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Making Thoughts Visible through Formative Feedback in a Mathematical Problem-Solving Process

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Abstract

This study aims to elicit the role of formative feedback in the development of students in a mathematical problem-solving process. For this purpose, the study's primary process is to investigate the development of elementary school students (aged 10 to 11) through feedback given during a problem-solving process. While visually engaged in the sub-processes expressing a problem situation and describing their thinking structures in writing, three different dimensions are addressed: communicating visually what they understood from the problem; expressing their thoughts about solution; and creating explanations regarding their solution process. The six-week embedded mixed method study reveals that students' explanations of their thinking processes developed towards the expectations. They were able to depict the problem and the relationships involved in the problem more clearly in their drawings to understand the problem. They made fewer mistakes in mathematical operations.

Key words: Explanation, Formative feedback, Mathematics, Problem solving, Visualization

Introduction

Problem-solving is an essential competence both for mathematics and for life. In both national and international mathematics curricula, problem-solving is addressed as one of the fundamental objectives (e.g., Ministry of National Education, 2017; National Council of Teachers of Mathematics, 2000). Altun (2001) describes a problem as a situation in which an individual wants to do something, but cannot immediately figure out the problem-solving to go about it. In problem-solving, students are intellectually active (Bayazit & Aksoy, 2009) Lester (1994) defines problem-solving as the most problematic area of learning, but at the same time, the most crucial goal of doing mathematics.

Mathematical problems, and especially word problems, are tools that help students enhance their thinking skills, and tools that help them gain the skills needed to solve problems they encounter, particularly in everyday life (Pimta, Tayruakham, & Nuangchalerm, 2009). Among all types, one of the most challenging types of problems for mathematics learners is word problems (Verschaffel, Schukajlow, Star, & Dooren, 2020). Students seeing a problem as a mass, made up of words and numbers, and thinking that they can solve it by performing four main operations with the numbers given to them. Therefore, understanding problems is difficult for them. This is not the only example as to why students have difficulties related to problem-solving. However, it is a fact that students are far from the desired level of success in problem-solving (Soylu & Soylu, 2006). From this aspect, although problem-solving seems like an old topic, the methodologies to be applied in the process details are not so simple (Reimers & Chung, 2016). Therefore, studies regarding, 'How students' problem-solving processes should be supported' remain popular.

The Problem-Solving Process

Research has addressed the elements of primary school students' problem-solving processes. Problem-solving practices should be based on students' active participation, and various problems should be provided for students (Yazgan & Bintaş, 2005). As another issue, enabling students to re-express what they have understood from the problem through visual elements, mathematical symbols and explanations, is useful since asking students to explain their thoughts supports their thinking and increases the accuracy of solution processes (Rittle-Johnson,

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2006). In addition, the incorporation of feedback into problem-solving seems essential in developing problem-solving skills. There are studies regarding the provision of feedback to students in the problem-solving process. These studies have shown that, during the problem-solving process, providing feedback is beneficial for student learning (Hattie & Gan, 2011; Luwel, Foustana, Papadatos, & Verschaffel, 2011; Mory 2004; Shute, 2008).

The problem-solving process is much more than merely asking students to solve problems. This process also requires teacher guidance. In the process of problem-solving, the feedback provided by the teacher can increase the efficiency of the learning process, as it will help students to both verify their knowledge and change it if necessary (Fyfe & Brown, 2020, Mory, 2004). Within this context, feedback is central to problem-solving (Cáceres, Nussbaum, González & Gardulski, 2019) and the development of this process. Although feedback is accepted as an essential factor in advancing learning, its effectiveness depends on specific situations (Cáceres, Nussbaum, González, & Gardulski, 2019). On the other hand, a lack of studies on how feedback affects primary school students' learning is highlighted in the literature (Cáceres et al., 2019; Schaeffer, Margulieux, Chen, & Catrambone, 2016; Van der Kleij, Feskens, & Eggen, 2015). Therefore, more research studies are required clarifying how and when feedback should be given during the problem-solving process in mathematics classes (Huxham, 2007), and particularly to identify the role of feedback in problem-solving at primary school level as well. Moving from the issues above, this study focuses on giving feedback during problem-solving processes in primary mathematics classes, and aims to investigate primary school students' development through formative feedback given during the problem-solving process. Students are engaged in the sub-processes of expressing a problem situation visually and describing their thinking structures in writing. The research question is as follows: *What is the role of formative feedback on development of students' performances in the problem-solving process?*

Conceptual Framework

Giving feedback as a supporting element can improve students' problem-solving processes. For both formal and informal learning environments, feedback is a common practice (Fyfe & Brown, 2020). Students need to check whether they are on the right track and receive feedback on their progress while going through the challenging process of problem-solving. Therefore, feedback is the essence of formative assessment (Núñez-Peña, Bono, & Suárez-Pellicioni, 2015).

The literature defines the basic characteristics of effective feedback. According to this, feedback should: be task-specific (Fyfe & Brown, 2020, Hattie & Yates, 2014); be detailed (Narciss et al., 2014; Van der Kleij, Feskens & Eggen, 2015); be process-based (Narciss et al., 2014); be given at the correct time (Brookhart, 2008; Fyfe & Rittle-Johnson, 2017); require active participation (Havnes, Smith, Dysthe & Ludvigsen, 2012); and facilitate reflection in students (Nicol and Macfarlane-Dick, 2006). Feedback should be task specific because, in this way, it provides students with information regarding their performance in a particular task (Hattie & Timperley, 2007). Moreover, it is useful to consider students' performance in feedback. (Hu, Li, Zhang, Roberts & Vitiello, 2021). This would help to bridge the gap between current and desired performance (Attali & Van der Kleij, 2017). In relation to performance, students' prior knowledge should be considered (Fyfe & Rittle-Johnson, 2016). As a last important issue, detailed feedback increases the contribution to learning (Van der Kleij, Feskens & Eggen, 2015) since elaborated type is the most effective one (Narciss et al., 2014)

Although feedback is found useful in developing students' performance, the effects of feedback are complex and related to many variables (Van der Kleij et al., 2015). In relation to different variables, such as the timing, type, and ability of the student (Butler, Godbole & Marsh, 2013), feedback can affect student performance and development differently. This situation is valid for studies investigating the effectiveness of feedback in the context of problem-solving (Cáceres, et.al., 2019). The effectiveness of feedback may differ even for different problems on the same topic (Chase & Klahr, 2017). Therefore, different mechanisms should be established for feedback that provide detailed information to students and which is directed towards their individual needs (Van Meeuwen, 2013).

