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Supporting Preservice Secondary Mathematics Teachers' Professional Noticing of Student Thinking

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*Abstract***:** *A growing body of evidence suggests developing the attention of preservice, secondary mathematics (PSMTs) teachers towards professional noticing of student thinking should feature in teacher education programs. There were two aims for this qualitative study: first, to explore the extent to which an Interview Module (IM) supported the development of PSMTs' ability to notice and make pedagogical decisions based on student thinking evidenced in video- and paper-based work samples. A secondary aim was to establish the viability of the IM in an Australian context. Overall, PSMTs regarded their involvement in the IM as beneficial to their development as teachers. Specifically, participants outlined that the IM helped to shift their beliefs about teaching and learning, and helped promote productive teacher dispositions. Furthermore, these shifts were enabled through opportunities to engage with authentic student work, and the access they were given to new forms of responding.*

Introduction

The practices of attending, interpreting, and responding to students' mathematical thinking, what Jacobs and colleagues (2010) refer to as professional noticing, are necessary components of ambitious teaching (Lampert et al., 2013). Grounded in a strengths-based view of students, ambitious teaching (also referred to as adaptive or responsive teaching), seeks to engage all learners in intellectually challenging mathematics by continually responding to the mathematics being learned and to the students as learners of that mathematics (Lampert et al., 2013; Richards & Robinson, 2016). Sherin et al. (2011) have also highlighted the clear connections between professional noticing and responsive teaching. Acknowledging that teacher noticing is an "active process, where teachers are actors in the instructional scene that they are observing" (Sherin, 2011, p. 5), these authors regard noticing as involving two main processes which are cyclical and interrelated. Specifically, responsive teaching demands that teachers attend to particular events in the instructional setting and then work to make sense of those events in relation to what they know about mathematics, students, and the broader learning context. What teachers notice and how they interpret what they have noticed becomes the basis for deciding how to respond. Walshaw and Anthony (2008, p. 539) have echoed this claim, amplifying that

> *Importantly, the way in which teachers manage multiple viewpoints is very much dependent on what they know and believe about mathematics and on what they understand about the teaching and learning of mathematics. A successful teacher of mathematics will have both the intention and the effect to assist pupils in making sense of mathematical topics. Moreover, the effective teacher is able*

to make sense of students' conceptual understandings and is able to determine where those understandings might be heading.

These characteristics of effective teachers coincide with recommendations from the Teacher Education Ministerial Advisory Group's (TEMAG) (2015, p. 12) for teacher education programmes to ensure that new teachers not only possess strong content knowledge but are also equipped with pedagogical strategies that will allow them to be effective from their first day. This is no easy task. During the course of their teaching degree, secondary preservice mathematics teachers (PSMTs) typically have limited opportunities to observe classrooms or to analyse student work (Anthony & Hunter, 2015; Simpson & Haltiwanger, 2017), leaving them ill-equipped to attend to and interpret thinking that may not be their own.

To address this lack of experience, the authors chose to implement an Interview Module (IM) in one secondary teaching methods course in an Australian university. The intervention, developed by the second author and colleagues, has been employed with secondary PSMTs across various universities in the United States (see Monson et al., 2020) with positive results. After engaging in the IM PSMTs showed gains in all three component skills of noticing (i.e., attending, interpreting, and responding) and importantly, were better able to craft responses that elicited or built on student thinking (Casey et al., 2018; Monson et al., 2020). Given similarities in teacher preparation practices at the researchers' universities (Lesseig & Hine, 2021), coupled with shared goals for responsive mathematics teaching in the US and Australia (Association of Mathematics Teacher Educators, 2020; National Council of Teachers of Mathematics, 2014; TEMAG, 2015), we hoped the IM would support similar advances in this new context. The primary aim of this study was to explore the extent to which the IM supported the development PSMTs' ability to notice and make pedagogical decisions based on student thinking. A secondary aim was to establish the viability of the IM in this new context.

Literature Review

The intervention employed in this study was an Interview Module designed to develop secondary preservice teachers' noticing abilities. Based on initial research, the IM was modified to incorporate activities to directly support PSMTs' abilities to respond to student thinking (Lesseig et al., 2016; Monson et al., 2020). Given the nature of this intervention, we first define professional noticing, discuss its theoretical roots, and highlight what research has revealed about the construct and its relationship to other aspects of teaching. We then review the literature surrounding teachers' typical ways of responding to student thinking that motivated the intervention and helped frame our analysis. Finally, we summarise characteristics of successful noticing interventions, to situate our investigation of secondary preservice teachers' noticing.

Professional Noticing of Student Thinking

Teacher noticing is rooted in Goodwin's (1994) articulation of professional vision as the distinctive ways in which members of a particular social group or profession see and understand events. In essence, teacher noticing is the process teachers engage in as they actively attend to, discriminate among, and make sense of the overabundance of sensory data available in an instructional situation (Sherin et al., 2011). The construct of teacher noticing necessarily positions teachers as active decision-makers who act in response to what is noticed (e.g., deciding what, how, and whose mathematical ideas are leveraged in classroom discussions). In his influential treatise on the discipline of noticing, Mason (2002) describes a process of sensitising oneself to notice salient aspects in-the-moment that will enable one to act freshly, rather than habitually, in future situations. Noticing should bring to mind a different way of responding. A key attribute of productive noticing therefore is the ability to hold open multiple, even contradictory, interpretations and to consider the implications of various possible actions (Mason, 2011).

