Insights into Learning Disabilities 18(2), 187-209, 2021

Through an Equity Lens: Teaching Practices for Children who are Bilingual With Learning Disabilities during Mathematics Discussions

Juanita M. Silva
Texas State University, USA

This study presents three students who are bilingual with learning dis/abilities in small group mathematics discussions. The three students showcased their abilities as problem solvers, justifiers, and evaluators of basic operation and fractional understanding. This paper presents the teaching practices used to support the mathematical agency of three students who were bilingual with learning dis/abilities. The students were able to exhibit mathematics agency by using their own math strategies, explaining their thinking, convincing peers of their ideas, and take risks during group discussions. Overall, this study provides a documentation of what is possible when teachers use the strengths of students who are bilingual with learning dis/abilities to support the learning of basic operations and fractions.

Keywords: **Learning Disabilities, Bilinguals, Teacher Practices, Mathematics Agency, Word Problems**

Introduction

Mathematics critical thinking in basic operations and fractions is essential for all children to be successful in Science, Technology, Engineering, and Mathematics (STEM) careers (Gottfried et al., 2014). Early grade children benefit from learning not only the basic facts but also from being able to understand concepts in basic operations and fractions (National Mathematics Advisory Panel, 2008). In particular, children with learning dis/abilities (LD) usually have more difficulties in fraction concepts within word problems (Mazzoco & Devlin, 2008), and this can hinder their success in later content areas like Algebra needed for graduation (Shifrer, Callahan, and Muller, 2013).

Special education studies have mostly focused on mathematical interventions of children with learning dis/abilities or learning difficulties. These studies usually center teaching practices through individualized instruction. And instructional interventions often document children's cognitive deficits (e.g., executive function, impaired language skills, or working memory) (Tan et al., 2019), instead of children's strengths and how these allow them to be successful in constructing mathematical knowledge. For example, Flores, Hinton,

Insights into Learning Disabilities is published by Learning Disabilities Worldwide (LDW). For further information about learning disabilities, LDW's many other publications and membership, please visit our website: www.ldworldwide.org.

and Strozier (2014) describe their teaching intervention practices during direct instruction as the teacher demonstrating "several problems for the students" followed by "guided practice" that allows the teacher to provide several prompts on how to solve these problems, and finally allowing students to do "independent practice in which the teacher instructed the students to solve a set of problems without guidance" (p. 550). Students are expected to follow a set of procedures or steps to solve problems which then limits their mathematics learning to memorizing mathematics vocabulary and recalling facts (Robinson & Temple, 2013, Gersten et al., 2009). These teaching intervention practices limit the opportunities for children with learning dis/abilities to participate in complex mathematics concept construction and are absent of experiences where social interactions with peers are prioritized during the learning of mathematics.

A Word on Terminology

The research literature in education in the United States uses different terms to refer to children whose first language is not English and those with learning disabilities. For this reason, it is important to use a non-deficit and person first terminology to refer to the children of this study. In this article, the term bilingual applies to children who speak English and Spanish and will only refer to English Language Lerner (ELL) when citing or quoting the work of other authors who use the term and policy documents used to identify participating children in school assessments. In addition, the article uses the term learning dis/ability (LD) to refer to any child identified with a "learning disability" to position the child from a neurodiversity lens. A neurodiversity lens recognizes that all human brains are highly variable, with no average learners (Baker, 2017; Hunt, Silva & Lambert, 2019; Silva, 2020). The term also points out the inequities when identifying a child with a "disability" through a societal construct (Tan & Kastberg, 2017). Thus, the term dis/ability refers to children having the ability and power to learn mathematics regardless of their identifications in and out of school.

The Teaching of Children who are Bilinguals with LD in Special Education Literature

In intersecting the research of children with learning difficulties, LD, and who are bilingual within math and special education, the teaching practices usually follow what math LD studies do within their interventions, namely direct instruction (Garcia & Tyler, 2010; Orosco, 2014a). In these studies, they often subject children to the same instructional interventions documenting teaching practices of rote memorization of facts, direct or explicit modeling, and vocabulary identifications (Garcia & Tyler, 2010). Some researchers have focused on teaching practices for word problems, but they usually consist of children learning to identify keywords within the story context of the problem (e.g., "total" or "in all" means to add) (Orosco, 2014b). Teaching keyword identifications can be problematic for many reasons including students taking the keyword out of context, thus producing the wrong operation and it does not necessarily help them understand the problem as supported in the Common Core Standards for Mathematics (CCSS, 2010; Karp et al., 2019). In another study on children identified as English Language learners (ELLs) with math learning difficulties, Orosco and Abdulrahim (2017) looked at the teaching practices of a special education teacher during problem solving instruction where they explain the importance of using peer collaboration. They explain that the teacher allowed children opportunities to collaborate with peers, but these were in the form of correct pronunciation of mathematical vocabulary terms or work collaboratively to solve problems only after they received explicit and direct instruction and practice independently (Orosco & Abdulrahim, 2017). Often, these studies describe children as able to interpret word problems within whole class settings using direct explicit instruction on an individual level but ignore the potential of what children can do within small group instruction that focus on student's strengths.

Promoting High Order Teaching Practices for Children who are Bilingual with LD

Research shows that taking part in mathematical practices of problem solving, having discussions with peers about the mathematical strategies, and justifying solutions supports children's conceptual development bolsters achievement in the classroom (Bodovski & Farkas, 2007; Webb et al., 2014). Common Core Standards for Mathematics (CCSS, 2010) recommends that all children in K-12 mathematics classrooms engage in "making sense of problems", "constructing viable arguments", and "critiquing the reasoning of others" (CCSS, 2010, pp. 6). If research and policy in education press on the importance of using these practices within the teaching of mathematics, then we need studies documenting how to advance our understanding of how these could be implemented in the classroom with children who are bilingual with LD.

