THE USE OF THE HISTORY OF MATHEMATICS IN TEACHING-LEARNING PROCESS: THE PERSPECTIVES OF FACULTY MEMBERS AND TEACHERS[1]

Abstract: The aim of this study was to investigate the faculty members' and the middle school mathematics teachers' perspectives regarding the use of the history of mathematics in the learning-teaching process of mathematics. As a phenomenological study, the qualitative data were collected through semi-structured interviews from 27 middle school mathematics teachers and seven faculty members and then, subjected to the content analysis. The findings revealed that both teachers and faculty members believed that using the history of mathematics is a worthwhile effort, with the potential to not only provide meaningful learning opportunities for students but also enrich teachers' professional development. However, it was also found that lack of historical perspective in the curriculum, teachers' inadequate knowledge, time constraint, no room for the history of mathematics in the textbooks and exams, overloaded curriculum and students' inadequate desire to learn were some of the reasons for rarely-use of the history of mathematics. Based on the overall findings of the study, it is concluded that teacher education (both pre-service and in-service), the structure of mathematics curriculum, teachers' and students' characteristics were the most important dynamics to integrate the history of mathematics into teaching effectively.

Keywords: the history of mathematics, mathematicians, mathematics teachers

Tan-Şişman, Gülçin, PhD
Assist. Prof. Dr.
Department of Educational Sciences
Hacettepe University
Turkey
Contact:
E-mail: gulcintans@gmail.com
ORCID: 0000-0002-3806-6086

Gençkaya, Şeyda, MsC
Mathematics Teacher
Şehit Meriç Alemdar Middle School
Turkey
Contact:
E-mail: seydauyar88@gmail.com
ORCID: 0000-0003-4970-8336

[1] The present study is partly based on the second author's master thesis under the supervision of the first author.

Psycho-Educational Research Reviews | Vol. 10, No. 2
London Academic Publishing, August 2021, pp 241–257
https://www.journals.lapub.co.uk/index.php/PERR

INTRODUCTION

Mathematics is a branch of science that manifests itself in every field from the smallest building blocks to the most complex formations in the universe. Striking reflections of mathematics can be seen in countless natural settings including the number of daisy flower petals, the intersecting spirals of pine seeds on cones, the elliptical orbits of the planets around the sun, the fixed value of division of the circumference by the diameter of all circles, or the helix curve shape of ivy around trees (Altun, 2008; Sertöz, 2011). Not only Galileo's description of nature as "a book written in the language of mathematics" (as cited in Topdemir, 2011, p.104) and also Sertöz's description mathematics as "creator's hints left in the nature" (2011, p.3) imply that mathematics has in fact existed in the universe since the genesis.

Since the first years of history, human beings started laying the foundations of mathematics in order to solve the daily needs, encountered problems, and to understand the universe. The earliest mathematical activity started approximately 5000 B.C. around Egypt, Mesopotamia, China, and India (Baki, 2014; Bell, 1992). The discipline gradually flourished as number systems were invented by the Babylonians, the Egyptians, the Chinese, and the Greeks fulfilling their daily needs; mathematical and arithmetic studies were carried out by mathematicians such as Pythagoras and Euclide; and Plato performed studies on irrational numbers and smooth polyhedrons (Abdulhay, 2014; Baki, 2014; Cajori, 1919/2014). In the following years, significant progress were made in the finest calculation of trigonometric values, in algebra, cubic equations, and logarithm, derivatives, integral, and complex numbers, eventually resulting in an exponential accumulation of mathematical knowledge (Baki, 2014; Cajori, 1919/2014; Zeki Bey, as cited in Demir, 2004). After the 19th century, developments in mathematics have spread across the world; as a result, significant numbers of mathematicians in different parts of the world have contributed to mathematics in various ways.

