The Mathematics Educator
2021 Vol. 30, No. 1, 3–24

Teacher Candidates' Reflections on Responding to Errors: Exploring Their Vision and Goals

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Responding to student errors is a complex practice that connects teachers' vision and goals around students, mathematics, and teaching. We explore teacher candidates' (TCs) reflections on responding to errors during rehearsals of whole-class discussion to gain insight into the vision and goals that might influence their thinking. We discuss five TCs' assessment of their practice based on video-elicitation interviews to infer their vision around responding to errors and their associated goals for practice. In particular, we attend to how their vision and goals interact in shaping TCs' reflections on practice. This work offers implications for considering TC development and support.

Keywords: *responding to errors, teacher candidates, video elicitation interviews, reflecting on practice*

Responding to student contributions during whole-class discussion is a complex and critical aspect of the work of teaching (Boerst et al., 2011). That work becomes particularly difficult when the contribution contains a mathematical error, as teachers need to consider how to respond to the error, position students in productive ways, and make progress toward established mathematical goals. Current research around errors and error-handling advocates for embracing errors as

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opportunities for learning (e.g., Borasi, 1994; Jacobs & Empson, 2016; Son, 2016). However, teachers often focus the discussion on correct contributions or try to correct errors immediately (Santagata, 2005; Silver et al., 2005).

Research has explored connections between teachers' practice and beliefs around responding to errors (Bray, 2011; Santagata, 2005), suggesting the importance of examining teachers' perspectives alongside their practice. In this study, we explore secondary teacher candidates' (TCs') vision (Hammerness, 2001) around responding to errors in the context of rehearsals of leading whole-class discussion. By eliciting TCs' reflections on their practice in this context, we gain insight not only into their potential vision for how to respond to errors generally, but also the particular goals they had in the moment. Exploring the relationships between vision and goals adds to the knowledge base around how TCs respond to errors and provides a new way of understanding the challenges TCs face when learning and enacting this complex practice.

Framing Our Study

Errors and Error-Handling

Errors are a natural and necessary part of mathematics teaching and learning. Following Brodie (2014), we see errors in mathematics as complex instances of student sensemaking that contribute to students' own mathematical learning process (Nesher, 1987). With the goal of viewing students' ideas as resources rather than having some sort of deficit (Nesher, 1987; Smith et al., 1994), we use the word "error" to position students' conceptions as valuable, even if they are not yet consistent with canonical mathematics. In particular,

> Errors are reasonable and show reasoning among learners; they are a normal and necessary part of learning mathematics; and learner errors give teachers access to learners' current thinking about and ways of doing mathematics and access to possibilities for future growth in

> their mathematical thinking and practices. (Brodie, 2014, p. 224)

From this definition, student errors are not instances of misspeaking or accidently computing something incorrectly, which might be called mistakes.

We, along with others (e.g., Borasi, 1994; Jacobs & Empson, 2016; Son, 2016), take the stance that errors should be embraced as opportunities for learning. As such, responding to errors—and doing so in ways that positions both errors and students in productive ways—is a critical part of the work of teaching. However, doing this in practice can be quite difficult. Some responses to errors frame them negatively and remove opportunities for sensemaking. Teachers may make corrections (e.g., Tulis, 2013), quickly highlighting the error and then introducing correct ideas. Teachers may also try to avoid having students contribute errors at all (Bray, 2011; Santagata, 2005; Silver et al., 2005) and steer the conversation toward correct contributions.

Productive teacher responses to errors support students' sensemaking and position students as capable doers of mathematics, consistent with recent recommendations for mathematics teaching (e.g., National Council of Teachers of Mathematics, 2014). This might include revoicing the student's idea, treating the contribution as worthy of consideration, or explicitly inviting debate about mathematical ideas. Teachers must identify opportunities to build on student thinking and support students to build their own mathematical understandings (Bray, 2011; Van Zoest et al., 2017).

Influences on Responding to Errors

Given the challenge of responding to errors productively, it is important to consider the array of factors that might influence the nature of those responses. Some scholars have explicitly attended to the role of teacher beliefs (Bray, 2011; Santagata, 2005) in responding to errors. For example, Bray (2011) explored how beliefs and knowledge influenced error-handling practices. She found that when teachers believed an emphasis on

errors might confuse students or embarrass those who contributed the error, teachers were less likely to make errors central to classroom discussion. In contrast, teachers who were intentional about addressing errors tended to believe that doing so provided critical learning opportunities for all students.