Using Children's Strengths in Mathematics Problem Solving

There exist equity asset-based teaching practices that have been successful for children who are bilingual in the mathematics classroom (Maldonado Rodríguez, Krause, & Adams-Corral, 2020; Moschkovich, 1999; Garcia et al., 2017). Moschkovich (1999) utilized practices such as re-voicing, clarifying questions, and gestures to help support children who are bilingual during mathematical discussions to promote mathematical understanding. Another highly successful teaching practice noted by Maldonado Rodríguez and colleagues (2020) is dynamic bilingualism, where teachers allow children to use "all their languages all the time" and to see "language as a resource" in helping them learn and create mathematical ideas (p.19). They documented that children who are bilingual learn better if they are emersed in using language flexibly during the learning process. They gain a sense of self and can contribute to class discussions. Not only do children who are bilingual can communicate with their native language during math instruction, but so do the practicing teachers. Krause, Silva and Aguilar (2020) showed the potential of teaching practices uncovered when pre-service teachers engaged in the practice themselves. Language flexibility between teacher and student helped teachers learn what to ask and how to prompt for specific questions, and that helped extend children's understanding of fractions. Although these studies were done with children who are bilingual, these teaching practices could also be beneficial to children who are bilingual with LD.

Similarly, a few researchers have documented asset-based teaching practices with children with learning dis/abilities or difficulties in mathematics (Hunt & Silva, 2020; Hunt & Empson, 2015; Lambert, 2015; Silva, 2020). Hunt and Empson (2015) interviewed a set of 10 third through fifth graders with LD and found impressive mathematical strategy conceptions of fractions using word problem tasks. Hunt and Silva (2020) recently documented children with LD can progress in their fractional understanding when teacher's use their prior knowledge to guide the instruction. They found that children have sophisticated ways of solving fractional problems. Children can advance in their conceptual reasoning when teachers provide responsive teaching moves like changing the context of the word problem, prompting to use a second strategy, or promoting the child to notice her "own ways of representing through revoicing" (Hunt & Silva, 2020, p. 344). Not only have researchers documented the benefits of these teaching practices, but also the drawbacks of teaching using direct individualized instruction. For example, Lambert (2015) documented the differences two children with LD experienced when a teacher used didactic teaching and problem solving during her math instruction. In this study, Lambert reports that these two children exhibited low self-esteem when the teaching was didactic and competence when the teaching was about problem solving.

Although work was been done separately to document how marginalized children populations can be successful when learning mathematics concepts, few studies have yet to document what children who are bilingual with LD can do (Silva, 2020, Lambert, 2015). Bridging the work of both bodies of mathematics research in bilingualism and LD, I hope to provide an example of what these children can do if provided with choices to show their prior knowledge, use their native langauge, and create their mathematical strategies.

Building Mathematical Agency Within Teaching Practices

In creating environments where teachers engage in these mathematical practices, children will not only gain conceptual understanding but also build *mathematical agency* (Empson, 2003; Turner et al., 2013), thus bolstering their competence levels as mathematical thinkers and doers of math ideas. Teachers could position children as power agents in their own mathematical learning.

Carpenter et al., (1998) argue that children who use invented strategies to "make sense" of the mathematics are more successful in achieving deeper understandings of the mathematics than the children who only use direct instruction strategies (e.g., standard algorithms). When children are given a choice to enact an invented strategy, this choice becomes essential in helping them develop *mathematical agency.* In this study the term *critical mathematical agency* is used, which Turner (2012) defines as:

> Student's capacity to (a) understand mathematics, (b) identify themselves as powerful mathematical thinkers, and (c) construct and use mathematics in personally and socially meaningful ways (p. 55).

Turner's definition highlights the importance of using problem solving around norms of choice (e.g., children have opportunities to select their solution strategies or how they communicate ideas), to support children's use of their prior knowledge to solve problems and thus see themselves as powerful mathematical thinkers who understand and construct mathematics.

A central role of enacting agency is when children have opportunities to socially negotiate ideas within the classroom, where they share and listen to others. In these spaces, children can see themselves as mathematicians who have expertise, can argue for their ideas, and can critique the ideas of others. Children can show competence in sharing their invented strategies with others and defend them if they need to.

This study sought to investigate the teaching practices that invited three children to develop mathematical agency when engaged in solving mathematics problems. Thus, centering the work around the research question: *How do children who are bilingual with LD respond to teacher moves that promote mathematical agency as sharing, defending, and taking risks when doing mathematics?* This study presents the case studies of three Latino/a children, Carlos, Jesus, and Thalia. These children were an example of elementary grade students who were bilingual with learning dis/abilities participating in a small intervention on problem solving discussions.

Method

Participants

The participants were third and fourth graders (aged 9 to 10 years old) attending one culturally diverse elementary school in an urban city in the southern United States. Approximately 50% English Language Learners (ELLs) and about 10% of the children were receiving special education services at this school. The three children were selected based on the following criteria: (1) Latino/a children with identified ELLs, (2) children who had identified math difficulties or LD, and (3) motivation to take part in the intervention in a pre-interview.

Children's math difficulties were Tier 3 identified under the response to intervention (RTI) model explained by Fuchs and Fuchs (2007) or a cognitively defined learning dis/ability. Children identified with cognitively defined LD were those who had individualized education goals in math (IEPs) and sustained low performance measures via the Woodcock Johnson test of achievement and tests of cognitive abilities. Carlos and Jesus were in fourth grade and Thalia was in third grade. The school identified Carlos and Jesus as having an LD and as being an ELL and Thalia as an ELL within a Tier 3 category under the response to intervention (RTI) model. Tier 3 consist of highly targeted individualized and intensive intervention, in which Thalia was being considered for special education but nothing formal had been initiated at the time of the study. All three children could discuss their mathematical thinking in English and Spanish at various degrees of proficiency (see Table 2 for more details). Carlos preferred to speak, write, and read in Spanish. Thalia and Jesus preferred to speak, write, and read in English. In their respective math classrooms, all children received grade level math problems and all of their IEP goals included needing remediation to solve problems that included place value operations, word problem solving, and operations. In a pre-assessment conducted by their math teachers, Carlos and Jesus solved double digit word problems using standard algorithms correctly, were as Thalia struggled to employ similar algorithms, often misinterpreting what the problem was asking her to do.

The assistant principal identified all three students in the school, showing they fit the criteria described above. Each participant's parent and guardian granted written consent for their child to take part in the intervention, along with written consent from each participating child. Participating children and parents had the option to opt out of the study at any time.