In the classroom, feedback may take many forms, such as a teachers' annotations, prompting questions, statements for verification, and direct results (Chin, 2006). Within this context, it is possible to classify feedback types as presenting results, presenting correct answers and giving detailed feedback (Shute, 2008). Therefore, feedback can be described as the 'consequence of the performance' (Hattie and Timperley 2007, p.81). As a main actor of this study, formative feedback is defined as 'information communicated to the learner that is intended to modify his or her thinking or behavior for the purpose of improving learning' (Shute, 2008, p.154).

Within this context, the feedback is expected to tell the student what to fix and provide comments or suggestions supporting how to do this (Black & Williams, 1998).

Giving feedback makes a difference compared to not giving feedback (Basu, Biswas, & Kinnebrew, 2017), and feedback is beneficial in specific ways (Alfieri, Brooks, Aldrich & Tenenbaum, 2011). Therefore, it is not enough to guide students in the learning process, but it is also necessary to give feedback in certain ways (Cáceres et. al., 2019). Teachers' expectations of students' success affect their performance (Rubie-Davies 2017). Specifying these expectations in a planned, systematic and direct way in the learning environment can contribute to students' understanding and acting accordingly (Fyfe & Brown, 2020). This situation also applies to problem-solving. For the effectiveness of feedback in the problem-solving process, teachers should clearly present expectations to the students on this issue (Fyfe & Brown, 2020). When teacher feedback is general, it may not be meaningful for students, while feedback given on their current performance may contribute to their understanding of the difference between anticipation and their own actions, and to taking steps towards reducing this difference (Li, Cao & Mok, 2016). Therefore, one way to achieve giving good feedback is to share rubrics containing expectations with students (Andrade, Du, & Wang, 2008; Toker, 2020). In addition, it is difficult to implement an individual feedback process as it requires writing notes for each student in large classes (Frank, Simper & Kaupp, 2018; Núñez-Peña et. al., 2015). Therefore, rubric containing typical feedback expectations may be used, as in the case of this study.

Good problem-solving is associated with the creation a mental representation (Marshall, 1995). Students expressing verbal problems visually, in writing or verbally is related to their understanding of problem structures (Anderson & Krathwohl, 2001). It is important to provide an environment for visualizing the problem, because such representations are of great importance in understanding the problem and developing solutions (Krawec, 2014), and it can even be said to be critical in problem-solving (Ho & Lowrie, 2014). While visual expressions and drawings help students make abstract concepts more concrete (Douville & Algozzine, 2004) and meaningful (Fuson & Willis, 1989), they also help them develop their conceptual understanding (Pape, 2003). Students can figure out which operations are needed to solve a problem with the help of visuals (Van de Walle, Karp, & Bay-Williams, 2013). When students draw appropriate representations to understand the problem, the possibility of solving the problem correctly increases (Boonen, Van Wesel, Jolles, & Van der Schoot, 2014). From this point of view, the visual representation of the problem is not a sufficient step in solving it, but it may be a necessary step in getting started (Ho & Lowrie, 2014). Therefore, visualization plays a role in facilitating problem-solving (Van Garderen, Scheuermann & Jackson, 2013).

Asking students to explain their ways of thinking while solving a problem provides benefit both for teachers and for students. Students explaining their ways of thinking to make their thoughts visible, in other words explaining their thoughts in writing, will help them during the problem-solving process (Steele, 2005). Students explaining their thinking involved in the problem-solving process elicits their reasoning processes. Furthermore, this contributes to the development of their communication and relating skills (Countryman, 1992). In this manner, formative feedback allows students to articulate a general view of their work, reflect on the processes they have followed, clarify their opinions and ideas (NTCM, 2000), and verbally explain their thoughts. For the students, having the necessary knowledge to use feedback effectively to improve their skills (Stone, 2000) is essential. For teachers, it allows them to recognize the sources of students' misconceptions, make new plans for instruction based on these misconceptions (Colton, 2010), and gain detailed information about their students' learning and conceptual understanding. In light of the basic issues discussed above, this study focuses on the elements of formative feedback, visualization, written explanation and the use of rubrics in making thought visible in the problem-solving process. A conceptual framework, summarizing the elements used in this study to make thoughts visible during the problem-solving process, is shown in Figure 1.

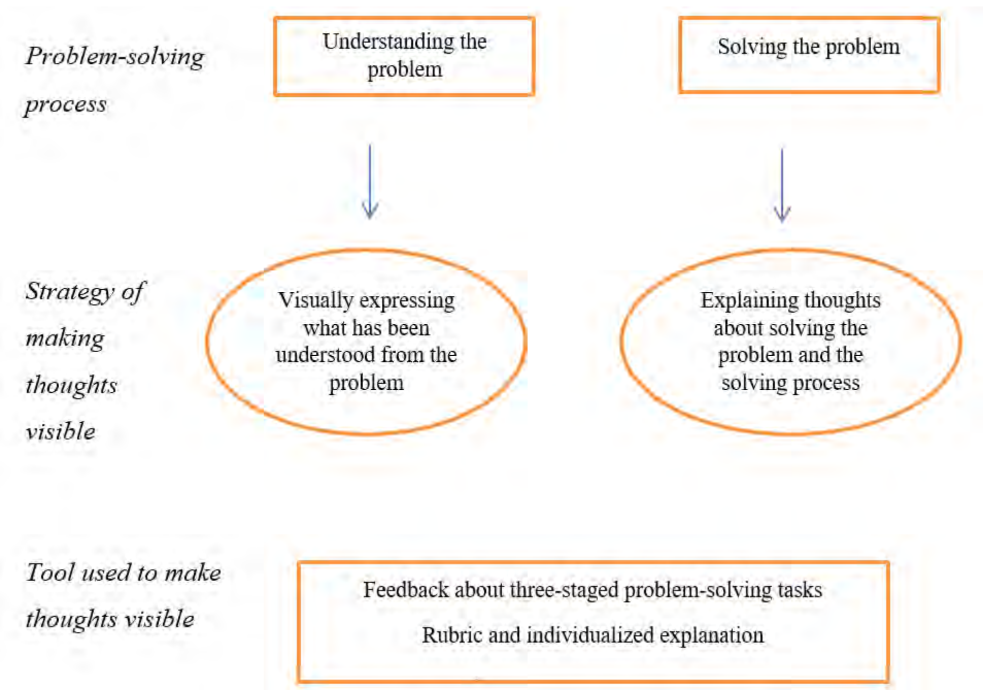


Figure 1. Framework for Making Thoughts Visible During the Problem-solving Process

Method

Research Design

This study covers the application, evaluation, and formative feedback processes over a six-week period investigating the role of formative feedback based on a rubric on the development of primary school students' performances. Their performance in visually expressing what they have understood from the problem and expressing their ways of thinking in written form during the problem-solving are considered. The entire study employs a classroom teaching experiment methodology (Cobb, 2000). However, in this embedded mixed method research report, the focus is on an investigation of the effect of formative feedback on students' writing and visualization. For this focus, one group pretest-posttest design is implemented and, in addition to pre/post test data, samples of students' written work are provided.

