromeo-Ferri, 2013; Yanık et al., 2017) and that MM activity should be chosen according to the level and readiness of the student (Işık, & Mercan, 2015; Urhan, & Dost, 2016). In this regard, the results show parallelism with the results of previous studies.

The challenges of MM in terms of the environment in solving daily life problems are expressed as classroom size and course time. Similarly, in the study conducted by Şahin and Eraslan (2019), it was found out that MM adversely affects classroom management in crowded classrooms and that the long-time activities may create a problem in practice. Additionally, the fact that the intensity in the curriculum is expressed as a factor that makes the use of MM in terms of time difficult in many studies (Akgün et al. 2013; İncikabı, & Biber, 2020; Özdemir, & Işık, 2015; Tekin-Dede, & Bukova-Güzel, 2013; Urhan, & Dost, 2016); Yanık et al., 2017; Yu, & Chang, 2009) support this study's results.

Teachers emphasized that MM is used for two purposes in the teaching environment, namely concretization and national examinations. First, the results of this study show parallelism with the results in the literature reporting that MM is mainly used for concretization in textbooks (Çavuş-Erdem et al., 2017; Tekin-Dede, & Bukova-Guzel, 2013), and teachers tend to use MM to concretize (Işık, & Mercan, 2015). Secondly, Şahin and Eraslan (2019) underlined that MM could be used in preparation for the Program for International Student Assessment (PISA) and Trends in International Mathematics and Science Study (TIMSS) exams. In this study, the results of the studies differ in terms of expressing the use of MM in preparation for national examinations. The basis of this difference may be due to the addition of MM skills to the curriculum (MoNE, 2018) and that, unlike the TEOG exam, the LGS is similar in terms of mathematical skills measured in PISA and TIMSS exams. (Kırnap Dönmez, & Dede, 2020). It can be said that this situation encourages teachers to use MM in classroom practices in solving daily life problems.

In the research results, it is critical to note that teachers use MM in their classroom practices at a minimal level. In different studies, it was argued that teachers do not have sufficient knowledge about MM, and they encounter many difficulties in practice (Akgün et al., 2013; Doğan-Temur, 2012). To this end, it can be deduced that teachers do not have sufficient knowledge and experience in practice about the application of MM in solving problems.

As a result, it was understood from the teachers' opinions that the MM approach has many benefits in solving daily life problems, albeit significant difficulties experienced or experienced in practice.

This study reflects the opinions of five mathematics teachers regarding the use of the MM approach in daily life problems and the extent to which teachers actually use MM processes in an application. In this study, indications were obtained that even if the teachers had MM education, they partially acted in accordance with the MM processes, tended to use the approaches they are accustomed to in daily life problems, and had difficulties in some MM processes. However, these indicators need to be tested in larger groups of teachers in different problem situations (with teachers with or without familiarity) and in the context of different modeling approaches. It is expected that the studies to be conducted in this direction will open new doors for the effective use of MM in problem-solving from the perspective of teacher training.

Acknowledgments

In this study, the authors have equal rights to the article. We thank all the teachers who voluntarily participated in the study.

References

Akgün, L., Çiltaş, A., Deniz, D., Çiftçi, Z., & Işık, A. (2013). Primary school mathematics teachers' awareness of mathematical modelling. *Adiyaman University Journal of Social Sciences*, (12), 1-34. https://doi.org/10.14520/adyusbd.410

Ang, K. C. (2001). Teaching mathematical modelling in Singapore schools. *The mathematics educator*, *6*(1), 63-75. https://repository.nie.edu.sg/bitstream/10497/49/1/TME-6-1-63.pdf

Ang, K. C. (2009). Mathematical modelling and real life problem solving. In B. Kaur, Y. B. Har & M. Kapur (Eds.), *Mathematical Problem Solving Yearbook 2009, Association of Mathematics Educators* (pp. 159-182). Singapore: World Scientific Publishing.

Anhalt, C.O., & Cortez, R. (2016). Developing understanding of mathematical modeling in secondary teacher preparation. *J Math Teacher Educ,* 19, 523–545. https://doi.org/10.1007/s10857-015-9309-8

Ärlebäck, J. B. (2009). On the use of realistic fermi problems for introducing mathematical modelling in school. *The Montana Mathematics Enthusiast*, *6*(3), 331- 364. https://scholarworks.umt.edu/cgi/viewcontent.cgi?article=1157&context=tme

Aztekin, S., & Taşpınar Şeker, Z. (2015). The content analysis of mathematical modelling studies in turkey: a meta-synthesis study. *Education and Science*, *40*, 139-161. http://egitimvebilim.ted.org.tr/index.php/EB/article/view/4125/1006

Berenger, A. (2018). Pre-service teachers' difficulties with problem solving. In J. Hunter, L. Darragh, & P. Perger (Eds.), *Mathematics Education Research Group of Australasia* (pp. 162-169). MERGA. https://files.eric.ed.gov/fulltext/ED592470.pdf

Berry, J., & Houston, K. (1995). *Mathematical modelling.* Bristol: J. W. Arrowsmith Ltd.