Our work deals with a narrow slice of noticing that Jacobs and colleagues (2010) refer to as professional noticing of children's mathematical thinking (hereafter professional noticing). Professional noticing is comprised of three interrelated, consequential practices: attending to the mathematics evidenced in student thinking, interpreting what that thinking reveals about student understanding, and deciding how to respond to the student based on this interpretation. Attending to details in students' mathematical contributions and interpreting student strategies in relation to learning trajectories and/or research on common student conceptions (or misconceptions) is often considered a precursor to productive responding. However, these three components are not necessarily distinct and in practice often occur simultaneously (Jacobs et al., 2011).

As an instantiation of teacher decision-making, professional noticing is naturally intertwined with teacher knowledge and orientations (Schoenfeld, 2011; Thomas et al., 2017). However, the relationship among these constructs is complex (Bray, 2011; Dreher & Kuntze, 2015) and is mediated by contextual and cultural factors (Ding & Dominquez, 2016; Yang et al., 2020). Research with elementary preservice teachers has consistently demonstrated that while strong content knowledge is necessary to productively interpret and respond to conceptual errors or alternative strategies, it is not sufficient (Bartell et al., 2013; Maher & Muir, 2013; Son, 2016). Research at the secondary level has also revealed some dependency on content knowledge; however, the ability to interpret and respond at higher levels appears to be more heavily influenced by teachers' knowledge (or lack of knowledge) of students' mathematical thinking (Sánchez-Matamoros et al., 2019) and beliefs about teaching (Dreher & Kuntze, 2015; Santagata, 2005; Son, 2013; Wieman & Webel, 2019).

Studies investigating relationships among the three component parts of professional noticing have demonstrated that teachers' ability to respond is often contingent on the degree to which they are able to detail student strategies and connect student thinking to important mathematics (e.g., Sánchez-Matamoros et al., 2019; Shin, 2019). However, attending and interpreting at high levels does not always lead to productive responses. Perhaps because of its complexity and potential co-dependencies (e.g., on one's ability to attend and interpret with some detail, knowledge of content and students, and beliefs about teaching) deciding how to respond has proven to be the most difficult of the three components for preservice teachers to enact with expertise (Krupa et al., 2017; LaRochelle et al., 2019; Lee & Choy, 2017; Sánchez-Matamoros et al., 2019; Simpson & Haltiwanger, 2017).

Difficulties in Responding to Student Thinking

The intervention employed in our investigation of Australian PSMTs' noticing was specifically designed to address reported difficulties in responding to student thinking and disrupt the teacher-centred approaches that continue to dominate mathematics classrooms (Nachlieli & Tabach, 2019). Reverting to the well-documented Initiate-Respond-Evaluate (IRE) pattern of interaction (Cazden, 2001) is even more common when responding to student errors or incomplete ideas. Rather than pose questions or next tasks that build on student thinking, teachers tend to respond to errors by giving answers or explaining procedures (Son & Sinclair, 2010; Son, 2016; Weiland et al., 2014). According to the work of Santagata (2005), there is scope for mathematics teachers to use student errors as a public opportunity for further elaboration of mathematics concepts or to consider the reasoning behind the errors.

The literature base suggests that PSMTs envision their role as one delivering instruction rather than listening and responding to students (Shin, 2019; Son, 2010; Son & Sinclair, 2010). To commence, Son and Sinclair (2010) investigated how elementary preservice teachers responded to a conceptual student error in a geometry task. The most common type of responses involved some form of showing, telling, or talking to the student generally about the related geometric properties. Other approaches included those where the teacher assumed the student needed to 'return to the basics' or at the opposite end of the spectrum, assumed that the student had the requisite knowledge, but had merely forgotten. In the former, responses entailed some dumbing down or over-simplifying the original task, whereas the later approach led PSMTs to provide information or reminders—conveying to the student that the situation required memory rather than understanding. In a similar study involving a ratio and proportion task, Son (2013) documented difficulties both elementary and secondary preservice teachers had in providing concept-based responses. Shin (2019) found that secondary PSMTs responded to what they noticed about students' interactions with a technological tool (TinkerPlots), rather than to students' statistical thinking. Across these studies, PSMTs' responses were more typically oriented toward procedural assistance than developing conceptual understanding.

Professional Noticing Interventions

There is widespread consensus in the field that developing PSMTs' attention to student thinking is not only achievable but should be a critical focus in teacher education programs (Anthony et al., 2015; Jacobs & Spangler, 2017). Research has documented how structured analyses of student work not only supports professional noticing, but also contributes to productive beliefs about mathematics teaching (Casey et al., 2018; Warshauer et al., 2015). Such structured interventions help PSMTs recognise the importance of moving beyond black and white interpretations of student understanding in order to allow student understanding to guide instruction (Busi & Jacobbe, 2014; 2018). For instance, McDuffie et al. (2014) designed a video-based intervention to hone PSMTs noticing on students' multiple mathematical knowledge bases. These researchers utilised four lenses (i.e., teaching, learning, task, and power and participation) to shift elementary PSMTs' noticing away from isolated teacher actions toward deeper interpretations of student thinking and awareness of the relationships between teaching and learning. In a more generalised noticing context with secondary PSMTs, Roller (2016) designed and conducted a video noticing intervention in a microteaching lab setting (concurrent with a teaching methods course) where participants received feedback from the university instructor and peers, and engaged in reflective class discussion. As a result of their participation, PSMTs showed developmental progress, demonstrating the ability to look beyond their own teaching manner to focus on student learning at a level beyond that typically demonstrated by novice teachers.