The study was conducted in a small tutoring group during each participating child's extra math time during school hours in a conference room and in a small resource room in the counselor's office. Each teaching session lasted approximately 50 minutes. The participants were given materials such as linking cubes, paper, pencil, and white-board and colored markers. Both rooms' layout consisted of a table in the center and large white board. All participants could communicate ideas in both English and Spanish throughout the study.

Data Collection

The qualitative study was conducted over one school semester with 12 tutoring sessions, and one semi-structured open-ended interview. Children's written work and video recording of their interactions were collected throughout the study for data analysis. The author served as the teacher-researcher and a graduate student helped with data analysis and interpretations.

During the semi-structured interview, the teacher-researcher used a prior assignment documented by (Chao et al., 2019) called *Numbers about Me*, where children share personal items about themselves using numbers. This activity helped the teacher get to know each child a little better, and thus create meaningful story contexts for the word problems presented in the tutoring sessions. There were no prior pre or post assessments given prior to beginning the intervention. The sessions were planned to include teaching moves during the problem-solving process around basic operations and fraction story problems (see Table 1 for a detailed description of each). The story problems and the teaching moves in the session were created using the frameworks from Carpenter et al., (2015) and Jacobs and Empson (2016).

Table 1. ***Description of Teacher Moves***

Teacher Moves	Description
Ensuring children are making sense of the problem	The teacher aids children in familiarize themselves with the story context. The teacher could ask specific children to describe specific details they know about the story problem (e.g., How many brownies does Juan have? And "How many brownies are inside each box?") and what the essential question is asking them to find.
Clarifying children's thinking	The teacher aids children in explaining the strategies used and provides prompts to help in clarifying what the problem is about. Also asks questions to help children link the story problem and the details of their current strategies.
Eliciting mathematical thinking	The teacher invites individual or pairs of children to explain the strategies used and attends to the details of the strategies used (e.g., I saw that you added six each time, why did you do that? How did that help you?)
Assigning competence to children's ideas	The teacher re-voices children's mathematical strategies and thinking, prompts children to justify their agreement with a peer's strategy, and invites children to evaluate their disagreement with a peer's strategy during the explore and discussion phase.
Extending children's thinking	The teacher will solicit different strategies, ask children to use a number sentence, or ask follow-up problems with challenging numbers during the explore and discussion phase.

The teaching of the sessions was planned using a problem-solving model with three phases: the launch, exploration and discussion (Stein et al., 2008). During the launch the teacher reads the story problem out loud to the small group of participants, asking questions (e.g., What is this problem about? What is the question asking us to solve? How many carrots does Frank have in the problem?) to help each child understand the problem. During the exploration, children are allowed to solve the problem in any way they would like to, using any strategy that makes sense to them. They may use manipulatives (e.g., linking cubes), paper and pencil to draw a strategy, use of fingers, or any other materials that could help them make sense of the mathematics within the story problem. The teacher walks around the group asking questions to understand how each child is solving the problems. The teacher asks questions like: How do you know to add 5 and 6 to get a solution? What in the story problem tells you needed to add? These questions help children make sense of their mathematical strategies to the word problem, where the teacher is looking for comprehension, rather than key word identifiers. Children are also given the choice to work individually or in partners to solve each word problem. During the discussion of the problem-solving model, the teacher asks children to share their strategies with the group, where they can share their ideas with their peers. Children usually participate by sharing their notebooks or writing their strategies on the whiteboard with the group. They then explain how they solved the problem and what they did to get a final solution. The teacher asks questions that invite children to notice each other's mathematical ideas (e.g., "Mark how did Jose solve the problem, can you explain it in your own words?", "What about Jose's strategy did you understand?", or "Do you agree or disagree with Jose's strategy and can you explain why?").

The sessions were collected to document how the teaching practices allowed children to develop or enact mathematical agency when participating in mathematical discussions during problem solving situations. Three primary data sources were collected: video recordings, field notes, and children's written work.

Data Analysis

In this study the data analysis was conducted using the constant comparison method of the teaching moves and mathematical agency codes evident in all 12 sessions. Followed by validity and reliability of the teaching moves and math agency codes.

In this study, the first author used constant comparison analysis (Glaser & Strauss, 1967) of the teaching moves for the first session using video, transcripts and children's work, which later were coded by a graduate research assistant independently and identified different categories in the video and transcripts. We identified how teaching moves promoted mathematical agency by examining the interactions of each case study across the 12 sessions. Before beginning data analysis, we first read through the transcriptions of all the sessions as a team. Next, we chunked the data into smaller sections (i.e., teacher moves pertinent to children's agency) looking for meaningful sections. All video transcripts were transferred and analyzed using MAXQDA 2018 software, where (n= 111) episodes where we identified of interactions and participation among the three children. We identified episodes as coherent interactions around an explanation of a single mathematical strategy in the exploration or discussion phase of the intervention (Corbin & Strauss, 2008). Once episodes were identified, the author and graduate student independently documented memo notes of each episode answering questions to find out when, who, what, and how children explained and showed their mathematical agency, and the teacher moves associated with these and noted any changes across the teaching sessions. Memo notes were used to identify what the teacher-researcher did to help children explain, elaborate, participate, and make sense of the mathematical ideas developed during problem solving. We coded each session individually for the teaching moves and met to confirm or disconfirm the presence of these teacher moves (e.g., assign children as experts, prompt children to share details in their strategies). Next, we met to create an initial codebook that we would use to code the rest of the sessions. We adapted and defined the categories deductively focused on the way children exhibited mathematical agency within the teacher moves. Throughout the constant comparative level of analysis, we compared each teacher move and mathematical agency code with previously coded data to ensure consistency. Subsequently, analyzing our codes and discussing any disagreements until we reached consensus. If new teaching moves were present, we conducted validity checks with subsequent sessions and met to peer debrief for agreement.