Through comparing U.S. and Italian teachers, Santagata (2005) found that culture and beliefs about student learning influenced teachers' responses to errors. She argued,

> Beliefs about learning (as a public and collective process versus a private individualistic one) and about appropriate ways to deal with mistakes (the positive aspect of exposing students to the discussion of mistakes made by others versus the negative effects on self- esteem of publicly discussing mistakes) inform practices in complex ways: single beliefs cannot be linked directly to single practices. (Santagata, 2005, p. 506)

This result further highlights the many factors that play a role in how teachers make decisions around responding to errors during whole class discussion.

The existing literature on responding to errors during whole-class mathematics discussion highlights some of the potentially competing or even conflicting factors that teachers may contend with while making decisions about how to respond. Teachers may have a desire to keep student reasoning central to the discussion while also not wanting students to feel bad or lose social standing because of an error. Teachers may want students to see themselves in the mathematics while also wanting to avoid "confusing" students with incorrect ideas. In the case of errors made during whole-class discussion, teachers must also navigate the potentially differing needs of the student who contributed the error with the needs of the other students in the class, who may or may not share that student's conception.

Vision and Goals Around Responding to Errors

The existing literature has shown the significant role beliefs and knowledge can play in influencing practice related to responding to errors. To further explore this practice,

particularly in the context of leading whole-class discussions, we turn to the idea of vision (Hammerness, 2001). The construct of vision helps us account for the way responding to errors is nested in the work of leading discussions.

Hammerness (2001) defined the notion of teachers' vision as "a set of images of ideal classroom practice for which teachers strive" (p. 143). For example, a teacher may have a vision related to the nature of classroom discourse during instruction. This vision could simply involve an idea that all talk should be about mathematics, or it could be a more detailed vision that talk should be conceptually oriented and should spawn new mathematical investigations (Munter & Correnti, 2016).

Hammerness (2001) also discussed how a vision of an ideal classroom practice may lead to disappointment, as other factors at times get in the way of practice playing out in this ideal manner. For example, a teacher may have a vision for student-centered discussions, but a lack of classroom management experience by the teacher may serve as a roadblock for this vision. A teacher's practice may also be influenced by a conflicting vision being promoted by district or school leaders.

We take Munter's (2014) perspective that the Hammerness (2001) idea of vision is distinct from Sherin's (2001) idea of professional vision. Munter noted that, "Hammerness argued for attending to a teacher's vision *of* instruction, whereas Sherin described what could be considered . . . vision *in* instruction" (2014, p. 588), with the latter describing the types of vision you have in the moment of teaching rather than an ideal set of practices. Munter also noted that this idea of vision is evolving rather than static, distinguishing vision from the idea of teacher beliefs:

> Whereas belief suggests a relatively static set of decontextualized ontological commitments, vision is intended to communicate a more dynamic view of the future (Hammerness, 2001; Senge, 2006). After all, it should not be surprising if teachers' talk about mathematics instruction is out ahead of their enactments. (Munter, 2014, p. 587)

We draw on this definition of vision *of* practice from Munter and Hammerness, with attention to how TCs' instructional vision may be associated with their practice of responding to errors in whole-class discussions as well as how they assess their practice retrospectively.

One way we consider the connections between instructional vision and practice is through recognizing the more specific and concrete ideas aligned with one's vision, what we call goals. In conceptualizing goals, we take a sociocultural perspective, specifically ideas from activity theory (e.g., Grossman et al., 1999; Leont'ev, 1981), that situate goals in the context of activity in particular settings. From this perspective, one's vision and goals for teaching are shaped by the settings in which one does the work (Grossman et al., 2000). This perspective is distinct from other conceptions of goals, such as that from Schoenfeld (2011), focused on cognitive aspects of decision-making as interacting with individual beliefs, knowledge, and preferences.

We attend to the interaction between vision and goals because while a TC's vision is not necessarily framed in a way that informs how to act in a specific situation, such as when a student contributes an error in whole class discussion, the TC would have a goal, or multiple goals, that represent ideas related to the specific situation. Those goals would then correspond to an action in practice, which may include the use of a culturally defined tool (Ghousseini et al., 2015). Certain circumstances in the classroom may present a conflict among goals, which creates a problem-solving situation for the TC (Grossman et al., 2000).