Prior to the study, a pretest was administered to determine the students' current level of problem-solving, with a focus on written and visual explanations. The classroom teacher administered the pre-test in the presence of the researcher during one class hour. After completion of the study, a posttest was used to gauge their development. Between these two applications, a total of four feedback cycles were conducted. The research process is summarized in Table 1.

Table 1.General Outlook of the Research Process

Process	Determination of readiness (Pretest)	1st Feedback Cycle	2nd Feedback Cycle	3rd Feedback Cycle	4th Feedback Cycle	Evaluation of development (Posttest)
Purpose	Determination of the students' current competence in the steps of problem-solving	Students solve problems and the teacher gives written feedback				Determination of the students' development in problem-solving

Participants

The research was conducted with a fourth-grade class that consisted of nineteen students at a primary school in Turkey. Of the participating students, eight were girls, and eleven were boys. As one of these students had to study many classes in different settings within the context of special education, the student was unable to participate in all parts of the study. Consent was gathered from all of the students and parents. Moreover, an approval form was taken from the administration.

The school where the research was conducted offers education at many schooling levels (primary school, middle school, high school), is affiliated with the Ministry of National Education, and administers an International Baccalaureate Primary Years Program (IB PYP). The students were mostly the children of the university's academic and administrative personnel that the private school is affiliated with. Although the general mathematics course achievements were defined as an average class throughout the school, it is possible to define the distribution of students in the class as heterogeneous. Table 2 shows the normality of the distribution.

Table 2. Normality distribution

	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
Pre-test	,178	18	,139	,958	18	,558
Post-test	,150	18	,200[*]	,925	18	,157

When the average score data of the pretest data is examined, it is found to be Sig. = .139> α = 0.05, so we can say with 95% confidence that the data shows a normal distribution. Similarly, since Sig. = .200> α = 0.05 for the posttest average score data, we can safely say that the data fits normal distribution with 95% confidence.

Data Collection Tools

The study employed the following data collection tools: the pretest developed to measure the students' level of readiness; the post-test having the same content as the pretest; the problem forms used in the problem-solving activities conducted over four weeks; and the rubric used in the feedback process.

Pretest and Posttest

At the beginning of the study, to determine the students' problem-solving capabilities, a pretest consisting of five problems was administered. The problems in the pretest were selected from the textbook used in the students' mathematics lessons. In the selection of the problems, the following criteria were used:

- They must be suitable for students to express their understanding of the problem visually in more than one way.
- They must allow different solving methods.
- Their solutions must involve more than one stage.
- They must include elements of daily life.
- They must require students to conduct operations with natural numbers.

In addition to this, the opinion of the classroom teacher, who had observed the students' problem-solving processes over a long period, was considered. The teacher was asked to evaluate the problems in terms of: (1) their suitability for the students' level; (2) sentence structure and complexity; and (3) the presence of terms with which the students were not familiar. The suitability of the problems for the students' level was also investigated by another academician working in the field of mathematics teaching, and they were all found to be suitable. The problems used in the pretest and posttest are presented in Appendix 1. The students were asked to find solutions to the given problems, and no further explanation was made. The problems used in the pretest were also used in the posttest after completion of the study. In both administrative processes, the students were asked to solve the questions themselves, but not to complete them by only writing an answer. No intervention was made by the teacher or the researcher in the process of solving the questions during pretests and posttests. Evaluation of the pretest and posttest results was conducted using a five-step graded scoring key. The answers given by the students to their problems were evaluated twice by the researcher at different times. In addition, 15% of the answers were subjected to independent evaluation by a researcher specializing in mathematics education. The steps in the graded scoring key, and information concerning the scores to be taken from the key, are given in Appendix 2.

Data Collection during Formative Feedback Sessions

As the data collection tools, a problem-solving form (Appendix 3) and the rubric were used in the four cycles of giving feedback. The problem-solving form prompted the students to produce a drawing related to the problem, a mathematical operation, and an explanation. The structure of the form was developed based on framework by Minton (2007) and was then adapted to Turkish. The language and format validity of the form was addressed by seeking an expert opinion. The form was piloted by two fifth-grade students, and statements in the form were found to be precise.

The questions used over the four weeks are shown in Table 3.

Table 3. Problems Used in the Feedback Cycles

1	The Şengör family bought a car. They agreed to pay the cost in installments for eighteen months. They have to pay 3,525 TL each month. How much is the car?
2	A shoe company takes an order for 490 pairs of sandals, 820 pairs of slippers, and twice as many trainers as sandals, to be delivered in May. What is the total number of items in the order?
3	There are 6,530 balloons in the store of a company that organizes various celebration ceremonies. Of these, 2,890 are yellow, and the rest are red. 3,800 red balloons are required from the company for a birthday celebration. Can the company meet this order? Explain.
4	The municipality has identified two plots for a children's park. The length of the short side of the first plot, which has a rectangular shape, is 18 m, and the longer side is 30 m. The circumference of the other plot, which has a square shape, is the same as the first plot. What is the length of one side of the second plot?

Regarding the suitability of these problems, the views of a mathematics education expert were received. Following completion of the pretest, the problems in the problem-solving test were administered in the classes in the researcher's presence. The students were given problem sheets and what was expected for each section of the problem-solving form was explained to them. After this, they were asked to draw pictures to represent the problem, solve the problem, and explain the process. For solving problems, they were given twenty minutes. Next, in the remaining twenty minutes of the forty-minute lesson, the students were given feedback individually by the researcher in light of the discussion regarding focus and content of feedback with the classroom teacher. They were then asked to act in line with the given feedback when solving the next problem. While the researcher was giving individual feedback, the classroom teacher gave the other students a task out of the particular lesson's content.

The teacher and the researcher conducted the whole process collaboratively. The test was administered by the classroom teacher in the presence of the researcher. The problems are selected together in light of the classroom teachers' experience and familiarity with the students' profiles. The administrative process of the test and feedback sessions in micro cycles were designed in collaborative discussion sessions. The focuses of feedback were identified by collaboration.