Results

The study yielded six themes regarding the teaching practices established that promoted mathematical agency among all three children. First set of themes were around the norms established around choice, allowing children to participate in ways that gave them the opportunity to choose (1) the language to communicate in, (2) the strategies to solve math problems, (3) how to work with others or individually, and (4) how to participate in discussions. The second set of themes were around the teaching moves during problem solving discussions that allowed children to exhibit mathematical agency. These moves include teacher prompts that (1) invite children to share specific details about their mathematical strategies, (2) ask children to critique other's mathematical ideas, (3) encourage children to take risks when participating in sharing their math ideas.

Problem Solving Norms Around Choice

When planning the intervention, the teacher provided opportunities for children to engage in elements of choice during math problem solving discussions. The goal was for children to have the freedom to express their mathematical ideas in ways that seemed natural to them and that did not restrict how they interacted with each other or how they participated in discussions. It was important that children had choices to communicate in any way that allowed them to make sense of the word problems and then engage in meaningful discussions about their mathematical thinking. The following paragraphs offer a glimpse of how the teacher established these norms in the small group discussions with Carlos, Thalia, and Jesus.

Choice of Language

When designing the study, the teacher purposefully allowed children to use their native language and English to communicate mathematical ideas. This choice was important because it allowed all children to communicate flexibly to share their ideas while learning mathematics. Carlos, Thalia and Jesus could use Spanish and English to communicate ideas. For example, during the beginning of Session five, the teacher read the word problem: *Messi has 24 soccer balls. He puts five soccer balls inside a bag. How many bags can he fill?* in both English and Spanish to help them interpret the context of the story, then asked a series of questions to help children make sense of the story. The following excerpt describes how Spanish and English were used to help them interpret the word problem.

Excerpt 1

Teacher: Quiero saber si ya me entendieron el problema. ¿De que se trata el problema? [I want to know if you already understood the problem. What is this problem about?]

Carlos: De [About] soccer balls

Teacher: Y que mas Jesús? [What else Jesús?]

Jesus: How much bags he needs

Teacher: Cuanto que? [How many what?]

Jesus: bags he needs

Carlos: Cuantas bolsas [How many bags]

Teacher: Cuantas bolsas necesita, verdad? ¿Ok, so cuantos balones tiene Messi en total? [How many bags does he need, right? OK, so how many soccer balls does Messi have in total?]

Jesus: Veinte cuatro [Twenty-four]

Teacher: Veinte cuatro balones de futbol tiene. ¿Y cuantos pone dentro de cada bolsa? [He has twenty-four soccer balls. And how many does he put inside each bag?]

Carlos: Cinco [Five]

Thalia: He puts five soccer balls

Teacher: Five soccer balls, so he puts five soccer balls inside

Thalia: So, he has a bag, and he puts one, two, three, four, five [Thalia uses gestures to count the five balls]

Teacher: That is right, so he puts one, two, three, four, five… Y que esta preguntando la pregunta, que queremos saber? [And what is the question asking us to find out?]

Thalia: Wha…What?

Teacher: What do we want to know? What is the question asking us to find out?

Jesus: How much bags…

Thalia: How many bags can he fill

Teacher: Hmhm, so we want to know the number of…

Carlos: bags

Teacher: So, we want to know the number of bags. Ok. So let's go ahead and go to our table and solve the problem in any way that makes sense to you. Hay que resolver el problema de cualquier manera que ustedes puedan entender.

In this session, and many others, children were encouraged to communicate ideas in both languages during their interpretation of any problem or when solving each problem. The teacher posed questions in both Spanish and English and translated any words or sentences they found confusing. In this excerpt, Thalia found it difficult to understand the question "*What is this question asking us to find out?*" in Spanish, and therefore the teacher translated it into English. This occurred throughout the sessions when participants found it difficult to remember a word in English or Spanish, and thus in the moment the teacher translated it to the language it was most accessible. The teacher would also remind children they could express their thinking in Spanish or English when explaining their math strategies during the problem-solving process and discussions. The teacher would prompt children by saying "*You can explain your thinking of your strategy in English or Spanish*" or invite them to share in both languages at the same time by asking "*How do you say pieces of chocolate bars in Spanish? Does that help you describe your strategy?*". The choice of language helped children communicate in ways that were accessible to them, therefore allowing them to concentrate on the mathematics content instead of the vocabulary or terms of the story problems. It allowed children to understand the context of each of the story problems.

Choice of Strategy to Solve Problems

Another norm established by the teacher at the beginning of the intervention was the choice of solving word problems using any mathematical strategy children thought would help them make sense of the mathematics. For example, children could use linking cubes to solve a problem about 12 pencils in five bags, by counting by ones, twos, fives, or tens. Also, if they used a standard algorithm (e.g., 12 x 5= 60), they were not discouraged to do so. Or if they used a combination of invented strategies with facts, they could also do so (e.g., 10X5 is 50 and 2X5 is 10 so it is 60 pencils). Basically, the teacher encourage any strategy they could think of using that would help them make sense of the mathematics. The choice of strategy was not only encouraged at the beginning of the problems but also during problem solving, as children struggled to explain their thinking of their strategy, sometimes they benefited from discussions with peers on how you can solve the problem using a different strategy.

With the ability to choose their strategy, the teacher encouraged children to use their sensory capabilities. For example, if children used markers and then linking cubes to come up with a strategy they were encouraged to do so. In particular, this benefited Thalia. She would often begin solving word problems with linking cubes, then she would transition to drawing sticks and boxes on her paper. She seemed to enjoy having multiple ways of representing her strategies, and it also helped her make sense of what she was doing in her strategy by having multiple sources of representations. For example, in Session two, Thalia solves the following problem: *Julia has three boxes of cookies. Each box has 10 cookies in* *it. How many cookies does Julia get altogether?* She begins by first counting out three sets of 10 linking cubes and counting all the linking cubes one by one until she arrived at 30, then she drew out the linking cubes as squares in her drawing, labeling each box of 10 cookies and counting out each square to arrive at 31. She then double checked her count with the linking cubes to see how many cookies she had arrived at a solution of 30. Had she not used the linking cubes, she probably would not have noticed that she created an extra square in her drawing for the third box of cookies (see Image 1).