Various successful interventions have emerged in the steadily growing literature base of professional noticing. Common among these interventions is the use of tools and resources including specific frameworks or lenses to focus PSMT noticing (McDuffie et al., 2013; Schack et al., 2013; Stockero et al., 2017), structured time for peer discussion and reflection (McDuffie et al., 2013; Roller, 2016), and feedback from university faculty and peers (Amador & Carter, 2018; Fernandez, 2020; Roller, 2016). Such interventions have included lesson study (Amador et al., 2016; Amador & Carter, 2018; Lee & Choy, 2017), clinical interviews (Lee, 2018; Schack et. al., 2013), video club (Stockero et al., 2017) and animation techniques (LessonSketch) (Casey & Amidon, 2020; Lee, 2020).

Research Design

This research project is based on an IM, which was developed by one of the authors and colleagues and has been used within the United States to demonstrate gains in PSMTs' noticing abilities. The IM is comprised of a pre-post video assessment, prescribed readings, a one-on-one interview with a secondary student, a sequence of responding assignments involving analysis of student work samples and a summative, written reflection paper (Monson et al., 2020). A timeline has been included in Table 1 below to indicate when PSMTs completed the activities of the IM. It should be noted that the face-to-face delivery of the course was disrupted by the COVID-19 pandemic from Week 5 onwards (of a 9-week course). In a similar manner to all Australian universities, all classes had to be offered online from that time thereafter. The 10-week practicum experience was also cancelled for all students enrolled in this course, and therefore the one-on-one interviews with secondary school students could not be held.

Week	Task(s) Completed
1	Introduction to Interview Module, Pre-Video Assessment, Reading 1
2	In-Class Responding Assignment (pp. 1-2)
3	Reading 2, In-Class Responding Assignment (pp. 3-4)
4	In-Class Responding Assignment (pp. 5-6)
5	Take-Home Responding Assignment: Analysis of Student Work Samples (Students A & B
6	Take-Home Responding Assignment: Analysis of Student Work Samples (Students C & D)
7	Take-Home Responding Assignment: Analysis of Student Work Samples (Student E)
8	Post-Video Assessment
9	Written Reflection

Table 1. Timeline of Interview Module Activities Undertaken by PSMTs

Participants and Context

Data for this paper were collected from a cohort of PSMTs enrolled in a secondary mathematics education course in their first or second year of a teacher preparation program at one Australian university. During this course, PSMTs attend 27 hours of instruction (9 three-hour classes) where key topics, ideas, and strategies about teaching mathematics in secondary schools are presented and explored. The role of secondary mathematics teachers, effective instructional techniques, and the importance of reflective practice are examined. National and state school curriculum documents are interrogated and applied to lesson planning and forward planning documentation. Pedagogical approaches, assessment practices, and the use of resources are considered from an age-appropriate perspective. Following the completion of the course, PSMTs are expected to apply the knowledge and skills acquired during a formal 10-week school experience, where they will plan, teach, evaluate and reflect upon a program of work. In particular, the data collected for this project came from two instruments embedded in the IM: the pre-post video assessment and the summative reflection paper PSMTs submitted at the end of the course. From a cohort of 31 PSMTs, 27 completed the pre-video assessment, 20 completed the post-video assessment, and 18 submitted a summative reflection paper. It is worth noting that the number of PSMTs completing both the post-video assessment and written reflection (activities completed and submitted during lockdown/online learning) was lower than the number of PSMTs completing the pre-video assessment. PSMTs were invited to upload their post-video assessments and written reflections to an online repository accessible only by the researcher.

Instruments

Pre-post Video Assessment

The researchers collected pre-post assessment data, which were based on PSMTs' written responses to short videos shown prior to and upon completion of the full IM. These videos show a Mathematics Teacher Educator conducting a task-based interview with a secondary student. In each video, the student is asked to solve two quadratic equations (one resulting in one real solution, the other with two imaginary solutions). The problems and prompts given to the student are provided below in Table 1.

Question 1:	Solve for x: x^2-4x+4=0
Probe:	Could you solve that another way?
Question 2:	Solve for x: x^2-2x+3=0
Probe:	Could you solve that another way?

Table 1: Pre-post video assessment problems and prompts

After watching the pre- and post-videos, the PSMTs were asked to independently respond in writing to three prompts, each corresponding to one component in the noticing framework: (1) What do you notice? (Attending) (2) How would you describe what this student understands? (Interpreting) and (3) Describe some ways you might respond to this student and explain why you chose those responses (Responding).

Summative Reflection Paper

Following completion of both the course and the IM, PSMTs were invited to draft and submit a summative reflection paper. The four questions guiding the reflection paper are outlined below in Table 2.

Question 1:	How has the in-class instruction and this take-home assignment (videos, student work examples, readings) improved your ability to respond?
Question 2:	How do the responses you craft now differ from those you gave prior to this instruction on responding? What strategies do you use to develop your responses?
Question 3:	What implications does learning how to notice and respond to student thinking have for you in your future work with
Question 4:	students?