In all the feedback cycles, a similar procedure was followed:

1. The students are presented with the problem of the week in a problem-solving form.
2. The students read the form and say if there is any term unknown to them.
3. The students engage individually in solving the problem.
4. The researcher gives feedback to the students individually based on the rubric.
5. The students reflect on the written feedback given and on understanding the problem, and the solving processes.
6. If they need to, they discuss the feedback and develop their performances with the researcher.

Formative feedback was given based on the students' answers to the problems. In this process, successive administration of the feedback processes had the potential to provide for the students' development. In this formative feedback process, hints and strategies regarding students' work were provided to enhance this development. As in this example, students' work was evaluated within the framework of the expectations defined in the rubric, and individual feedback was provided to each student regarding the level of realizing these expectations.

In evaluating the findings for the first week, it was found that there were certain common points in the students' drawings, solutions, and explanations; that is, in general, there were similarities in their mistakes and missing parts. The written feedback statements were then converted to the graded scoring key, and during the following weeks, the students were given their feedback based on this table. The students were given formative feedback (as exemplified in Figure 2) and provided explanations related to their answers using the rubric. The rubric consisted of both explanations and points. After dealing with the week's problem the completed table was given to the students for them to refer to in the next problem-solving process. The constructed feedback table is shown in Table 4.

Table 4. Rubric used in the cycles of giving feedback

	4 points	3 points	2 points	1 point	0 point
Drawing a picture to understand the problem	Drawing a picture to understand the problem with all elements -elements -their relationship -what is asked	Drawing a picture to understand the problem with two of the elements	Drawing a picture to understand the problem with one element	Drawing a picture to understand the problem partly including one element	Not drawing
Writing the mathematical sentence explaining the solution	Writing the mathematical sentence explaining the solution, correct operation selection, correct process, and correct result.	Writing the mathematical sentence explaining the solution-Correct operation selection, correct process.	Writing the mathematical sentence explaining the solution-one of them: Correct operation selection, partially correct process.	Writing the mathematical sentence explaining the solution-one of them: Correct operation selection.	Did not find the correct operation to solve the problem
Explaining the process followed to find the solution	Explaining the process followed to find the solution -why -what -how	Explaining the process followed to find the solution-two-and one is partially appropriate -why -what -how	Explaining the process followed to find the solution-two -why -what -how	Explaining the process followed to find the solution- one is appropriate -why -what -how	Did not write what you did, why you did it, or what you found as a result

The students were allowed to look back at their work once more based on the problem-solving form's written feedback. In addition, for each student, it was explained and exemplified which anticipation was at each stage accompanied by this rubric. They were also allowed to discuss, if they needed, how they could individually develop their performances.

Data Analysis

The paired sample t-test was used to analyze the students' development in problem-solving from pretest to posttest. Additionally, for the data collected during the micro-cycles, the three dimensions (drawing a picture to understand the problem, writing a problem sentence explaining the solution, and explaining the process followed to find the answer) revealed the points for which feedback was given. The distribution and change of the scores taken by the students across the weeks were analyzed to gain insight into the effectiveness and utilization of the feedback points.

For each micro-cycle, student responses were examined one by one, and the data was coded by a second encoder familiar with the subject. For the coding compatibility, a Cohen's Kappa coefficient was calculated and found to be .86, indicating substantial agreement. In all of the cycles, the researcher's role throughout the research was to follow the students' question-solving during the course, grade the students' work, and provide oral and written feedback in the light of the issues discussed with the classroom teacher. The classroom teacher took a role in every part of the process. Starting from selecting problems at the beginning to the administration of the posttest at the end, she was in close collaboration with the researcher and involved in the entire process. In addition to classroom practices, each week the researcher and the teacher collaborated on the process.

Results

The results of this study are presented in two stages. In the first stage, the pretest and the posttest results are compared, and the application's effectiveness is then determined. In the second part, in each micro-cycle of the implementation process, the students' mean scores in three dimensions are presented with frequency and rates. In addition, as the weeks progressed, the results relating to the kind of formative feedback provided in the process, with examples from student work, are given. Additionally, it is determined how the feedback has resulted in changes during the process.

Pretest and Posttest Results

The students' pretest scores were evaluated out of twenty and then converted into percentage values. The percentage values for the pretest and posttest results are given in Table 5.

Table 5. Pretest and Posttest Mean Scores

	1	2	3	4	5
Pretest	3,72	2,33	2,06	3,44	3,78
Posttest	4,00	2,83	2,89	3,33	3,89

When the pretest data is analyzed from normality, the mean score data is found to be Sig. = 00139> α = 0.05, meaning that the data shows normal distribution with 95% confidence. Similarly, since the posttest average score data is Sig. = 00200> α = 0.05, it can be said that the data fits normal distribution with 95% confidence. Since the pretest and posttest data shows normal distribution, the tests' relationship was examined with a paired sample t-test. Table 6 shows the result of the analysis relating to the t-test.

Table 6. Results of Paired Sample T-test for Pretest and Posttest

Paired Differences					t	df	Sig.(2-tailed)
Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
			Lower	Upper			
-,32222	,30785	,07256	-,47531	-,16913	-4,441	17	,000

The results of the t-test show that, with 95% confidence, there is a statistically significant difference between the mean scores before and after the teaching experiment.

Formative Feedback Session Results

First Dimension: Drawing a Picture to Understand the Problem

The students' scores from the dimension of drawing a picture over the weeks were 2.11, 3.17, 3.61, and 3.72. While the scores taken at the beginning of the study varied between 0 and 4, in the second week there were no children who scored 0 or 1 point, and when it came to the final week, the scores were mostly 3 and 4. Throughout the study, a continuous increase was observed in the means of the drawing scores. Table 7 shows the frequencies and percentages of the scores taken from the drawing dimension of the study.

Table 7. Scores Taken from the Drawing Dimension over the Weeks

	1st week		2nd week		3rd week		4th week	
	# of students	Percent (%)	# of students	Percent (%)	# of students	Percent (%)	# of students	Percent (%)
4 P.	5	27,78	7	38,89	13	72,22	13	72,22
3 P.	4	22,22	7	38,89	3	16,67	5	27,78
2 P.	2	11,11	4	22,22	2	11,11	-	-
1 P.	2	11,11	-	-	-	-	-	-
0 P.	5	27,78	-	-	-	-	-	-

When the changes in the students' work were examined in terms of content, it was found that in the first week of the study, while only 27.78% of the students drew pictures that included all the elements of the problem, this percentage increased to 72.22% in the third and fourth weeks. While initially, nearly one-third of the students did not draw pictures that could help solve the problem, or did not draw any pictures at all, as of the second week, all of the students drew at least certain elements of the problem.