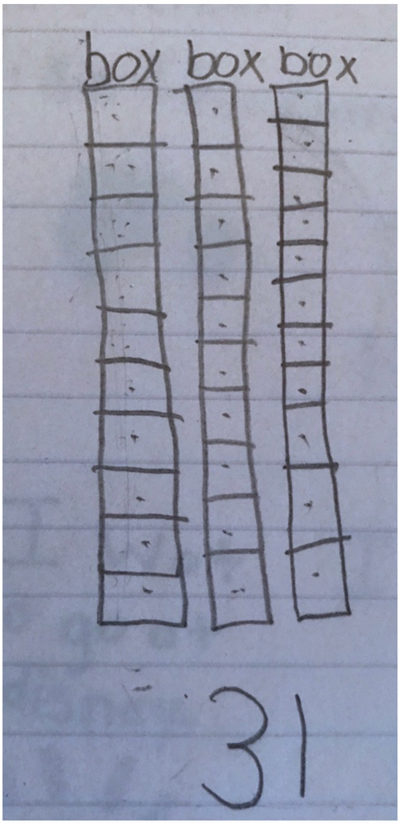

Image 1. **Thalia's strategy for total cookies in 3 boxes of 10 cookies each**

Choice to Work with Others or Individually

Central to the intervention was designing sessions to be less restrictive in how children collaborated with others during their problem-solving process. The teacher intentionally allowed children to work with others, share ideas, and observe each other's strategies. It helped children explain their thinking and make sense of their strategies. Although it may be thought that allowing children to share and see each other's strategies would invite children to take their peers' ideas as theirs, this was not the case. Throughout the sessions, children rarely took each other's ideas as theirs, and often their strategies were different in one way or another. This in part was due to encouraging children to explain their thinking for their strategies, which discouraged them from taking other's ideas as theirs. For instance, Jesus and Carlos would work together throughout most of the sessions, and bounced ideas of each other, but ultimately created different mathematical strategies. For example, in Session eight, Jesus noticed Carlos had solved a problem about *8 chocolate bars shared among three people*, by sharing two whole chocolate bars with each person and cutting the two leftovers into three parts. Jesus noticed that his strategy of cutting the two leftovers into half would not work because there were three kids in the word problem. Ultimately, this observation helped him make sense of a second problem the teacher posed. The second problem involved *sharing seven chocolate bars among five people*, which Jesus solved by sharing out one whole chocolate bar to each person and partitioned the two leftovers into five parts each (see Image 2).

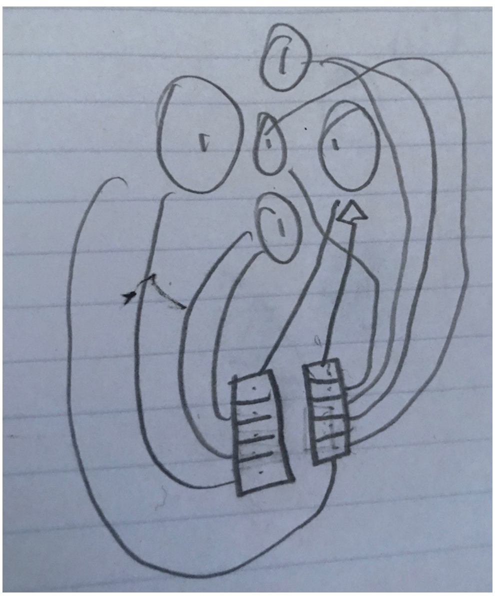

Image 2. Jesus's strategy for five people share seven chocolate bars

Table 2. ***Description of Participants***

Children	Age	Grade	School Identification	English Proficiency
Carlos	9 yrs.	3rd	Learning Disability & Emotional Disturbance Behavior Disorder	Intermediate
Thalia	10 yrs.	4th	RTI Tier 3	Intermediate
Jesus	10 yrs.	4th	Learning Disability	Advanced

Note. Woodcock-Johnson IV Test, Woodcock Munoz III Cognitiva, and the English Language Proficiency Assessments

Choice on How to Participate in Discussions

Throughout the sessions, the teacher wanted children to make sense of the math so it was essential to not restrict the way they could participate during discussions. For example, she encouraged children to participate in different ways when sharing their strategies. Children's ways of participation included sharing strategies in English or Spanish, silently drawing on the whiteboard their strategy, or simply sharing what they had written on their paper. It was not expected that they explain their solution verbally, in either Spanish or English. This was important, as sometimes children knew how to solve the problem when explaining their thinking to the teacher while engaged in problem solving but struggled in explaining it to peers during the discussion part of the sessions. For example, in Session eight, Carlos did not want to verbally share with the group how he had solved the problem but invited the teacher to share his journal with the group. He had written his strategy on the journal and was happy to share it with the group. The teacher shared his strategy by saying "*Are you OK, with me sharing your strategy?*" to which he nodded, and she continued with "If I say something that does not relate to your strategy will you let me know?" and he proceeded with a second nod. Although his participation was silent, he could still contribute to sharing his strategy during group discussions.

Teaching Moves That Promoted Mathematical Agency

Along with creating norms that allowed choice in how children participated, shared and made sense of the basic operation and fractional word problems, the teacher created specific questions and prompts that would allow children to express their mathematical agency as doers of mathematics, justifiers of mathematical ideas and as risk takers. At the start of the sessions, it was important that all three children share the specific details of their strategies so that they could later engage in critiquing the mathematical ideas of their peers and thus justify why their solutions were appropriate. These practices were essential, as they allowed the teacher to prompt children to take risks in sharing or defending their mathematical ideas with peers. The teaching goal was to encourage children to show mathematical agency when taking part in problem solving discussions.