Our study explores secondary TCs' vision and goals around the practice of responding to student errors in whole-class discussions. We address the following research question: How do TCs' vision and goals interact in the context of reflecting on responding to errors in whole-class discussion? In contrast to other studies, this work looks explicitly at TCs rather than practicing teachers. Thus, implications can be closely tied to recommendations for initial teacher preparation.

Methods

This study took place in a secondary mathematics methods course over two years at a large, public university. As part of the methods course, TCs rehearsed leading sorting discussions, where students sorted cards into examples and non-examples of a mathematical object in order to develop and refine a definition (Baldinger et al., 2016, 2020). The purpose of these rehearsals was for TCs to engage in and reflect on the approximated practice (Grossman et al., 2009) of leading whole-class discussions. The rehearsals were designed to provide opportunities for TCs to gain experience with responding to student errors, either through the use of "planted errors" (Campbell et al., 2020) or other errors that were contributed independently by TCs acting as students during the rehearsals. During the rehearsals, the teacher educator (second author) provided in-the-moment feedback to the rehearsing TC and the TC had the opportunity to pause the rehearsal and ask questions.

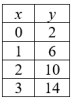
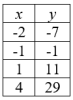
Data Collection

Three TCs had the opportunity to rehearse leading sorting discussions each year with the other TCs taking on the role of students. There were 16 TCs in year one, and 12 TCs in year two. All of the rehearsals focused on defining linear functions and each rehearsal used a different representation of linear functions (Baldinger et al., 2020)—one used cards with symbolic notation, one used cards with graphs, and one used cards with tables. All rehearsals were video recorded. Following the rehearsals, the first author conducted video elicitation interviews with each rehearsing TC to gain insight into their reasoning and interpretation of moments in the video (Goldman et al., 2007).

Each interview focused on two video clips: one with a correct student contribution and one that contained an error (see Table 1).

Table 1

Student Contributions Discussed in Each Video Clip

Year	TC	Correct contribution	Contribution containing an error
2016	Sophia	$y = x^2$ "This is not a linear function because if you graph it, it's not a line."	$y = 17 - 5x$ "This is a linear function because 17 is the slope and 5 is the y-intercept."
2016	Todd	 "This is a linear function because it's going up by 5 each time." [a]	 "This is a linear function because you're dividing by 2 every time."
2016	Geoff	 "This is a linear function because it's definitely a straight line."	 "This is a linear function because it's a straight line, so that's what makes it linear."
2017	Tessa	 "This is a linear function because it's going up by 4 each time."	 "This is NOT a linear function because it goes up by a different amount every time."
2017	Greg	 "This is a linear function because it's a straight line and it also has a constant rate of change."	 "This is a linear function because it's a straight line."

Note. All names are pseudonyms.

[a] This is the direct quote given by the student in the clip, which the rehearsing teacher accepted as correct. However, we acknowledge that, from a mathematical perspective, this is insufficient evidence to identify this as a linear function. In this case, the rehearsing TC didn't feel the need to push for further evidence in the moment.

In all rehearsals, there was no more than one student contribution that contained an error. One of the rehearsals did not contain an error; that interview was therefore excluded from the analyses presented in this paper. The remaining student contributions in each rehearsal were correct. The shortest clip of the interaction around a correct contribution from each rehearsal was chosen for convenience as all correct contributions had similar features, including the initial student contribution, a revoicing move from the teacher, and an opportunity for students to provide a reason for disagreement, though no students did so. The clips contained the whole-class discussion of each contribution and any coaching from the teacher educator.

The interviewer did not label the clips as having or not having errors, nor were the participants asked to identify whether each clip had an error. However, all participants organically made comments indicating they recognized that one of the clips contained an error. For each clip, the interviewer asked why the TC responded the way they did, and if they would respond that way again. After discussing both clips, TCs were asked if they noticed any differences in their responses and if they intended for their responses to be similar or different.