Examples of the students' drawings are given in Figures 2.1, 2.2, and 2.3.

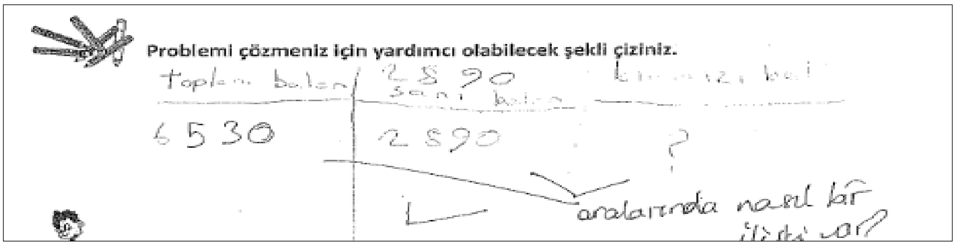


Figure 2.1. Drawing of student 2 (3 points)

Drawing that shows students' understanding of the problem -with two of the elements

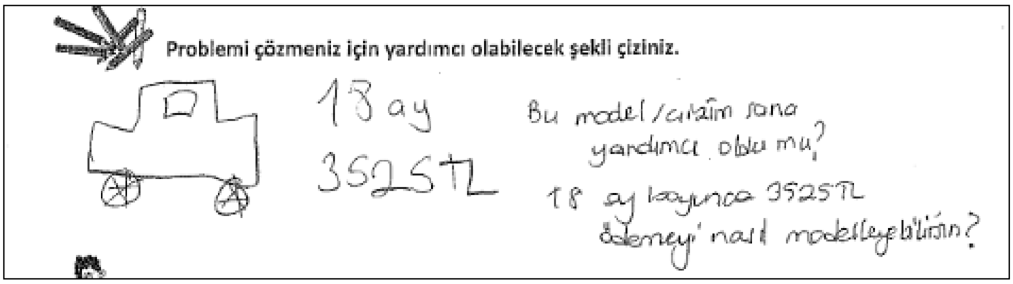


Figure 2.2. Drawing of student 8 (1 point)

Drawing that shows students' understanding of the problem- including one element partially

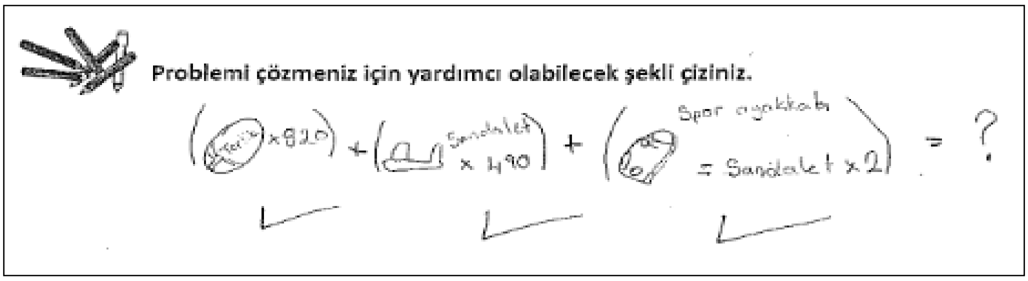


Figure 2.3.Drawing of student 13 (4 points)

Drawing that shows students' understanding of the problem with all elements

As shown in Figures 2.1, 2.2, and 2.3, while the students did not experience much difficulty expressing each element in the problem, they experienced great difficulty in visually representing the relationships between the components. The focuses of the feedback that emerged in the first dimension, 'Drawing a picture to understand the problem' were:

1. Inclusion of elements making up the problem in the drawing produced for the problem.
2. Inclusion of the relationships between the parts of the problem.

Considering the drawings of students regarding this dimension from the first week to the last week, in the beginning, a number of students only expressed the elements in the problem visually. Some students did not specify what the images represent, including the visual component for only a part of the problem, and did not have the elements' relationships. It can be seen that the students' drawings became more and more appropriate in all respects. The students both shaped their previous studies with the individualized formative feedback given to them and became more aware of these new studies' expectations.

Second Dimension: Writing a Mathematical Sentence

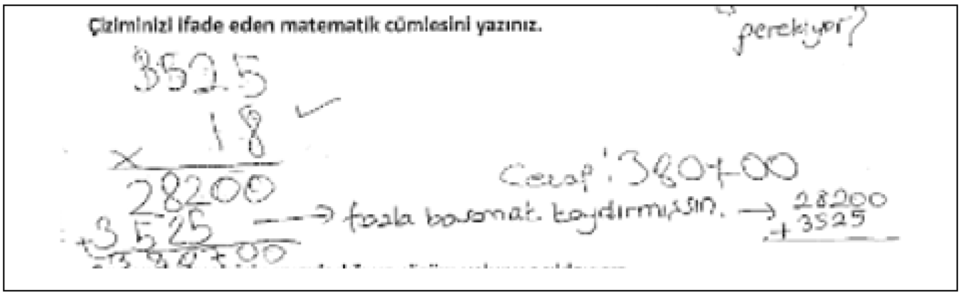
The students' mean scores from the drawing a picture dimension across the weeks were 2.72, 3.00, 3.78, and 3.94, while in the first week of the study, the students' scores were mostly 0, 3, and 4. In the second week, the number of students who received scores of 0 and 3 decreased, and the number of students with scores of 1, 2, and 4 increased. In the third week, only one student had a score of 2, and in the fourth week, almost all of the students received a score of 4. A continuous increase was observed in the students' mean scores for using a drawing expressing the problem situation. In Table 8, the distribution of the scores taken from the dimension 'Writing a mathematical sentence' by using a drawing expressing the problem is given as frequencies and percentages.