Berry, J., & Nyman, M. (1998). Introducing mathematical modelling skills to students and the use of posters in assessment. *Primus*, *8*(2), 103-115. https://doi.org/10.1080/10511979808965888

Blum, W., & Borromeo-Ferri, R. (2009). Mathematical modelling: can it be taught and learnt?. *Journal of Mathematical Modelling and Application,* *1*(1), 45-58. https://bu.furb.br/ojs/index.php/modelling/article/view/1620

Blum, W., & Leiß, D. (2007). How do students and teachers deal with modelling problems? In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical modelling: Education, engineering and economics* (pp. 222–231). Chichester: Horwood.

Bukova-Güzel, E., Tekin-Dede, A., Hıdıroğlu, Ç. N., Kula-Ünver, S. & Özaltun-Çelik, A. (2016). *Mathematical modelling in mathematics education: for researchers, educators and students.* Ankara: Pegem Akademi Publishing

Christensen, L. B., Johnson, R. B., & Turner, L. A. (2015). *Research methods: design and analysis* (Translation Editor: Ahmet Aypay). Ankara: Anı Publishing.

Çakmak-Gürel, Z., & Işık, A. (2018). Investigation of the competences of pre-service secondary school mathematics teachers related to the mathematical modelling. *E-International Journal of Educational Research*, *9*(3), 85-103. https://www.e-ijer.com/tr/pub/e-ijer/article/477651

Çavuş-Erdem, Z., Doğan, M. F., Gürbüz, R., & Şahin, S. (2017). The reflections of mathematical modeling in teaching tools: textbook analysis. *Adıyaman University Journal of Educational Sciences*, *7*(1), 61-86. https://dergipark.org.tr/tr/download/article-file/320383

De Corte, E., Verschaffel, L., & Greer, B. (2000, November). Connecting mathematics problem solving to the real world. *In Proceedings of the International Conference on Mathematics Education into the 21st Century: Mathematics for living* (pp. 66-73). http://dipmat.math.unipa.it/~grim/Jdecorte.PDF

Demir, N., Ertem-Akbaş, E., & Gök, M. (2021). The views of lecturers about the renewed elementary mathematics teacher undergraduate program views. *YYU Journal of Education Faculty*, *18*(1), 70-105. https://dergipark.org.tr/tr/pub/yyuefd/issue/59626/859490

Deniz, D., & Akgün, L. (2017). High school mathematic teachers' views about mathematical modelling method and applications. *Journal of Social Sciences of Mus Alparslan University*, *5*(1), 95-117. https://dergipark.org.tr/tr/pub/anemon/issue/27858/272677

Deniz, D., & Akgün, L. (2018). Examining prospective mathematics teachers' abilities of mathematical modeling. *Mediterranean Journal of Educational Research, 12*(24), 294-312. https://doi.org/10.29329/mjer.2018.147.16

Doerr, H. M., & Pratt, D. (2008). The learning of mathematics and mathematical modelling. In M.K. Heid & G. W. Blume (Eds.), *Research on technology and the teaching and learning of mathematics: Vol. 1. Research syntheses* (pp. 259-286). Charlotte, NC: Information Age Press.

Doğan-Temur, Ö. (2012). Analysis of prospective classroom teachers' teaching of mathematical modeling and problem solving. *Eurasia Journal of Mathematics, Science and Technology Education*, *8* (2), 83-93. https://doi.org/10.12973/eurasia.2012.822a

Doruk, B. K., & Umay, A. (2010). The effects of mathematical modeling on transferring mathematics into daily life. *Hacettepe University Journal of Education*, *41*, 124-135. http://www.efdergi.hacettepe.edu.tr/shw_artcl-685.html

Duran, M., Doruk, M., & Kaplan, A. (2016). Mathematical modeling processes of mathematics teacher candidates: the example of tortoise paradox. *Cumhuriyet International Journal of Education*, *5*(4), 55-71. https://dergipark.org.tr/en/download/article-file/314336