Invite Children to Share Specific Details of Their Strategies

At the start of the tutoring sessions, it was difficult to get children to share the details of their mathematical thinking. For instance, Carlos struggled to explain the details of his math strategies. In Session 1, the teacher asked Carlos to explain his strategy of adding 12 and four together to get a total of 16 in the subtraction word problem: *Jasper has 4 carrots. His friends gave him some more carrots. Now Jasper has 12 carrots. How many carrots did Jasper's friends gave him?* he simply shrugged his shoulders and said, "I don't know". The teacher suspected Carlos was not used to explaining his thinking and that he misinterpreted what the problem was about, so she pressed with questions like "*What do the 12 and four represent in the story problem?*", but even then, it was not enough to get him to explain why he had added 12 and four together. Had Carlos explained his thinking, then perhaps he may have noticed that the problem was actually inviting him to subtract the two quantities. This struggle continued in Session two, when the teacher asked him to share his strategy for finding the total number of cookies in three boxes with 10 cookies in each. He again shook his head and said "hmm, I don't know". But the teacher continued to press for an explanation by asking, "*I see that you are doing something. What are you doing there?*" This question finally encourages some explanation of the details of his thinking. He responded by saying, "*I see that there are three boxes of cookies and there are 10, and I have…, the total is going to be 30.*" Although he did not at first explain how he got to 30 using the linking cubes, he does eventually explain that he had counted each group of 10 linking cubes to get to 30.

As the sessions progressed, Carlos not only began explaining the details of his strategies to the teacher but also with his peers, and often not needing prompts to share the details of his thinking. For example, in Session 11, Carlos shared that his solution to a problem about *7 people sharing 11 candies*, by saying "*I drew seven kids and seven candy bars, and then I gave seven candy and I had four leftovers, and then I cut them in half, and then I had one leftover and cut into 7 parts*". He shared his thinking without hesitation and explained the details of his strategy, explaining how he had cut the last four leftover candies. He explained he cut each leftover in half, noticing that the last half in his leftover parts needed to be cut into seven parts to share it equally among seven people.

Similarly, Jesus and Thalia, became more confident in sharing details of their strategies when the teacher continued to press for explanations of their ideas, asking questions such as "*How did you know you needed to cut each candy into four parts?*" and "*Why did you start to change your strategy from cutting your leftovers from half, to cutting each leftover by seven?*". These questions and similar ones promoted children to not only share their mathematical strategies with others but also helped each child understand other's strategies and solidify their knowledge about basic operation and fraction problems.

Prompt Children to Engage in Critiquing Other's Mathematical Ideas

Throughout the sessions, the teacher realized it was not enough to have them share their strategy details with each other in order to support the understanding of their mathematical solutions, but that it was also important for children to critique and convince others of their mathematical ideas. Therefore, the teacher created teaching moves around this goal. Early in the sessions, the teacher encouraged children to agree or disagree with each other's strategies and to offer justifications for their agreement or disagreements. As the sessions progressed, children did not always need prompts from the teacher to explain their disagreements or agreements with their mathematical ideas. For instance, in session 12, Thalia and Jesus engaged in discussions about the problem: *Janie's mom brought 4 brownies to share with 6 kids. She wants to share the brownies, so that everyone gets the same amount. How much brownie can each child get?* were Jesus tries to explain Thalia's strategy. In the following excerpt, Thalia had solved the problem by cutting three of her bars in halves and the last leftover into 12 equal parts. Before the teacher could compare Thalia 's strategy with Jesus's and Carlos's strategy, Jesus volunteered to explain the differences between their strategies and Thalia's.

Excerpt 2

Jesus: [Jesus had interrupted the teacher] So, then this one is going to be [Jesus gets up from his chair and points at Thalia's strategy on the board. The teacher turns around and looks at Thalia's strategy with Jesus] two out of two… [Jesus points at two of the halves Thalia's had in one of the bars labeled 1 and 2. He was trying to quantify the share for one of Thalia's sharers]
Carlos: Whaaat?
Teacher: Two out of two
Jesus: Yeah, this one is two?
Thalia: No!
Jesus: Well I don't know
Teacher: Why do you think it's no? [Teacher turns and asks Thalia]
Carlos: So, each of them gets 2?
Thalia: I split them in halves [Thalia is referring to the first three wholes] and I gave one to each people [Thalia grabs the marker from the teacher and draws a circle representing a person and draws a line from one of the halves to the circle] and two to the other people [Thalia draws two lines pointing to two of the 12 pieces in her last bar to the same circle]
Teacher: Ahh!
Jesus: Ah.. mmm.. [Places a finger on his mouth]
Thalia: And to the other people [Thalia continues giving halves and two parts of 12 to new people (circles)]
Jesus: Yeah, …

Jesus offered his own interpretations on Julia's strategy without it being prompted. Thalia defended her mathematical strategy when Jesus had incorrectly suggested her strategy for one person's share was "two out of two". Thalia had understood that they were discussing each person's share because as she defended her strategy and drew the share for one person as one half and two parts of the total 12 parts of the last leftover. Jesus had interpreted Thalia's solution as "two out of two" and was attentive to her explanation. The exchange of ideas between Jesus and Thalia in critiquing and defending their mathematical ideas was important, as it allowed both children to exhibit mathematical agency and understand each other's thinking.

Encourage Children to Take Risks When Participating in Sharing Their Math Ideas

Another effective teaching practice evident in the sessions was promoting children to take risks to show their incomplete ideas or disagreements. For example, in Session 10, Thalia and Jesus were solving the problem: *Ms. Rodriguez bought 20 cookies to share with 7 kids. She wants to share the cookies so that everyone gets the same amount. How much cookie can each child get?* Jesus and Thalia had attempted several strategies such as giving each person one cookie and splitting the six leftover cookies into two parts (Thalia's strategy) then four parts (Jesus's strategy). In the excerpt below, the teacher uses several prompts to encourage Thalia and Jesus to break apart a difficult problem by focusing on one of the six leftovers. Thalia takes a risk by sharing her ideas on how to cut one of the six leftovers for Jesus's strategy.

Excerpt 3

Teacher: If you have one giant cookie and you want to share it with 7 people

Thalia: Let's see… [Thalia directs question to Jesus] what do you think?

Jesus: [Jesus shakes his head] I don't know

Teacher: Sometimes its ok to think about it, you don't have to give a right answer all the time… you can brainstorm… you can talk to each other…

Jesus: Split them into eights?

Teacher: Split them into eights… so you are going to give each of those to each person [Teacher taps and points on the table]

Jesus: There is going to be one leftover [Thalia looks pensive]

Teacher: There is going to be one last leftover, what if it's a giant piece, how can we split it up?