Table 8. Scores Taken from the Dimension of Writing a Mathematical Sentence Across the Weeks

	1st week		2nd week		3rd week		4th week	
	# of students	Percent (%)	# of students	Percent (%)	# of students	Percent (%)	# of students	Percent (%)
4 P.	7	38,89	10	55,56	15	83,33	17	94,44
3 P.	7	38,89	3	16,67	2	11,11	1	5,56
2 P.	-	-	1	5,56	1	5,56	-	-
1 P.	-	-	3	16,67	-	-	-	-
0 P.	4	22,22	1	5,56	-	-	-	-

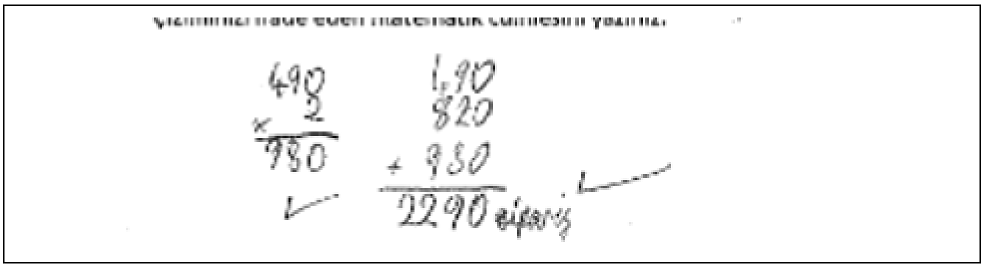
It was found that most of the students were able to use the correct operation to solve the problem primarily as of the first week. In the first work, one student's method of solving the problem did not include the steps leading to the solution, only the result. After the student was given feedback, this changed in the following solutions. In the third and fourth weeks especially, in addition to finding the correct operation, the students did not make mistakes in the intermediary stages and found the right answers. Therefore, progress was observed in their expressions of mathematical sentences.

Examples of the students' mathematical sentences produced in the problem-solving process are given in Figures 3.1, 3.2, and 3.3.



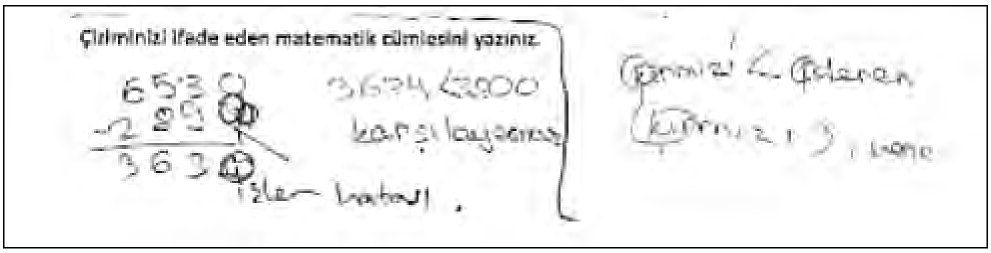
Mathematical sentence explaining the solution-one of them: correct operation selection, partially correct process

Figure 3.1 Mathematical sentence of student 4 (2 points)



Mathematical sentence explaining the solution correct operation selection, correct process, correct result

Figure 3.2 Mathematical sentence of student 7 (4 points)



Mathematical sentence explaining the solution- correct operation selection, correct process

Figure 3.3 Mathematical sentence of student 10 (3 points)

The focuses of feedback that emerged in the second dimension, ‘Writing a mathematical sentence’ were:

1. Writing the numbers correctly in the mathematical sentence,
2. Planning the operations correctly in the mathematical sentence,
3. Conducting the operations correctly in the mathematical sentence,
4. Conducting the operations in the correct order in the mathematical sentence, and
5. Finding the correct answer to the mathematical sentence.

In general, the students' constructed their mathematical sentences correctly. On the other hand, operational mistakes stemming from a lack of information in the necessary applications related to basic operational skills in certain of the problems were observed. For instance, a number of students performed multiplications without shifting the numbers one under the other by one digit in the calculation process. In addition, certain calculation errors, stemming from the students' carelessness were observed. Through the feedback given to the students, such calculation errors were reduced.

Third Dimension: Explaining the Process Followed to Find the Solution

While the students' mean scores were low in general, a continuous increase was observed and, in the last week, this ratio increased by a considerable extent. The scores were 0.83, 1.56, 3.33 and 3.58 for four weeks. When the changes in the students' scores taken from the dimension of writing an explanation were examined, it was found that, particularly in the first week, no student ultimately met expectations. In the work for the first week, while half of the students received 1 point, nearly half received 0 points. Only two students wrote what they did to solve the problem and why they did it, but they did not report what they had found as a result. In the following weeks, a continuous improvement was observed in their ratio of writing explanations and meeting expectations in their descriptions. In the final week, two-thirds of the students completely satisfied the expectation regarding writing a reason; that is, they could explain what they had done, why they had done it in the problem-solving process, and what they had found as a result. In Table 9, the distribution of the scores taken in the dimension ‘Explaining the process followed to find the result’ is given as frequencies and percentages.

Table 9. Scores in the Dimension of Writing an Explanation across the Weeks

	1st week		2nd week		3rd week		4th week	
	# of students	Percent (%)	# of students	Percent (%)	# of students	Percent (%)	# of students	Percent (%)
4 P.	-	-	1	5,56	12	66,67	12	66,67
3 P.	2	11,11	5	27,78	2	11,11	1	5,56
2 P.	-	-	2	11,11	3	16,67	3	16,67
1 P.	9	50,00	5	27,78	-	-	2	11,11
0 P.	7	38,89	5	27,78	1	5,56	-	-

Examples of the students' explanations are shown in Figures 4.1, 4.2, and 4.3.