Eraslan, A., & Kant, S. (2015). Modeling processes of 4th-year middle-school students and the difficulties encountered. *Educational and Sciences: Theory and Practice, 15*(3), 809-824. https://files.eric.ed.gov/fulltext/EJ1067424.pdf

Borromeo-Ferri, R. (2013) Mathematical modeling – the teacher'sresponsibility. In B. Dickman & A. Sanfratello (Eds.), *Proceedings from the Teachers College Mathematical Modeling Oktoberfest* (pp. 26-31). New York: Teachers College Columbia University. https://doi.org/10.7916/jmetc.v0i0.660

Gök, M. (2020). Mathematical mystery in a cultural game. *World Journal of Education*, *10*(6), 64-73. https://doi.org/10.5430/wje.v10n6p64

Haines, C. R., & Crouch, R. M. (2007). Mathematical modelling and applications: Ability and competence frameworks. In W. Blum, P. L. Galbraith, H. W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education* (pp. 417-424), New York, Springer. https://doi.org/10.1007/978-0-387-29822-1_46

Han, S., & Kim, H. M. (2020). Components of mathematical problem solving competence and mediation effects of instructional strategies for mathematical modeling. *Education and Science*, *45*(202), 93-111. http://dx.doi.org/10.15390/EB.2020.7386

Hartono, Y. (2020, March). Mathematical modelling in problem solving. In *Journal of Physics: Conference Series* (Vol. 1480, No. 1, p. 012001). IOP Publishing. https://iopscience.iop.org/article/10.1088/1742-6596/1480/1/012001/pdf

Hıdıroğlu, Ç. N. (2012). *Analysing mathematical modelling problems solving processes in the technology-aided environment: An explanation on approaches and thought processes.* Master Thesis. Dokuz Eylul University, İzmir.

Hıdıroğlu, Ç. N., Tekin Dede, A., Kula, S. & Bukova Güzel, E. (2014). Examining students' solutions regarding the comet problem in the frame of mathematical modeling process. *Mehmet Akif Ersoy University Journal of Education Faculty, 1*(31), 1-17. https://dergipark.org.tr/tr/pub/maeuefd/issue/19404/206140

Hıdıroğlu, Ç. N., Tekin-Dede, A., Kula-Ünver, S., & Bukova-Güzel, E. (2017). Mathematics student teachers' modelling approaches while solving the designed eşme rug problem. *EURASIA Journal of Mathematics, Science and Technology Education*, *13*(3), 873-892. https://doi.org/10.12973/eurasia.2017.00648a

Heymann, H. W. (2003). *Why teach mathematics? A focus on general education*. Dordrecht: Kluwer.

Işık, A., & Mercan, E. (2015). Analysis of the views of secondary school maths teachers on model and modeling. *Kastamonu Education Journal, 23*(4), 1835-1850. https://dergipark.org.tr/tr/pub/kefdergi/issue/22597/241360

İncikabı, L., Ayanoğlu, P., & Uysal, R. (2020). Sixth-grade students' procedural and conceptual understandings of division operation in a real-life context. *International Electronic Journal of Elementary Education*, *13*(1), 35-45.https://www.iejee.com/index.php/IEJEE/article/view/1158

İncikabı, S., & Biber, A. Ç. (2020). Middle school mathematics teacher candidates' evaluation of mathematical model eliciting activities. *Medeniyet Eğitim Araştırmaları Dergisi*, *4*(1), 10-27. https://dergipark.org.tr/tr/pub/mead/issue/56310/741341

Kapur, J.N. (1982). The art of teaching the art of mathematical modeling. *International Journal of Mathematical Education in Science and Technology*, *13*(2), 185-192. https://doi.org/10.1080/0020739820130210

Karaci Yasa, G., & Karatas, I. (2018). Effects of the instruction with mathematical modeling on pre-service mathematics teachers' mathematical modeling performance. *Australian Journal of Teacher Education*, *43*(8), 1-14. http://dx.doi.org/10.14221/ajte.2018v43n8.1

Karataş, İ., & Güven, B. (2010). Examining *High School Students Abilities of Solving Realistic Problems*. *Erzincan University Journal of Education Faculty*, *12*(1), 201 - 217. https://dergipark.org.tr/en/pub/erziefd/issue/5999/79996

Kertil, M. (2008). *Investigating problem solving abilitiy of pre-service mathematics teachers in modeling process.* Master Thesis. Marmara University, İstanbul.