Table 2: Summative Reflection Paper Questions

Data Analysis

Both researchers used an *a priori* coding scheme adapted from the module developers (Krupa et al., 2017), and independently coded PSMTs' ability to attend to, interpret, and respond to student thinking on the pre- and post-video assessments. Each question was coded as demonstrating either no evidence (0), limited ability (1), or emerging ability (2). Initially, each researcher coded responses (pre- & post-video) from eight participants before meeting virtually to discuss any discrepancies in coding. After resolving all coding differences and reaching consensus, the researchers coded the responses from the remaining 19 participants before meeting virtually again to agree on a consensus score for each response. Reliability in coding was enhanced by the creation and maintenance of an operative codebook with examples and non-examples of responses at each level (Miles et al., 2013), and any discrepancy was discussed in reference to the codebook until consensus was reached.

For the summative reflection paper, the researchers analysed all written reflection data according to a framework offered by Miles et al. (2013) which comprises four key steps: data collection, data reduction, data display, and conclusion drawing/verification. In turn, each of these steps comprises the components: coding, memoing, and developing propositions. After all written reflection data were collected, the researchers developed an *a posteriori* coding scheme to analyse collected data. Following a similar process to the pre- and post-video assessment data, the researchers initially coded a sample of written reflections (six each) before meeting virtually to discuss generated codes and the application of those codes to raw data. After reaching consensus on a coding scheme, and the application of this scheme to the initial sample of written reflections, the researchers independently coded the remaining 12 reflections. The researchers met virtually again to reach consensus on the remaining written reflections before continuing with the selected analytical framework. While there was some overlap in participant responses, the coded responses for Question 1 were categorised as Benefits; responses for Questions 2 and 3 were categorised as Developing Responses; and those for Question 4 were Implications. These categories are explored further in the Discussion section.

Results

Pre-post Video Assessment

The pre-post video assessment data provided the researchers with an opportunity to look for growth in PSMTs' attending, interpreting, and responding skills. The scores from pre-post video assessment for the 20 participants who completed both assessments are presented in Table 3. The scores represent the following: 0 = No evidence, 1 = Limited Ability, 2 = Emerging Ability. Tables 4, 5 and 6 outline verbatim participant responses according to *attending*, *interpreting,* and *responding* skills, respectively, together with the video phase (pre-/post-) and the consensus score (0, 1, 2) given to the response by the researchers (all participant names are pseudonyms). While not necessarily meant as exemplars, these responses have been included to indicate a qualitative range and to illustrate the researchers' coding levels.

	Attending		Interpreting		Responding	
	Pre	Post	Pre	Post	Pre	Post
Score of 2	5	3	3	7	3	6
Score of 1	12	14	15	10	10	11
Score of 0	3	3	2	3	7	3

Table 3: Pre-post video assessment scores

Changes in PSMTs' Noticing in the Pre-post Video Assessment
Attending

As a group, PSMTs predominantly received scores of 1 (limited) for attending in both the pre-video and post-video assessment with scores remaining relatively stable. When looking at individual changes (see Appendix), the majority of participants maintained the same score (11 of 20), four improved their scores, and five participants received lower scores on the post-video assessments. For the four participants who improved, two moved from the *no evidence* level (a score of 0) to the *limited evidence* level (a score of 1), and two increased from the *limited evidence* level to the *emerging ability* level (a score of 2). Presented in Table 4 are several verbatim participant responses to illustrate the researchers' coding levels for attending. Harry's response received a score of 0 as he was unable to name and detail a method/procedure the student used to solve the problem. The pre-video assessment response from Addy (scored as 1) included several named methods/procedures, provided some evaluative comments on the student's procedure, and considered the role of the interviewer in some detail. In contrast, Addy's post-video assessment response (scored as 2) named and detailed various methods/procedures, provided some evaluative comments on the student's procedure, and included commentary on graphing.

Participant	Response
Harry (Pre-video, 0)	Her method is correct but her understanding of the maths is lacking. She's just following the steps without understanding. She recognised the perfect square.
Addy (Pre-video, 1)	Factors first; other way - quadratic formula; knew when to factor and when to do quadratic formula; unsure on answer when [it is] $\sqrt{-8}$, no solution (says "0"). Teacher asked to talk through; asks her why she is doing those steps.
Addy (Post-video, 2)	Student understands how to factorise and use factor pairs and uses this as the step for both of the equations. Student understands that when she cannot factorise to use the quadratic formula. Thinks of graphing the equations to solve for x - knows what a quadratic looks like. Stops when there is a negative square root.

Table 4: Examples of participants' attending responses

Interpreting

Again, the most common score for interpreting was 1 (limited) on both assessments. However, as a whole there was a slight gain, with the average score moving from 1.05 to 1.2. In addition, seven PSMTs received a score of 2 (emerging) on the post-video assessment, while only three did so on the pre-video assessment. Sixteen participants either improved upon (7 of 20) or maintained (9 of 20) the same score from the pre-video to post-video assessments. For the seven participants who improved, two moved from the *no evidence* level to the *limited evidence* level, and five increased from the *limited evidence* level to the emerging ability level. Presented in Table 5 are several verbatim participant responses to illustrate the researchers' coding levels for interpreting.