In the 21st century, understanding of mathematics has of utmost importance since learning mathematics provides individuals to develop scientific thinking skills, apply them to different situations, produce original ideas, make research, gain self-regulation skills, and develop self-confidence (Hattatoğlu, 2010; Işık, Çiltaş & Bekdemir, 2008; Karakurumer, 2003; National Council of Teachers of Mathematics, 2000; Rizki & Priatna, 2019). Putting emphasis on the essence of mathematics literacy, Ojose (2011, p.91) stated that "Mathematics is so entwined with today's way of life that we cannot fully comprehend the information that surrounds us without a basic understanding of mathematical ideas". According to the framework of the Programme for International Student Assessment (PISA) 2021, mathematical literacy is "an individual's capacity to reason mathematically and to formulate, employ, and interpret mathematics to solve problems in a variety of real world contexts. … It assists individuals to know the role that mathematics plays in the world …" (OECD, 2018; p.7). Tekin and Tekin (2004) also argued that mathematically literate person has knowledge and skills about the following areas: (a) content area literacy; (b) mathematical thinking process; (c) up-to-date knowledge of mathematics, and (d) historical evolution of mathematics. In a similar vein, Steen (2001) acknowledged "Cultural Appreciation" as one of the fundamental elements for the concept of quantitative literacy, and defined as "Understanding the nature and history of mathematics, its role in scientific inquiry and technological progress, and its importance for comprehending issues in the public realm" (p.8). In this respect, being mathematically literate requires understanding of the historical side of mathematics including not only evolution of mathematics as a discipline but also the leading mathematicians and their contributions. Comparing the perspectives from past to present, Ding (2019) also emphasized the essence of the history of mathematics as follow:

> "In the past, it was generally believed that the study of mathematics history was an elegant, sunny and white snow-like model. However, in today's era, mathematics teaching … pays more attention to the cultivation of ideological methods and emotional attitudes and values. The history of mathematics has thus got rid of the situation … and it has gradually been valued …" (p.783)

The history of mathematics is a field of study that put forwards past obstacles and difficulties which mathematicians have overcome in the development of mathematics; reveals mathematics' dynamic nature (Liu, 2003) and "shows the evolutionary and progress of mathematical knowledge through civilizations" (Baki, 2014, p.3). In other words, history of mathematics is a comprehensive area that deals with the growth

processes of mathematics, the lives, works, achievements or failures of leading figures who have contributed to mathematics, the social and cultural dimension of mathematics, and development and progression of mathematical knowledge (Bidwell, 1993; Burton, 2003; Eves, 1990; Katz, 1993; Otte, 2007; Pepe, & Guerraggio, 2017; Yee and Chapman, 2011). Studies on the use of history of mathematics in mathematics education first appeared in the 1970s. The research in this area is now being supported by various world-wide institutions and organizations concerned with mathematics education (e.g. the International Commission on Mathematical Instruction [ICMI]) (Clark, Kjeldsen, Schorcht, Tzanakis and Wang, 2016; Fauvel and Maanen, 2002; Fried, 2001).

The findings of various research studies pointed out that the use of history of mathematics in learning and teaching process bears potential contributions to both students and teachers. Specifically, it is stated that the history of mathematics helps students to comprehend the formation of mathematical thinking, improve problem-solving skills, assess mathematical topics in a comparative way between the past and present, establish relationships between mathematical topics and other disciplines, and appreciate that mathematics is a constantly evolving discipline (Alpaslan, 2011; Ho, 2008; Jankvist, 2009; Lim & Chapman, 2015; Liu, 2003; Sullivan, 1985; Wilson and Chauvot, 2000). Besides, the history of mathematics has a supporting role for teachers to gain different perspectives, to comprehend mathematical facts unnoticed before, and to move from product-oriented instruction to process-oriented (Radford, 2014). Teachers, while blending their qualified knowledge about the history of mathematics with in-class activities, can develop their creativity and also acknowledge the reason for teaching each specific topic. As a result, their teaching skills might improve (Furinghetti, 1997; Guillemette, 2017; Haile, 2008; Kjeldsen, 2011; Liu, 2003; Nataraj & Thomas, 2009; Pengelley; 2002). Bidwell (1993, p.461) notes that students think mathematics "as a closed, dead, and emotionless island; where teachers can rescue them for replacing them on an alive, open, full of emotion, and always interesting mainland" when they integrate history of mathematics in the learning and teaching of mathematics. In the literature, there have been numerous studies emphasizing that the integration of mathematics history into the learning and teaching process might provide more meaningful and real-life connected learning environment, yet there have been some factors affecting the development of such environment. According to the literature, such issues as no room for the history of mathematics in the curriculum, lack of instructional resources and/or materials, teachers' limited or lack of knowledge about the history of mathematics, etc. (Baki & Yıldız, 2010; 2016; Başıbüyük, & Şahin, 2019; Sözen, 2013; Fried, 2001; Siu, 2007; Niitsuma & Nagaoka, 2014; Tan-Şişman & Kirez, 2018).