Thalia: Into sevens [Thalia gets up and begins to draw a rectangle on the board and partitions it into seven pieces]

In this excerpt, the teacher used prompts like "*sometimes it's ok to think about it*" to help children see that mathematical ideas do not have to be rushed. She also used follow-up questions such as "*you don't have to give a right answer all the time*", to encourage children to share strategies even if they are wrong. This helped both Thalia and Jesus realize they can give wrong answers and still try until they have something that works. Thalia takes the initiative by not only suggesting that they cut the last leftover into sevens but begins drawing a cookie representation on the whiteboard and partitions it into seven parts. In creating an environment that promotes a productive struggle the teacher not only helped Julia take a risk on Jesus' strategy, but promoted the idea that mathematics does not have to be static (e.g., right or wrong answers), that they can engage in trial-and-error strategies to come up with a solution that works.

Discussion

This study sought to investigate how children who are bilingual with LD respond to teaching practices that support their mathematical strengths and use of prior knowledge. The teaching practices focused on providing language resources, offer elements of choice, and give children the opportunity to show their agentic roles while solving mathematics basic operation and fraction word problems (Garcia & Kleifgen, 2010; Maldonado Rodríguez, 2020; Stein et al, 2008; Turner et al., 2013). This study shows how three children who are bilingual and have learning dis/abilities are competent in their mathematics understanding. These children were capable of doing and understanding math when the teacher focused on their strengths. Thalia could make sense of her strategies and take risks in providing ideas to her peers when solving word problems. Carlos explained the details of his mathematical strategies with peers when collaborating in group work. And Jesus gained confidence in critiquing the strategies of his peers. The children exhibited mathematical agency while taking part in math discussions.

Limitations

Given the right tools and resources, a teacher can support the mathematical learning of children who are bilingual and identified with LD in small group discussions. Yet because this study was not done within a whole class setting, the question of whether Thalia, Jesus and Carlos would have exhibited the same type of mathematical agency remains unanswered. Thus, studies need to investigate what these teaching practices would look like in whole class settings. Second, the teaching practices used in the study were done by a teacher who is bilingual. Thus, the study was limited to the practice of a teacher who is bilingual within math discussions for children who are bilingual with LD. Therefore, more studies should be conducted with monolingual teachers with similar settings with these children. Finally, the study's goal was to look at these children's mathematical agency interactions over time and was not focused on performance but on the learning the three children exhibited during the intervention. Thus, other studies might look at how these children advance in their learning and performance with the use of pre and post assessments along with how that relates to their mathematical agency when instruction centers on their strengths.

Implications for Practice

Despite these limitations, teachers of mathematics and special education of children who are bilingual with learning dis/abilities should reflect upon the instruction that they currently use. Does this instruction further hinder and marginalize this population, or does it offer opportunities for these children to exhibit mathematical agency? Teachers should learn more about these children's dispositions and interests, as this helps create more meaningful mathematics learning experiences. Finally, teachers should consider the tools (e.g., teacher moves, high cognitive demanding tasks, and norms of choice) explained in this study to provide opportunities for children to show ownership and power of their mathematical ideas.

If we want to prepare children for real life experiences, then we must abandon the old traditional instructional practices of teaching only for rote memorization in favor of critical thinking and peer collaboration of solving problems. Scientists, engineers and mathematicians solving real life problems in their jobs do not solely rely on memorizing or recalling facts, they also focus on the process of critical thinking through ill-constructed problems with the help of colleagues. As Harry and Klinger (2007) expressed, "rather than devoting extensive resources to finding out whether [children] "have" disabilities, we should devote those resources to assessing" children's abilities (p. 16). Instead, we need to teach all our students, including those with identified dis/abilities, how to problem solve, collaborate with others, innovate, and come up with new solutions not yet visible in a textbook or curriculum.

References

Baker, D. (2017). *The politics of neurodiversity: Why public policy matters.* Lynne Rienner Publishers.

Bodovski, K., & Farkas, G. (2007). Mathematics growth in early elementary school: The roles of beginning knowledge, student engagement, and instruction. *The Elementary School Journal, 108*(2), 115–130. doi:10.1086/525550

Carpenter, T. P., Franke, M. L., Jacobs, V. R., Fennema, E., & Empson, S. B. (1998). A longitudinal study of invention and understanding in children's multi-digit addition and subtraction. *Journal for Research in Mathematics Education, 29*(1)3–20.

Carpenter, T.P., Fennema, E., Franke, M.L., Levi, L., & Empson, S.B. (2015). *Children's Mathematics: Cognitively guided instruction.* (2nd ed.). Portsmouth, NH: Heinemann.

Chao, T., Maldonado, L. A., Kalinec-Craig, C., & Celedón-Pattichis, S. (2019). Getting ready: Activities for preparing prospective elementary mathematics teachers to critically engage with the TEACH MATH modules. In T. G. Bartell, C. Drake, E. E. Turner, J. M. Aguirre, A. Roth McDuffie, & M. Foote (Eds.), *Transforming mathematics teacher education: An equity-based approach* (pp. 135–146). Springer. doi.org/10.1007/978-3-030-21017-5_11

Common Core State Standards Initiative. (2010). *Common Core State Standards for Mathematics.* National Governors Association Center for Best Practices and the Council of Chief State School Officers.

Corbin, J. M., 1942, & Strauss, A. L. (2008). *Basics of qualitative research: Techniques and procedures for developing grounded theory* (3rd ed.). Sage.

Empson, S. B. (2003). Low-performing students and teaching fractions for understanding: Aninteractional analysis. *Journal for Research in Mathematics Education, 34*(4), 305–343.

Fuchs, L. S., Fuchs, D., & Hollenbeck, K. N. (2007). Extending responsiveness to intervention to mathematics at first and third grades. *Exceptional Children, 22*(1), 13–24. http://doi.org/10.1111/j.1540-5826.2007.00227

García, O., Ibarra Johnson, S., & Seltzer, K. (2017). *The translanguaging classroom: Leveraging student bilingualism for learning*. Caslon Publishing.

Garcia, O., & Kleifgen, J. (2010). *Educating English language learners as emergent bilinguals: Policies, programs and practices for English language learners*. Teachers College Press.