Participant	Response
Joanna (Pre-video, 0)	Makes mistakes —> corrects self —> reflection. Knows fundamental rules. Probing questions. Instead of telling her the answer, gets her to think about it more and to draw these out.
Tony (Pre-video, 1)	Can factorise in simple form for x^2. Understands the $\pm$ and x part. Knew about quadratic formula but couldn't use it. Knows about getting the square root of a negative number. Good addition and subtraction with negative numbers. Knows quadratic equation but not how it looks.
Peta (Post-video, 2)	This student understands how to solve for x by factorising and then how to check whether her answer(s) are correct by using FOIL. The student also understands that the quadratic formula can be used to solve for x and knows how to do [this] using the quadratic formula. However, it can be seen that the student does not arrive at the correct final answer as it seems though she doesn't understand that you cannot take the square root of a negative number.

Table 5: Examples of participants' interpreting responses

The response from Joanna received a score of 0 as there was no or limited evidence detailing what the student does or does not understand. Earning a score of 1, Tony named various mathematical ideas/relationships used by the student, and mentioned the student's strengths and weaknesses. The post-video assessment response from Peta received a score of 2, as there were various mathematical ideas/relationships identified, several student strengths and one weakness mentioned - with the weakness, the students' understanding of the square root of a negative numbers, noted in specific detail.

Responding

PSMTs exhibited the greatest gains in responding scores. Initially, the average score for responding was 0.8 (the only average below 1) but increased to 1.15 on the post-video assessment. Seven participants initially scored 0 (no evidence) whereas only three did so on the post-video assessment. The number of participants scoring at the emerging ability level increased from 3 to 6, almost matching gains in the interpreting scores. All participants except two either maintained (11 of 20) or improved (7 of 20) their score from the pre-video to post-video assessment. Of the seven participants whose scores improved, three moved from the *no evidence* level to the *limited evidence* level, two moved from the *limited evidence* level to the *emerging ability* level, and two increased by two levels moving from no *evidence* to the *emerging ability* level. The scores for participants Yorke and Yvonne moved down one level each. Presented in Table 6 are several verbatim participant responses to illustrate the researchers' coding levels for responding.

Participant	Response
Julian (Pre-video, 0)	Depends on the teaching method you are looking to implement. However, the teacher should correct the mistakes made during the 1st and 2nd question.
Jeanne (Post-video, 1)	This student seems confident when factorising, I would give her some more difficult questions involving factorising to challenge her. I would help her revise on what it means when you have a negative number inside of a square root symbol (as she thought you were still able to solve it).
Damon (Post-video, 2)	I would ask her to draw up a small table for the 1st/2nd question to assist with drawing the graph. She is saying there is going to be 2 zeroes but not drawing the graph correctly, and there is only one zero point. Doing a small table will help her actually draw the graph out and realise what she is looking at - this will assist with realising there is no solution for the second question. I would also compliment her on her understanding of solving algebraic equations and following a good process (and checking her work) - this helps her make sure she doesn't make any silly or unnecessary errors in the process. I think it's important you provide positive feedback when something is done correctly rather than just pick up on the things that need improvement. The main issue here is not being able to relay the algebra with the graphical notation of the equations to make sense of answers.

Table 6: Examples of participants' responding responses

According to the researchers' coding, Julian's response was scored as a 0 due to the overall lack of a mathematics education trajectory, as well as any specific details regarding the actions the teacher might take in "correcting the mistakes". The response from Jeanne received a score of 1 as within her response there was evidence of offering further questions to probe or extend student thinking. Damon's responding response (scored as 2) was well connected to his interpreting response, and overall there was a solid mathematics education trajectory. Damon offers a specific instructional move to confront an identified weakness that builds on student thinking and can lead to increased understanding.

Summative Reflection Paper

The findings from the summative reflection paper have been organised according to three categories, namely: Benefits, Developing Responses, and Implications. A summary of codes, code descriptions, number of quotations, and number of PSMTs have been tabulated for each of the key findings in Tables 7, 8, and 9, respectively.

Benefits

Half of the participants (9 of 18) stated that the IM benefitted them through the provision of examples of student thinking, and through being exposed to a range of ways students may approach problems. To illustrate, Rosie shared:

> *It has allowed me to see actual students' work and thinking which has been amazing practice in how to understand how students think and have time and support to craft a useful*
> *response for the student. It showed me that it is easier to respond if the student talks through what they have done and why they got stuck instead of just assuming what they got confused about.*

In a similar vein, Damon outlined how the IM

…has shown me how varied student performance over one task can be and how the problem can be interpreted in a number of different ways by a student and can be understood (or misunderstood) through a variety of techniques or approaches.

A number of participants (7 of 18) also described how the Interviews Module provided them with a useful model for how to respond to students. Jack explained that

> *Prior to being exposed to [the IM] I would have had limited ability to help point students in the right direction without telling them the full answer. This would mean I wouldn't be allowing for students to think for themselves and would be limiting their learning.*

In support of this statement, Ewen highlighted that he had benefited as the IM "made me think about how my responses need to be crafted to aid the students' thought process, not to tell them the answer". Implicit in these comments by Jack and Ewen is the notion that the teacher's role is not to "tell" but rather to facilitate the development of students' thinking. As evidenced in the next section, this theme is echoed in PMSTs' descriptions of how they would respond to students in the future, and one we elaborate on in the discussion. Other frequently mentioned responses included the provision of student misconceptions or errors, as well as the provision of an authentic experience undertaken by mathematics teachers.