Data Analysis

To prepare data for analysis, all interviews were transcribed. Each interview was analyzed holistically, using multiple readings to generate and revise claims related to our research question. We identified each TC's vision through the lens of their reflection on practice. We used analytic memos (Miles et al., 2013) to capture the ways in which TCs described how they felt they responded to each contribution. In particular, we noted whether TCs intended to respond in the same or different ways in the two clips and their apparent satisfaction with their approach in each clip (and across the two clips).

As TCs discussed their intentions and their satisfaction (or lack of satisfaction), they often provided evidence of their vision and goals that were at play in their instruction. We defined vision as an approach to teaching or a guiding principle that seems to apply to their practice outside of this rehearsal. If a TC discussed

an action that they wanted to take in the context of these rehearsal clips, we could categorize that as a goal for that TC. In some cases, a goal was clearly linked to a vision. Our inferences of TCs' vision and goals, and their potential connection, emerged from additional phases of analytic memo writing, building on the initial pass capturing TCs' assessment of their own practice.

For instance, if a TC said they want to keep whole-class discussions student-centered, this would be considered a vision. If this same TC stated that they opted not to correct an error to turn the discussion back to the students, then not correcting errors would be considered a goal associated with the vision of student-centered discussions. We did not necessarily feel a need to connect each goal to a vision, and in some cases, such a link was either implicit or not evident.

We also considered whether TCs spoke about their vision and goals tentatively or with regularity. For example, if a TC mentioned student-centered discussions on multiple occasions, we categorized them as speaking about this idea with regularity. On the other hand, if a TC mentioned that they wish to treat errors and non-errors differently, but used phrases such as "maybe" or "I don't know," we categorized this as speaking about that idea tentatively. We also used a TC's assessment of their practice to adjust determinations about the regularity or tentativeness with which they spoke about a particular goal or philosophy. For example, if we had inferred that a TC spoke with regularity about the idea that errors do not need to be resolved immediately, but in their assessment of practice they expressed regret that they did not make more moves to resolve an error, this self-assessment would seem to conflict with that idea. This would lead us to re-categorize the discussion of that idea as being tentative.

Findings

Through our analysis of TCs' assessment of practice, we identified three distinct themes that characterize the relationship between the ways TCs assessed their practice, the nature of their vision and goals, and the tentativeness or regularity in which

they spoke about their vision and goals. Each theme provides insight into the interaction between these constructs and their practice that we attempted to further understand through this work.

Vision and Goals in Alignment: A Goal to Respond Consistently

Greg and Geoff each held a vision of keeping discussions student-centered. Additionally, this was associated with a goal of responding consistently to errors and non-errors. They each expressed satisfaction with their practice in their respective rehearsals, particularly because they responded to the error and non-error contributions in consistent ways.

For Geoff, it was important to keep his talk moves consistent whether or not the contribution contained an error. He said,

> I feel like for students, it's pretty obvious when they didn't give the answer that I wanted and I want to steer them—like even if I don't explicitly tell them, it's like, "okay . . ." then let's move on. . . . I was just trying to let it sit and have another student talk about it without any input.

It was important to him to "give students more control" in the "mathematical conversation" by having an atmosphere where it was okay to challenge another student and for contributions to not always have to go through the teacher. Geoff described his moves as recording what the student was saying while "trying to not put any sort of judgment on it." He also said he did not feel a need to correct the error in the moment as he felt "that's appropriate at the end of the discussion, but telegraphing yes or no at the beginning I think would be detrimental to the discussion."

Geoff assessed his own practice as being successful in responding consistently to both contributions selected for the interview. The only concern he had was a minor one—that his tone of voice might have indicated uncertainty when responding to the error, and that he might have hesitated when a student made an incorrect contribution.

Greg also felt that he successfully treated the two contributions the same, by asking in each case for agreement or disagreement. Despite using similar talk moves in each case, he noted that his probe for agreement or disagreement elicited more discussion around the error, due to more disagreement existing. Greg reflected on his choice to at least momentarily table the discussion of the error, and recalled advice from the teacher educator that the error need not be resolved immediately, saying, "you know, you can leave it and then like, you can come back to it if need be." Greg expressed satisfaction that the error was resolved later in the discussion (after the clip used for the interview). He expressed that he was much more comfortable leaving the error unresolved, at least temporarily, than he would have been saying that the student was wrong, or giving the impression that the student was wrong:

> I mean, it just completely shoots them down. Like, I feel like that's, you know, if they're the first person you called on and if you like, say "you're wrong," they probably won't want to participate for the rest of the class I'd assume. And it probably would just make them feel horrible, like making them shut down. Like, I've seen it so many times.