García, S. B., & Tyler, B. (2010). Meeting the needs of English language learners with learning disabilities in the general curriculum. *Theory into Practice, 49*(2), 113–120. doi:10.1080/00405841003626585

Gersten, R., Chard, D. J., Jayanthi, M., Baker, S. K., Morphy, P., & Flojo, J. (2009). Mathematics instruction for students with learning disabilities: A meta-analysis of instructional components. *Review of Educational Research, 79*(3), 1202.

Flores, M. M., Hinton, V. M., Strozier, S. D., & Terry, S. L. (2014). *Using the concrete representational abstract sequence and the strategic instruction model to teach computation to students with autism spectrum disorders and developmental disabilities. 49*(4), 547–554.

Glaser, B., & Strauss, A. (1967). *The discovery of grounded theory: Strategies for qualitative research*. Aldine.

Gottfried, M. A., Bozick, R., Rose, E., & Moore, R. (2014). Does career and technical educationstrengthen the STEM pipeline? Comparing students with and without disabilities. *Journal of Disability Policy Studies, 26*(4)1–13.

Harry, B., & Klingner, J. (2007). Discarding the deficit model. *Educational Leadership, 64*(5), 16–21.

Hunt, J. H., & Empson, S. B. (2014). Exploratory study of informal strategies for equal sharing problems of students with learning disabilities. *Learning Disability Quarterly, 38*(4), 208–220. doi:10.1177/0731948714551418.

Hunt, J. H., & Silva, J. M. (2020). Emma's negotiation of number: Implicit intensive intervention. *Journal for Research in Mathematics Education, 51*(3), 334–360.

Hunt, J. H., Silva, J. M., & Lambert, R. (2019). Empowering students with specific learning disabilities: Jim's concept of unit fraction. *Journal of Mathematical Behavior, 56*(2), 100738. https://doi.org/10.1016/j.jmathb.2019.100738.

Jacobs, V. R., & Empson, S. B. (2016). Responding to children's mathematical thinking in the moment: an emerging framework of teaching moves. Zentralblatt für Didaktik der Mathematik, 48(1), 185-197. https://doi.org/10.1007/s11858-015-0717-0.

Karp, K. S., Bush, S. B., & Dougherty, B. J. (2019). Avoiding the ineffective keyword strategy. *Teaching Children Mathematics, 25*(7), 428–435.

Krause, G., Silva, J.M., & Aguilar, J. (2020). Bilingual pre-service teachers and their opportunities to learn. *Investigations in Mathematics Learning, 12*(4), 289–303. https://doi.org/10.1080/19477503.2020.1836458.

Lambert, R. (2015). Constructing and resisting disability in mathematics classrooms: A case study exploring the impact of different pedagogies. *Educational Studies in Mathematics, 89*, 1–18.

Maldonado Rodríguez, L. A., Krause, G., & Adams-Corral, M. (2020). Flowing with the translanguaging corriente: Juntos engaging with and making sense of mathematics. *Teaching for Excellence and Equity in Mathematics Journal, 11*(2), 17–25.

Mazzocco, M. M., & Devlin, K. T. (2008). Parts and 'holes': Gaps in rational number sense among children with vs. without mathematical learning disabilities. *Developmental Sci-*

ence, 11(5), 681–691.
Moschkovich, J. (1999). Supporting the participation of English language learners in mathematical discussions. *For the Learning of Mathematics, 19*(1), 11–19.
National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel.* U.S. Department of Education.
Orosco, M. J. (2014a). A math intervention for third grade Latino English language learners at risk for math disabilities. *Exceptionality, 22*(4), 205–225.
Orosco, M. J. (2014b). Word problem strategy for Latino English language learners at risk for math disabilities. *Learning Disability Quarterly, 37*(1), 45–53.
Orosco, M., & Abdulrahim, N. (2017). Culturally responsive professional development for one special education teacher of Latino English language learners with mathematics learningdisabilities. *Insights into Learning Disabilities, 14*(1), 73–95.
Robinson, S. J., & Temple, C. M. (2013). Dissociations in mathematical knowledge: Case studies in Down's syndrome and Williams syndrome. *Cortex*, *49*(2), 534–548. https://doi.org/10.1016/j.cortex.2011.11.011
Shifrer, D., Callahan, R.M., & Muller, C. (2013). Equity or marginalization? The high school course-taking of students labeled with a learning disability. *American Educational Research Journal, 50*(4), 656–682.
Silva, J. M. (2020). Exploring the mathematical agency of a multilingual child with an identified learning disability. *Teaching for Excellence and Equity in Mathematics, 11*(3), 25–31.
Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning*, *10*(4), 313–340.
Tan, P., & Kastberg, S. (2017). Calling for Research Collaborations and the Use of Dis/ability Studies in Mathematics Education. *Journal of Urban Mathematics Education*, *10*(2), 25–38.
Tan, P., Lambert, R., Padilla, A., & Wieman, R. (2019). A disability studies in mathematics education review of intellectual disabilities: Directions for future inquiry and practice. *Journal of Mathematical Behavior*, *54*, 100672. https://doi.org/10.1016/j.jmathb.2018.09.001
Turner, E., Dominguez, H., Maldonado, L., & Empson, S. (2013). English learners' participation in mathematical discussion: Shifting positioning's and dynamic identities. *Journal for Research in Mathematics Education, 44*(1), 199–234
Turner, E. (2012). Critical mathematical agency in the overcrowding at Francis Middle School project. In Tan, E., Barton, A. C., Turner, E., & Gutiérrez, M. V. (Eds.), *Empowering science and mathematics education in urban schools* (pp. 51–75). University of Chicago.
Webb, N. M., Franke, M. L., Ing, M., Wong, J., Fernandez, C. H., Shin, N., & Turrou, A. C. (2014). Engaging with others' mathematical ideas: Interrelationships among student participation, teachers' instructional practices, and learning. *International Journal of Educational Research, 63*, 79–93. doi:10.1016/.2013.02.001

Authors' Note

Correspondence concerning this article should be addressed to Juanita M. Silva, Ph.D., Texas State University, Department of Curriculum and Instruction, 601 University Drive, San Marcos, TX 78666, USA, Email: jsilva@txstate.edu