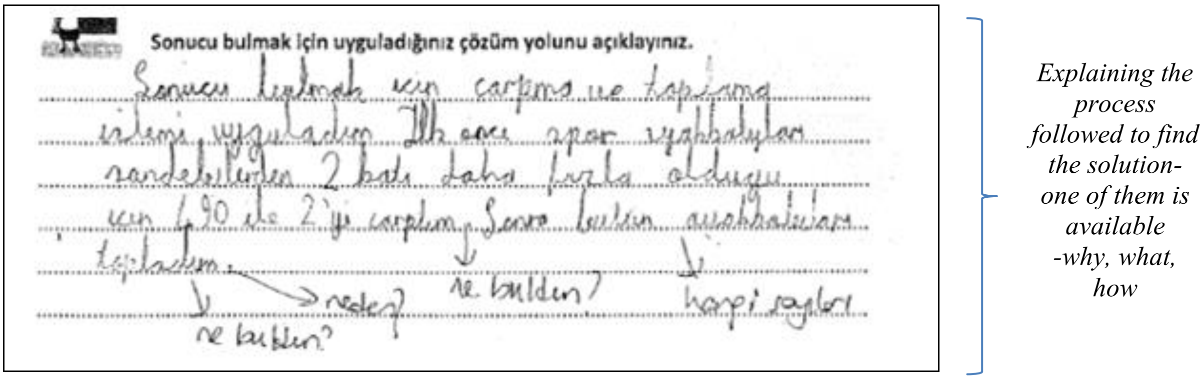
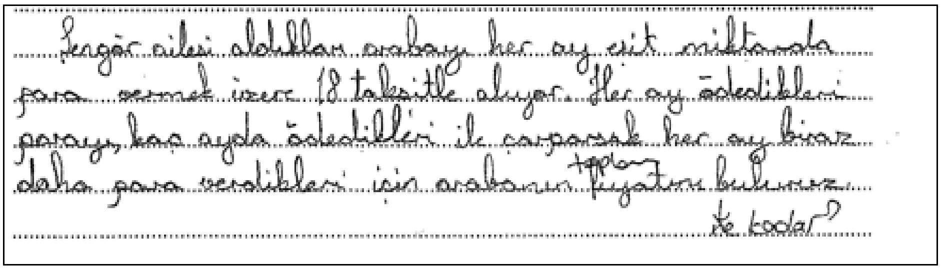
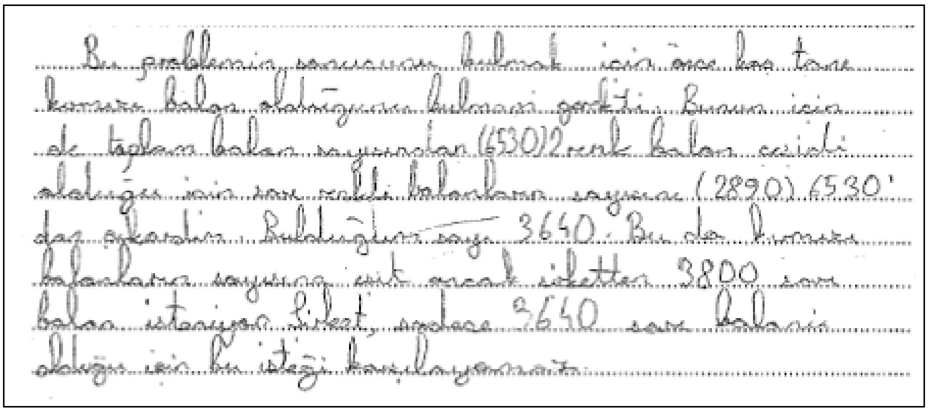


Figure 4.1. Explanation of student 9 (1 point)



Explaining the process followed to find the solution-two-and one is partially appropriate -why, what, how

Figure 4.2. Explanation of student 14 (3 points)



Explaining the process followed to find the solution -why, what, how

Figure 4.3. Explanation of student 10 (4 points)

In Figure 4.1, student 9 mentions only one aspect, which is the operation to be used for the solution, without indicating the numbers, reason for using this operation, or how to do it. In Figure 4.2, student 14 explains the process by indicating how to calculate and why this did not provide the answer. In Figure 4.3, the student explains by answering how what, and why questions and received 4 points. The focuses of feedback emerging in the third dimension, 'Explaining the process followed to find the result' were:

1. Explaining what they had found as a result of the solution.
2. Explaining why they had used the numbers and operations in the solution.
3. Explaining which numbers and operations they had used in the solution.

The students tended to write 'what they had done' when explaining the process and finding the solution. While a number of students used highly generalized expressions, such as 'First I added and then I multiplied' others explained which operations they had used with which numbers in a detailed manner, such as, "I subtracted 2,890 red balloons from the 6,570 balloons". The elements less mentioned in their explanations are related to 'why they did what they did' and 'what they found as a result'. In their first week's work, the students usually did not mention why they had used certain operations in their explanations. Following the feedback cycles, they paid greater attention to explaining what they had done and why they had done it and what they had found.

Discussion

The purpose of this study is to elicit the role of formative feedback in the development of fourth-grade students' problem-solving performance. They were engaged in the sub-processes of expressing a problem situation visually and describing their thinking structures in writing. Through this connection, the study reveals the focuses of formative feedback in making students' thoughts visible in the problem-solving process. In Table 10, the points of feedback are summarized.

Table 10. Focuses of Formative Feedback

Problem-solving process	Formative Feedback points
Drawing a picture to understand the problem	In the picture drawn to represent the problem, inclusion of the elements making up the problem Inclusion of the relationships between the elements of the problem

Writing a mathematical sentence explaining the solution	Writing the numbers correctly in the mathematical sentence explaining the problem Planning the operations correctly in the mathematical sentence explaining the problem Conducting the operations correctly in the mathematical sentence explaining the problem Conducting the operations in the correct order in the mathematical sentence explaining the problem Finding the result correctly in the mathematical sentence explaining the problem
Explaining the process followed to find the solution	Explaining what is found in the solution followed to find the result Explaining why these numbers and operations are used in the solution followed to find the result Explaining which numbers and operations are used in the solution followed to find the results

The results of the study, in which the students performed drawings to understand problems, found solutions and explained their methods, shows that the students made progress in the problem-solving process. This finding is consistent with the results of other studies in the literature pointing out that feedback is beneficial (e.g. Hattie & Gan, 2011; Luwel, Foustana, Papadatos, & Verschaffel, 2011; Mory 2004; Rittle-Johnson, 2006; Shute, 2008). It can be said that the students' overall performance towards the development of problem-solving has increased and that they created products that are more suitable for teacher expectations.

The results of the current study show that the students improved their written expressions regarding their thinking structures involved in the problem-solving process as of the first week. This may indicate that feedback is a useful tool for making students' thoughts visible in the problem-solving process. On the other hand, students should need to have the necessary knowledge to use feedback effectively to improve their work (Stone, 2000). The feedback was given to the students to reflect on what, why, and how they did what they had done and what they had found. As a result, the problem-solving process over time encouraged the students to initiate a reflective process independently and to carry out this process without assistance.

In addition to the improvements observed in written expression, the results show that the students progressed from producing visual drawings, including essential elements, while visually expressing what they understood from the problem toward creating more complex visuals that better explain the whole structure and the relationships involved in the problem. When a drawing was expected regarding a problem, the students developed their visual expressions according to expectations. In many implementations in the literature, the same feedback is provided without considering about the degree of accuracy (Attali & van der Kleij, 2017). However, contrary to the feedback given to the students, irrespective of the correctness of their solutions or regardless of their prior knowledge, when individualized formative feedback is provided, as in this study, the visual and written explanations of the students regarding the problem-solving processes can be improved.