Although there have been various research studies conducted with mathematics teachers on why's and how's of the mathematics history, to our knowledge, there is no research study on the views of mathematicians regarding the use of the history of mathematics. Ding (2019) argued that neither theory (the pure knowledge of mathematics history) nor practice (the pure knowledge of how to teach mathematics) has produced better ways to integrate the history of mathematics. In this respect, the purpose of the present study was to investigate the views of faculty members' and mathematics teachers' regarding the use the history of mathematics in the learning-teaching process of mathematics. More specifically the study seeks to answer the following research questions:

1. What are the faculty members' opinions about the use of the history of mathematics in learning and teaching process?
2. What are the the mathematics teachers' opinions about the use of the history of mathematics in learning and teaching process?

It is believed that portraying comparatively the views of faculty members, who have deep theoretical knowledge of the mathematics discipline, and the mathematics teachers, who are practitioners of the learning-teaching process, about why and how to use the history of mathematics is essential to promote theory-enhanced practice for the integration of the history of mathematics. It is also expected that the results of this study will be valuable for many stakeholders. Firstly, the findings will contribute to mathematics teachers who are one of key actors in interpreting and transforming written curriculum into the learning and teaching process. In addition to the mathematics teachers, the findings of this study may also be worthwhile for curriculum developers to provide comprehensive bases of why and how the history of mathematics should be used in mathematics education in terms of the different points of view. Besides, this study might

provide helpful insights and implications, emerged from not only the experiences of the mathematics teachers but also from the suggestions of the faculty members with their in-depth content knowledge, both for curriculum development and implementation process.

METHOD

RESEARCH DESIGN

In this qualitative study, the phenomenological approach that focuses on "to seek reality from individuals' narratives of their experiences and feelings, and to produce in-depth descriptions of the phenomenon" (Yüksel & Yıldırım, 2015, p.1) was used. Since the focus of this study was on the insights of faculty members and mathematics teachers about why and how the history of mathematics should be used in teaching and learning process; the phenomenological research design was employed to understand how a phenomenon, which in this case was the use of mathematics history in learning and teaching processes, comprehended among different stakeholders.

PARTICIPANTS

The study was conducted with seven faculty members working at different Turkish public universities and 27 mathematics teachers working at the public middle schools located in six central districts of Ankara. The faculty members were selected by convenient sampling. Through searching on the official web pages of the Turkish state universities, the researchers listed the faculty members who were working as a full-time faculty at the departments of mathematics or mathematics education and were interested in the history of mathematics. Then, the faculty members were selected on the basis of their accessibility and convenience to the researchers and they were invited to the study via e-mail. As given in Table 1, totally seven faculty members from five different state universities volunteered to take part in this study. Among seven faculty members, four of them were working as a full-time faculty at the department of mathematics education and three were at the department of mathematics. With regard to the K-12 level teaching experience, three of them have experience of 1-5 years, three have no experience and one has experience of 5-10 years. In addition, while the majority of the faculty members (n=6) did not take any course about the history of mathematics during their undergraduate or graduate education, four of them lectured the history of mathematics course at undergraduate and/or graduate level.