Kertil, M., Erbaş, A., & Çetinkaya, B. (2017). Pre-service elementary mathematics teachers' ways of thinking about rate of change in the context of a modeling activity. *Turkish Journal of Computer and Mathematics Education (TURCOMAT), 8*(1), 188-217. https://dergipark.org.tr/en/pub/turkbilmat/issue/27238/304212

Kırnap Dönmez, S. M., & Dede, Y. (2020). Analysis of mathematics questions in the exams for transition to secondary education (2016, 2017 and2018 years) in terms of mathematical proficiency. *Başkent University Journal of Education*, *7*(2), 363-374. http://buje.baskent.edu.tr/index.php/buje/article/view/327

Korkmaz, E. (2010*). Middle school prospective maths and elementary school prospective teachers' views about mathematical modelling and their mathematical modelling competency*. PhD Thesis. Balıkesir University, Balıkesir.

Lesh, R. & Zawojewski, J. (2007). Problem solving and modeling. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp.763-804). Charlotte, NC: Information Age Publishers.

Lincoln, Y.S., & Guba, E.G. (1985). *Naturalistic inquiry*. Beverly Hills, CA: Sage.

Lingefjärd, T., & Holmquist, M. (2005). To assess students' attitudes, skills and competencies in mathematical modeling. *Teaching Mathematics and Its Applications: International Journal of the IMA*, *24*(2-3), 123-133. https://doi.org/10.1093/teamat/hri021

Mayring, P. (2011). *Introduction to Qualitative Social Research. A guide towards qualitative thinking*, A. Gümüş and M. S. Durgun (Trans.), BilgeSu, Ankara.

Mason, J. (1988). Modelling: What do we really want pupils to learn? In D. Pimm (Ed.), *Mathematics, Teachers and Children* (pp. 201-215). London: Hodder & Stoughton.

Merriam, S. B. (2013). *Qualitative case study research* (Trans.: E. Karadağ). S. Turan (Ed.), in *Qualitative Research: A Guide to Design and Implementation* (3 ed.). Ankara: Nobel.

Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis: An expanded sourcebook*. Thousand Oaks, CA: Sage

Milli Eğitim Bakanlığı [MEB] (2018). *Mathematics curriculum (Primary and secondary school 1, 2, 3, 5, 6, 7 and 8th grades).* Ankara: MoNE-Board of Education and Discipline.

Muşlu, M., & Çiltaş, A. (2016). The impact of the mathematical modeling method at teaching the subject of the process on natural numbers on the student success. *Journal of Bayburt Education Faculty*, *11*(2). https://dergipark.org.tr/tr/pub/befdergi/issue/28762/307845

National Council of Teachers of Mathematics [NCTM] (2000). *The principles and standards for school mathematics.* Reston, VA: NCTM.

Özdemir, G., ve Işık, A. (2015). Teachers' views on the instruction of area and volume concepts in solid objects with mathematical models and mathematical modeling method. *Kastamonu Education Journal*, 23(3), 1251-1276. https://dergipark.org.tr/tr/pub/kefdergi/issue/22598/241407

Özer, A. Ö., & Bukova-Güzel E. (2016). Mathematical modelling problems from the viewpoint of students, prospective teachers and teachers. *Manisa Celal Bayar University Journal of the Faculty of Education*, *4*(1), 57-73. https://dergipark.org.tr/en/pub/mcbuefd/issue/54502/742770

Peter-Koop, A. (2004). *Fermi problems in primary mathematics classrooms: pupils' interactive modelling processes.* Proceedings of the 27th Annual Conference of the MERGA. Queensland, Australia. http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.512.6509&rep=rep1&type=pdf

Polya, G. (1973). *How to solve it: A new aspect of mathematical method* (2. Baskı). Princeton, New Jersey, USA: Princeton University.

Saldaña, J. (2011). *The coding manual for qualitative researchers*. Thousand Oaks, CA: Sage

Schroeder, T. L., & Lester, F. K. (1989). Understanding mathematics via problem solving. In P. Trafton (Ed.), *New directions for elementary school mathematics* (pp. 31-42). Reston, VA: National Council of Teachers of Mathematics.