Another dimension intended to be supported with feedback in the current study is, ‘Writing a mathematical sentence’, and it was concluded that the feedback given to the students in this regard was useful. This study finding coincides with the finding of Fyfe and Rittle-Johnson (2017). Therefore, feedback provided during the application helped students perform better. The students' mathematical operation mistakes were reduced over time in the process. Lee (2006) claims that feedback is useful only when students use it effectively. Therefore, it can be said that the students in this study used their existing knowledge that they benefitted during the formative feedback process to scaffold their new learning.

In this study, the formative feedback presented to students is seen to support their performance in problem-solving. Detailed feedback was given for task-based performances, and given in correct time as similarly claimed in different literature sources (e.g. Brookhart, 2008; Fyfe & Brown, 2020, Fyfe & Rittle-Johnson, 2017; Hattie & Yates, 2014; Narciss et al, 2014). In this process, the quality of both the accuracy and appropriateness of the students’ answers, their drawings and their explanations of the problems increased. At this point, it is important for students to understand the rubrics used and the expectations in practice. Students benefited from the rubrics to both confirm the accuracy of their performances and to provide the changes of necessary elements in the problem-solving process. They need to know of the teacher expectations through the rubrics provided for them with the opportunity to learn about their performance and make changes regarding it (Fyfe & Brown, 2020; Mory, 2004).

The improvement in performance is at the same level for every student and for every problem, which coincides with the results of previous findings in the literature. Variables, such as students' prior knowledge (Fyfe &

Rittle-Johnson, 2016), ability (Butler, Godbole & Marsh, 2013), and the content of the subject (Chase & Klahr, 2017) may have caused students' performances to be different and developed at different levels. In many implementations, the same feedback is provided regardless of the answer's correctness (Attali & Van der Kleij, 2017). However, contrary to the feedback given to students, irrespective of the correctness of their solutions and regardless of their prior knowledge, when individualized formative feedback is provided, as in this study, the visual and written explanations regarding the problem-solving processes of students can be improved.

It is shown in the meta-analysis study of Alfieri, Brooks, Aldrich, and Tenenbaum (2011) that feedback is beneficial in specific ways, and that scaffolding is beneficial in the process. The results of the current study are intended to reveal the role of feedback in making students' conceptions visible in different aspects to achieve improvement in different dimensions of the process. The items on how the formative feedback process focuses on three dimensions, summarized in Table 10, are the most used focuses in giving individual feedback. At this point, it can be assumed that scaffolding was provided by way of formative feedback.

It can be said that the students showed less improvement in the fourth problem compared to the other problems. This finding coincides with the conclusion expressed by Chase and Klahr (2017) that the effectiveness of feedback may differ in different problems within the same subject area. Furthermore, the students did not encounter a problem from the learning area of numbers and operations, unlike with the other three problems. This problem also includes a context related to geometry and measurement learning areas that may have caused students to perform differently than with other problems.

The effect of feedback may vary based on the type of activity, a particular context, the type of information required, or the students' current knowledge (Chase & Klahr, 2017; Fyfe & Rittle-Johnson, 2016, Fyfe & Brown, 2018). In addition, the use of visualization in problem-solving may not be considered for all word problems. Feedback may also differ among students and may require different processes for different learning stages (Stevenson, 2017). In this study, a number of students showed faster improvement in their problem-solving performance than others. On the other hand, they improved in all three dimensions from the beginning to the end of the study. In this respect, the results of this study can provide an example of how feedback may be used to guide students towards goal-oriented work, based on the task given in problem-solving (Hattie & Yates 2014).

Conclusion

One of the critical results regarding the problem-solving process in this study is that the students used both the feedback given to improve their current work and to consider what they learned in subsequent problem-solving processes. Therefore, as a practical implication of the study, detailed information can be provided to students through this kind of formative feedback process. The students can be guided with regard to what type of work they should do for their development. In addition to this, teachers can use the problem-solving form to make students' thoughts visible in different problem-solving processes in the curriculum. Furthermore, the rubric and focus points in the feedback process can be used in classes to give verbal or written feedback in problem-solving processes. In this regard, the results of the current research may shed light on mathematics and primary school teachers seeking to develop their students' habits in problem-solving.

This study has the potential to contribute to the literature in terms of clarifying how, when and for which purpose formative feedback should be given in the problem-solving process in mathematics. As a limitation, this study was carried out with nineteen students over a six-week period. Different studies conducted at different grade levels may yield different results. Different applications may take place in conducting similar studies in more crowded classrooms. In addition, studies investigating how students reflect their acquisitions on subsequent problem-solving processes can be conducted. Therefore, future research may investigate the effects of the routine use of the process tried in this study for longer periods of time, at different class levels, in other learning areas and problem situations, in various school structures, and with different student profiles. Moreover, empirical studies could be conducted to reveal how feedback can be provided in the problem-solving process with different variables.

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Appendix 1

Problems Used in the Pretest and Posttest

The distance between City A and City B is 1,124 km. A bus leaving City A has traveled 137 km. How many kilometers more must it travel to arrive at City B?
Every day, an athlete runs 3,000 meters around a piece of land in the shape of a square the side length of which is 200 meters. The athlete has completed three circuits around the land. How many more meters should he run to complete 3,000 meters?
A mining company will transport ore by way of rail from Turkey to Kazakhistan. Each of the freight cars can carry eighteen tons of cargo. The train has seven freight cars. How many times should it travel to carry 250 tons of coal?
In a 4×400 flag race run in teams of four, when the third runner gives the flag to the fourth runner, he has completed 1200 meters. How many meters will the last runner run?
A toy seller, Cevdet, counted the toys in his store at the end of the year. He found that there are 275 toy cars, 148 dolls, 1,023 balls, and 816 kites. In total, how many toys are there in his store?

Source: . (Öztürk, Kişi, Öztaş, & Oruç, 2011).

Appendix 2.

Pretest and Posttest Graded Scoring Key

Assessment criteria	Score
Finds the correct operation to solve the problem, finds the intermediary stages correctly, and finds the correct result.	4
Finds the correct operation to solve the problem, finds the intermediary stages correctly, but makes mistakes in finding the correct result.	3
Finds the correct operation to solve the problem, but makes mistakes in the intermediary stages.	2
Finds the correct operation to find the solution.	1
Leaves empty.	0

Appendix 3.

Problem-Solving Form

Problem-solving form

Name (First and Last): Class: Date:
Problem

Draw a picture that can help you solve the problem.
Write a mathematical sentence explaining your solution.
Explain the process you followed to find the solution.