Table 1. The Faculty Members' Profiles

Faculty Members' Profiles	F1	F2	F3	F4	F5	F6	F7
Gender	Male	Male	Male	Male	Male	Male	Male
Age	40-45	61-65	50-55	50-55	50-55	55-60	61-65
Title	Asst. Prof. Dr.	Assoc. Prof. Dr.	Prof. Dr.	Prof. Dr.	Prof. Dr.	Prof. Dr.	Prof. Dr.
University	A	B	C	D	B	E	C
Department	Math	Math Ed.	Math Ed.	Math	Math Ed.	Math Ed.	Math
K-12 teaching experience	-	5-10	1-5	-	1-5	1-5	-
Taking HoM* course	No	No	No	No	No	Yes	No
Lecturing HoM course	No	Yes	No	No	Yes	Yes	Yes

*HoM: The history of mathematics

Further, the mathematics teachers were selected through maximum variation sampling method that allows researchers to collect in-depth information and mirror divergent perspectives rather than making a generalization (LeCompte, Preissle, & Tesch, 1993; Patton, 2015). In this scope, 27 math teachers from 12 different middle schools located in Çankaya, Yenimahalle, Etimesgut, Keçiören, Pursaklar, and Altındağ districts of Ankara took part on a voluntary basis. Besides, the central district where the school located, the population of the schools, the types of schools (double-shift [DS]/single-shift [SS]), years of experience in teaching, and educational background was taken into account while determining the participants. As seen

in Table 2, 23 female and four male mathematics teachers were participated to the study. With regard to teachers' years of experience in teaching, it ranges from one to 35 years, while the majority (*n*=9) has experience of 1-5 years. In addition, 22 of the participants graduated from faculty of education and five from faculty of science.

Table 2. The Mathematics Teachers' Profiles

Teachers	Gender	Graduation	District	Teaching experience	School population	Type of schooling
T1	Male	F.ED*	Keçiören	26 and above	1000-1499	DS
T2	Female	F.ED	Keçiören	11-15	1000-1499	DS
T3	Female	F.ED	Etimesgut	11-15	1500-1999	DS
T4	Female	F.ED	Altındağ	1-5	1000-1499	DS
T5	Female	F.ED	Yenimahalle	11-15	1000-1499	DS
T6	Female	F.ED	Pursaklar	1-5	1000-1499	DS
T7	Female	F.ED	Etimesgut	6-10	500-999	SS
T8	Female	F.ED	Etimesgut	16-20	500-999	SS
T9	Female	F.ED	Yenimahalle	11-15	1500-1999	DS
T10	Female	F.ED	Altındağ	1-5	500-999	DS
T11	Male	F.SC**	Yenimahalle	16-20	1000-1499	DS
T12	Female	F.ED	Keçiören	1-5	1000-1499	DS
T13	Female	F.SC	Keçiören	26 and above	1000-1499	DS
T14	Female	F.ED	Altındağ	1-5	500-999	DS
T15	Female	F.ED	Pursaklar	1-5	1500-1999	DS
T16	Female	F.SC	Altındağ	1-5	1000-1499	DS
T17	Female	F.ED	Çankaya	6-10	500-999	DS
T18	Female	F.ED	Pursaklar	1-5	1500-1999	DS
T19	Female	F.ED	Çankaya	11-15	500-999	SS
T20	Female	F.ED	Yenimahalle	11-15	1000-1499	DS
T21	Female	F.SC	Yenimahalle	21-25	1500-1999	DS
T22	Female	F.ED	Pursaklar	11-15	1500-1999	DS
T23	Male	F.ED	Çankaya	21-25	500-999	SS
T24	Female	F.ED	Çankaya	16-20	500-999	SS
T25	Female	F.SC	Çankaya	16-20	500-999	SS
T26	Female	F.ED	Yenimahalle	6-10	1500-1999	DS
T27	Female	F.ED	Pursaklar	1-5	1000-1499	DS