Code	Code Description	Number of Quotations	Number of PSMTs
BEN-EX	Provided examples of student thinking; exposed them to range of ways students may approach problems	9	9
BEN-MOD	Provided models for how to respond to students	7	7
BEN-ERR	Provided PSMTs with student misconceptions/errors	6	5
BEN-AUT	Provided PSMTs with an authentic experience-recognised as something they need to do in real teaching	5	4

Table 7: Summary of Benefits to Participants

Developing Responses

As a result of participating in the IM, all PSMTs were able to identify at least one way that they would respond differently to students in future teaching/learning opportunities. More than half of the participants (13 of 18) stated that they would now build on or ask about student thinking rather than giving the answer, with the code RES-ST being applied to 23 statements. To commence, Isaiah noted "I think that it is really important not just to tell the student the answer, [we] need to make sure we are challenging the student's thinking and not giving them answers". Similarly, Celine stated

> *My responses to student work initially just explained how to do the question. None of my responses were focused in asking student to think and problem solve for themselves. Now I feel my responses encourage students to learn rather than just help them to work through that specific question.*

A majority of participants (10 of 18) expressed that in the future, they would ask students open-ended questions. Following on from the above-mentioned response, Celine added:

> *I now ask open-ended questions that make the student contemplate why they have had trouble or remind them of something they may have forgotten. I use open-ended questions such as "Why did you think that it is the wrong answer?" Instead of, "This is where you have gone wrong, do this instead". It encourages*

the students to think more critically and may help them remember what they have potentially forgotten or missed when completing a question.

A similar response was proffered by Yorke, who wrote that

Wherever possible, I will try to frame questions so that the student needs to think about the answer and be able to justify or explain their response. Ideally, by doing this, the student is the one who discovered the next step rather than just being told. Closed questions are useful to confirm a level of understanding and set something of a reference point, while open-ended questions are used to try and encourage the student what they are doing and why they are using that approach or process.

Other commonly expressed responses included PSMTs planning on taking the time and reflect on students' questions before responding, and to provide more specific responses.

Code	Code Description	Number of Quotations	Number of PSMTs
RES-ST	Will build on or ask about student thinking rather than telling answers	23	13
RES-Q	Will ask open-ended questions	14	10
RES-DIF	Will ask them to think about different representations	8	7
RES-REF	Will take time and reflect before responding	8	6
RES-SPE	Responses will be more specific now	7	7

Table 8: Summary of Future Responding Approaches for Participants

Implications

When asked to comment on the extent to which learning to notice and respond to student thinking will influence their future work with students, all PSMTs were able to identify at least one implication. The most popular coded response was IMP-REF, where half of the participants (9 of 18) expressed that they now see reflecting on student thinking and responding appropriately as what mathematics teachers need to do. For example, Julian explained that the IM "… will allow my responses to student thinking and learning develop over time to a point where my responses will provide opportunities for students to learn and develop their craft based off of these responses". Yorke also highlighted how reflection and responding were concomitant processes, where for him

The first point is to try and understand what particular learning style is preferred by the student, so that I can respond in a manner that best aligns with this. When I hear a question or an explanation from a student, by replaying it to them, I am checking that I have understood that they have asked or explained, which is important to ensure that we are aligned.

Half of the participants also mentioned how responding to student thinking can lead to students' deeper learning, whereas simply telling students answers promotes rote learning. To illustrate, Celine stated that "Responding appropriately is critical to encouraging students to problem solve and evaluate their work. Just giving them the answer explaining how to do the question again will likely not result in the student learning anything". This response was amplified by Damon, who shared

I think it is crucial, it will allow me to deepen my connection with each student and more effectively instruct and help guide them in mathematics. I think a big *problem, and where a lot of students struggle, is that they understand how to do something and then the rote learning method applies. Then, when something is different to what they've seen or they make an error that don't have enough a deep enough understanding of what they doing to 'gut-check' or to try to prove they have determined the right answer. I think being able to notice when students have an issue, and then respond in a way that is in line with their level of understanding and 'where they are at' allows a teacher to more appropriately answer and assist students.*

Other commonly registered responses included boosting student confidence and building problem-solving skills, and to build teacher-student relationships.

Code	Code Description	Number of Quotations	Number of PSMTs
IMP-REF	Similar to RES-REF, see reflecting on student thinking and how to respond as what teachers need to do	10	9
IMP-DEEP	Sees that this other (modelled) way of providing responses can lead students to deeper learning - as opposed to just telling which leads to rote learning	9	9
IMP-DIFF	Differentiation: students learn differently and it is necessary to meet the needs of different students	5	4
IMP-AFF	Responding in new ways can also boost student confidence (or other affective benefits) and build problem solving skills	4	3
IMP-REL	Responding can help build relationships with students	4	3

Table 9: Summary of Implications for Participants' Teaching

Discussion

Despite only slight improvements in the post-video assessment scores, PSMTs' written reflections indicate that the IM was a worthwhile experience. The module activities not only opened PSMTs' eyes to the range of thinking they might expect from students, but also gave them opportunities to practice responding in a way that honours and extends that thinking. Perhaps most importantly, the module activities raised PSMTs' awareness of the limitations of feedback that is solely focused on correct answers or a set procedure; hence changing PSMTs' views on their role and the type of responses they need to provide. In line with the secondary aim of this project we are encouraged by the results and contend that the IM is an effective intervention that can be readily implemented in differing contexts. We see this work as a critical step toward preparing future secondary teachers for responsive teaching.