It was important to Greg to have a discussion regardless of errors being present or not. While he wanted the error resolved eventually, he felt that it did not need to happen right away.

Geoff and Greg both wished to respond consistently to errors and non-errors. This goal of responding consistently was associated with a vision of keeping discussions student-centered. Further, Geoff and Greg did not feel a desire to address and resolve errors in the moment, which we inferred was associated with the success they expressed around responding consistently.

Geoff felt it was important to keep his response "as neutral as possible in order to let the students have the discussion." Responding this way "gets more at what the mathematical discovery process should be" rather than students accepting something as true because of the authority of the teacher. Greg similarly felt that his role was strictly as a "facilitator" of discussion and that students should be leading the discussion so

they can learn from each other. Both Geoff and Greg expressed these thoughts frequently and with regularity. Geoff and Greg felt that responding differently to errors and non-errors would imply a sense of judgement from the teacher. They felt that this sense of judgement would not be productive towards keeping the discussion student-centered. It appears that there was no conflict or tension between the goals for each of these TCs and this is likely the reason for their satisfaction with their practice.

Vision and Goals in Conflict: A Goal to Respond Consistently and Resolve Errors Quickly

Like Geoff and Greg, Sophia and Todd expressed a vision of keeping discussions student-centered during their interviews. Additionally, this was associated with a goal of responding consistently to errors and non-errors, just as it was for Geoff and Greg. However, in contrast to Geoff and Greg, Sophia and Todd each did not feel successful in achieving this goal.

Sophia observed that even though both her responses were moves to orient students to one another's thinking, she heard wording differences that might lead students to realize "there's something wrong here."

> I definitely seem to be more . . . like leaning in the direction that "yes, this is correct," and then when [the error] came up, I was like "oh, is this—do we agree or disagree?" So, it already was putting this like, there's something wrong here. So I guess, that's something good to point out because especially with students, they're so focused on getting the right answer but not really knowing why they got to the right answer, so regardless, if the student is correct, again asking, you know, if we agree or disagree on their answer, or even saying, "Can we elaborate more? Does someone want to elaborate on what he said?" Like, just using consistency [in my moves] so that each student's work is validated regardless of if it's correct or wrong.

Overall, while Sophia felt comfortable with part of her response—that she did not immediately tell the student that the contribution was incorrect—she did not feel as though she

responded consistently across contributions. She felt that using a consistent agree or disagree move when responding to the correct contribution would have been an improvement. Like Geoff and Greg, Sophia also had a goal of responding consistently.

Additionally, while considering how Sophia discussed responding consistently, it initially appeared as if she was discussing this goal with regularity. However, her further self-assessment caused us to re-categorize her talk as tentative. She reflected on further moves she could have made to improve her response to the error. For example, she felt she could have asked a question such as "Could someone tell a story about this equation?" as a way to potentially highlight and resolve the error through other students' idea. This idea appears to lie in conflict with the idea of responding consistently to errors and non-errors. It was through analyzing the tentativeness of Sophia's discussion that we were able to identify that Sophia had multiple goals that seemed to contradict when confronted with a student contributing an error in whole class discussion. While she wanted to respond consistently, like Geoff and Greg, she also had a goal to elaborate on the error in a way that would encourage students to notice and correct the error. There was no such move related to the contribution without an error, leading us to infer that Sophia felt a strong pull to resolve the error quickly despite her goal of responding consistently.

Todd also felt unsuccessful, as he responded differently despite his goal of being consistent. Like Sophia, he observed that his responses in both cases were orienting talk moves but he noticed that his wording might "give it away," which he did not want to do. Todd said,

> I'm not really sure what I could have said that doesn't give it away, because if I said what I said the first time, [when] I said, "Does anyone agree with this?" and the problem is that I don't want people to agree with this, like I want to hear people that aren't agreeing with it, so I don't know how to—you know, I don't know if there's a better line that I can't think of but I would like to know if there is.

Todd seemed to be searching for language that would not reveal whether or not a contribution was an error though he was uncomfortable with the idea of an error being supported through discussion. He felt a strong pull to resolve the error in the moment. Todd noted that his "ultimate goal was [to] have people say 'this is not an example.'"