F.ED*: Faculty of Education F.SC**: Faculty of Science

DATA COLLECTION INSTRUMENTS

In this study, Faculty Members Interview Form (FMIF) and Teachers Interview Form (TIF) were developed by the researchers and used as the main data collection instruments. While developing the interview schedules, first of all the literature was reviewed in detail to outline the important issues regarding the use of the history of mathematics in teaching and learning process. Through synthesizing the information gathered from the literature, the draft interview schedules were written in line with the research questions. Afterwards, the drafts went through revision with the help of expert opinions gathered from two faculty members from the department of Educational Sciences and two mathematics teachers. Based on feedbacks received from the experts, the wording of some questions was changed and some follow-up questions and prompts were either added or removed in order to make questions more clear and understandable. Then, the draft TIF was piloted with three mathematics teachers and FMIF was piloted with two faculty members to determine whether the questions make sense to the interviewees and as a result, no changes were applied to the drafts. In this respect, the final interview schedules were composed of the questions about demographic information (e.g. age, educational background, etc.) and the use of history of mathematics in learning and teaching process (e.g. "What is your opinion about the integration of the history of mathematics in teaching and learning of mathematics?", "What do you think about the current situation of the use of the history of mathematics in classrooms?", "What should be done to use the history of mathematics effectively?").

DATA COLLECTION AND ANALYSIS

Before the collection of the data, the necessary permissions from the Ethics Commission of Hacettepe University and the Ministry of National Education were obtained. In order to conduct interviews with mathematics teachers, the researchers contacted with the principals of the selected schools for scheduling the short meetings to introduce the study. Then, the interviews were conducted with 27 mathematics teachers who had volunteered to participate in the study. The faculty members were invited to the study via e-mail and totally seven faculty members from five different state universities volunteered to take part in the study. All interviews were audio recorded and lasted approximately between 30 to 45 minutes.

The collected data were subjected to content analysis. The steps followed during the content analysis process were: (1) transcribing the raw data, (2) organizing and preparing data for analysis, (3) reading through all data, (4) coding the data, (5) generating themes or description, (6) interrelating themes/description and (7) interpreting the meaning of themes/descriptions (Creswell, 2013; Yıldırım & Şimşek, 2013). In this respect, first of all, the interviews were transcribed word by word on a word document. Then transcribed data were read thoroughly for a few times considering the research problems to identify the conceptual framework. The coding was done by considering approximately every expression used by participants not missing any important details. Finally, the codes focusing/implying the similar ideas were combined under categories and themes. The teachers were coded as T1 through T27; while the faculty members were represented with F1 to F7. The opinions of the participants were reported with direct quotations.

TRUSTWORTHINESS

Trustworthiness of a research is characterized as provisions or actions that establish reliability and persuade readers about accuracy of the findings (Lincoln & Guba, 1985). To enhance trustworthiness of the present study, several strategies were used such as credibility, transferability, dependability, and confirmability. To establish credibility, triangulation via data sources (Shenton, 2004) was carried out among the teachers' and field experts' perspectives. The codes in data analysis were submitted to one faculty member from the department of Educational Sciences, who have knowledge of the research problem and qualitative research methods, as required by peer debriefing for credibility. In addition, interviews were recorded with a recording device with participants' consent and the recordings were verified by participants at the end of the interviews for member checks. Further in order to help transferability of the study, the scope and limitations of this research were set out clearly and the methodology of the study was explained thoroughly. Besides, the data were enriched by maintaining quotes from participants as well as displayed in details in order to provide thick description for the sake of trustworthiness of the study. During the study confirmability, namely "the qualitative investigator's comparable concern to objectivity" (Shenton, 2004; p.72), were provided by making use of audit trail and triangulation. The detailed methodological descriptions were made to allow integrity of research results to be investigated. In addition, the data were reported by reducing the effect of researcher bias. Lastly, Miles and Huberman's (1994) formula (Reliability = Number of agreements / (Agreements + Disagreements) x 100) was used for reliability analysis. In this respect, two randomly selected transcriptions of the participants (two from teachers, two from faculty members) were coded by one researcher and one expert separately and independently. The agreement between coders was found as .87 for TIF and .84 for FMIF. To solve the disagreements, both sets of data were compared, and through discussion, the disparities were reconciled to reach a consensus.