There was no dramatic increase in pre-post video assessment scores. In fact, while the majority of PSMTs' scores for attending and interpreting improved or remained the same, there were a number of PSMTs who scored lower on the post-video assessment (5 for attending, 4 for interpreting). Such inconsistencies in noticing are perhaps not surprising given the relatively short time frame of the intervention and the fact that for most PSMTs this is the first time they have been asked to do this type of work (Simpson & Haltiwanger, 2017). As mentioned earlier, these results may also simply be a consequence of the mid-term disruption that occurred due to the COVID-19 pandemic. Unlike the pre-assessment, PSMTs completed the post-assessment in a fully online environment when they may have also been experiencing an increased level of stress. That said, there was a noticeable improvement in responding with the number of PSMTs who scored 0 (no evidence) decreasing from seven to only three. Six PSMTs scored a 2 (emerging ability) on the post assessment whereas only three did so prior to completing the Module. As evidenced in Julian's response above, initial responses tended to focus on correcting procedural errors (Sánchez-Matamoros et al., 2019). In contrast, in the post-video assessment PSMTs more often suggested responses that validated the student thinking that was presented and focussed on helping students make connections and build conceptual understanding. These improvements in responding are especially encouraging given research documenting that responding is the most difficult of the three component skills for both practicing and preservice secondary teachers (LaRochelle et al., 2019; Lesseig et al., 2016; Sánchez-Matamoros et al., 2019). Perhaps more importantly, these types of responses are indicative of a shift in PSMTs' view of their role in promoting student understanding.

Our analysis of the written reflections further revealed ways in which the IM supported a shift in PSMTs' understanding of the overall purpose of teaching along with the development of productive dispositions and professional noticing skills. PSMTs commented on how the authentic examples of student work, together with models of good responses, would help them respond differently in the future. Relatedly, PSMTs' reflections indicated a new awareness of the need to respond differently—in ways that moved beyond evaluation and led to deeper learning. We elaborate on these themes below and discuss implications for teacher educators.

Opportunities to Engage with Authentic Student Work

The student work offered in the responding components of the IM revealed the variety of ways students might approach a task—ways that were often markedly different than PSMTs own preferred methods. Exposure to authentic student work helped PSMTs in our study realise that students have differing strengths and weaknesses. As a consequence, PSMTs saw how important it was for them to gather evidence of student thinking and interpret that thinking in order to meet the needs of individual students. Specifically, PSMTs talked about aligning their responses with the student's learning style and current understandings (Yorke) and not just "making assumptions" about what students understand (Rosie).

PSMTs saw the need to value approaches that differed from their own and to look beyond the correct answer when attending to and interpreting student thinking. These findings reinforce those in Simpson and Haltiwanter's (2017) study and highlight the benefits of providing preservice teachers with structured opportunities to analyse student work. Similar to Sánchez-Matamoros and colleagues (2019) we argue that PSMTs would benefit from exposure to a range of student responses in order to see that there will be variation and to recognise the importance of teasing out those differences. These varied examples, coupled with ensuing discussions about how to best respond, inspired PSMTs to think differently about strategies they would employ in the future. More than two-thirds of PSMTs professed that they would now make sure to build on or ask questions about student thinking, rather than just provide answers. They were committed to asking open-ended questions and providing space for student thinking. Given the predominance of the IRE pattern of interaction in secondary classrooms, this result is particularly noteworthy and leads to our second theme.

Access to New Forms of Responding

In their reflections, PSMTs not only discussed specific ways in which their responses would differ, but also indicated an awareness that they *needed* to respond differently. PSMTs saw limitations associated with simply providing correct answers or procedural explanations. Such responses, they concluded, lead to rote learning and do little to help the student solve problems in the future. PSMTs discussed how, in contrast, asking questions or building on the students thinking processes could lead to deeper student learning. PSMTs also noted how this way of responding might boost student confidence and problem-solving skills as well as promote more positive student-teacher relationships. In short, PSMTs moved from what Walshaw and Anthony (2008) describe as path-smoothing or product-help assistance to consider the long-term benefits of process-help assistance. This result bodes well for what PSMTs may be capable of in the future as Walshaw and Anthony's (2008) report highlights how teachers' ability to differentiate among student responses and provide assistance based on process rather than product correlates with more effective pedagogies that supported student learning.

Just as critical, the module activities supported positive shifts in PSMTs' views of students. In moving away from a focus on correct answers, PSMTs shifted toward a more strengths-based approach to attending to and interpreting student work (Jilk, 2016; Kalinec-Craig et al., 2020). Comments in the written reflection evidenced a belief that students are capable and that student ideas are worthy of attention. Responding in ways that build on student thinking, rather than redirect that thinking, requires particular skills and tools. Providing PSMTs with models of such responses, and a vision of what this type of responding looks and sounds like is a first step toward developing those skills. Second, and perhaps more importantly, professional noticing of student thinking, and responsive teaching more generally, requires that teachers adopt productive dispositions toward student-centred instruction (Dreher & Kuntze, 2015; Schoenfeld, 2011; Walshaw & Anthony, 2008).