Sekerak, J. (2010). Phases of mathematical modelling and competence of high school students. *The Teaching of Mathematics*, *13*(2), 105-112. http://teaching.math.rs/vol/tm1323.pdf

Suh, J., Matson, K., Seshaiyer, P., Jamieson, S., & Tate, H. (2021). Mathematical *Modeling as A Catalyst for Equitable Mathematics Instruction: Preparing Teachers and Young Learners with 21st Century Skills*. *Mathematics*, *9*, 162. https://doi.org/10.3390/math9020162

Şahin, N., & Eraslan, A. (2019). Middle-school prospective mathematics teachers' opinions on the use of modeling activities at the course of mathematics applications. *Turkish Journal of Computer and Mathematics Education (TURCOMAT)*, *10*(2), 373-393. https://doi.org/10.17762/turcomat.v10i2.207

Şahin, S., Doğan, M. F., Cavus Erdem, Z., Gürbüz, R., & Temurtas, A. (2019). Prospective teachers' criteria for evaluating mathematical modeling problems. *International Journal of Research in Education and Science*, *5*(2), 730-743. https://www.ijres.net/index.php/ijres/article/view/680

Takaoğlu, Z. B. (2015). The effect of physics courses mathematical modelling used on prospective teachers' interests and how they associate physics with real life and other courses. *YYU Journal Of Education Faculty*, *12*(1), 223-263. https://dergipark.org.tr/en/pub/yyuefd/issue/25854/272581

Tanju, B. (2020). *Investigation of representation and making connections skills of pre-service mathematics teachers in mathematical modeling process.* Master Thesis. Hacettepe University, Ankara.

Tekin Dede, A., & Yılmaz, S. (2013). Examination of primary mathematics student teachers' modelling competencies. *Turkish Journal of Computer and Mathematics Education*, *4*(3).

Tekin Dede, A., & Bukova Güzel E. (2013). Mathematics teachers' views concerning model eliciting activities, developmental process and the activities themselves. *Bartın University Journal of Faculty of Education*, *2*(1), 300-322. https://dergipark.org.tr/tr/pub/buefad/issue/3812/51104

Türker, B., Sağlam, Y., & Umay, A. (2010). Preservice teachers' performances at mathematical modeling process and views on mathematical modeling. *Procedia-Social and Behavioral Sciences*, *2*(2), 4622-4628. https://doi.org/10.1016/j.sbspro.2010.03.740

Ural, A. (2014). Examining prospective mathematics teachers' abilities of mathematical modeling. *Dicle University Journal of Ziya Gökalp Faculty of Education*, (23), 109-140. https://dergipark.org.tr/en/download/article-file/786765

Urhan, S., & Dost, Ş. (2016). The use of mathematical modelling activities in courses: teacher perspectives. *Electronic Journal of Social Sciences*, *15*(59), 1279-1295. https://doi.org/10.17755/esosder.263231

Verschaffel, L., Greer, B., & De Corte, E. (2000). *Making sense of word problems*. Lisse, The Netherlands: Swets & Zeitlinger.

Vijayan, V., & Joshith, V. P. (2018). Reflection of problem solving skill in life and mathematics education through modeling and applying. *i-manager's Journal on Educational Psychology*, *12*(2), 1-6. https://doi.org/10.26634/jpsy.12.2.15141

Wilkerson, M.H., Bautista, A., Tobin, R.G., Brizuela, B. M., & Cao, Y. (2018). More than meets the eye: Patterns and shifts in middle school mathematics teachers' descriptions of models. *J Math Teacher Educ, 21*, 35–61. https://doi.org/10.1007/s10857-016-9348-9

Yanagimoto, T. (2005). Teaching modelling as an alternative approach to school mathematics. *Teaching Mathematics and Its Applications*, *24*(1), 1-13. https://doi.org/10.1093/teamat/hrh011

Yanık, H. B., Bağdat, O., & Koparan, M. (2017). Investigating prospective middle-school teachers' perspectives of mathematical modelling problems. *Journal of Qualitative Research in Education (JOQRE)*, *5*(1), 80-101. https://dx.doi.org/10.14689/issn.2148-2624.1.5c1s4m

Yu, S. Y., & Chang, C. K. (2009). What did Taiwan mathematics teachers think of model-eliciting activities and modeling? In G. Kaiser, W. Blum, R. Borromeo-Ferri ve G. Stillman (Eds.), *Trends in teaching and learning of mathematical modelling international perspectives on the teaching and learning of mathematical modelling* (pp. 147-156). Springer. https://doi.org/10.1007/978-94-007-0910-2_16

Zbiek, R. M., & Conner, A. (2006). Beyond motivation: Exploring mathematical modeling as a context for deepening students' understandings of curricular mathematics. *Educational Studies in mathematics*, *63*(1), 89-112. https://doi.org/10.1007/s10649-005-9002-4

Zulkarnaen, R. (2018, October). Why is mathematical modeling so difficult for students?. In *AIP Conference Proceedings* (Vol. 2021, No. 1, p. 060026). AIP Publishing LLC. https://doi.org/10.1063/1.5062790