RESULTS

THE VIEWS OF THE FACULTY MEMBERS REGARDING THE USE OF HISTORY OF MATHEMATICS

The findings revealed that all of the faculty members expressed that the history of mathematics should be used in learning and teaching process. They believed that students could benefit from using the history of mathematics in the classroom as it provides opportunity to (a) internalize the idea that mathematics is a human creation, (b) enhance their learning, (c) understand the process of mathematical knowledge formation, (d) gain a rich perspective on mathematics and (e) appreciation of mathematics. In addition, the faculty members stated that making use of the history of mathematics might help for mathematics teachers

to enhance their content knowledge and support to implement student-centered instruction. In the Table 3, the results were summarized with quotations from the faculty members' views.

Table 3. The Benefits of Use the History of Mathematics

Benefits for Students	*f*	*Faculty Members' Views*
Internalizing the idea that mathematics is a human creation	3	*"…Most of the students leave the lessons with a wrong impression like this: 'What I've learned right now … is came out of the minds of a mathematician or a few mathematicians … as a whole like the way the teacher taught me'. In fact, what one needs to know what stages have been through for bearing that theorem, its simple forms, the issues inspired that theorem."* F7
Enhance students' learning	2	"*…According to the Fuzzy logic, there are gray areas between zero and one. You need to rate them, too. So it's been a lot of application to technology. For example, dishwashers, you set the program as very dirty, less dirty or even lesser dirty… That's Fuzzy logic! Now, if this kind of information is given during the lesson or on textbook or a teacher draws students' attention by mentioning them in two or three minutes; s/he can transform abstract structure of mathematics into concrete one."* F5
Understanding the process of mathematical knowledge formation	1	"*You know, students always ask about mathematics 'What does that do? Where does it come from?' It is taught in a very abstract way. When it is taught with the historical perspective, students can be able to see that in reality, every concept was born by a need, or this need may be both the need of physics and math or came as a need in mathematics itself, but nothing contains an abstract nature in the sense that students think. Abstract but that abstraction surely solves a problem responding to something. That is the biggest advantage of the historical perspective."* F4
Gaining a rich perspective of mathematics	1	"*One day I asked; "Who is this Binomial, what is the nationality?" etc. Some of the students said German, some said British. They think Binomial is a mathematician! There is no mathematician called Binomial!... Then I felt that we should look at the history of mathematics. For instance, the Pythagoras Theorem, who is Pythagoras? Is Pythagoras a human? etc. Instead of focusing only the memorization of the theories, we need to provide students with a deep and broad view of mathematics.*" F2
Appreciation of mathematics	1	"*No matter which country you go to around the world, you see that mathematics and mathematicians are always perceived differently. Mathematics is always regarded as difficult, like in our country, but doing math is considered as the indicator of a much higher standard. It is written in many articles, mathematics has a door opener position for professional career, it is true for not only in our country, but for the whole world. This means that through the history of mathematics, students can realize that how mathematics is valuable and important.*" F5
Benefits for Teacher	*f*	*Faculty Members' Views*
Enhancing the content knowledge	1	"*The strength of the use of history of mathematics in terms of a teacher, s/he can teach better. Because s/he gains knowledge about where the concepts come from, how the problems are related to.* "F4
Supporting student-centered instruction	1	"*The history of mathematics is one of the best medium of instruction for the teachers who would like to design student-centered learning environment; by making use of the examples chosen from the history of mathematics, s/he might group students to work together and implement problem solving method.*" F6

Some of the faculty members also mentioned such restricting-issues related to the use of history of mathematics as "loss of prestige and trust" and "extra workload" due to teachers' poor/lack of knowledge. In Table 4, the results were summarized with quotations from the faculty members' views.

Table 4. The Restricting-issues about Use of the History of Mathematics

Teacher-related issues	*f*	*Faculty Members' Views*
Loss of prestige and trust	2	"*While teaching Binominal Theorem, one of the students asks, 'When did Binomial live, my teacher?' if the teacher doesn't know, s/he is ruined there! Ruined!"* F2
Extra workload	1	"*The history of mathematics is a completely different area. A math teacher is not normally expected to know too much about the history of mathematics... It is an additional burden for her/him to deal with the history of mathematics… Spending extra efforts on it spending more time on..."* F4

Further, the results indicated that all faculty members thought that the history of mathematics is almost never integrated in current mathematics lessons due to several reasons as summarized in Table 5.