Todd has a goal of responding to errors and non-errors consistently. Like Sophia, he discussed this tentatively using phrases such as "I don't know." Also, like Sophia, the contribution of a student error brought about a conflict of multiple goals. Along with his goal of responding to errors and non-errors consistently, he also has a goal of not supporting the error during discussion by asking for agreement. It was this goal that seemed to cause him to be tentative about responding consistently. During his self-assessment, it appears that Todd was unable to find a way to respond in a way that satisfies both goals, further showing his struggle in navigating this practice.

Vision and Goals in Alignment: A Goal to Respond Differently

Tessa's situation differed from the other four, as she held a goal of treating errors and non-errors differently. This goal interacted with a vision more focused on the mathematical richness of the mathematical discussion, which foregrounded different ideals than those of the other TCs, who were more focused on discussions being student-centered. When asked to assess her practice, Tessa was satisfied with her practice overall, with some small things she would change. When there was no error present, Tessa asked for agreement or disagreement and received none. When there was an error present, she asked if anyone had anything to add. Tessa noted that this different phrasing was intentional on her part. Asking for agreement or disagreement was meant to serve as a quick check and she viewed asking if anyone had anything to add was more of a discussion starter. It is notable that she felt more comfortable spending more time on the contribution containing an error even though this might be "giving it away" in the way Todd and Sophia wished to avoid. However, Tessa did not mention any

concern about this and felt quite comfortable with any differences in her responses:

> Would I continue to treat them [errors and non-errors] differently? I think it would depend on the students. Yeah, right? And so, they would say there was an error . . . and so the students are like—if there is an error then I would probably continue to treat them—maybe. Maybe not. It probably depends on a lot of other factors, so like, time and what's the point of the lesson. What are my goals in the lesson?

Tessa's response suggests that situations exist where contributions with and without errors should be treated differently. It is ultimately the goals of the lesson that drives how Tessa responds, and not so much a goal to respond consistently. This contrasts with the other four TCs, who all expressed the intent to be consistent and neutral in their responses in order to keep the discussion student-centered. Additionally, we felt that Tessa spoke tentatively about treating errors and non-errors differently, using phrases such as "maybe not" and "yeah, right?" This means that there may be situations where she would see responding in those ways as not appropriate.

Ultimately, in the situation that unfolded in her rehearsal, Tessa felt it was appropriate to respond to the two contributions differently because it was more important to have a discussion around an error than a correct contribution. She felt that it is "really hard to have a rich mathematical discussion" when there is not an error. She viewed the error as an opportunity for discussion, not a deterrent. Checking for agreement or disagreement after a correct contribution was not meant to create robust discussion, while that was the intention of asking students to add on to a contribution that was an error.

With this evidence, it appeared that Tessa held a vision of wanting rich discussions. This was associated with a goal of treating errors and non-errors differently, when appropriate, since errors are an opportunity for rich discussion. For this particular scenario, she spoke with regularity about responding differently being the appropriate response. However, her

tentativeness related to treating errors and non-errors differently emphasizes how important she felt context was in determining if responding differently was her intended and preferred action.

We inferred that Tessa held the vision of having "rich discussions," with the goal that a teacher should respond differently to non-errors and errors in service of having moments of the lesson that were worthy of discussion. Even though this might "give it away," it was more important to Tessa to spend more time on the errors, not necessarily to correct them but to use them as an opportunity to advance student learning. Tessa spoke with regularity about the idea of having rich discussion while she spoke more tentatively about the idea of responding to errors and non-errors, given her stance that these decisions would depend on the context and the teacher's goals for a given discussion.

Discussion

These interviews revealed the potentially competing goals that TCs were navigating as they made decisions about responding to errors. TCs talked about responding to errors and non-errors consistently, keeping discussion student-centered, leaving or tabling errors, resolving errors quickly, and using errors to have rich discussions. Some of the TCs in our study held a set of vision and goals in ways in which they were viewed as competing with one another when assessing their practice, while others saw their goals and vision as more aligned.