Implications for Teacher Education

Results from this study, coupled with our ongoing work as mathematics teacher educators, lead us to make the following recommendations:

1. Provide repeated opportunities for preservice teachers to analyse authentic student work samples. The samples should include varied approaches and show a range of understandings (Sánchez-Matamoros et al., 2019). Student work in which the answer is incorrect but the explanation is correct (or vice versa) is particularly powerful in terms of generating productive analysis (Warshauer et al., 2015) and can disrupt the tendency to conflate procedural aptitude with conceptual understanding.
2. Model alternative ways of responding to students. If we expect the next generation of teachers to respond differently (i.e., shifting beyond evaluation to clarify, validate, and extend student thinking) then we need to demonstrate what that looks and sounds like at the secondary level. Coupled with examples of teacher responses, we recommend the use of structured protocols for attending, interpreting, and responding to student thinking similar to those developed for elementary methods courses (e.g., McDuffie et al., 2014; Schack et al., 2013). In our case, the list of four characteristics of a good response (Monson et al., 2020) provided specific criteria to guide discussion and self-reflection.

3. Motivate the need to respond differently. Disrupting long-standing patterns of interaction that have typically centred the teacher's, rather than students', ideas is no easy task. It demands that teacher educators not only equip future teachers with new tools and skills, but also support the development of productive dispositions and beliefs about students' capabilities. The responding components of the IM sparked frequent discussions in which the limitations of attending and responding only to procedures (whether correct or incorrect) were made explicit. These discussions effectively shifted PSMTs' beliefs about the importance of taking up student ideas and the teachers' role in facilitating deeper learning.

While these recommendations originated from our work in mathematics methods, we contend that the general principles can be extended to other content areas, or to address other desired teaching practices. We encourage all secondary methods instructors to consider ways in which they: expose preservice teachers to authentic student work; model expected teaching practices—especially when the desired teacher actions run contrary to what the preservice teachers themselves may have experienced as students; and make the benefits of the novel teaching practice(s) explicit.

Conclusion

The purpose of this study was to explore the extent to which an IM (Monson et al., 2020) impacted PSMTs' ability to notice and make pedagogical decisions based on student thinking. Having participated in a majority of components comprising the IM, PSMTs on the whole regarded their involvement as beneficial to their development as teachers. Principally, the IM effectively shifted PSMTs' beliefs about teaching and learning, and helped promote productive teacher dispositions. According to PSMTs' testimony, these shifts were enabled through opportunities to engage with authentic student work, and the access they were given to new forms of responding. While the researchers see these gains as important first steps towards responsive teaching, they re-affirm scholars' claims that enacting the practices of attending, interpreting, and responding in real time is a complex task for teachers (Jacobs et al., 2010, Schoenfeld, 2011). Moreover, Schoenfeld (2011, p. 233) emphasised that

> *Noticing is essential, but it does not suffice by itself. It takes place within the context of teachers' knowledge and orientations; and the decisions that teachers make regarding whether and how to follow up on what they notice are shaped by the teachers' knowledge (more broadly resources) and orientations.*

With the complex nature and affordances of professional noticing in mind, we underscore the implications of this study's findings. Despite widespread recognition that professional noticing opportunities are valuable (Anthony & Hunter, 2015), it is our contention that preservice teachers need continued access to intentional responsive teaching modules across their degree programs. Doing so would better prepare them for the profession through development of pedagogically appropriate instructional strategies and cultivation of productive dispositions.

Appendices

Participant	Attending score of 0, 1, 2 (Pre)	Attending score of 0, 1, 2 (Post)	Interpreting score of 0, 1, 2 (Pre)	Interpreting score of 0, 1, 2 (Post)	Responding score of 0, 1, 2 (Pre)	Responding score of 0, 1, 2 (Post)
Addy	1	2	1	2	1	1
Alex	0	1	1	0	0	1
Bart	1	1	1	1	0	1
Bryan	0	1	1	2	1	2
Chelsy	2	1	1	2	1	2
Damon	1	1	1	1	0	2
Ewen	2	1	1	1	1	1
Harry	0	0	1	2	0	2
Isaiah	1	1	0	1	1	1
Jack	1	1	1	1	1	1
Jeanne	1	1	1	1	1	1
Julian	1	1	1	1	0	1
Lauryn	1	1	0	1	0	0
Peta	1	2	2	2	2	2
Rosie	2	2	2	2	2	2
Siana	1	1	1	1	1	1
Stan	1	0	1	0	0	0
Tony	1	1	2	1	1	1
Yorke	2	0	1	0	1	0
Yvonne	2	1	1	2	2	1

Key				
	Increased 1 from Pre- to Post-	Attending = 4/20	Interpreting = 7/20	Responding = 5/20
	Increased 2 from Pre- to Post-	Attending = 0/20	Interpreting = 0/20	Responding = 2/20
	Stayed the Same from Pre- to Post-	Attending = 11/20	Interpreting = 9/20	Responding = 11/20
	Decreased 1 from Pre- to Post-	Attending = 4/20	Interpreting = 4/20	Responding = 2/20
	Decreased 2 from Pre- to Post-	Attending = 1/20	Interpreting = 0/20	Responding = 0/20

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