The possibility of aligned or competing goals and vision was associated with whether TCs spoke about their ideas tentatively or with regularity. This had implications for how TCs assessed their practice. For example, Todd had a goal of responding consistently to errors and non-errors. However, while assessing his practice, he also spoke of a goal of resolving the error quickly and was worried about the error being supported through discussion if anyone agreed with the error. When speaking about responding consistently, Todd spoke tentatively because of his concern that using his non-error response for the error would promote agreement. This tentative talk gave us insight into how Todd navigates these competing goals and how he has yet to find

a way to respond consistently without promoting agreement. This idea of competing goals is one example of how responding to errors is a complex practice and TCs need support as they navigate any goals they view as competing.

While Sophia's tentative talk gave us similar insights to Todd's, Tessa's tentative talk gave a different insight. Specifically, Sophia's and Todd's assessments revealed seemingly competing goals, and Tessa did not feel that her goals were competing. Her tentativeness instead highlighted how Tessa appreciates and recognizes the complexity of responding to errors. She felt very comfortable with her vision and goal in the particular context that she was assessing and felt successful with her practice. However, she felt that the goals she set would not necessarily be appropriate in a different context. This also highlights how responding to errors is a complex practice and TCs need support to navigate the variety of contexts they may encounter. Further, we begin to wonder what we might have observed related to Tessa's vision and goals had she been confronted with a different set of conditions.

Lastly, we did not feel that Geoff or Greg spoke tentatively about their vision or any of their goals. They offered no evidence that they felt any goals were competing and did not speak about doing anything differently in any other context. This was in part because their vision and goal was not challenged by an error, due to the fact that they felt no pull to resolve the error quickly. The regularity with which they spoke about their vision and goals may imply that they have grown more comfortable in sorting through their vision and goals around responding to errors. We have highlighted how navigating the practice of responding to errors is a complex task and it is possible that Geoff and Greg are further along in this navigation and have found ways to resolve some of the issues that Sophia, Todd, and Tessa were considering.

Implications

Previous research has highlighted the natural and necessary role that errors play in mathematics teaching and learning, while recognizing that responding to errors is a difficult and complex

practice for teachers (e.g., Bray, 2011; Brodie, 2014; Santagata, 2005). Attending to TCs' vision and goals for responding to errors during a rehearsal opened up new windows into understanding what TCs grapple with when responding to errors. Through applying the construct of vision as a lens for analyzing these interviews, we were able to see the potential roadblocks TCs face when learning to respond to errors. The reflection on practice helped clarify why there existed such a distance, at times, between TCs' vision and practice. Some TCs' had goals that seemed to impede achieving their broader vision. The goals and vision these TCs articulated could be compatible, but they viewed them as incompatible. The work then for teacher educators is to support TCs to identify the ways in which seemingly contradicting vision and goals related to responding to errors might actually be complementary.

Some of the TCs in this study felt conflicted in how to respond, in part because the error brought additional concerns that prevented them from leading the discussion in a way that aligned with their vision. Teacher educators should provide opportunities for TCs to identify these potential conflicts and support these TCs as they navigate them towards resolution. This may involve bringing attention to the moments of conflict and talking through potential strategies that may attend to the error in a way in which they are comfortable, while at the same time not compromising their vision.

Our work should be interpreted with respect to the limitations of a small sample of TCs from a single institution discussing their assessment of practice in responding to errors for one instructional activity. We make no generalization claims, rather we put forward that these three themes can exist among TCs, each providing examples of how different sets of vision and goals, and confidence in their vision and goals, have implications on TCs' perceptions of practice. We also acknowledge that these TCs were assessing their error-responding practices by reflecting on rehearsals with their peers. Results may have been different if TCs were reflecting on their actual classroom practice. Lastly, these goals around responding to errors were discussed in the context of whole-class discussion. These TCs may have had different goals, and may have assessed

their practice differently, in a different context, such as tutoring a student one-on-one.

Our study explored how TCs' vision and goals interact in the context of responding to errors during whole-class discussion. We investigated this by having TCs reflect on their responses to errors and non-errors while they viewed video clips of their rehearsals. Our findings suggest that while TCs sought to lead rich, student-centered discussions, the nature of their vision and goals associated with that idea may have influenced their satisfaction or dissatisfaction with their practice. By analyzing these ideas about responding to errors in whole-class discussion, we have identified important considerations that can support TCs in navigating this complex practice.

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